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Stellar astrophysics – Hand out

Basic ideas:

Stellar astrophysics is the study of stars, their associated physics and classification.

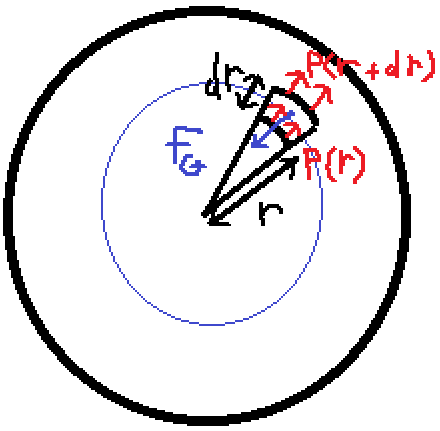
Generally speaking, a star is a near spherical ball of hot plasma held together by its own gravity. Plasma is sometimes described as the 'fourth state of matter'. It's sort of like a gas formed of ionised matter (separated nuclei and electrons), though its inner working is even more complicated than that of a gas due to the particles being charged and the presence of strong electric and magnetic fields.

Within stars, a constant war is waging between the force of gravity, trying to contract the star, and the force of thermal pressure, trying to expand the star. For the vast majority of a stars life time these forces balance out, allowing for a stable star. It's when one factor gets the upper hand thing get really interesting…

But what is it exactly that generates this thermal pressure?

Due to the huge gravitational pull inside of sufficiently large stars, nuclei within the plasma undergo a process called nuclear fusion, in which smaller nuclei are combined to form larger nuclei. When this process occurs, the product nuclei has less mass than the combined mass of the initial nuclei. In accordance with special relativity, this lost mass is converted into energy. The amount of energy produced is given by perhaps the best know equation in science; $E = mc^2$, where m is the lost mass E is the energy released and c is the speed of light ($300{,}000{,}000\ ms^{-1}$). You will notice the speed of light squared is a number of order 10^{16} (very big!!), so a small amount of mass can produce as very large amount of energy. This energy is the source of thermal pressure within the star.

Forces inside stars:

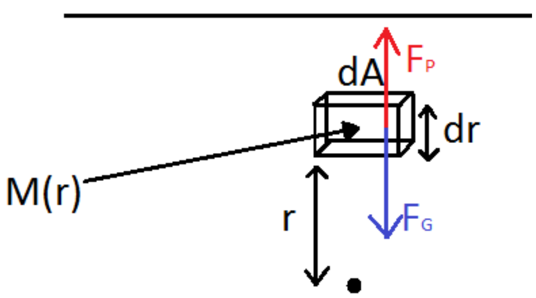


We will briefly be considering the forces at work within a star during a stable point in its evolution (i.e. it is neither expanding nor contracting). Trying to squish the star inwards, we have the gravitational force. Trying to blow the star outwards, we have the thermal pressure due to the heat of nuclear fusion.

Before we proceed, it is worth noting a useful theorem proved by Newton in the 1700's. The theorem states, if we have a small test mass a distance r from the centre of a large spherical mass distribution, the only mass that contributes a net gravitational effect on the test mass, is that contained within a sphere with radius r from the centre of the large sphere (Search 'Shell theorem').

We consider a small rectangular region of the star, a distance r from the centre, with face area dA and height dr. We say the mass density in this small region is $\rho(r)$, and the pressure at the two ends of the rectangle are $P(r)$ and $P(r + dr)$.

From this we can say the mass of this region is $\rho(r)\ dr\ dA$. We can also deduce the net force due to pressure is $[P(r) - P(r + dr)]dA$.



Equating the gravitational and pressure forces, and applying Newton's shell theorem, we find;

$$[P(r+dr)-P(r)]dA = -\frac{GM(r)\rho(r)drdA}{r^2}$$

Note; $M(r)$ is the effective mass, as given by Newton's shell theorem.

We can now cancel the dA's, and divide both sides be dr, leaving;

$$\frac{[P(r+dr)-P(r)]}{dr} = -\frac{GM(r)\rho(r)}{r^2}$$

From what we learnt previously about the derivative from first principles, it should be clear that as we make out box infinitesimally small (take $dr \rightarrow 0$) we have;

$$\lim_{dr \rightarrow 0}\left[\frac{P(r+dr)-P(r)}{dr}\right] = \frac{dP}{dr}$$

So we have;

$$\frac{dP}{dr} = -\frac{GM(r)\rho(r)}{r^2}$$

This is called the equation of hydrostatic equilibrium, and the first of the equations of stellar structure (of which there are usually considered to be four). We're not going to much anything else with this equation, but its derivation is a good exercise, and a useful thing to be able to follow.

Life of a star:

Birth of a star

Stars are born in sufficiently dense regions of nebulae (gas clouds), when gravity start sucking matter together. These newly formed blobs are known as protostars, they have not yet begun fusing hydrogen in their cores.

When fusion begins, the protostar blows away the in falling material by increased radiation pressure, thus forming a pre main sequence star. This soon stabilises and contracts into a main sequence star.

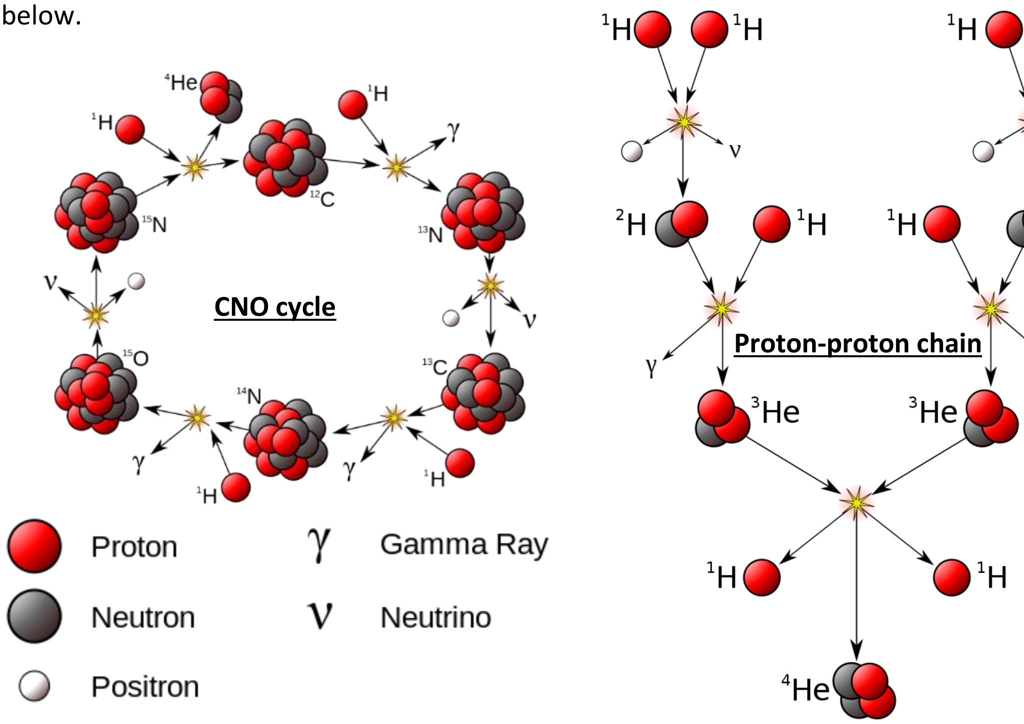
However, some small stars don't make it past this stage. Those that are too small to achieve Hydrogen fusion remain as bodies called 'brown dwarfs'.

Main sequence

Stars spend the majority of their life times in the main sequence phase. During this period of their life stars convert Hydrogen into Helium inside their core.

There are two main ways this is done, the first (which is dominant in smaller stars) is the proton-proton chain reactions, where by Hydrogen nuclei are transformed into Helium as the diagram (Wikipedia) below shows.

The second (which is dominant in larger stars) is called the CNO cycle (Carbon-Nitrogen-Oxygen cycle). In this process, a Carbon-12 nuclei is built up to a Nitrogen-15 nuclei through multiple fusion reactions, this Nitrogen-15 nuclei then fuses with a single proton and subsequently decays into a Helium nuclei and a Carbon-12 nuclei. Hence the Carbon-12 nuclei here behaves like a catalyst of the sort you will have encountered in chemistry. This process is illustrated on the diagram (Wikipedia) below.



Beyond the main sequence

The next stage of a stars evolution depends very much on its mass. Stars leave the main sequence stage when they have exhausted all their supply of Hydrogen in the core.

For small main sequence stars (roughly less than half the mass of the sun), we don’t actually have any observational data. This is due to the entire 13.8 billion year life span of the universe not being long enough for such small stars to exhaust their supply of Hydrogen. (It is useful to bear in mind, smaller stars are much longer lived than larger stars. Although they have less fuel, they burn it at a far slower rate.) It is believed such stars will very slowly contract into white dwarf stars (see next page) as they use up the Hydrogen in their core.

Medium sized main sequence stars (roughly between a third the mass of the sun, and eight times the mass of the sun) are destined to become red giants. When the core runs out of Hydrogen to fuse, the thermal pressure that previously countered the collapsing force of gravity is removed, this causes the core of the star to contract. This contraction generates heat, which in turn can restart the fusion process, causing three Helium nuclei to fuse to one Carbon-12 nucleus (triple alpha process). Then fusion of successively heavier elements from Carbon upwards then ensues. After the core becomes mostly the next element up, fusion decreases, and the core contracts again. This continues to happen until the star reaches the highest atomic mass element it can fuse. Which element this happens to be depends on the mass of the star (up to a point).

Once the star can no longer fuse nuclei within its core, the only force stopping continued contraction is a curious quantum mechanical effect known as the 'Pauli exclusion principle'. The principle states that no two like fermions (particles with half integer spin, e.g. the electron and positron) can occupy the same quantum state. Trying to contract all the electrons in the star into the same point would violate this principle, so a substantial force (called the electron degeneracy pressure) pushes back against the collapse. Stars that are supported by this pressure are known as white dwarfs. White dwarf stars cannot exist above a certain mass known as the 'Chandrasekhar limit' (about 1.4 times the mass of the sun), above this limit, the gravitational force is too large for the electron degeneracy pressure to support, thus the star collapses further forming other types of stellar remnant. White dwarf stars have a density roughly corresponding to the mass of the sun squashed into the volume of then Earth.

The exclusion principle is also the reason that electron orbitals can only fit in two electrons, and those electrons have opposing spins. The quantum states of the orbiting electrons are uniquely defined by their energy level, their spin, and two other quantities called the magnetic and azimuthal quantum numbers. Electrons in the same orbit have the same energy level, magnetic quantum number and azimuthal quantum number. Hence, in order to have non identical states, their spins must be different. As spin can either up or down, the spins must be opposite in the same orbital (but enough chemistry…).

The most massive main sequence stars will follow the same process as their medium sized counterparts, however if they are massive enough to fuse Iron in their core, regardless of their mass beyond that requirement, no stars can fuse elements heavier than Iron. This is because the fusion of elements heavier than Iron is no longer an exothermic process, it requires more energy to force the nuclei together than is released when they fuse. Once the core of a star is mostly Iron, it once more begins to cool (as described previously for medium sized stars). And will contract into either a white dwarf star, or something even more interesting.

If the stellar core is more massive the Chandrasekhar limit, the star will collapse beyond the densities of white dwarfs, into even denser objects. The least dense of these hyper dense objects is a neutron star. When the core contracts to such a high density, electrons and protons are forced to combine in a process called inverse beta decay (beta decay, but in reverse!) forming neutrons. These neutrons exert a pressure much like that inside of white dwarfs, called neutron degeneracy pressure, again due to the exclusion principle. Neutron stars are incredibly dense, equivalent to having roughly two suns compressed to about the size of a city.

If the core of the star is too massive even to be supported by the neutron degeneracy pressure, it continues to contract, and in fact never stops. This perpetual collapse down to a single dimensionless point, is what we call a black hole (as discussed in the 'Astrophysics' handout).

From death to birth

During the cores collapse into a white dwarf or neutron star, stars undergo a process called supernova (specifically a type two supernova in the context we are discussing). Where by the outer layers of the star are blown away in a violent explosion. This material spreads out into space, forming a nebula, the same sort of structure that gave birth to the star initially!

Nebulae are some of the most beautiful structures in the cosmos, for those interested I recommended taking a look at the Crab nebula and the Carina nebula.