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**Astrophysics – Handout**

Newton's law of gravity:

Newton's law of gravitational attraction, states that the gravitational force between to masses, M and m, separated a distance r apart is given be the following:

$$F = \frac{GMm}{r^2} \qquad G = 6.67 \times 10^{-11} Nm^2kg^{-2}$$

G is Newton's gravitational constant. Note how small it is, gravity is a comparatively weak for small masses.

Newton's equation of gravity is an example of an inverse-square law, as the force of attraction is inversely proportional to the square of the distance of separation. (Inverse square laws and forces often go hand in hand)

G was first accurately measured by Henry Cavendish in the late 1700's by measuring the angle of deflection between large metal spheres suspended by rope.

Gravitational potential energy and escape velocity:

$W = Fd$ gives the work done by a force F acting over a distance d.

Given the gravitational force is given by $F = \frac{GMm}{r^2}$, it can be reasoned that the gravitational potential energy is given by $U = \frac{GMm}{r}$ (This is a bit of fudge... but will suffice)

Let us contend that this is an expression for the gravitational potential energy. What this formula really gives us, is the amount of kinetic energy a mass m has after falling to a height r above a planet of mass M from an infinitely great distance.

By the conservation of energy, this figure is also the amount of kinetic energy needed to put into the smaller mass at a distance r, to get it to go back off to an arbitrarily large distance away from the planet.

We define the escape velocity to be the speed the mass m must be fired off at, such that it will not return back to the surface of M. (It will go off to infinity) Hence, we can equate;

$$\frac{1}{2}m{v_{esc}}^2 = \frac{GMm}{r} \rightarrow v_{esc} = \sqrt{\frac{2GM}{r}} \quad Note\ this\ expression\ is\ independant\ of\ m$$

Black holes:

Most black holes are the result of a sufficiently massive star collapsing in on itself once it has run of nuclear fuel in its core.

Black holes can exist at (almost) any mass, as we will show.

A black hole produces an incredibly strong gravitational field, such that even light cannot escape its interior.

The distance from the centre of a black hole, to its outer boundary (called the event horizon) is called the Swartzchild radius.

Consider the surface of a black hole. Just in from the outer surface, note even light is travelling fast enough to escape. Hence, as the very rim of the black hole, the speed needed to escape is the speed of light. Plugging this into our escape velocity formula:

$$c = \sqrt{\frac{2GM}{r_s}} \rightarrow r_s = \frac{2GM}{c^2}$$

This formula gives the radius a blob of matter must be compressed to, to become a black hole. Which, rather surprisingly, implies a black hole has no minimum size! (Classically anyway..)

Black holes are believed to be crucial to the formation of galaxies, with a supermassive black hole at the centre of most large galaxies.

The expansion of the universe:

Cosmological principal: The Universe is homogenous and isotropic. (to be explained)

Hubble's law: $v = H_0 D$ The speed, v, at which two galaxies recede away from each other is proportional to the distance between them, D, by a factor of $H_0 = 71 kms^{-1} Mpc^{-1}$ (Hubble's constant)

$[1\, pc\ (parsec) = 3.26\, ly\ (light\ years) = 3 \times 10^{16}\, M]$

Consider a galaxy of mass m, a distance D away from the centre of an expanse of total mass M.

Now consider the escape velocity of this smaller galaxy:

Note, the galaxy is receding with speed: $v = H_0 D$ from the centre of the cluster. Therefore we have;

$$\frac{1}{2} m (H_0 D)^2 = \frac{GMm}{D}$$

Let's say the space inside the sphere radius D has average density $\rho$, therefore as $M = V\rho$ we have;

$$M = \frac{4}{3}\pi D^3 \rho\, , substituting\ this\ into\ the\ above\ equation\ \ldots$$

$$\frac{1}{2}(H_0 D)^2 = \frac{G}{D}\frac{4}{3}\pi D^3 \rho \rightarrow \rho = \frac{3{H_0}^2}{8\pi G}$$

This density is actually called the critical density of the universe.

<u>Exoplanets:</u>

Exoplanets are planets outside of our own solar system.

As planets are generally obscured by the comparative brightness of their parent stars, direct observation can be tricky. For this reason, a number of cunning tricks have been developed to detect these bodies, a few of the more interesting ones are discussed briefly below.

1. Transits: When an exoplanet passes between its parent star and our line of sight, it blocks a portion of the light that would otherwise reach us from that star. This intensity time plot of this even can be used to estimate the size of the star. We can also infer a lot of information about the structure of the planet's atmosphere from the transit, by considering the absorption spectra of the atmosphere.

2. Pulsar timing: Pulsars are rapidly spinning neutron stars that act as precise natural clocks. If a rapidly spinning pulsar is orbited by a planet, the planet can cause the pulsar to 'wobble' about the systems centre of mass, causing a small variable delay in the regularity of pulses received.

3. Microlensing: Very handy, as it allows detection of objects that emit little to no light. Makes use of the relativistic effect of 'gravitational lensing'. A faint foreground star can magnify the image of a distant star (sometimes by more than a factor of ten!), if the foreground star also has planets in orbit, they two can amplify the light of the distant star, but for a much shorter period. The duration of this amplification is a function of the mass distance and velocity of the lensing body.

The diversity of the planetary systems we've discovered is staggering.

Gliese 436 b: Planet covered in ice, with a surface temperature of over 500K. The ice is maintained in a solid state by the extremely pressure. Note, there are over 17 different phases of solid water ice…

Kepler 22 b: A planet within the habitable zone of the Kepler 22 system in the constellation of Cygnus. About 2.4 times the size of Earth, with a year of 289 days.