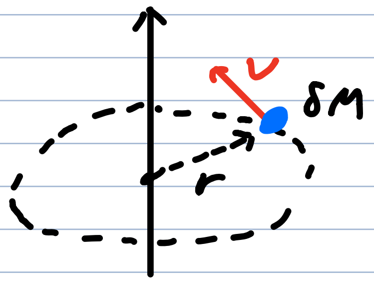
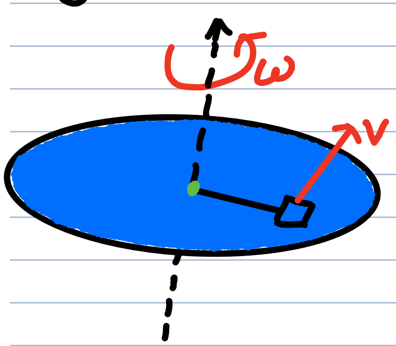
- Angular momentum of a particle undergoing circular motion:



$$L_{particle} = rP = r(\delta M)v$$

- Angular momentum of a 2D plane:



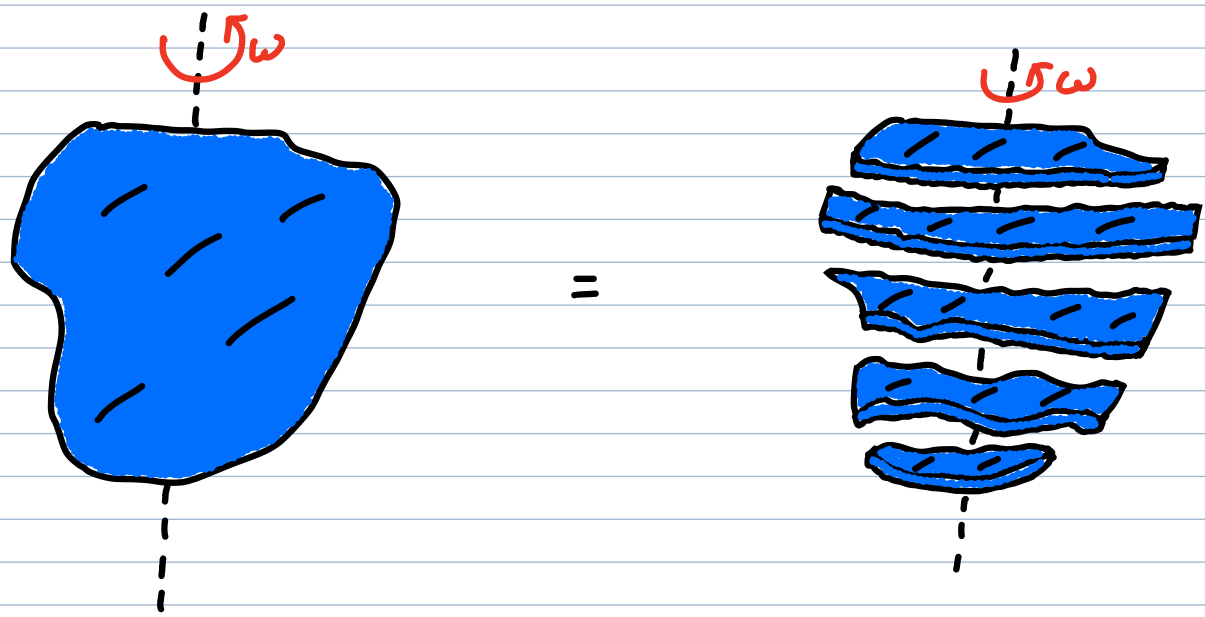
$$L_{plane} = \sum_{\substack{\text{sum over} \\ \text{all particles}}} (\delta M) r v$$

sum over all little pieces of plane

as $\delta M \to dM$, $\sum \to \int$

$$L_{plane} = \int r v \, dM$$

- Angular momentum of a 3D object:

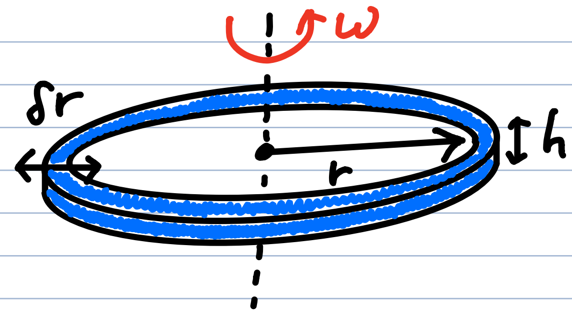


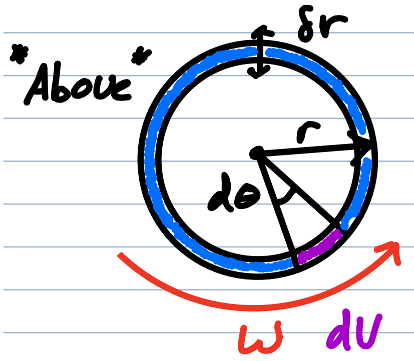
3D object is just many 2D planes stacked ontop.

$$L_{3D\,object} = \sum_{\substack{\text{sum over}\\\text{all planes}}} L_{plane}$$

Thin Ring

$$dM = \rho\, dV = \rho h\, dA$$
$$= \rho h r\, d\theta\, (\delta r)$$





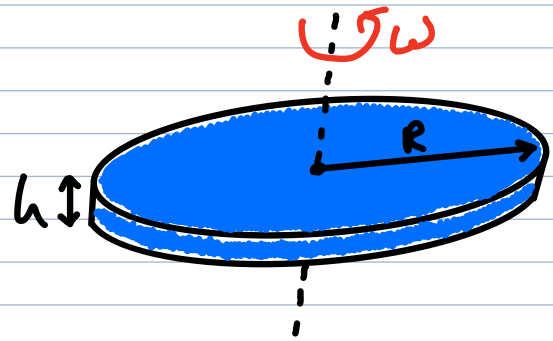
$$v = \omega r$$

$$L_{seg} = (dM)\, v\, r = (dM)\, \omega r^2 = \rho \omega h (\delta r) r^3 d\theta$$

$$L_{Ring} = \sum_{\substack{\text{All} \\ \text{Segments}}} L_{seg} = \int_0^{2\pi} \omega r^2 \rho (\delta r) h\, r\, d\theta$$

$$= 2\pi \rho \omega r^3 (\delta r) h$$

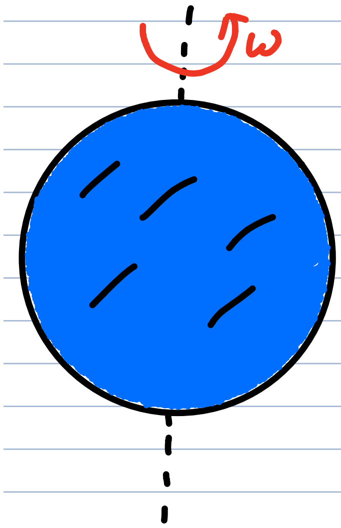
Disk



$$L_{Disk} = \int_0^R L_{Ring} = \int_0^R dr\, 2\pi \rho \omega r^3 h = \frac{1}{4} R^4 2\pi \rho \omega h$$

$$= \frac{1}{2} \pi \rho R^4 \omega h$$

3D Sphere



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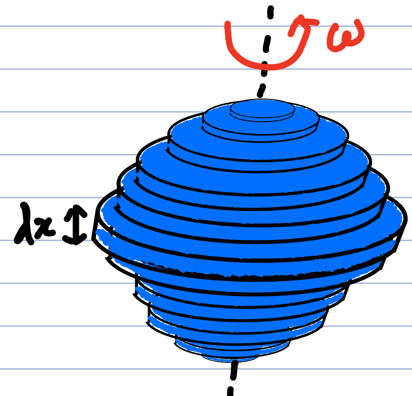
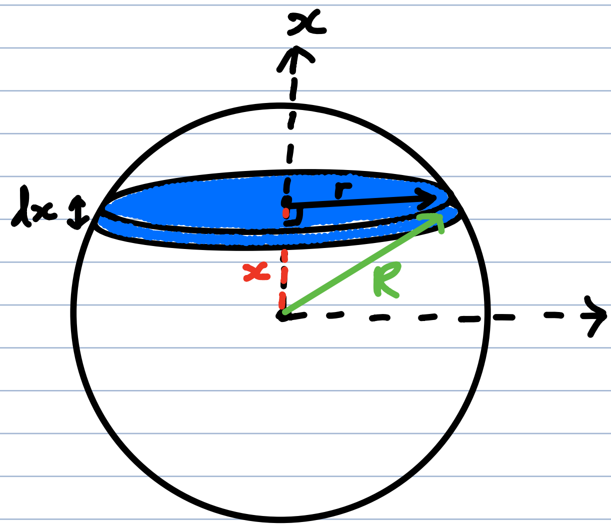


$$L_{Disk} = \frac{1}{2}\pi\rho R^4 \omega\,(dx)$$



$$r^2 = R^2 - x^2$$

$$L_{sphere} = \underbrace{\sum}_{\text{sum of all disks}} L_{Disk} \rightarrow \int L_{Disk} = \int_{-R}^{R} \frac{1}{2}\pi\rho\omega r^4\,dx$$

using $r^2 = R^2 - x^2 \Rightarrow r^4 = (R^2 - x^2)^2$

$$L_{sphere} = \frac{1}{2}\pi\rho \int_{-R}^{R} \omega\,(R^2 - x^2)^2\,dx$$

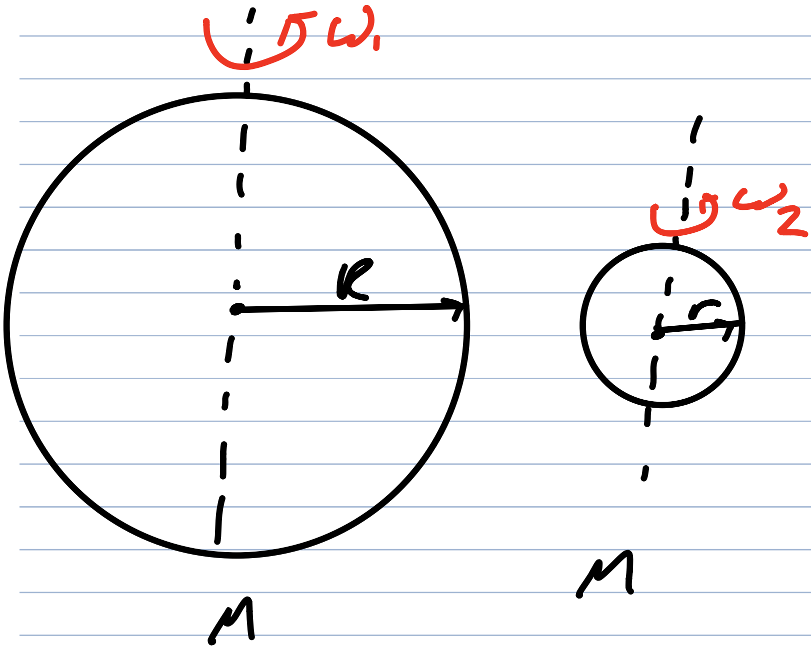
$$= \frac{1}{2}\pi\rho\omega \int_{-R}^{R} (R^2 - x^2)^2 \, dx$$

$$= \frac{1}{2}\pi\rho\omega \left[x R^4 + \frac{1}{5}x^5 - \frac{2}{3}R^2 x^3 \right]_{-R}^{R}$$

$$= \pi\rho\omega \left[R^5 + \frac{1}{5}R^5 - \frac{2}{3}R^5 \right]$$

$$= \frac{8}{15}\pi\rho\omega R^5 = \left(\frac{4}{3}\pi R^3 \rho\right)\frac{2}{5}R^2\omega = \frac{2}{5}MR^2\omega$$

$$L_{sphere} = \frac{2}{5}MR^2\omega$$

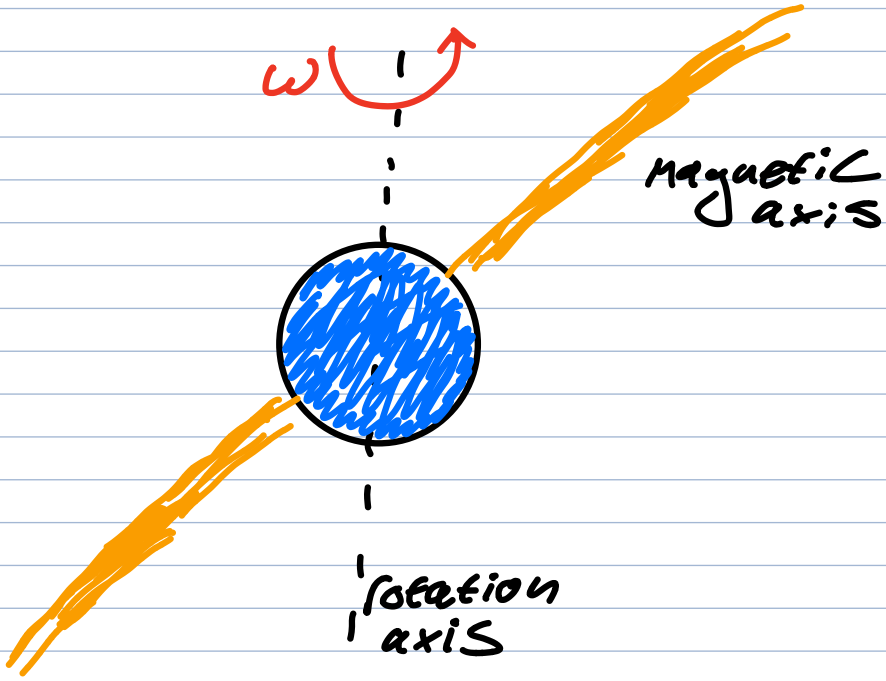


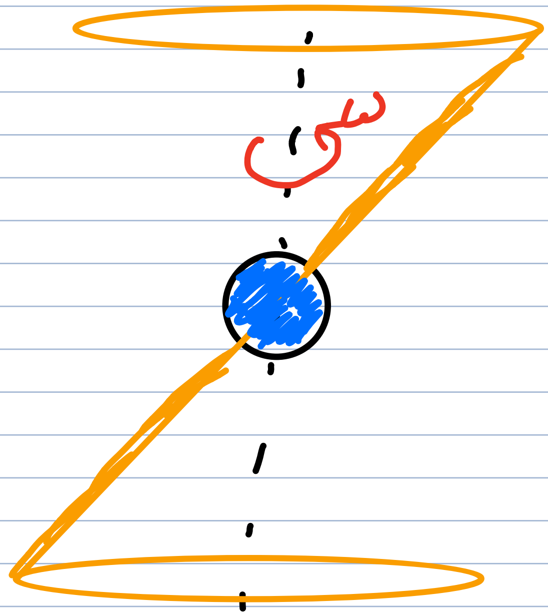
$$L_{before} = \frac{2}{5}MR^2\omega_1^2$$

$$L_{After} = \frac{2}{5}Mr^2\omega_2^2$$

$$L_{before} = L_{After}$$

$$\Rightarrow \quad \omega_2 = \left(\frac{R}{r}\right)^2 \omega_1$$


ω
magnetic axis
rotation axis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