Rotational Dynamics

There are two basic ideas you need to succeed with rotation-based physics problems:

1. Rotational dynamics are strongly analogous to linear dynamics.
2. Rotational & linear motion often need to be treated simultaneously!

First, #1:

$F \to N$: $\vec{N} = \vec{r} \times \vec{F}$ ($N = rF$ if $r \perp F$) $\quad m \to I$: $I = mr^2$

$a \to \alpha$: $\alpha = a/r$

$v \to \omega$: $\vdots$ $\quad KE_{rot} = \frac{1}{2} I \omega^2$.

$x \to \theta$:

Let's take an example:

A ball (m, R) is gently dropped on a surface while spinning with angular velocity ω_0. Once it is rolling without slipping, how fast is it moving? No energy is lost to the surface. $\left(I_{sphere} = \frac{2}{5} mR^2\right)$

Let's take an energy approach. First,

$$E_0 = \frac{1}{2} \cdot \left(\frac{2}{5} mR^2\right) \omega_0^2$$

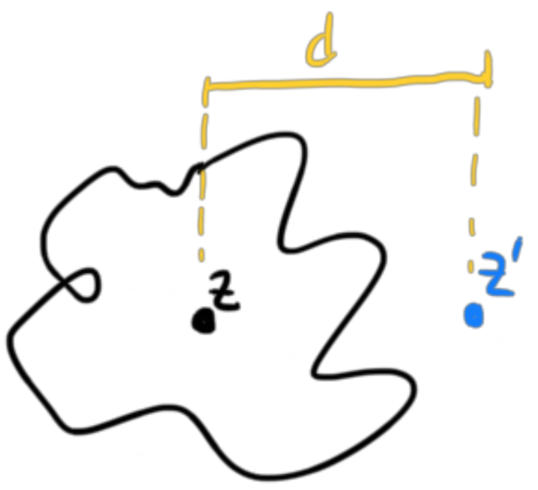
When it's rolling w/o slipping, $v = \omega R$. So,

E_{final} takes the form $$E_f = \frac{1}{2} mv^2 + \frac{1}{2} \cdot \left(\frac{2}{5} mR^2\right) \omega^2 = \frac{1}{2}\left[mv^2 + \frac{2}{5} mR^2 \frac{v^2}{R^2}\right] = \frac{7}{10} mv^2.$$

No energy is lost, so $E_f = E_0 = \frac{1}{5} mR^2 \omega_0^2$. So,

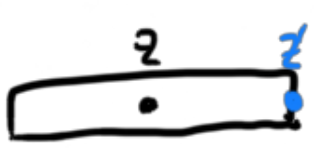
$$v^2 = \frac{10}{7\cancel{m}}\left(\frac{1}{5} \cancel{m} R^2 \omega_0^2\right) = \frac{2}{7} R^2 \omega_0^2 \Rightarrow \boxed{v = \sqrt{2/7}\, R\omega_0}$$

Parallel axis theorem



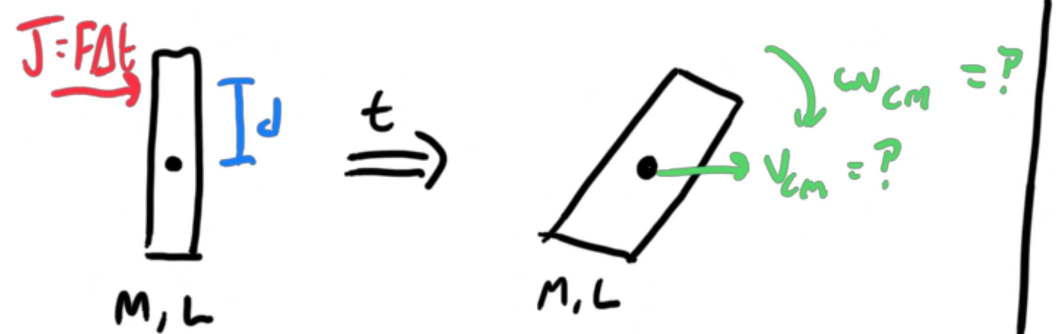
$I_{z'} = I_z + md^2$ for any two parallel axes.

Ex. rod about center vs. edge.



M, L $\quad I_{z'} = \frac{1}{12} ML^2 + M\left(\frac{L}{2}\right)^2 = \frac{1}{3} ML^2.$

Now, let's do a Walter Lewin problem from MIT 8.01x.



on a frictionless table.

First, let's do the linear component.

$$J = F\Delta t = Ma\Delta t = Mv_{cm} \Rightarrow \boxed{v_{cm} = J/M}$$

this is <u>indepent</u> of d!

Next, the rotational component.

$$\underbrace{J_N}_{= Jd} = F\Delta t\, d = I\alpha\Delta t = I\omega_{cm} = \tfrac{1}{12}ML^2\omega_{cm}$$

$$\Rightarrow \boxed{\omega_{cm} = 12Jd/ML^2}$$

One of your classmates (Ben) asked a good question about this problem.
"Does d affect the energy imparted on the rod since v_{cm} is indepenent?"
Let's find out!

$$\Delta E = F\Delta y \quad \text{(total displacement)}$$

$$= \frac{J\Delta y}{\Delta t}$$

For short impulse (small Δt), $\Delta y \simeq d\Delta\theta + \Delta x$.

If $\alpha = \frac{Jd}{I\Delta t}$ & $a = \frac{J}{M\Delta t}$, kinematics says:

$$\Delta\theta = \tfrac{1}{2}\alpha\Delta t^2 = \frac{Jd\Delta t}{2I}, \qquad \Delta x = \tfrac{1}{2}a\Delta t^2 = \frac{J\Delta t}{2M}$$

$$\Rightarrow \Delta y = \frac{J\Delta t}{2M}\left(1 + \frac{d^2}{L^2}\right)$$

So, $\boxed{\Delta E = \frac{J^2}{2M}\left(1 + \frac{d^2}{L^2}\right)}$

Which has <u>quadratic</u> dependence on d!