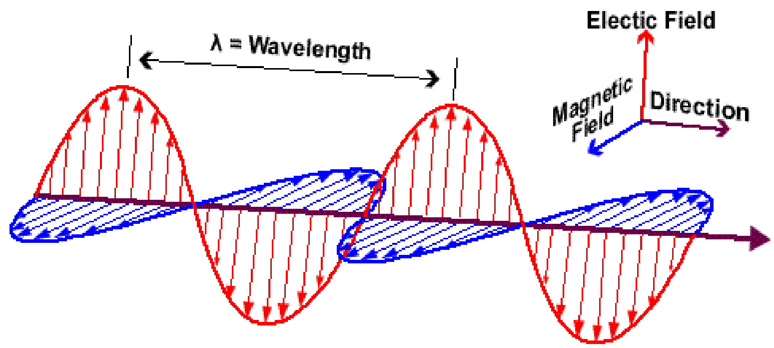
Robert Clemenson June 2017

Special relativity – Hand out



Maxwell's equations of electro-magnetism:

Light is a series of self-propagating electric and magnetic fields, at right angles to each other.

A condition drawn from Maxwell's equations, is that the speed of the wave must be a constant.

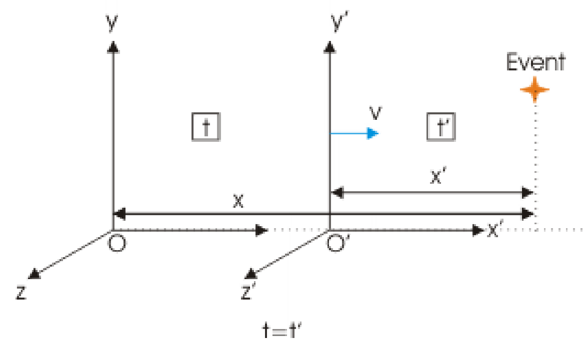
$$v_{wave} = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} = c = 3 \times 10^8\ ms^{-1}$$

Where ε_0 and μ_0 are constants associated to the electric and magnetic fields respectively.

This is a law of physics!

Galilean relativity:

Velocities add as expected. If two cars are colliding, and an observer standing still relative to the street measures one card to be travelling 30mph to the right, and the other 40mph to the left. Both cars will see the other approaching at 70mph.



Relative positions can also be found easily.

An inertial reference frame, is the frame of an observer moving with constant velocity (zero acceleration).

Galilean principle of relativity: **Newton's** laws are all obeyed in inertial frames. If you were in a non-accelerating (very quiet) train carriage with no windows (worrying prospect), you would not be able to tell you were moving, as the laws of motion you would experience and observe would be no different than if the train was stationary.

Special relativity:

Special principle of relativity: **All** laws of physics are obeyed in inertial frames. This is Einstein's extension of Galileo's principle of relativity, and the foundation of Einstein's theory of special relativity.

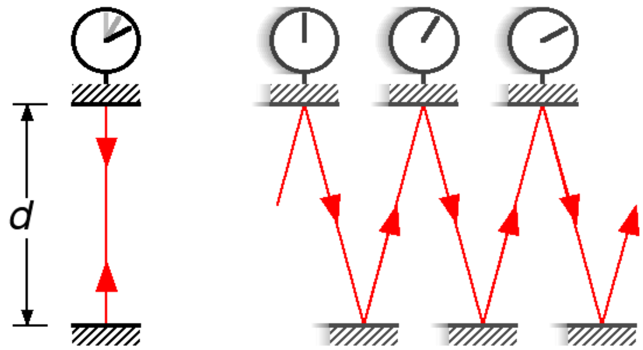
The law that the speed of an EM wave in a vacuum must be $3 \times 10^8\ ms^{-1}$ must also be unchanged in all inertial reference frames.

This implies, no matter how fast you run at a beam of light pointed at your face, you will always measure its speed to be $3 \times 10^8\ ms^{-1}$.

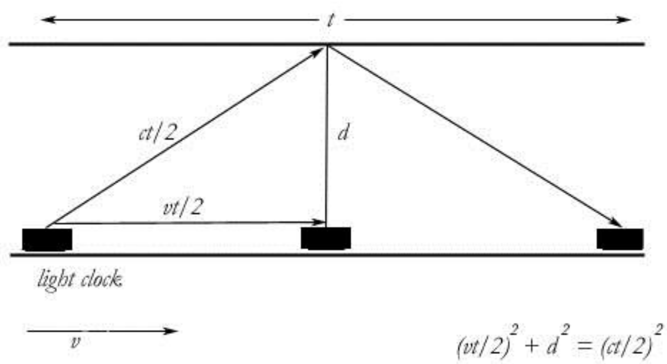
This can only be true if your perception of time slows down, such that you observe the light ray in slow motion in order for its speed to shrink back down to $3 \times 10^8\ ms^{-1}$.

<u>The light clock:</u>

By imagining a hypothetical clock, formed of two parallel mirrors a distance ‘d’ apart, with one ‘tick’ of duration $\Delta\tau$ taken to be the time for the light ray to return to the bottom of the clock where it started. (For more information on why τ is used, look up ‘proper time’)



We now imagine the perspective of someone who sees the clock to be moving with a velocity v to the right. We can denote the time for a ‘tick’ for this observer as Δt.



$d = \frac{c\Delta\tau}{2}$ - As should be fairly obvious.

From the diagram to the left, we have:

$$\left(\frac{v\Delta t}{2}\right)^2 + \left(\frac{c\Delta\tau}{2}\right)^2 = \left(\frac{c\Delta t}{2}\right)^2$$

Which simplifies to:

$$(\Delta t)^2 = \frac{c^2(\Delta\tau)^2}{c^2 - v^2}$$

Which simplifies further to:

$$\Delta t = \frac{\Delta\tau}{\sqrt{1-\frac{v^2}{c^2}}}$$

So, the time between each tick of the light clock is demonstrably dependent on the relative motion of the observer to the clock. Qualitatively, fast clocks run slow.

This effect is not one of mechanical fault in time keeping devices.
This is real, fundamental physics. Time really does slow down, the faster you go.

This effect is called time dilation, and it is only one of the many counter intuitive phenomena predicted and explained by Einstein’s theory of (special) relativity. Some of the others include:

Length contraction; the observed distance between two points gets smaller as you move faster relative to the points.

Mass increase; The faster you move, the greater your mass. This is the reason objects with mass cannot travels faster than (or even at) the speed of light. As mass increases, the amount of energy needed to increase the kinetic energy gets greater and greater, until it becomes practically infinite as the objects speed gets close to the speed of light. Objects can asymptotically approach the speed of light, but can never reach it.

Note: Special relativity is so named because it applies to the ‘special case’ that the frame is non-accelerating. Accelerating reference frames require a higher theory, general relativity, which is exceptionally complex. Frames inside gravitational fields are therefore not inertial, as the equivalence principle tells us that accelerations and gravitational fields are essentially indistinguishable. However, when the gravitational field is not very strong, special relativity will suffice.