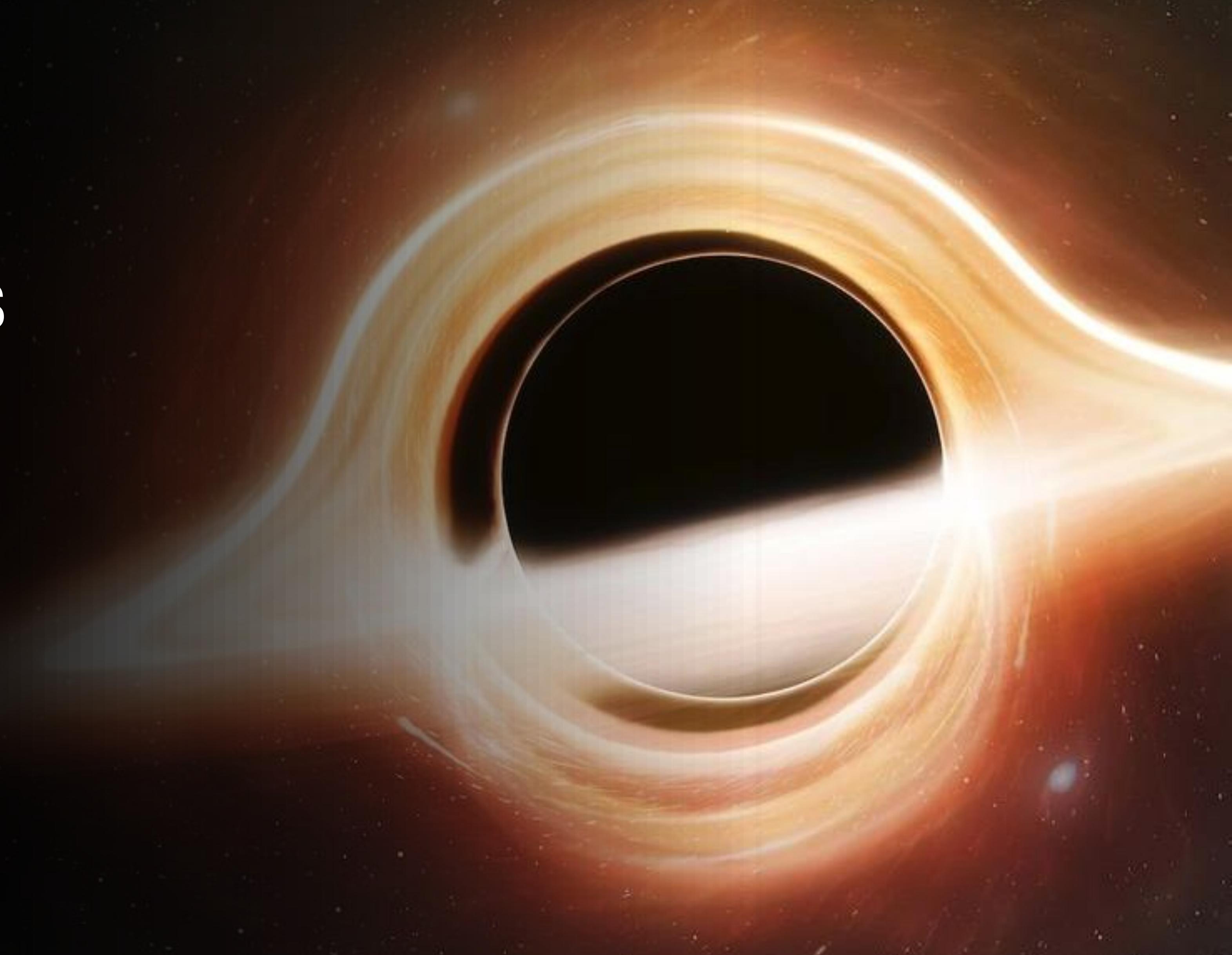
Black Hole Basics

Cosmic Sink Holes & Natural Time Machines



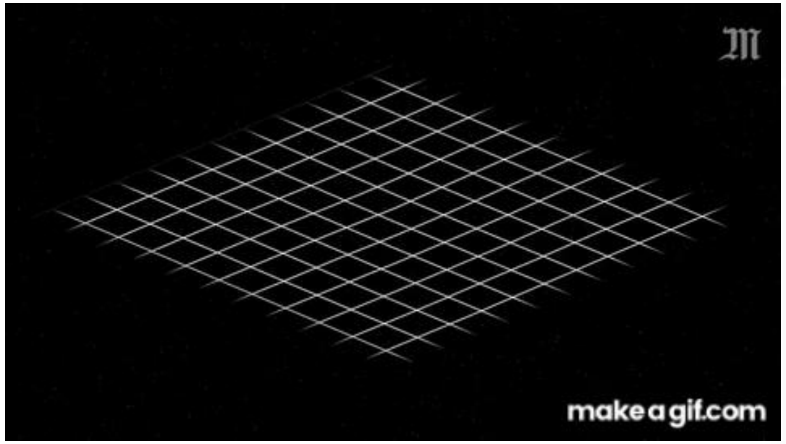
Robert Clemenson

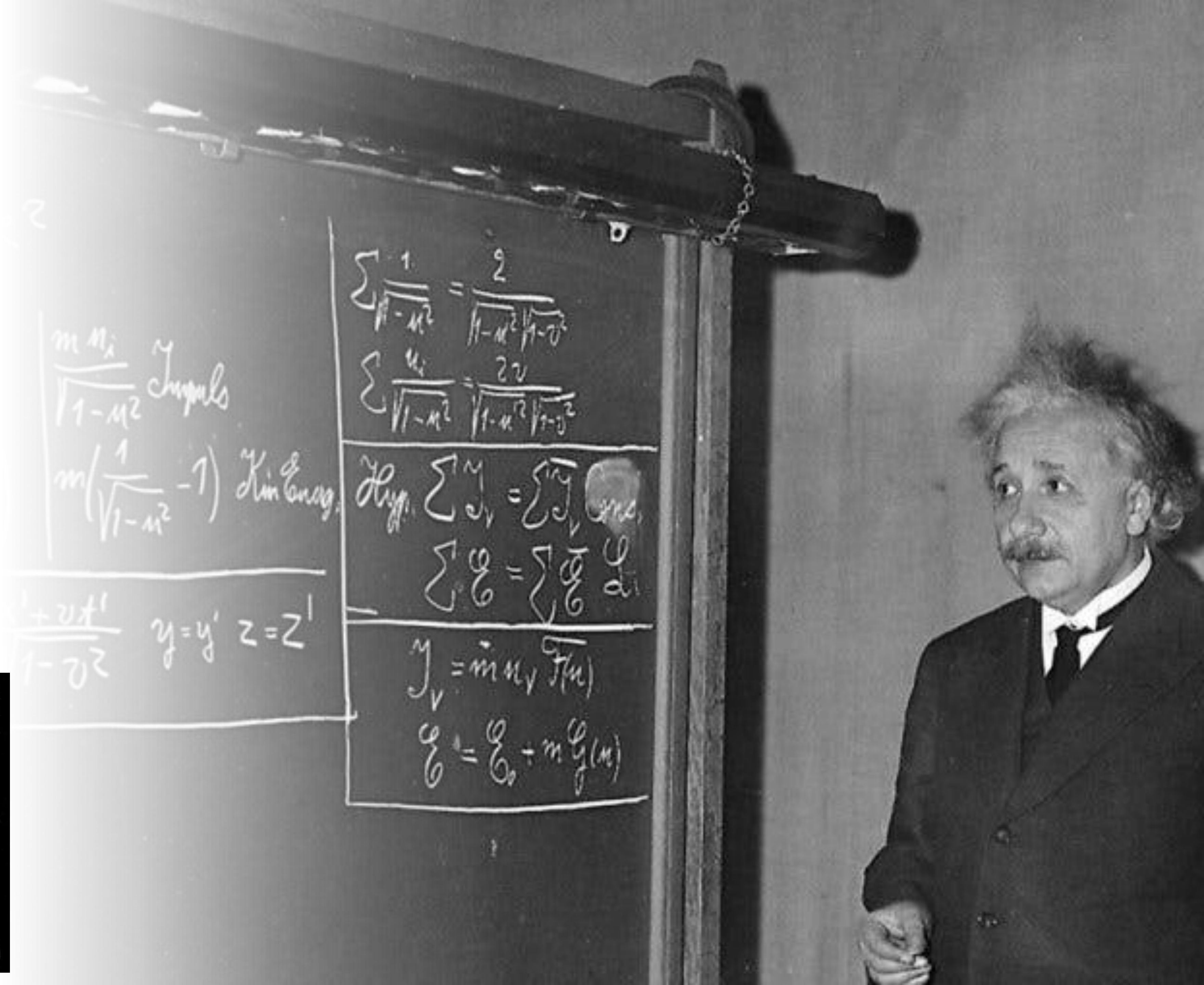
(Sussex U & Royal Holloway UoL)

Southend City Day – 01.03.2025

Einstein’s Breakthrough

1915: Einstein publishes his theory of gravity, called ‘*General Relativity*’.

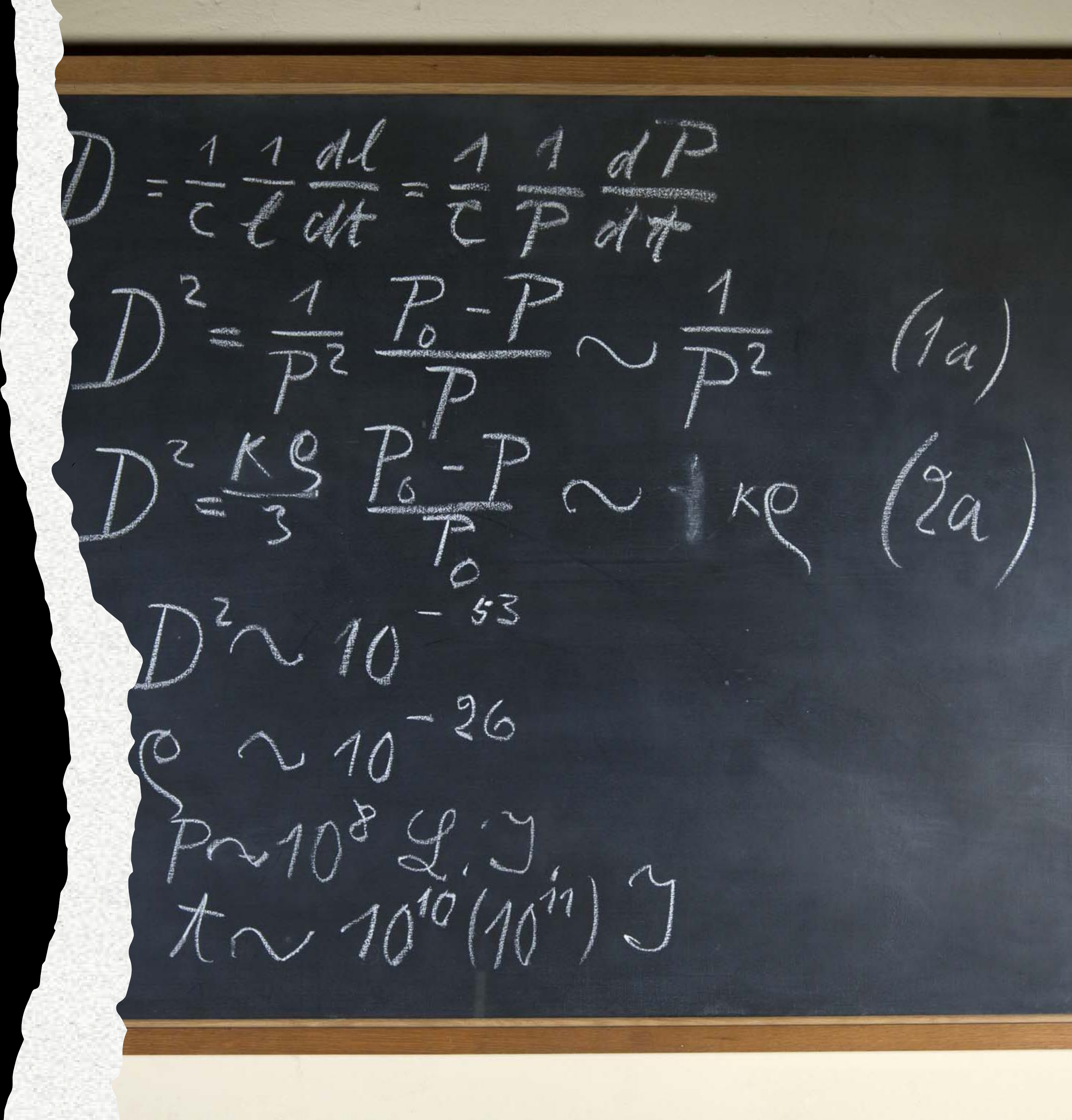




A Hidden Monster in the Equations

1916: Karl Schwarzschild is the first to solve Einstein’s new equations of gravity.

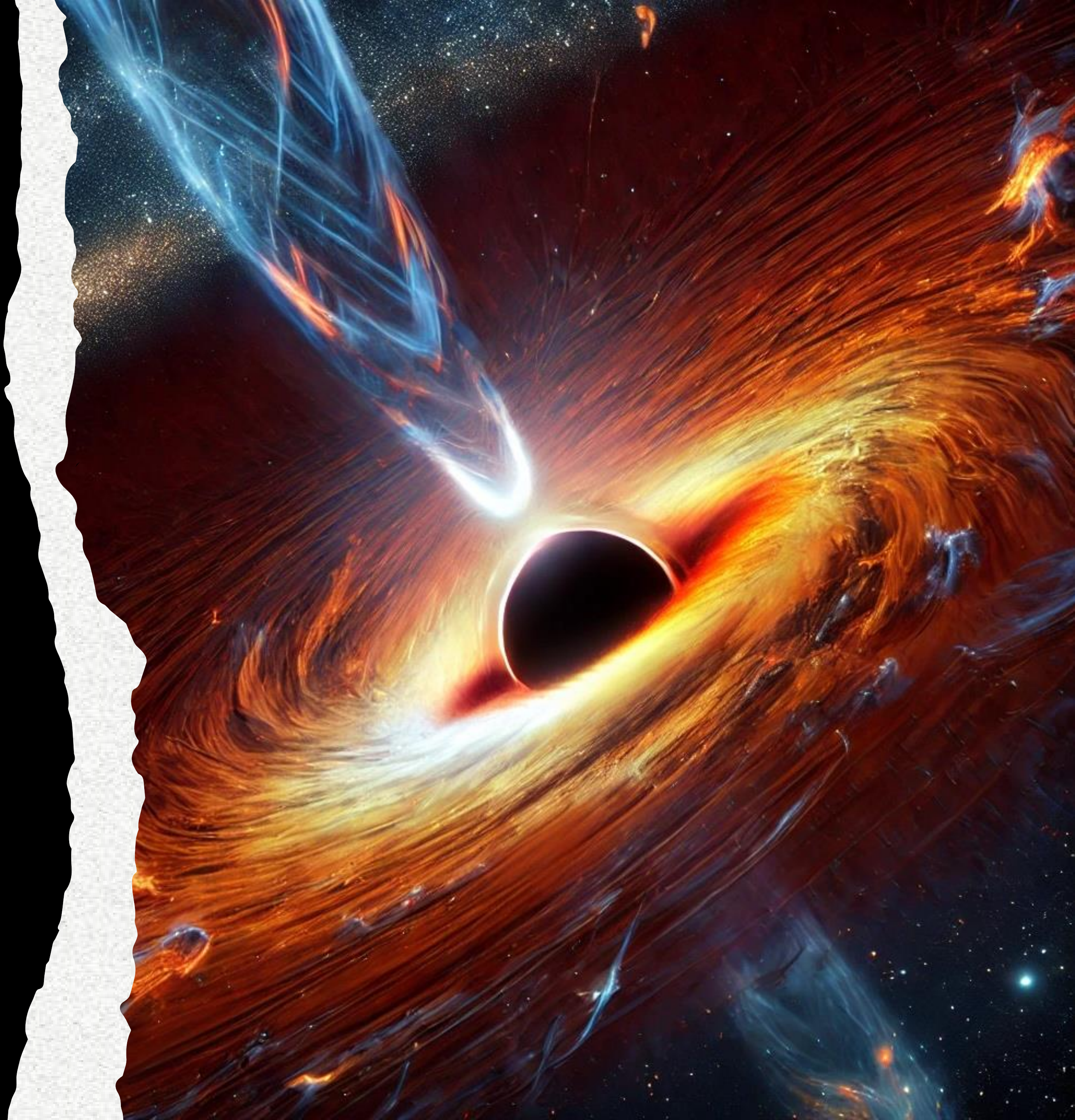
$$r_s = \frac{2GM}{c^2}$$



A Hidden Monster in the Equations

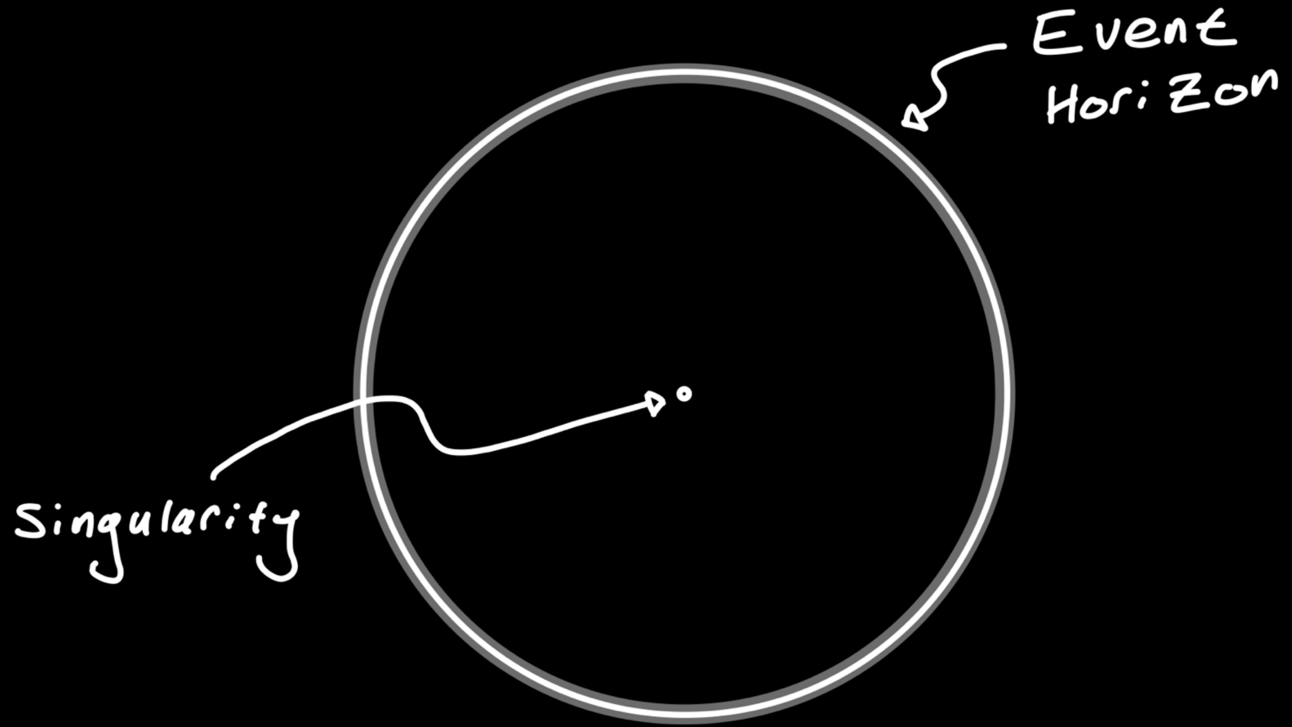
Schwarzschild's solution describes the extreme end point of the collapse of matter.

Einstein himself did not believe in these ‘*singularities*’.



Anatomy of a Black Hole

Describing a black hole is simple enough, as they have very few identifying features.



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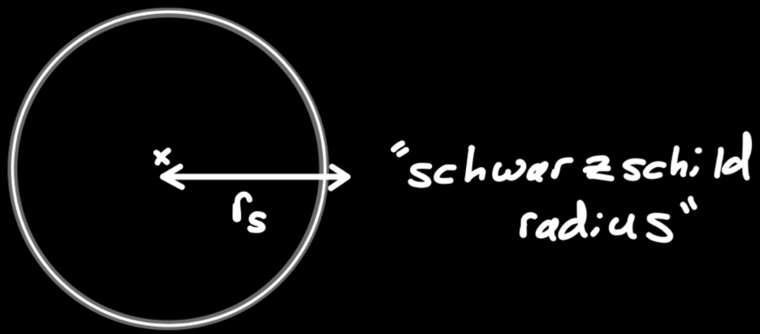
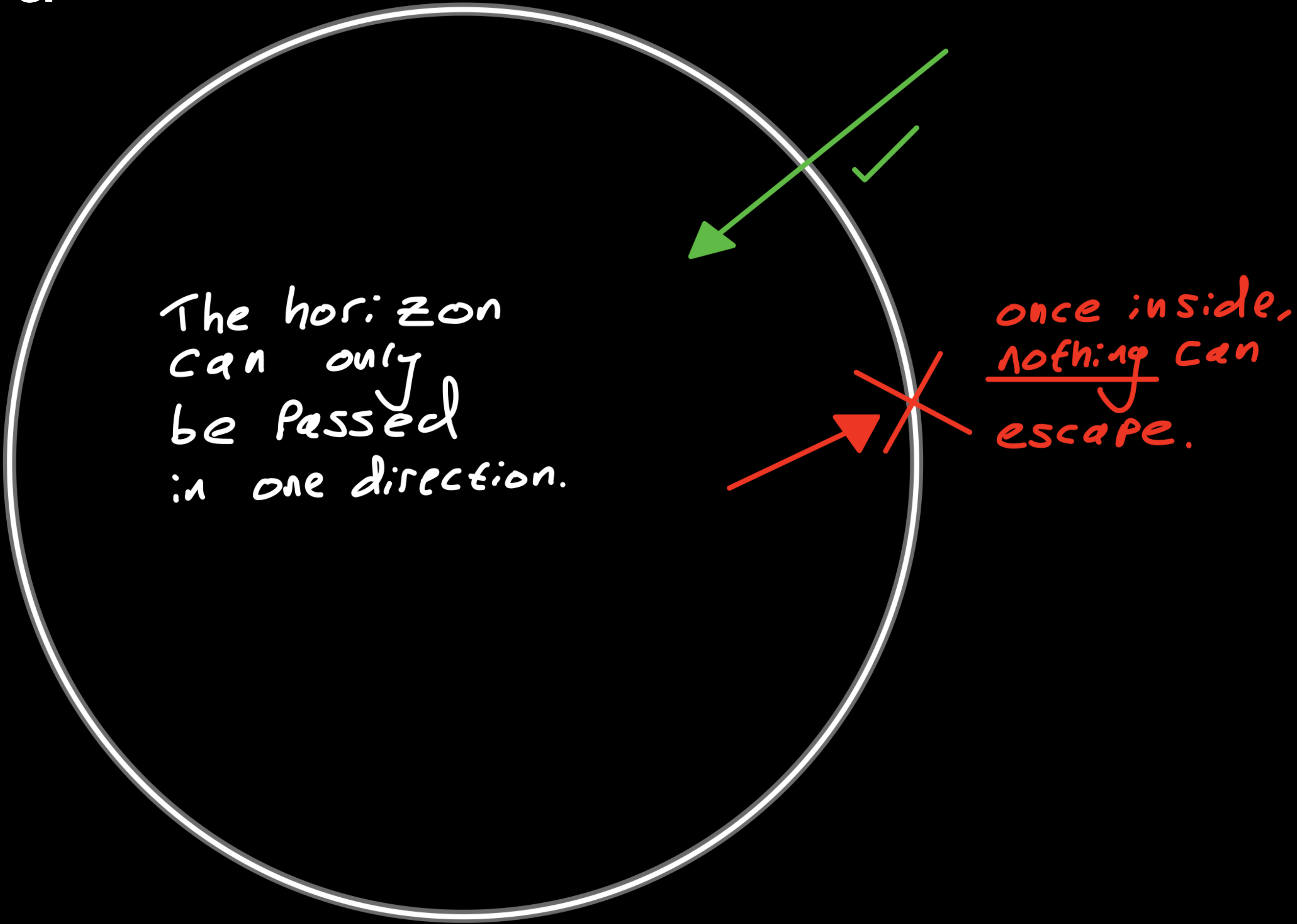
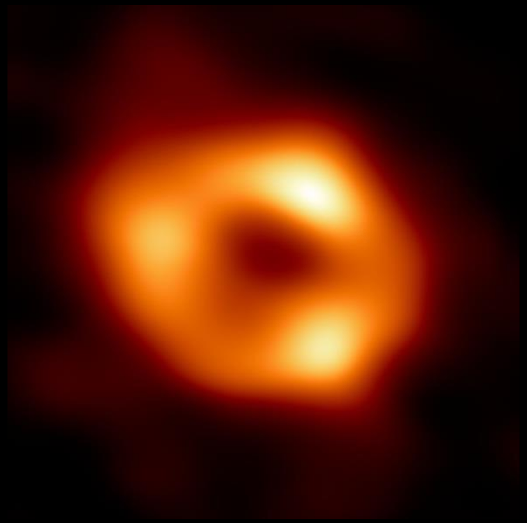


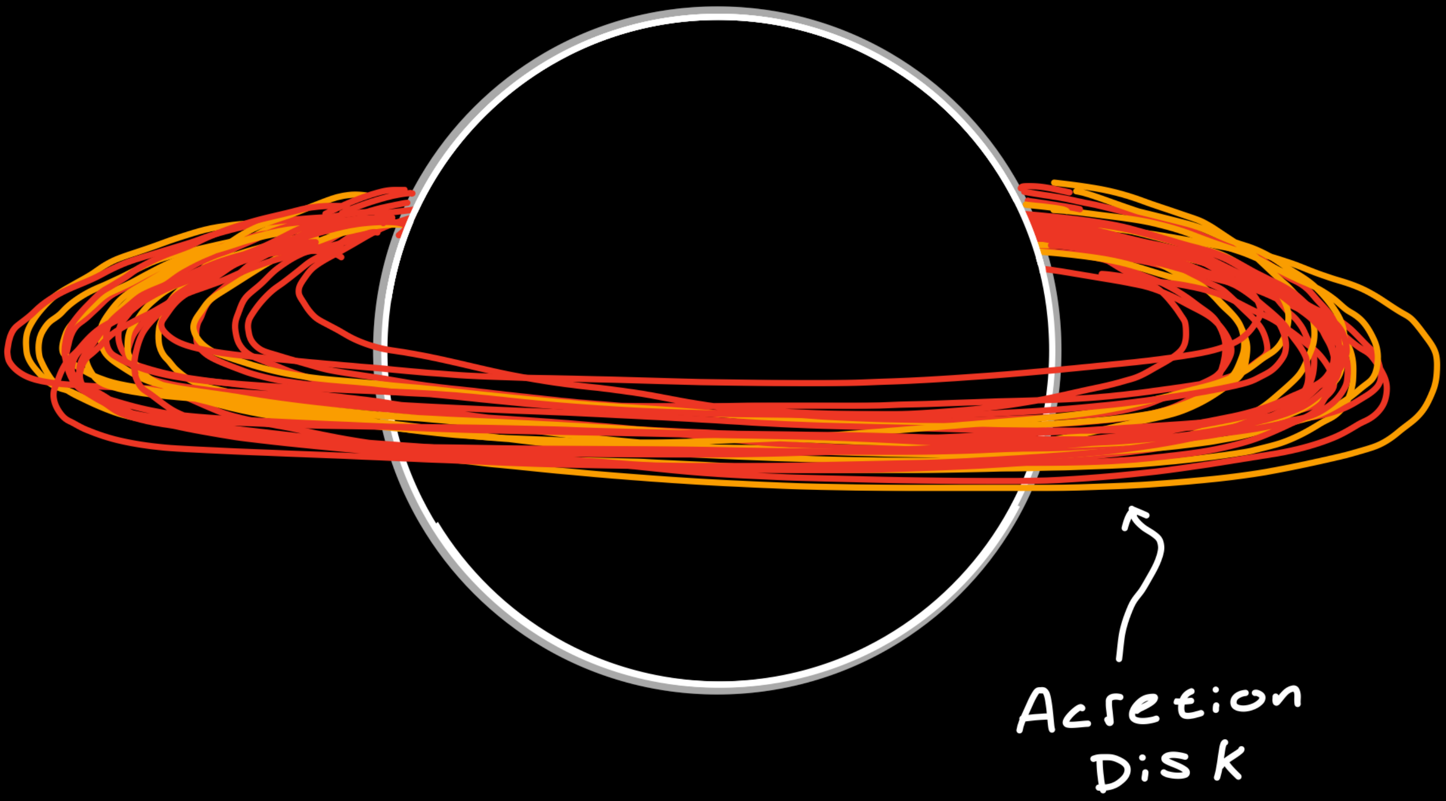
Anatomy of a Black Hole



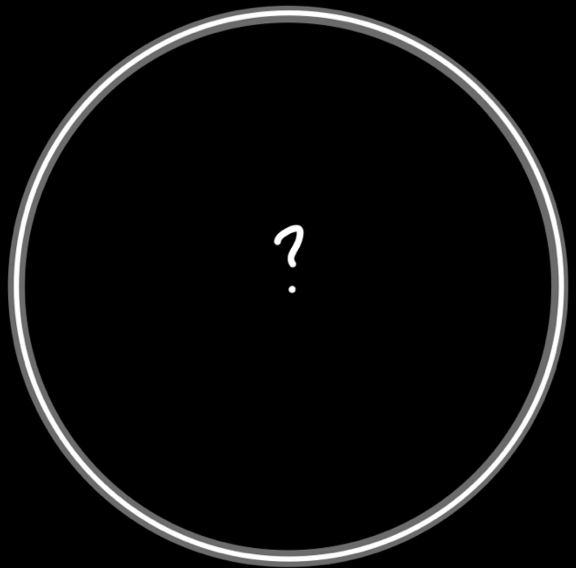
Anatomy of a Black Hole



Event Horizon Telescope
image of Sagittarius A*.



Anatomy of a Black Hole

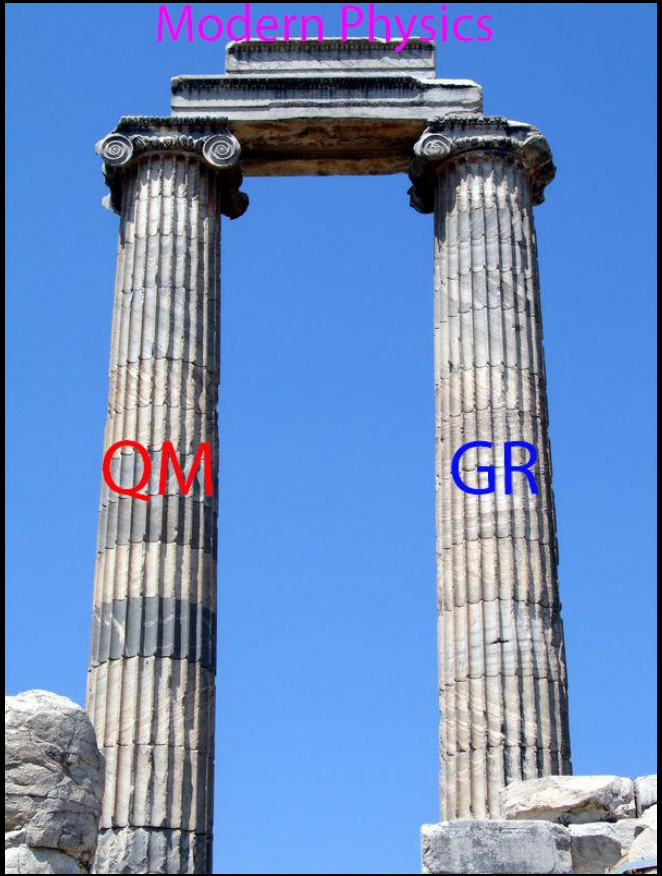


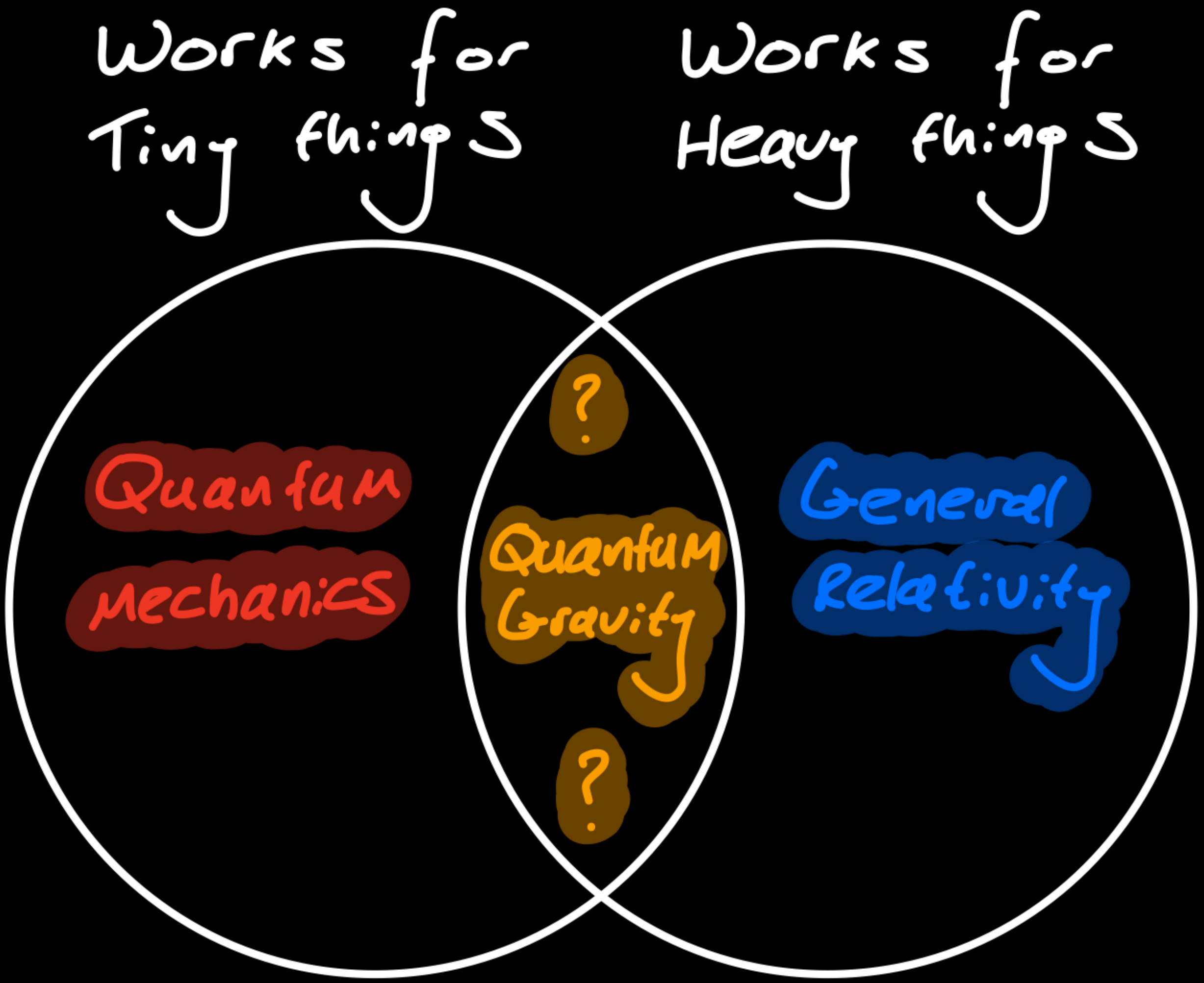
The Singularity:

- The point at the centre of a black hole.
- 'Infinitely Dense'.
- The laws of physics break down.

Anatomy of a Black Hole

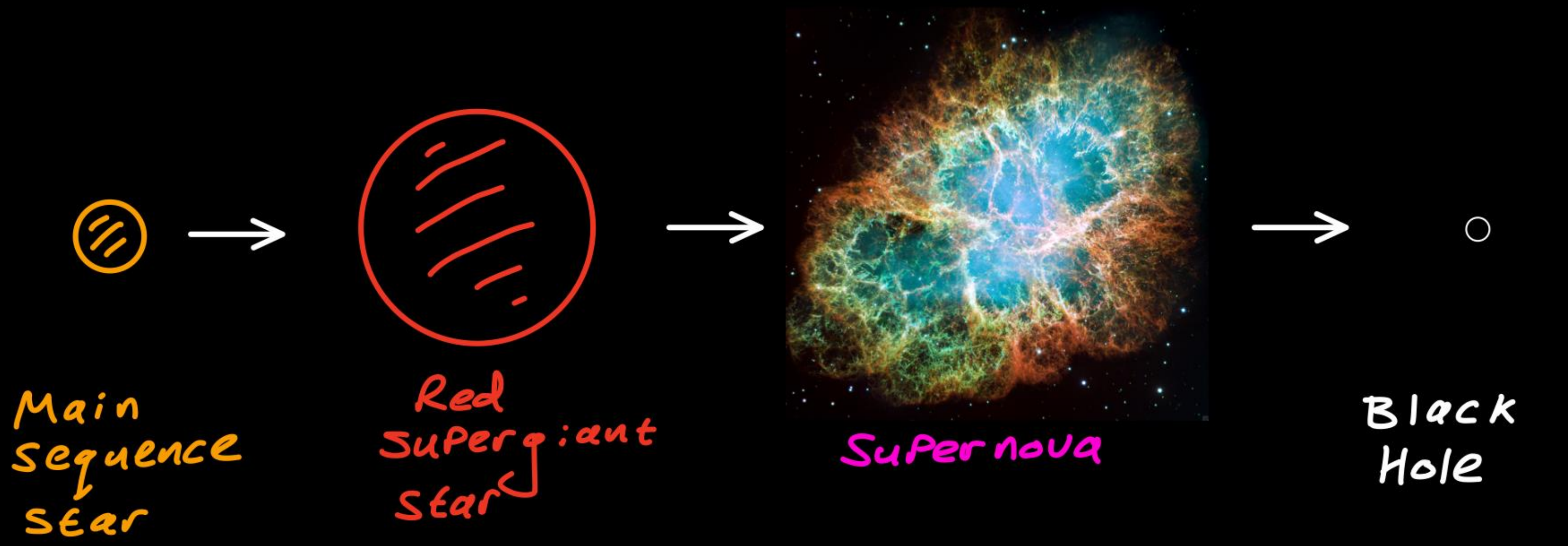
Why is the singularity so hard to understand?





Black Hole Formation

Black holes form when giant stars run out of fuel, and collapse under their own weight.

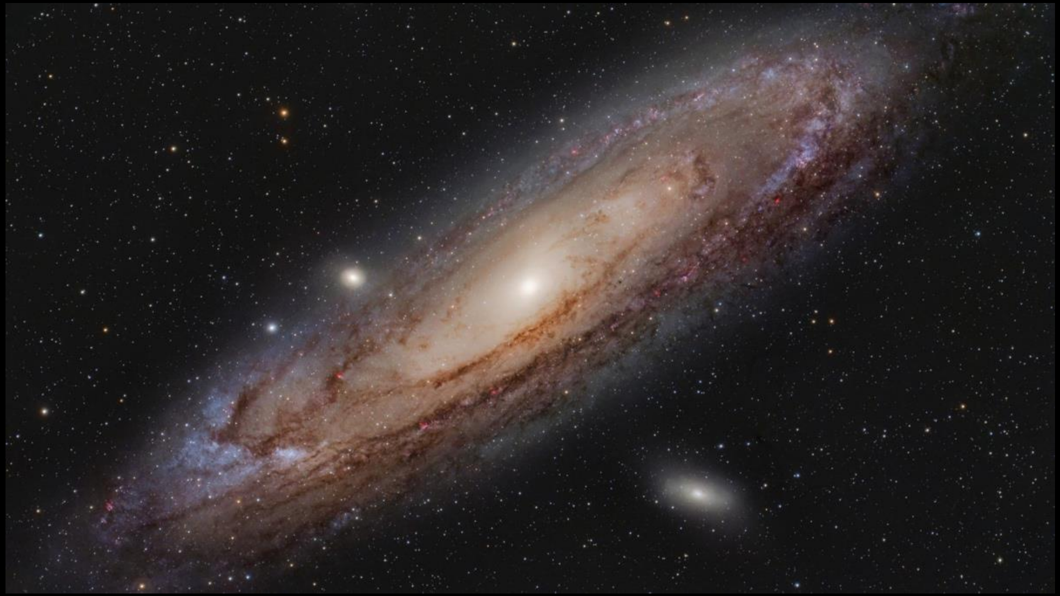


Roger Penrose and Stephen Hawking bridged the gap from Schwarzschild’s theoretical solution of Einstein’s equations, to show that black holes can form in real life (imperfect) conditions.

Supermassive Black Holes

Every large galaxy has a Supermassive Black Hole at its center.

An artist's impression of a Quasar, a particularly energetic kind of galactic nuclei.



The Andromeda Galaxy. The nearest neighboring galaxy to our own.

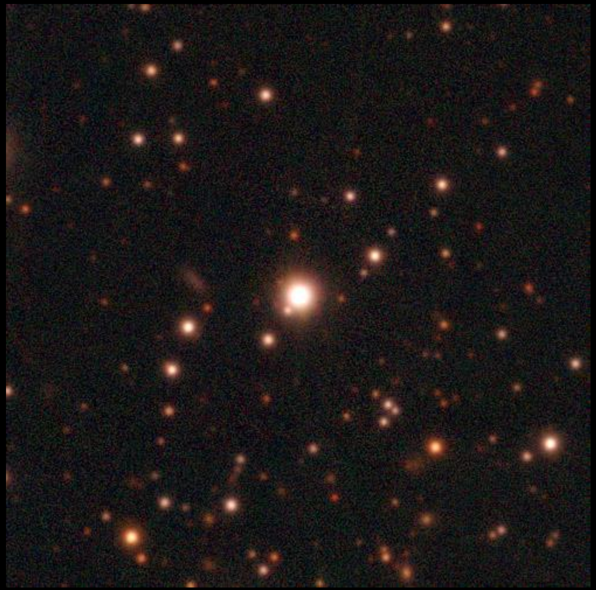
Black Holes are thought to be crucial to the formation of galaxies, in ways we don’t yet fully understand.

Note: Supermassive = Between one hundred thousand, and 1 billion times the mass of the sun.

Our Nearest Neighbor

1560 light-years away, a star very similar to our own sun orbits around a black hole.

This black hole has a mass nearly ten times greater than our sun.



Pan-STARRS image of the Gaia BH1 system.

Gaia BH-1

$$M = 9.62\, M_\odot = 9.62 \times 1.99 \times 10^{30}$$

$$= 1.91 \times 10^{31}\ \text{kg}$$

$$r_s = \frac{2GM}{c^2} = \frac{2 \times 6.67 \times 10^{-11} \times 1.91 \times 10^{31}}{(3 \times 10^{8})^2}$$

$$= 28310.4\ \text{m}$$

$$= 28.3\ \text{km}$$

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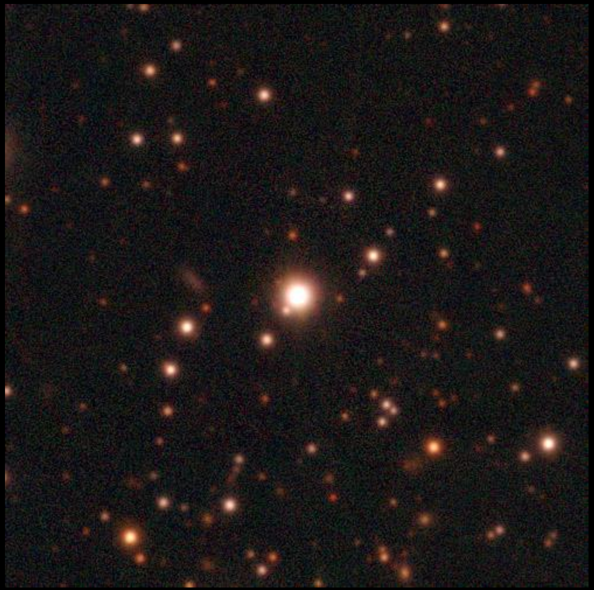


Pan-STARRS image of the Gaia BH1 system.

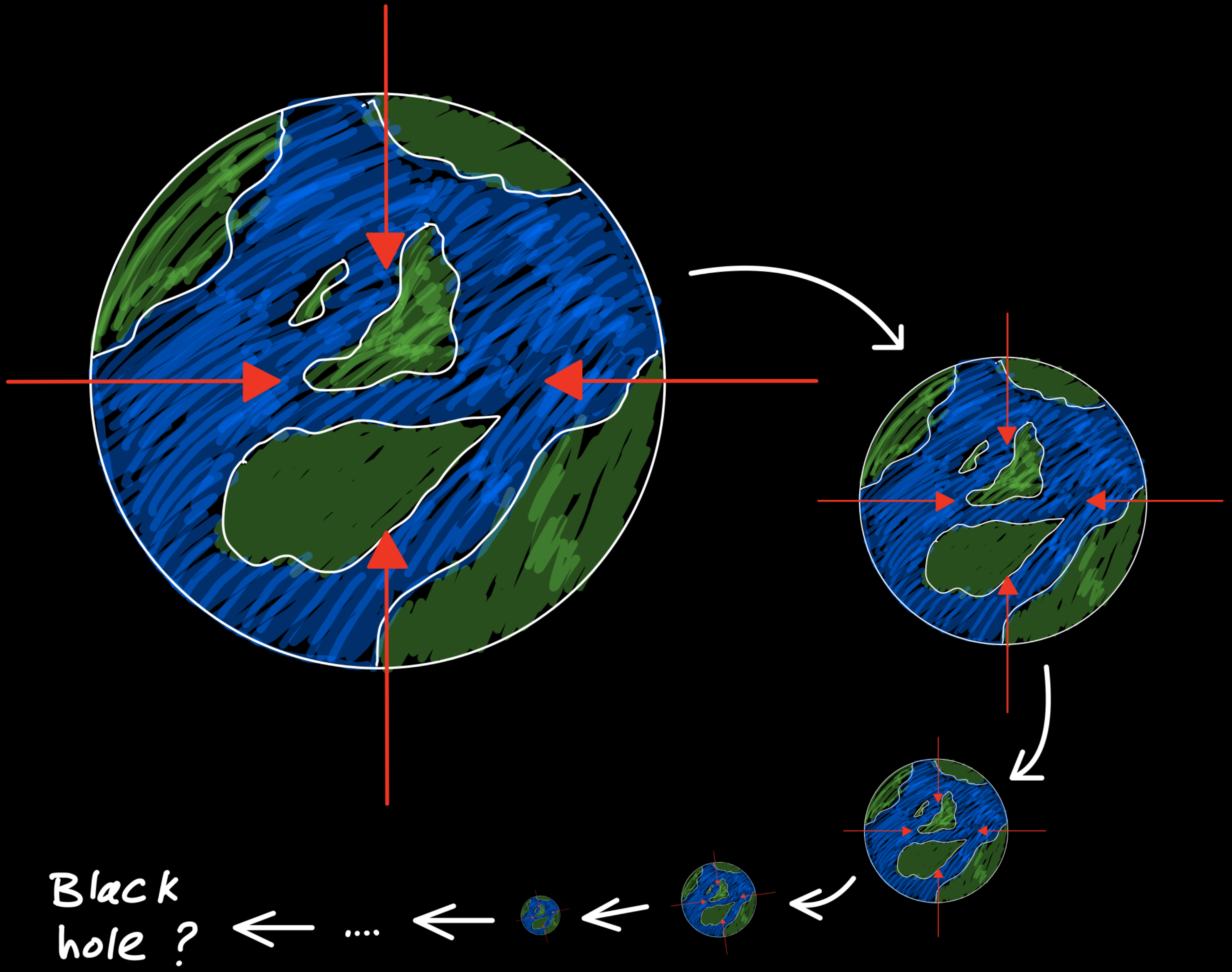
This is the size of our nearest black hole (Gaia BH1), compared to Essex.

The radius stretches roughly from the Beecroft Gallery to the Dartford Bridge.

A Sense of Scale

Let's squash the Earth down until it turns into a black hole.

How far would we need to squash it?



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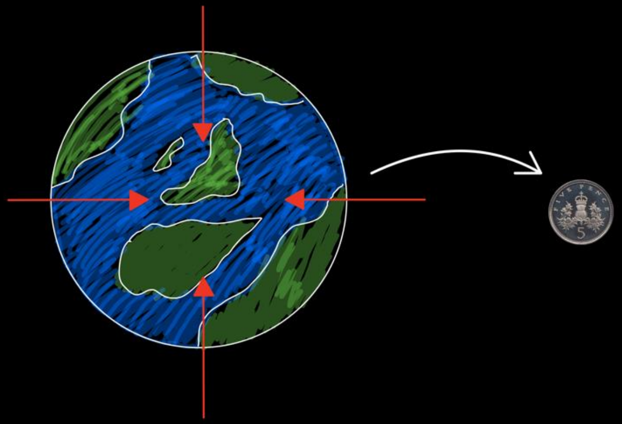


Earth Mass Black Hole

$$M_{Earth} = 5.97 \times 10^{24}\ \text{kg}$$

$$r_s = \frac{2GM}{c^2} = \frac{2 \times 6.67 \times 10^{-11} \times 5.97 \times 10^{24}}{(3 \times 10^8)^2}$$

$$= 8.85 \times 10^{-3}\ \text{m}$$

$$= 8.85\ \text{mm}$$

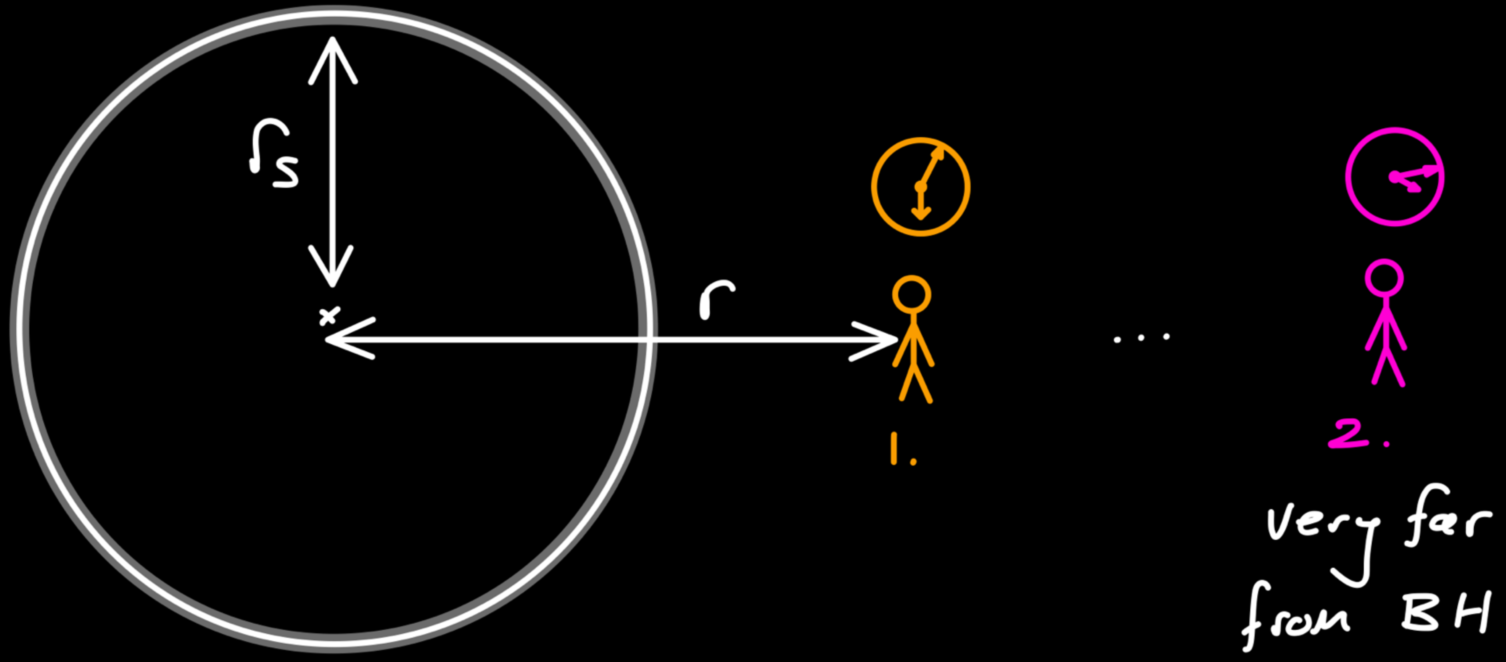
$$\Rightarrow \text{Diameter} \simeq 18\text{mm}$$

About the same as a 5p coin.

Natural Time Machines

Perhaps the strangest property of black holes, is how the bend and stretch time for those nearby...

$$\boxed{\Delta t_{near} = \Delta t_{far}\sqrt{1 - \frac{r_s}{r}}}$$



Person 2's clock will tick much faster than Person 1's.

Let's put this into perspective using Gaia BH1, the nearest known black hole to the Earth.

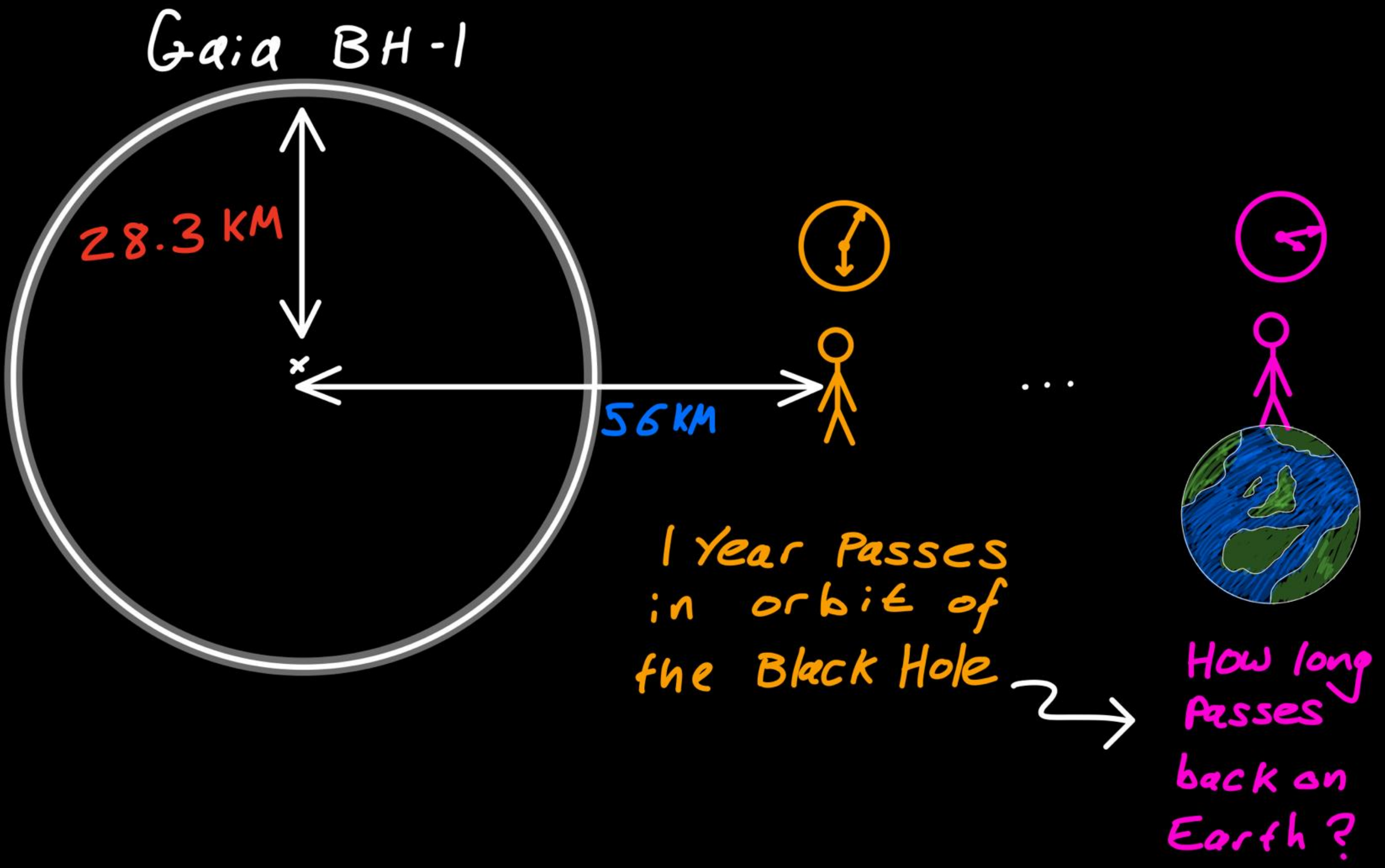
Natural Time Machines

Recall, Gaia BH1's radius stretched out to Dartford.

Suppose we get into an orbit that would stretch out to central London.

Natural Time Machines

Spend 1 year orbiting our nearest known black hole, and you will have travelled forwards in time by 5 months compared to everyone home on Earth.



$$\Delta t_{Earth} = \frac{1\ yr}{\sqrt{1 - \frac{28.3\ KM}{56\ KM}}} = 1\ yr + 5\ Months$$

How do we know this?

“How can you possibly know this is correct? Given we’ve never been to a black hole!”

We (physicists) did the maths!

“But how do you know the maths works? You can’t test this in a lab.

There are many predictions we can test in the real world.

E.g. GPS

$$\mathcal{L} = -\left(1-\frac{2M}{r}\right)\dot{t}^2 + \left(1-\frac{2M}{r}\right)^{-1}\dot{r}^2 + r^2\dot{\theta}^2 + r^2\sin^2\theta\,\dot{\phi}^2$$

where $\dot{y} \equiv \frac{dy}{d\lambda}$

a)

(i) Euler Lagrange equations: $\frac{d}{d\lambda}\left(\frac{\partial\mathcal{L}}{\partial\dot{x}^\mu}\right) = \frac{\partial\mathcal{L}}{\partial x^\mu}$

- $\mathcal{L}$ is independant of time

$\Rightarrow \frac{\partial\mathcal{L}}{\partial t} = 0$

$\Rightarrow \frac{\partial\mathcal{L}}{\partial\dot{t}}$ is constant along geodesics

$\frac{\partial\mathcal{L}}{\partial\dot{t}} = -2\left(1-\frac{2M}{r}\right)\dot{t} = \text{constant}$

Let $E := \left(1-\frac{2M}{r}\right)\dot{t}$

This conserved quantity is associated with the particles energy.

- $\mathcal{L}$ is independant of ϕ.

$\Rightarrow \frac{\partial\mathcal{L}}{\partial\dot{\phi}} = \text{constant}$

$\frac{\partial\mathcal{L}}{\partial\dot{\phi}} = 2r^2\sin^2\theta\,\dot{\phi}$

Let $J := r^2\sin^2\theta\,\dot{\phi}$

J is associated with conserved angular momentum.

- $\mathcal{L}$ is itself conserved.

The interval,

$$ds^2 = -\left(1-\frac{2M}{r}\right)dt^2 + \left(1-\frac{2M}{r}\right)^{-1}dr^2 + r^2\left(d\theta^2 + \sin^2\theta\,d\phi^2\right)$$

is invariant, and has no dependance on the affine parameter λ. Therefore $\mathcal{L}$ is a constant along the geodesic.

$\mathcal{L} = -1$ for timelike geodesics parameterized by proper time τ, and $\mathcal{L} = 0$ for null geodesics.

(ii)

Consider the Euler Lagrange equation for θ:

$\frac{\partial\mathcal{L}}{\partial\theta} = 2\sin\theta\cos\theta(\dot{\phi})^2$

$\frac{\partial\mathcal{L}}{\partial\dot{\theta}} = 2r^2\dot{\theta}$

$\Rightarrow \frac{d}{d\lambda}\left(2r^2\dot{\theta}\right) = 2\sin\theta\cos\theta(\dot{\phi})^2$

We can orient our coord system initially at $\lambda = 0$, such that $\theta = \frac{\pi}{2}$ and $\left.\frac{d\theta}{d\lambda}\right|_{\lambda=0} = 0$ without loss of generality.

Making this choice

$\Rightarrow \frac{d}{d\lambda}\left(2r^2\dot{\theta}\right) = 0 \quad \text{at } \lambda = 0$

$\frac{d}{d\lambda}\left(2r^2\dot{\theta}\right) = 4r\dot{r}\dot{\theta} + 2r^2\ddot{\theta}$

at $\lambda = 0$, this becomes

$2r^2\ddot{\theta} = 0$

$\Rightarrow \dot{\theta}$ will remain constant

So if initially $\dot{\theta} = 0$, and $\theta = \frac{\pi}{2}$, the geodesic will remain in the $\theta = \frac{\pi}{2}$ plane.

b) Along a radial geodesic:

$\dot{\phi} = \dot{\theta} = 0$

$\Rightarrow \mathcal{L} = -\left(1-\frac{2M}{r}\right)\dot{t}^2 + \left(1-\frac{2M}{r}\right)^{-1}\dot{r}^2$

Defining $K = -\mathcal{L}$, so $K = +1$ for timelike curves parameterized by τ, and $K = 0$ for null curves, and using $E = \left(1-\frac{2M}{r}\right)\dot{t}$,

we can write:

$-K = -\left(1-\frac{2M}{r}\right)^{-1}E^2 + \left(1-\frac{2M}{r}\right)^{-1}\dot{r}^2$

$\Rightarrow \dot{r}^2 = E^2 - K\left(1-\frac{2M}{r}\right)$

with $K = +1$ for timelike curves (with $\lambda = \tau$)

$K = 0$ for null curves

c)

M (Mass) — (Satellite) — $r = R \gg 2M$ Alice

$\dot{r}^2 = E^2 - K\left(1-\frac{2M}{r}\right)$

(i) for Alice:

$\dot{r} = \frac{dr}{d\tau_A} = 0$

$\Rightarrow E^2 = \left(1-\frac{2M}{R}\right) \Rightarrow E = \left(1-\frac{2M}{R}\right)^{\frac{1}{2}}$

E is conserved along Alice's world line with $E = \left(1-\frac{2M}{R}\right)\dot{t}$

$\Rightarrow \frac{dt}{d\tau_A} = E\left(1-\frac{2M}{R}\right)^{-1}$

$= \left(1-\frac{2M}{R}\right)^{-\frac{1}{2}}$

$\Rightarrow d\tau_A = \left(1-\frac{2M}{R}\right)^{\frac{1}{2}}dt$

$\tau_A = 0$ when $t = 0$

$\Rightarrow \tau_A = \left(1-\frac{2M}{R}\right)^{\frac{1}{2}}t$

$\Rightarrow \tau_A = t\left(1-\frac{M}{R}+O\left(\left(\frac{M}{R}\right)^2\right)\right)$

fine to do as $R \gg 2M$

$\Rightarrow \tau_A = t + O\left(\frac{M}{R}\right)$

(ii)

M (Mass) — $t = t_0$, $r = r_0$ (Satellite) — $r = R \gg 2M$ Alice

$\dot{r}^2 = E^2 - K\left(1-\frac{2M}{r}\right)$

Along null geodesic of signal, we now have:

$\left(\frac{dr}{d\lambda}\right)^2 = E^2$

$\frac{dr}{d\lambda} = E \quad \leftarrow$ Taking +ve root as signal is outwards going

$\frac{dr}{d\lambda} = \left(1-\frac{2M}{r}\right)\frac{dt}{d\lambda}$

$\Rightarrow dt = \left(1-\frac{2M}{r}\right)^{-1}dr$

$\int_{t_0}^{t_1} dt = \int_{r_0}^{R}\left(\frac{r}{r-2M}\right)dr$

$t_1 - t_0 = \int_{r_0}^{R}\frac{r-2M+2M}{r-2M}\,dr$

$= \int_{r_0}^{R}\left[1 + 2M\frac{1}{r-2M}\right]dr$

$= R - r_0 + \left[2M\log(r-2M)\right]_{r_0}^{R}$

$= R - r_0 + 2M\log\left(\frac{R-2M}{r_0-2M}\right)$

$\log\left(1-\frac{2M}{R}\right) \sim -\frac{2M}{R} + O\left(\left(\frac{M}{R}\right)^2\right)$

$\Rightarrow 2M\log\left(\frac{R-2M}{r_0-2M}\right) \simeq 2M\log\left(\frac{R}{r_0-2M}\right)$

$\Rightarrow t_1 - t_0 = R - r_0 + 2M\log\left(\frac{R}{r_0+2M}\right)$

using: $\tau_A = t + O\left(\frac{M}{R}\right)$

$\tau_1 = t_1 + O\left(\frac{M}{R}\right)$

$\Rightarrow \tau_1 = t_0 + R - r_0 + 2M\log\left(\frac{R}{r_0-2M}\right) + O\left(\frac{M}{R}\right)$

d) $E \rightarrow$ Energy of satellite geodesic

$\Rightarrow E = \left(1-\frac{2M}{r}\right)\frac{dt}{d\tau_S}$

M (Mass) — $t = t_0$, $r = r_0$ (Satellite) — $r = R \gg 2M$ Alice

M (Mass) — $t = t_2$, $r = r_2$ (Satellite) — $r = R \gg 2M$ Alice

How do we know this?

“How can you possibly know this is correct? Given we’ve never been to a black hole!”

We (physicists) did the maths!

“But how do you know the maths works? You can’t test this in a lab.

There are many predictions we can test in the real world.

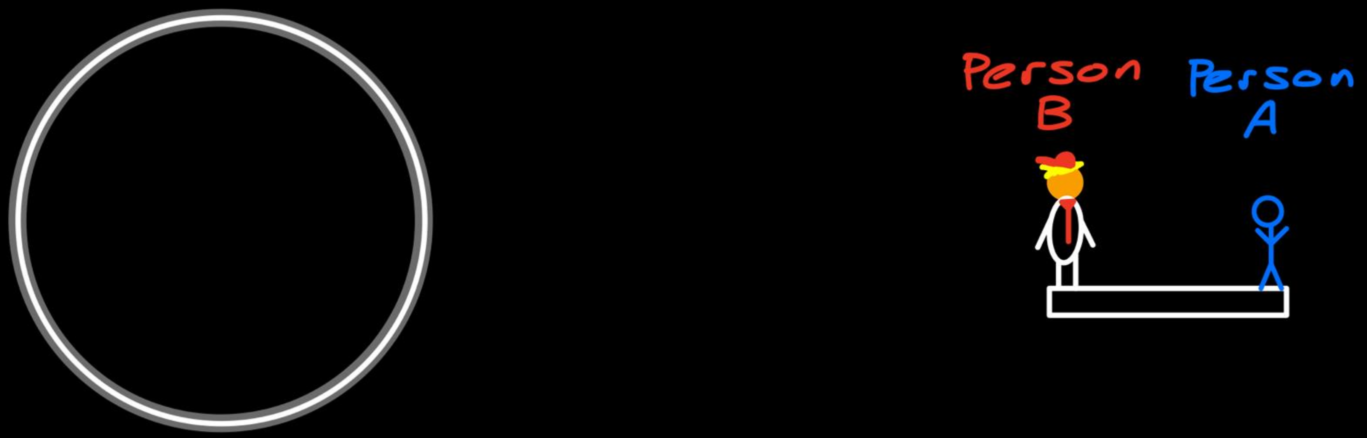
E.g. GPS

GPS satellites calculate location by pinging a signal between themselves and devices on the Earth. Extremely accurate clocks are needed to do this.

General relativity has to be accounted for in order for GPS to work.

Death by Black Hole

Person A pushes Person B into a black hole.



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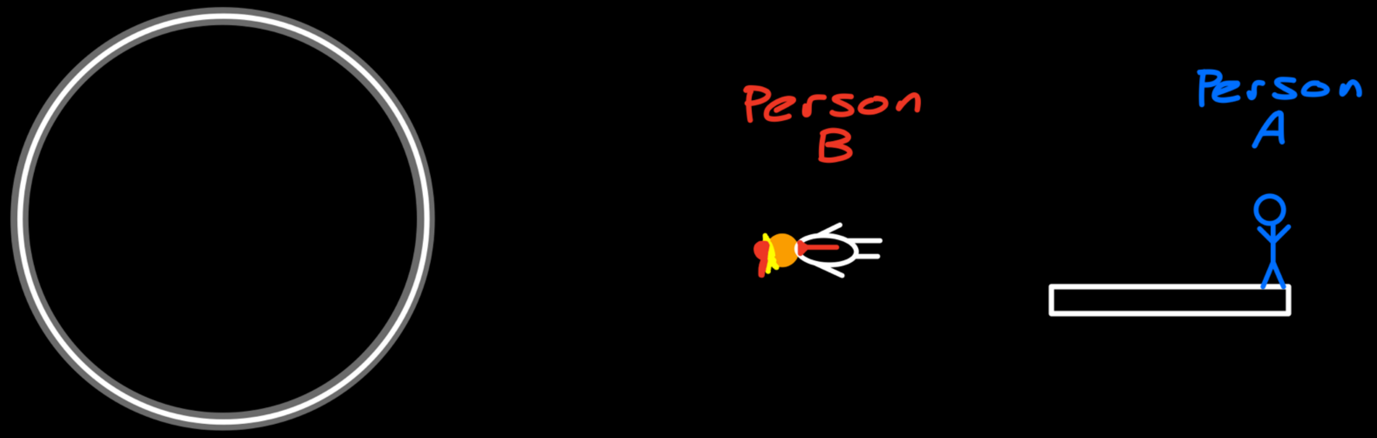


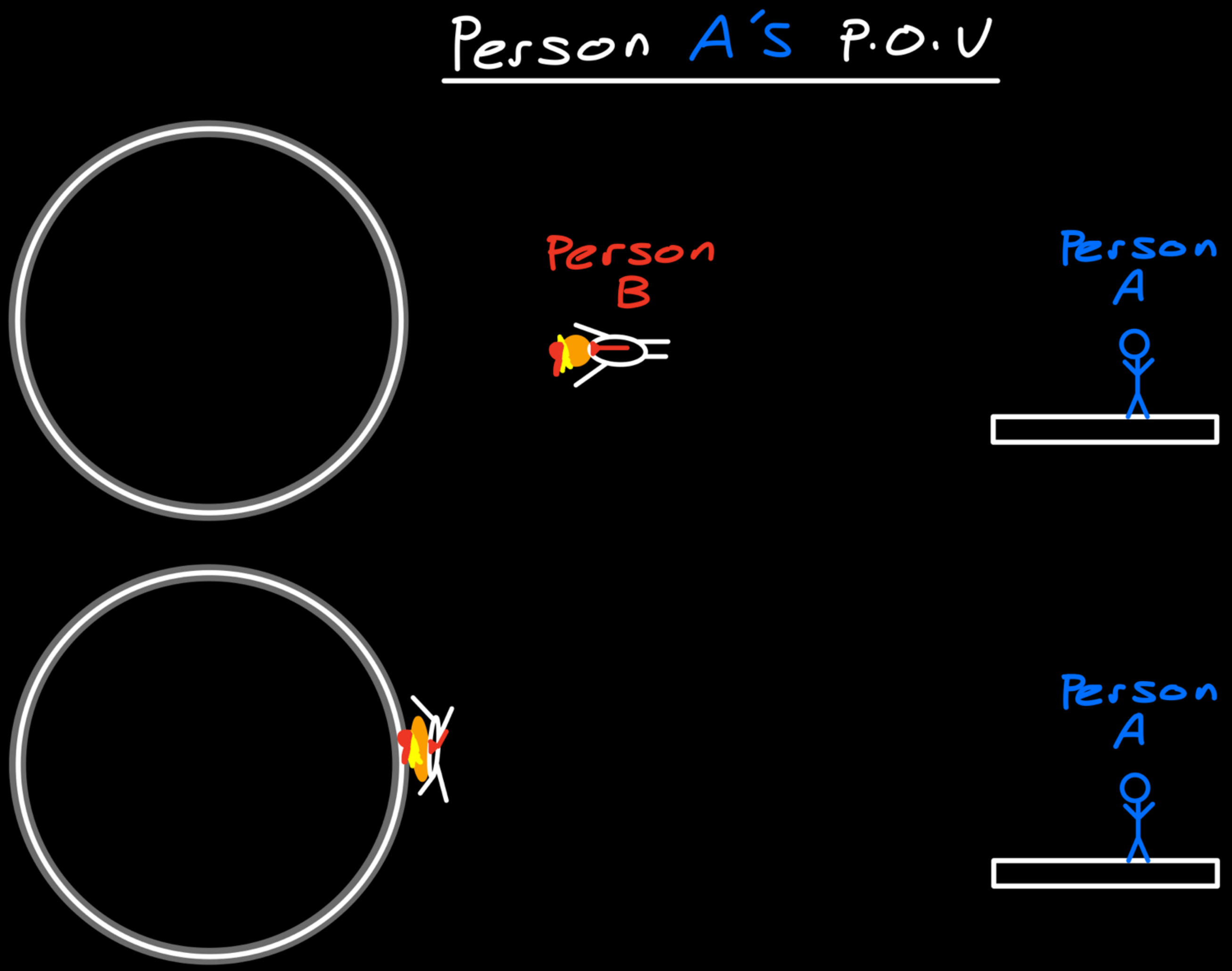
Let's consider how this looks to each person involved...

Death by Black Hole

Person A's POV

From Person A's POV, person B never crosses the event horizon.

Instead, Person B smears along the horizon, become flatter and flatter over billions of years.



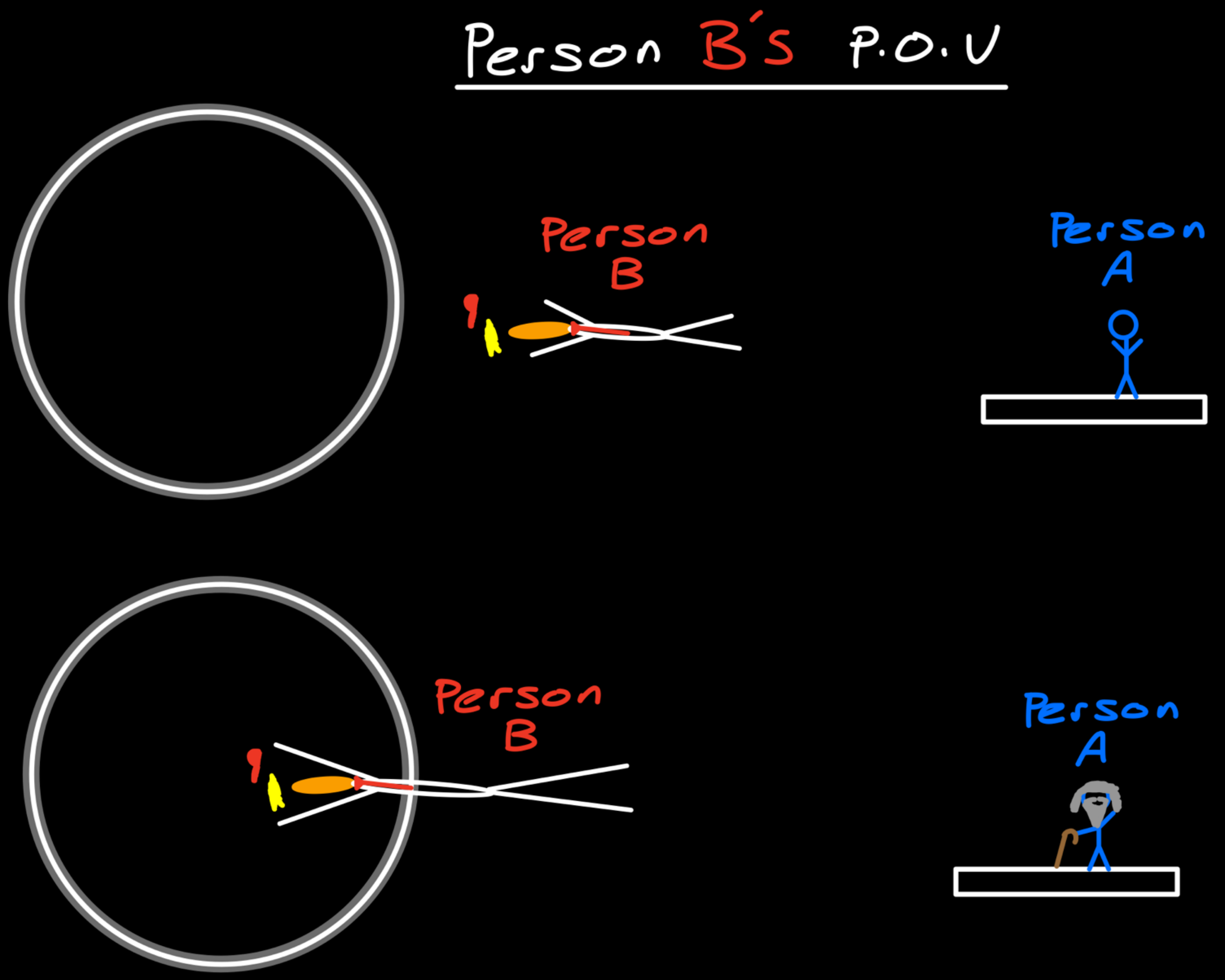
Death by Black Hole

Person B's POV

From Person B's POV, they pass through the horizon without much difficulty.

If the black hole is very small, they will be 'spaghettified' by the fall.

As Person B looks back at Person A, A begins to age rapidly.



Questions!

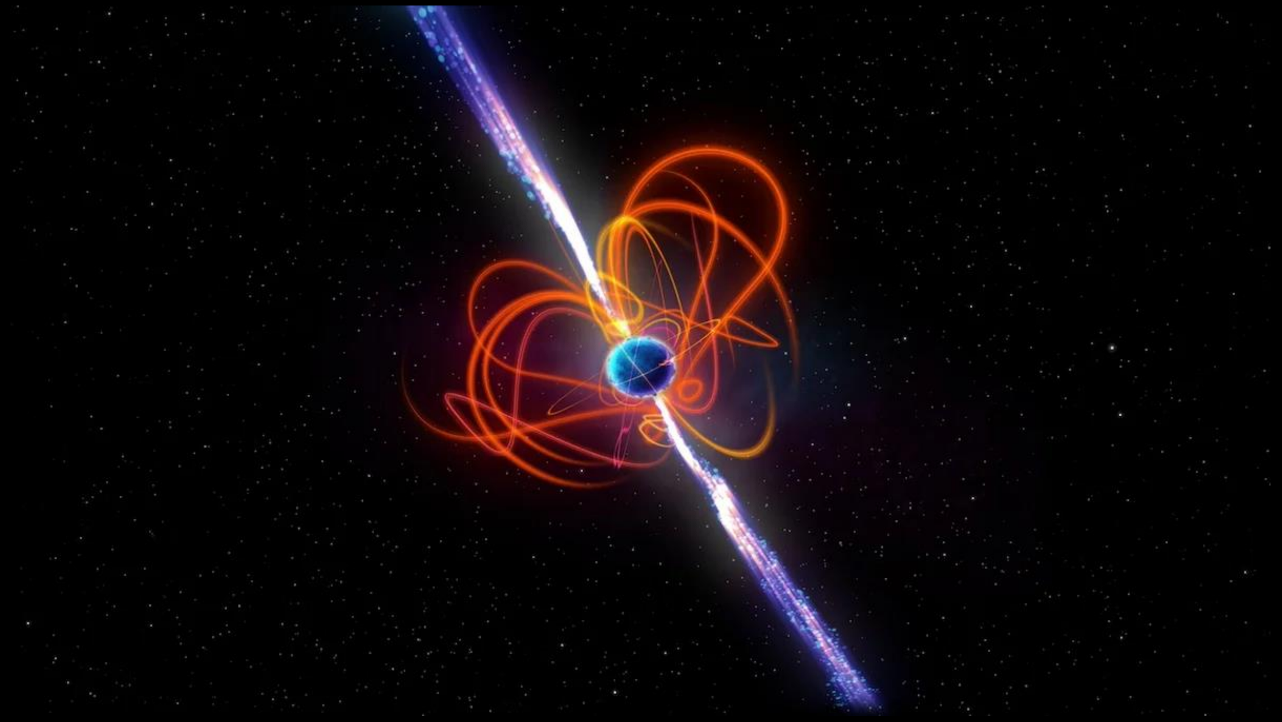
Coming Up…

Pulsars Lecture

April 16th 1pm @ The Beecroft Gallery Lecture Theatre

What does a pulsar have in common with a figure skater? Why did astronomers initially mistake these bizarre objects for alien life? How are physicists using the cosmos' most accurate clocks to learn more about the early Universe?

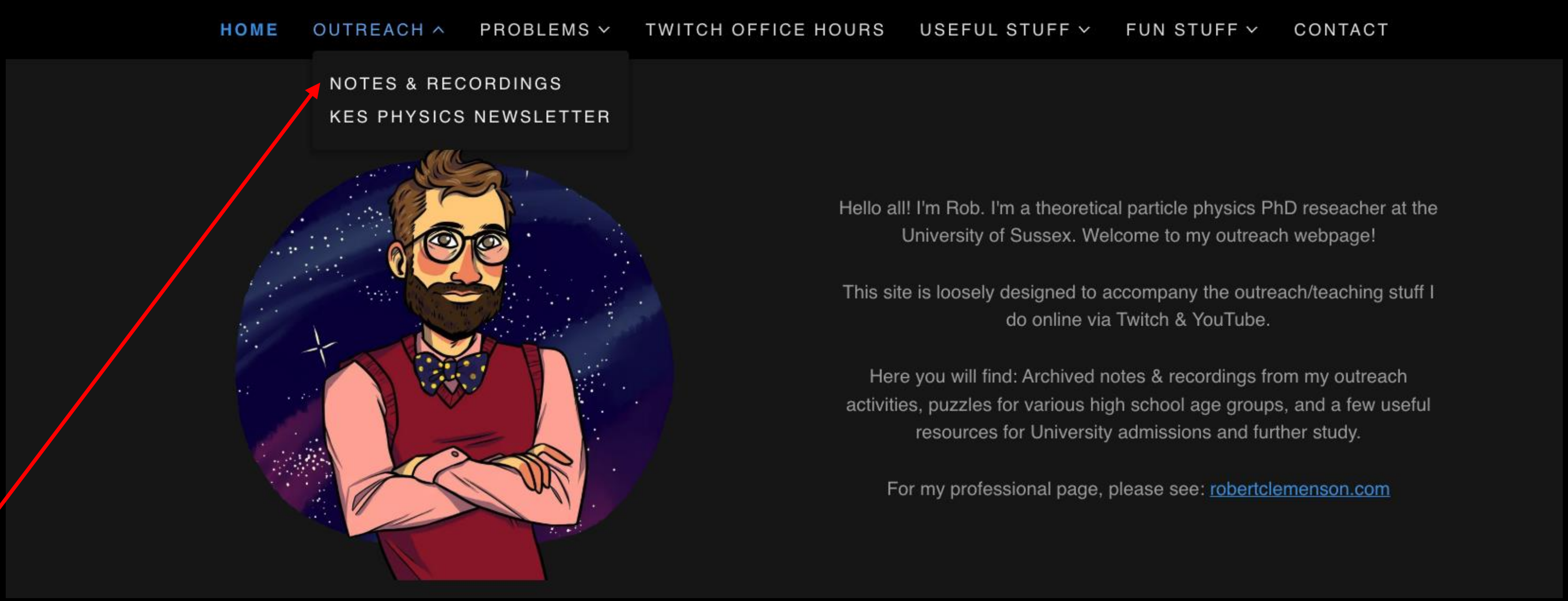
In this talk we will dive into the fascinating history leading to the discovery of pulsars, and get to grips with some of the science behind these ultra fast spinning neutron stars.



"The Science of Space: A Physicists Guide to the Galaxy"

Lecture series running throughout May.

Lecture Slides



These lecture slides are available on my outreach website:

CosmicConundra.com