

## Pendulums

Q: Anyone know the oscillation frequency of a simple pendulum?

$$\sqrt{g/L}$$



Let's derive that two ways.

Dimensional analysis: the frequency has units  $1/s$  and the only parameters are  $m$ ,  $L$  &  $g$ .

How do we make  $s^{-1}$  out of

$m: kg$   $L: m$   $g: m/s^2$ ? Inspection says  $\sqrt{g/L}$ .

This is an excellent way to eliminate wrong answers & check work.

Forces: For a small  $\theta$ ,  $\sin \theta \approx \theta$ .

Trigonometry says that the component of  $F_T$  along the path  $F_{T\theta} = F_T \sin \theta \approx F_T \theta$ , with  $F_T \approx -F_g$ . So,  $F_{T\theta} \approx -F_g \theta = -F_g s/L$ .

This is a restoring force, since  $F(s) \propto -s$ . Thus, using Hooke's Law:  $F = -kx \Rightarrow \omega = \sqrt{k/m}$ .

So, our  $k = mg/L \Rightarrow \omega = \sqrt{\frac{mg/L}{m}} = \sqrt{g/L}$ .

