Mathematics in Ancient Mesopotamia

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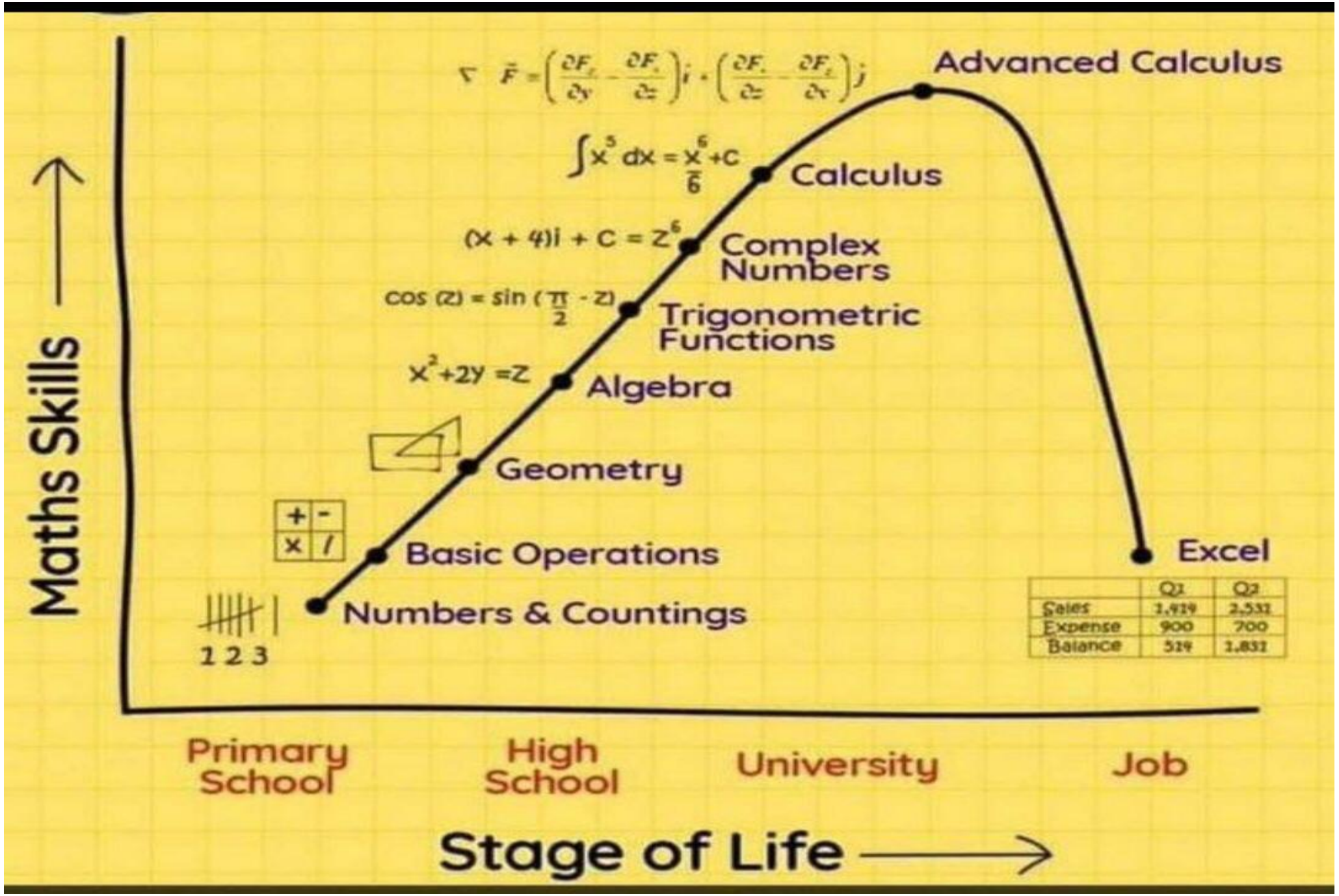

Maths Skills
Advanced Calculus
Calculus
Complex Numbers
Trigonometric Functions
Algebra
Geometry
Basic Operations
Numbers & Countings
1 2 3
Excel
Q1 Q2
Sales 1,939 2,531
Expense 900 700
Balance 529 1,831
Primary School
High School
University
Job
Stage of Life

At the beginning

- **Sumer** (a region of Mesopotamia) was the birthplace of:
  - writing,
  - the wheel,
  - agriculture,
  - the arch,
  - the plow المحراث ,
  - the saw,
  - irrigation,
  - beer,
  - the spear الرمح,
  - money,
  - calendar
  - and many other innovations and is often referred to as the Cradle of Civilization.

At the beginning

- The Sumerians developed the earliest known writing system – a pictographic writing system known as cuneiform script النصوص المسمارية , using wedge-shaped characters شكل الاوتاد inscribed on baked clay tablets – and this has meant that we actually have more knowledge of ancient **Sumerian and Babylonian mathematics** than of early Egyptian mathematics.

Old Mesopotamia

Mathematics

- **Mathematics** is a field of science that deals with the logic of reason, numbers, quantity, arrangement, sequence, and space/shape. (∫)
- **Mathematics** is involved in almost everything we do today, whether we know about it or not, and is really the mother of most sciences.
- Who discovered or invented mathematics?
- *Galileo*, said that mathematics is the language of science and expected that its grammar explains experimental results and even predicts novel phenomena

Concepts

- The **Sumerians** developed the **concepts** of addition, subtraction, multiplication and division.
- They also had a lot of knowledge about fractions and equations, and in fact, it is believed that many of the **math concepts** we have today **were** first thought of in Ancient **Sumer**.

Mathematical tablet

Sumerian Mathematics

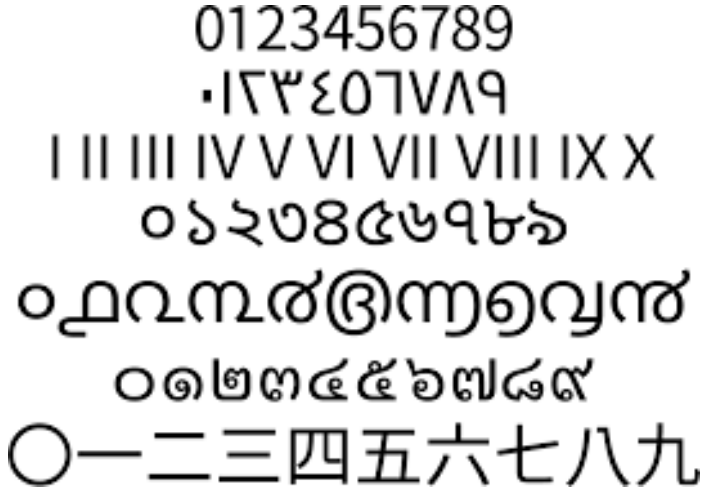
- Sumerian mathematics initially developed largely as a response to bureaucratic needs when their civilization settled and developed agriculture (possibly as early as the 6th millennium BCE) for the measurement of plots of land, the taxation of individuals, etc.
- In addition, the Sumerians and Babylonians needed to describe quite large numbers as they attempted to chart the course of the night sky and develop their sophisticated lunar calendar.

Sumerian Mathematics

They were perhaps the first people to assign symbols to groups of objects to make the description of larger numbers easier.

They moved from using separate tokens or symbols to represent sheaves of wheat, jars of oil, etc, to the more abstract use of a symbol for specific numbers of anything.

Number Systems



The basic concepts of numbers began to appear when the world's first civilization, the Sumerian civilization, established its roots in Mesopotamia more than 5,000 years ago.

The numerical sexagesimal system النظام الستيني used the positional property for the first time (meaning that the value of the symbol used depends on its position in the number).

The Sumerians had a complex variety of numerical systems, and each city had its own local way of writing numbers.

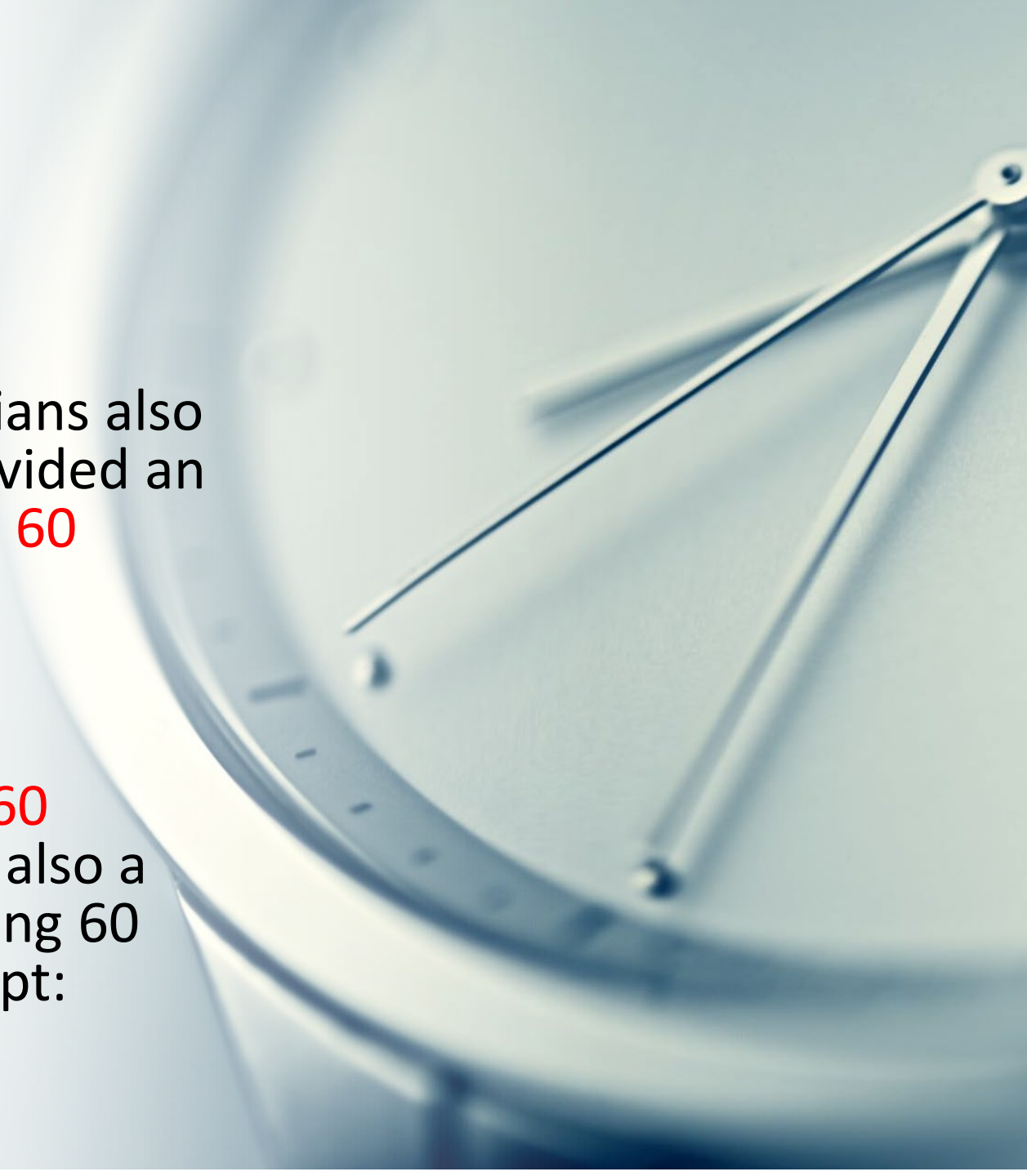
Time and Geometry

- **Time:**

The Sumerians formed the base for the system/concept of time and the Sumerians also divided time into groups of 60. They divided an hour into 60 minutes and a minute into 60 seconds.

- **Geometry and other concepts:**

The Sumerians divided the circle into 360 degrees, which (is not a coincidence) is also a multiple of 60. The concept of multiplying 60 by 6 is also part of a modern-day concept: multiplication.



Range of Sumerian Mathematics

- The Sumerians developed a complex system of metrology علم القياس from 3000 BC.
- From 2600 BC onwards, the Sumerians wrote multiplication tables on clay tablets and dealt with geometrical exercises and division problems.
- They were able to divide into fractions and multiply into the million, to calculate roots or raise numbers several powers. ...
- The 360-degree circle, the foot and its 12 inches, and the "dozen" as a unit, are but a few examples of the remains of their mathematics, still evident in our daily lives.

YBC 7289 Tablet

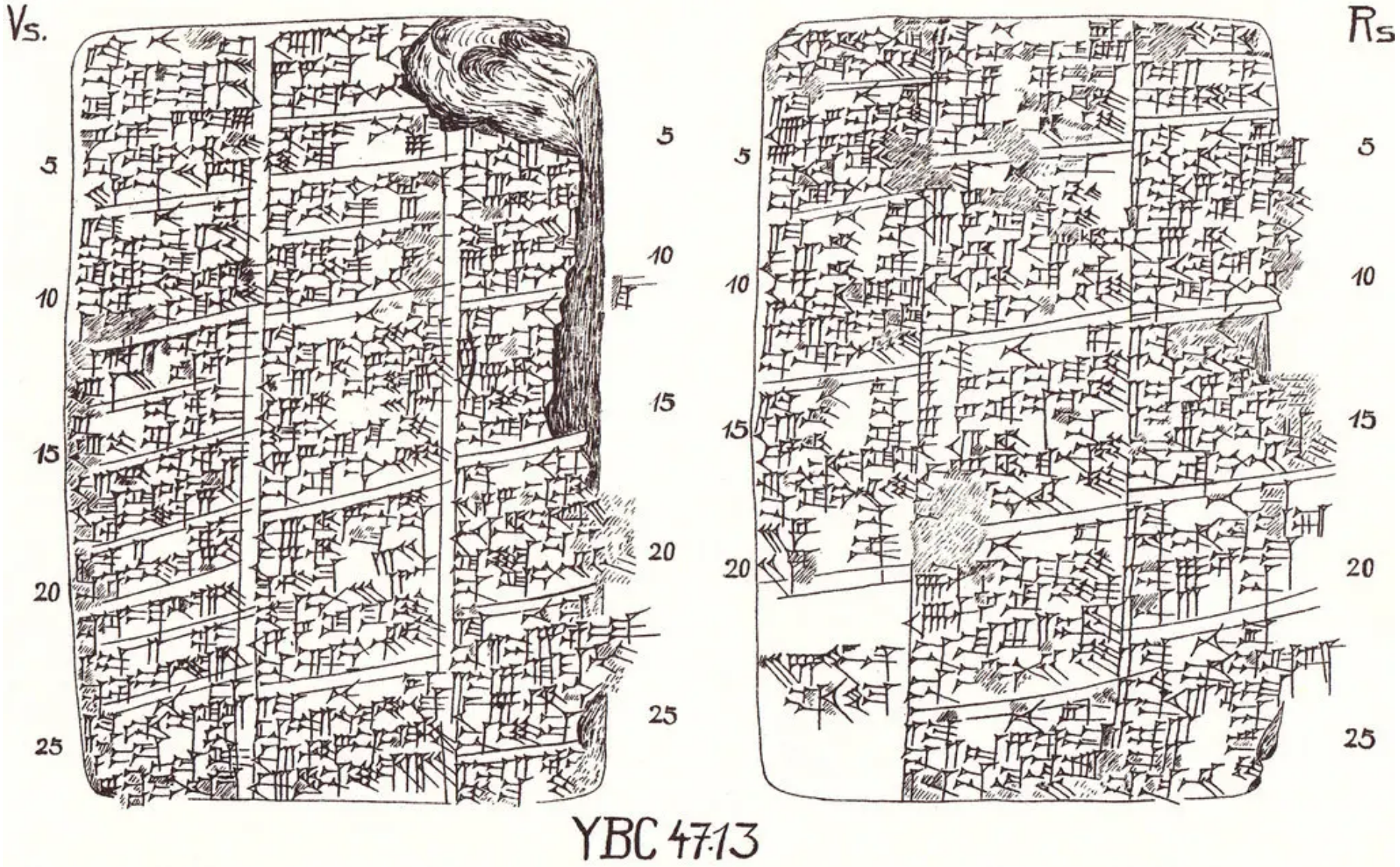
Square root of 2

- YBC 7289 is a small clay disc containing a rough sketch of a square and its diagonals.
- Across one of the diagonals is scrawled 1,24,51,10 sexagesimal number that corresponds to the decimal number 1.41421296.
- It can be recognized at once as the square root of 2.
- In fact, it's an approximation, a very good one, to the true value, 1.41421356.

Plimpton 322 Tablet

This table lists two of the three numbers in what are now called Pythagorean triples, i.e., integers a, b, and c satisfying $a^2 + b^2 = c^2$.

YBC 4713 Tablet



Sumerian Clay Cones

Sexagesimal System

- At that time, the Sumerians developed unique number systems, using the radix or base of sixty in scientific terms, and this system is called sexagesimal.
- The system base was 60, while the base of ten is what we use today.
- The Sumerians counted things with sixty as a unit. They had the same symbol for the numbers 1 and 60.

Sexagesimal System

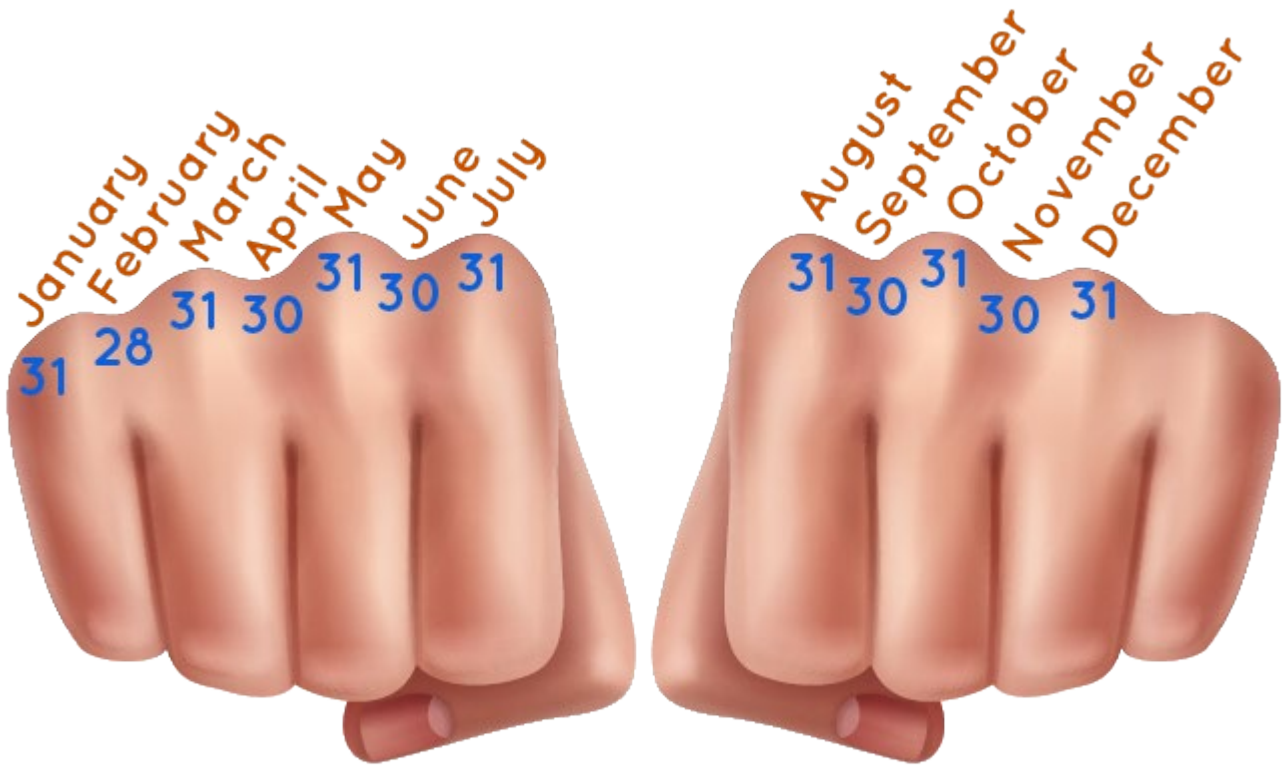
- The Sumerians used the sexagesimal numbers not only because the number 60 has many denominators or is countable on the fingers of both hands but because 60 is the lowest common multiple المضاعف المشترك of the number of fingers of both hands and the number of months in the year.

Time and Degrees

- Today, we use the base ten, not sixty, decimal number system as the basic unit, but that does not mean that the Sumerians' invention has become obsolete. As a matter of fact, it still plays a crucial role in our daily life.
- For example, why an hour is 60 minutes, and a minute has 60 seconds? Have you ever thought about why a full circle has 360 degrees? As it turns out, this was how the Sumerians kept track of their time, and that was how a full circle is defined.
- The Babylonians divided the day into twenty-four hours, every hour into sixty minutes, and one minute into every sixty seconds. This form of counting has survived for four thousand years to the present day.

Why 12?

- The Sumerians used their knuckles عظام الكفين to calculate the duodecimal system الاثني عشرية (12), and divided the day, from sunrise to sunset, into 12 parts, so the day and night were divided together into 24 parts.



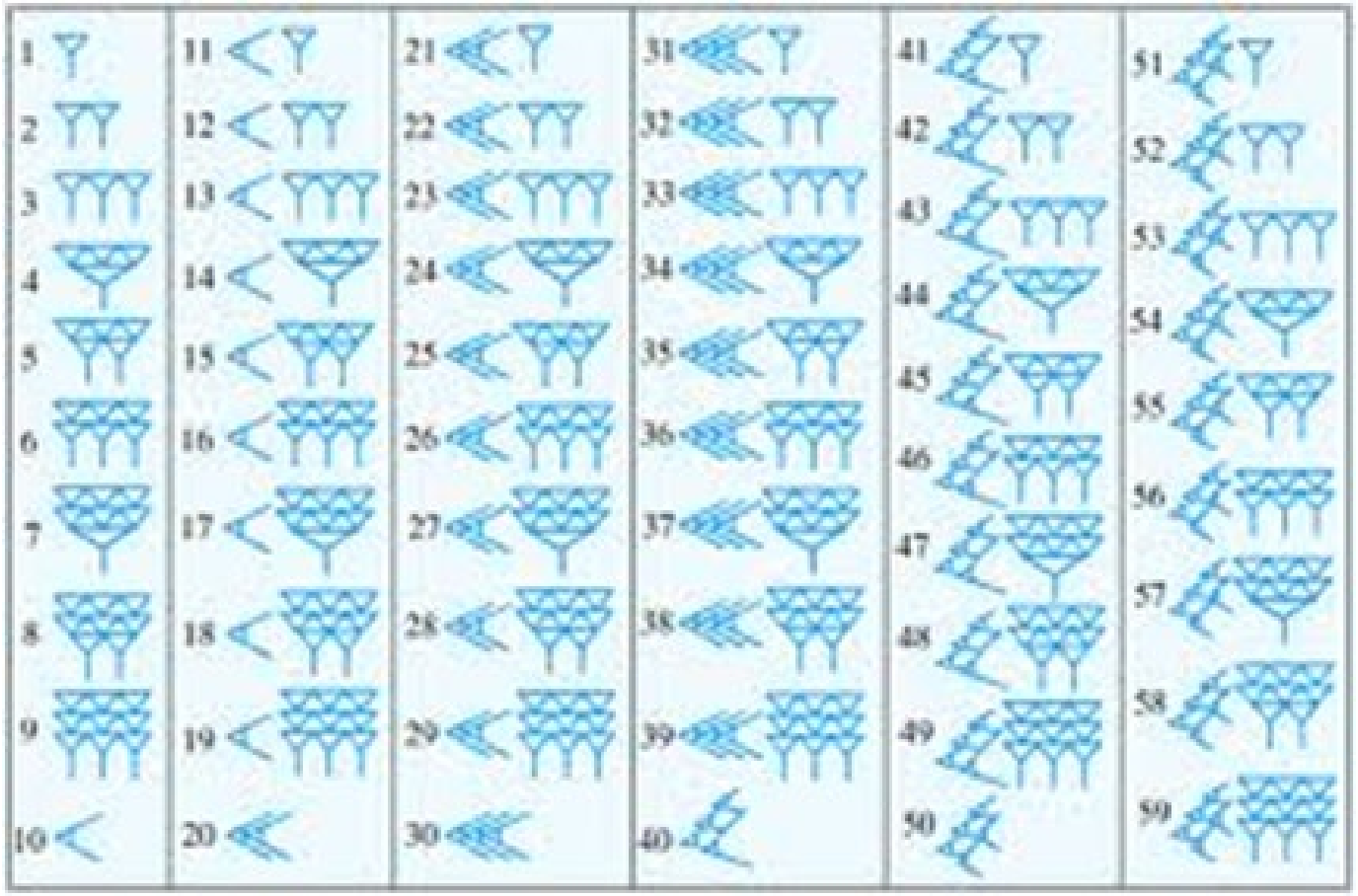
Old Mathematics Relics

- Even before the ancient Babylonian period, around 2000 BC, there was fully advanced mathematics.
- Thousands of mathematical and economic tablets have been discovered, demonstrating an impressive knowledge of
  - arithmetic,
  - linear and quadratic equations, and
  - numerous geometric and arithmetic constructions.

Old Mathematics Relics

- There are:
  - times tables,
  - tables of areas,
  - square roots, and
  - common constants.
- There are lists of math problems that teachers have created, and the solutions that they give to students.
- By this time the number system for arithmetic had settled on the sexagesimal, or sixty-rule, and it was easier for them to calculate fractions.

Sumerian Numerals Representation



Writing numbers

- Writing four 
- Writing 20 
- Writing 47 
- Writing 64 

Arithmetic

- The Babylonians used pre-calculated tables to assist with arithmetic. For example, two tablets found at Senkerah on the Euphrates in 1854, dating from 2000 BC, give lists of the squares of numbers up to 59 and the cubes of numbers up to 32.
- The Babylonians used the lists of squares together with the formulae:

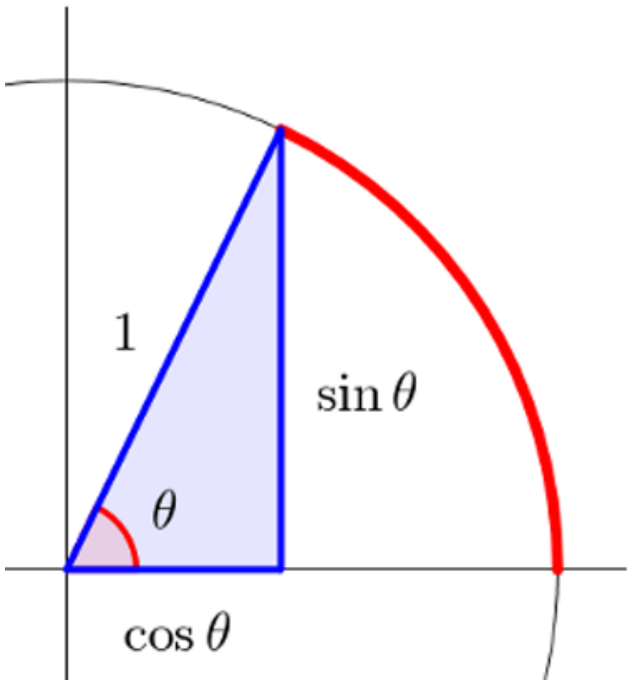
$$ab = \frac{(a+b)^2 - a^2 - b^2}{2}$$

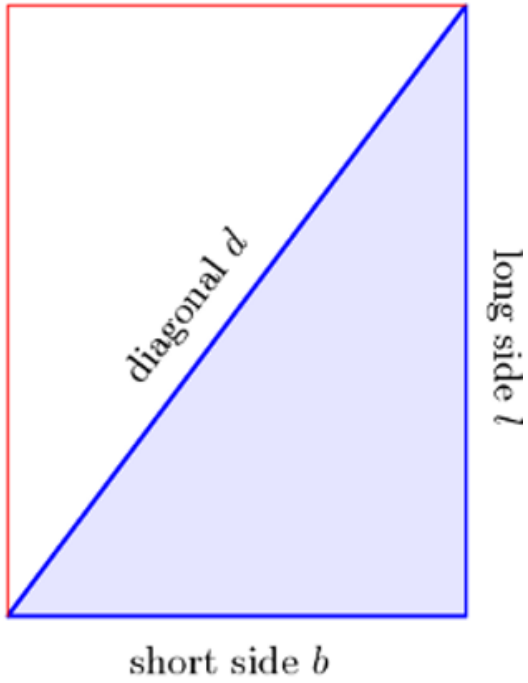
$$ab = \frac{(a+b)^2 - (a-b)^2}{4}$$

Babylonian trigonometry

- The Babylonians discovered their own unique form of trigonometry during the Old Babylonian period (1900-1600BCE), more than 1,500 years earlier than the Greek form.
- Remarkably, their trigonometry contains none of the hallmarks of our modern trigonometry - *it does not use angles and it does not use approximation.*
- The Babylonians had a completely different conceptualisation of a right triangle. They saw it as half of a rectangle, and due to their sophisticated sexagesimal (base 60) number system they were able to construct a wide variety of right triangles using only exact ratios.

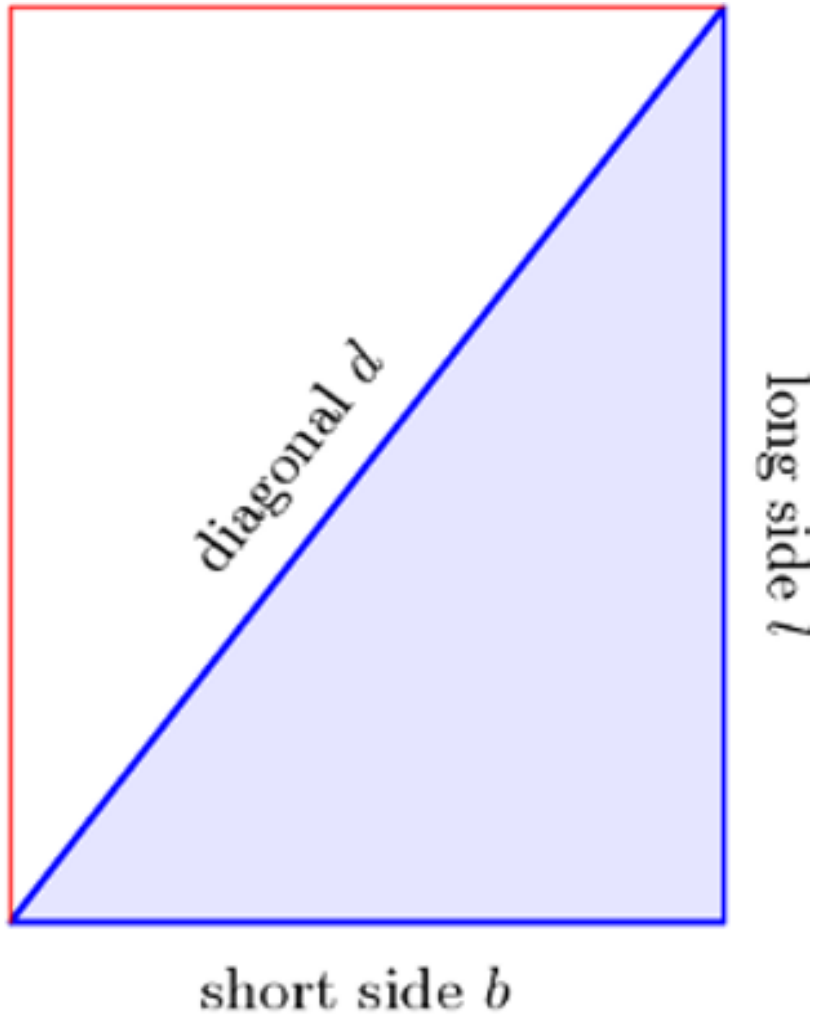
The Greek (left) and Babylonian (right) conceptualisation of a right triangle. Notably the Babylonians did not use angles to describe a right triangle.





b/l Ratio

- This precise arithmetic of the Babylonians also influenced their geometry, which they preferred to be exact. They were able to generate a wide variety of right-angled triangles within exact ratios b/l and d/l, where b, l and d are the short side, long side and diagonal of a rectangle.
- The ratio b/l was particularly important to the ancient Babylonians and Egyptians because they used this ratio to measure steepness.



The Plimpton 322 tablet

- We now know that the Babylonians studied trigonometry because we have a fragment of a one of their trigonometric tables.
- Plimpton 322 is a broken clay tablet from the ancient city of Larsa, which was located near Tell as-Senkereh in modern day Iraq.
- The tablet was written between 1822-1762BCE.

Trigonometric Tables

- The surviving fragment of Plimpton 322 starts with the Pythagorean triple 119, 120, 169. The next triple is 3367, 3456, 4825. This makes sense when you realise that the first triple is almost a square (which is an extreme kind of rectangle), and the next is slightly flatter. In fact the right-angled triangles are slowly but steadily getting flatter throughout the entire sequence.

b/l	d/l	Information relating to b/d or d/b			row
		$(d/l)^2$	b	d	
0.99166666	1.40833333	1.98340277	119	169	1
0.97424768	1.39612268	1.94915855	3367	4825	2
0.95854166	1.38520833	1.91880212	4601	6649	3
0.94140740	1.37340740	1.88624790	12709	18541	4
0.90277777	1.34722222	1.81500771	65	97	5

The first five rows of Plimpton 322, with reconstructed columns and numbers written in decimal.

Precision

- The most remarkable aspect of Babylonian trigonometry is its precision. Babylonian trigonometry is exact, whereas we are accustomed to approximate trigonometry.
- It is difficult to say why this approach to trigonometry has not survived. Perhaps it went out of fashion because the Greek approach using angles is more suitable for astronomical calculations.
- Some contend that the Babylonian approach is also much simpler because it only uses exact ratios. There are no irrational numbers and no angles, and this means that there is also no sin, cos or tan or approximation.

Geometry

- Babylonians knew the common rules for measuring volumes and areas. They measured the circumference of a circle as three times the diameter and the area as one-twelfth the square of the circumference, which would be correct if **π is estimated as 3.**
- They were aware that this was an approximation, and one Old Babylonian mathematical tablet excavated near Susa in 1936 (dated to between the 19th and 17th centuries BCE) gives a better approximation of π as 25/8 = 3.125, about 0.5 percent below the exact value.
- The volume of a cylinder was taken as the product of the base and the height.

The Pythagorean theorem was also known to the Babylonians.

Geometry

- The "Babylonian mile" was a measure of distance equal to about 11.3 km (or about seven modern miles). This measurement for distances eventually was converted to a "time-mile" used for measuring the travel of the Sun, therefore, representing time.
- The ancient Babylonians had known of theorems concerning the ratios of the sides of similar triangles for many centuries, but they lacked the concept of an angle measure and consequently, studied the sides of triangles instead.
- The Babylonian astronomers kept detailed records of the rising and setting of stars, the motion of the planets, and the solar and lunar eclipses all of which required familiarity with angular distances measured on the celestial sphere.

Geometry

- Tablets kept in the *British Museum* provide evidence that the Babylonians even went so far as to have a concept of objects in an abstract mathematical space.
- The tablets date from between 350 and 50 B.C.E., revealing that the Babylonians understood and used geometry even earlier than previously thought.
- The Babylonians used a method for estimating the area under a curve by drawing a trapezoid underneath شبه منحرف, a technique previously believed to have originated in 14th century Europe.

Babylonian Programming-1

- The Babylonian mathematicians were not limited simply to the processes of addition, subtraction, multiplication, and division; they were adept at solving many types of algebraic equations.
- But they did not have an algebraic notation that is quite as transparent as ours; they represented each formula by a step-by-step list of rules for its evaluation, i.e. by an algorithm for computing that formula.
- In effect, they worked with a "machine language" representation of formulas instead of a symbolic language.

General Procedure

- The calculations described in Babylonian tablets are not merely the solutions to specific individual problems: they actually are general procedures for solving a whole class of problems.
- The numbers shown are merely included as an aid to exposition, in order to clarify the general method.
- **Note also** the stereotyped ending, "*This is the procedure*," which is commonly found at the end of each section on a tablet.
- Thus, the Babylonian procedures are genuine algorithms, and we can commend the Babylonians for developing a nice way to explain an algorithm by example as the algorithm itself was being defined.

Conclusions

- Not enough recognition in the world of the role of Sumerians and Babylonians in creating mathematics but they were really original thinkers
- Some of the fundamental concepts of mathematics were developed by the Sumerians and Babylonians
- A lot of excavations and research needs to be done to understand clearly their methodologies for all mathematical operations
- Unfortunately a lot has been lost of their relic due to writing on clay rather than stone.

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