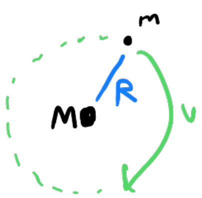
Orbits

Orbit-based problems are pretty common on the F=ma. The simplest kind, circular orbits, are simply when the radial force matches centripetal acceleration.

Ex. gravitational circular orbit.



$$a_{cent} = V^2/R$$

$$= a_{grav} = GM/R^2$$

So, for a given R, $\boxed{V = \sqrt{GM/R}}$.

What's the period of orbit?

At constant V & R, each orbit is $2\pi R$ long, so

$$T = 2\pi R/V = \boxed{2\pi\sqrt{R^3/GM} = T.}$$

What about other forces?

Orbits are stable when $F \propto R^{-2}$, so they'll all take the same form. Let's take $F = fR^{-2}$ as this general form:

← Proof beyond our scope.

$$fR^{-2} = F = mV^2/R.$$

So, $\boxed{V = \sqrt{f/mR}}$.

For gravity $f = GMm$, so $\boxed{V = \sqrt{GM/R}}$

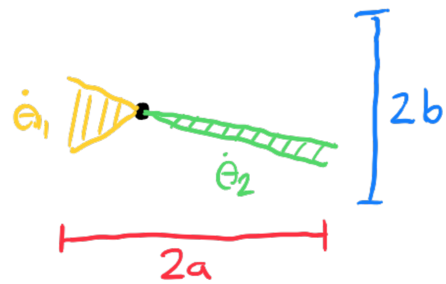
Kepler's Laws & Non-Circular Orbits.

I. all bound orbits are elliptical with the orbited body at a focus.

Corollary: circular orbits are elliptical with the two foci at the same point in space.

II. a line joining orbited and orbiter sweep equal area in equal time.

$$T \cdot {r^2}/{2} \; {d\theta}/{dt} = \pi a b.$$



$A_1 = A_2$

Since $\dot{\theta}_1 > \dot{\theta}_2$

even as $r_1 < r_2$.

III. $T^2 \propto R^3$, as derived earlier.

It can be generalized to elliptical w/o much difficulty. (HW!)

Non-circular orbits.

Range of v:

$0 \to \sqrt{GM/R}$ elliptical (R = apogee, maximum distance)

$\sqrt{GM/R}$ circular

$\sqrt{GM/R} \to \sqrt{2GM/R}$ elliptical (R = perigree, minimum distance)

$\sqrt{2GM/R}$ parabolic (escape speed)

$\sqrt{2GM/R} \to \infty$ hyperbolic