**Question 3**

I considered matrices $m_1, m_2$

And now I should consider matrices $m_1, m_2$

Like you said, but can I consider nests of matrices with upper even vertices and down even vertices but different.

Like this one?

The number of upper vertices $\neq$ down vertices.
Matrix $M_{14}$

With this nest the matrix is:

Matrix $A$

Matrix $B$

This is much easier to solve, in fact the color of lines starting from the group of vertices that are even would never fail, because the number of that lines is odd.

Same principle as before $\rightarrow$ as soon as the number of red dots $\neq$ number of black dots in a line you have a solution.

But here the friendliness is not sure...
How to assure the friendliness in the matrix nest \( m_1 m_2 m_3 \)?

Let's report here matrix B from previous page:

\[
\begin{array}{cccc}
V_1 & V_2 & V_3 & V_4 \\
\bullet & \bullet & \bullet & \bullet \\
\bullet & \bullet & \bullet & \bullet \\
\bullet & \bullet & \bullet & \bullet \\
\end{array}
\]

\[
\begin{array}{c}
U_1 \\
\bullet \\
4 = 4 + 0 \\
U_2 \\
U_3 \\
\end{array}
\]

**Solution**

**Horizontal strip** \( V_1 \):

One black and all other red.

**Horizontal strip** \( V_2 \):

Mirror of previous of \( V_1 \).

**Horizontal strip** \( V_3 \):

All dots are the same color that made the lonely dot before, in this case was black.
Let's try this system solution with a bigger nest like... G >= 3.

In this matrix, I used the mirror-complement and it messed up everything!
8-7

<table>
<thead>
<tr>
<th>V₁</th>
<th>V₂</th>
<th>V₃</th>
<th>V₄</th>
<th>V₅</th>
<th>V₆</th>
<th>V₇</th>
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</thead>
<tbody>
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</tbody>
</table>

40 = 26 + 14
$\begin{array}{cccccccc}
V_1 & V_2 & V_3 & V_4 & V_5 & V_6 & V_7 & V_8 \\
\hline
U_1 & & & & & & & \\
U_2 & & & & & & & \\
U_3 & & & & & & & \\
U_4 & & & & & & & \\
U_5 & & & & & & & \\
U_6 & & & & & & & \\
U_7 & & & & & & & \\
\end{array}$

$U_5 = \text{HALF} + 2$

$U_6 = \text{HALF} - 1$

$U_7 = \text{HALF} - 1$
In order to complete the matrices $A_{m \times n}$ and keep the friendliness (that is, the bigger concern since it is not more automatic like in matrix $A_{n \times m}$), it is better to fill up using the method of one of the complements until the last 3 strips.
The important is that every strip up to the last 3 has to have its own complement.
HOW TO FILL A MATRIX $M, M$

1. BEGIN ALWAYS FROM THE VERTICES THAT ARE ODD
2. 1 BLACK - ALL OTHER RED
3. COMPLEMENT OF PREVIOUS
4. FILL UP THE LAST 3 TOGETHER
5. 3RD FROM END \( \frac{\text{even vertices}}{2} \) RED
   AND REMAIN OTHER COLOR
6. 4TH FROM END \( \frac{\text{even vertices}}{2} - 1 \) RED
   AND REMAIN OTHER COLOR
7. SAME AS PREVIOUS... (LAST)
Looking at this picture we see that if we fill up the lines starting from the set of odd number of vertices and just paying attention of not taking an X, everything is fine.

In fact it is not possible to take any X in the lines starting from the top/upper vertices since those odd lines are always odd, so is impossible to have an X.
To solve a nest with one set of a even number of vertices and the other odd \( k \), \( m \) is easier.

When the number of vertices on the two sets in opposition were even for both, we had to take care of avoiding \( x \), since the friendliness was taken care by the fact that every vertex had its own complement.

Now that the number of vertices is not always odd, it is not possible to have a complement for every vertex so in the \( k \), \( m \), \( m \) the biggest concern is to keep the friendliness.

In order to keep the friendliness we begin to fill up the nest (or the matrix) from the set of vertices that are odd in number (see previous page to see why).

Then we proceed by filling up the first vertex in any way we like just avoiding \( x \).
The second vertex has to be the complement of the previous one.

The 3rd vertex will be filled out in any way that avoid an X.

The 4th vertex has to be the complement of the previous one.

The 5th vertex has to be filled out in any way that avoid an X.

The 6th vertex has to be the complement of the previous one and so on...

This way of proceeding will keep the friendliness that is our bigger concern right now.

Since we proceed in a way that keeps the friendliness, we cannot leave the last vertex (friendliness concerns an even number) but we are working on a set of odd vertices, so we will have:

\[ 2x + 1 \]

Last

We cannot proceed 2 by 2 with these because the second be the complement of the first one.
Because the last left in order

\[# \text{ of odd vertices} \#

\[ 2x + 1 \]

\[ \text{odd} \]

To be filled will need an equal number of red and an equal number of black (since we kept the friendliness until here using the system of alternating complements) and we will have an \( \times \).

So we solve this problem to change from

\[ 2x + 1 \]

\[ \rightarrow \]

\[ 2x + 3 \]

This is still an odd number so...
After applying the system of the alternating components to keep the friendliness we stop when we have 3 left (see pag. 62).

We can fill up the last 3 vertices belonging to the set of odd this way:

1. 3rd vertex from the end
   - Total number of even vertices: 2

2. 2nd vertex from the end
   - Total number of even vertices: 2

3. Last vertex
   - Total number of even vertices: 2

The algebraic $+2-1-1 = 0$

Assure the friendliness.