

Pendulums

Q: Anyone know the oscillation frequency of a simple pendulum?

$$\sqrt{g/L}$$



Let's derive that two ways.

Dimensional analysis: the frequency has units $1/s$ and the only parameters are m , L & g .

How do we make s^{-1} out of

m : kg L : m g : m/s^2 ? Inspection says $\sqrt{g/L}$.

This is an excellent way to eliminate wrong answers & check work.

Forces: For a small θ , $\sin \theta \approx \theta$.

Trigonometry says that the component of F_T along the path $F_{T\theta} = F_T \sin \theta \approx F_T \theta$, with $F_T \approx -F_g$. So, $F_{T\theta} \approx -F_g \theta = -F_g s/L$.

This is a restoring force, since $F(s) \propto -s$.

Thus, using Hooke's Law: $F = -kx \Rightarrow \omega = \sqrt{k/m}$.

So, our $k = mg/L \Rightarrow \omega = \sqrt{\frac{mg/L}{m}} = \sqrt{g/L}$.

