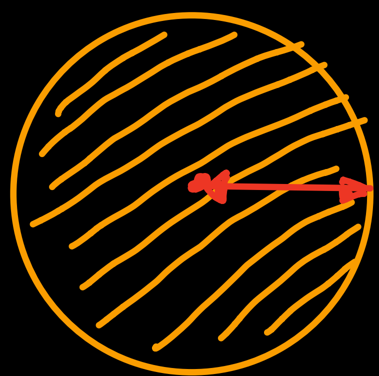
Stellar Structure

• Our Sun:



Radius, $R_\odot = 6.96 \times 10^{8}\,m$

Mass, $M = 1.99 \times 10^{30}\,kg$

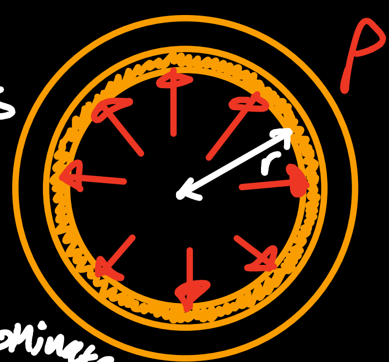
• Forces present in the sun:

– Outward Pressure:

Thermal Pressure – Dominates MS

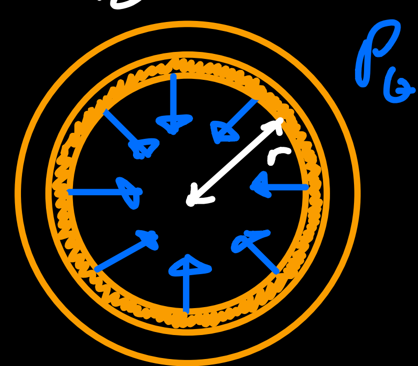
Radiation Pressure – Dominates $>100\,M_\odot$ *

White D's and neutron stars only → Degeneracy Pressure *



– Inward Pressure:

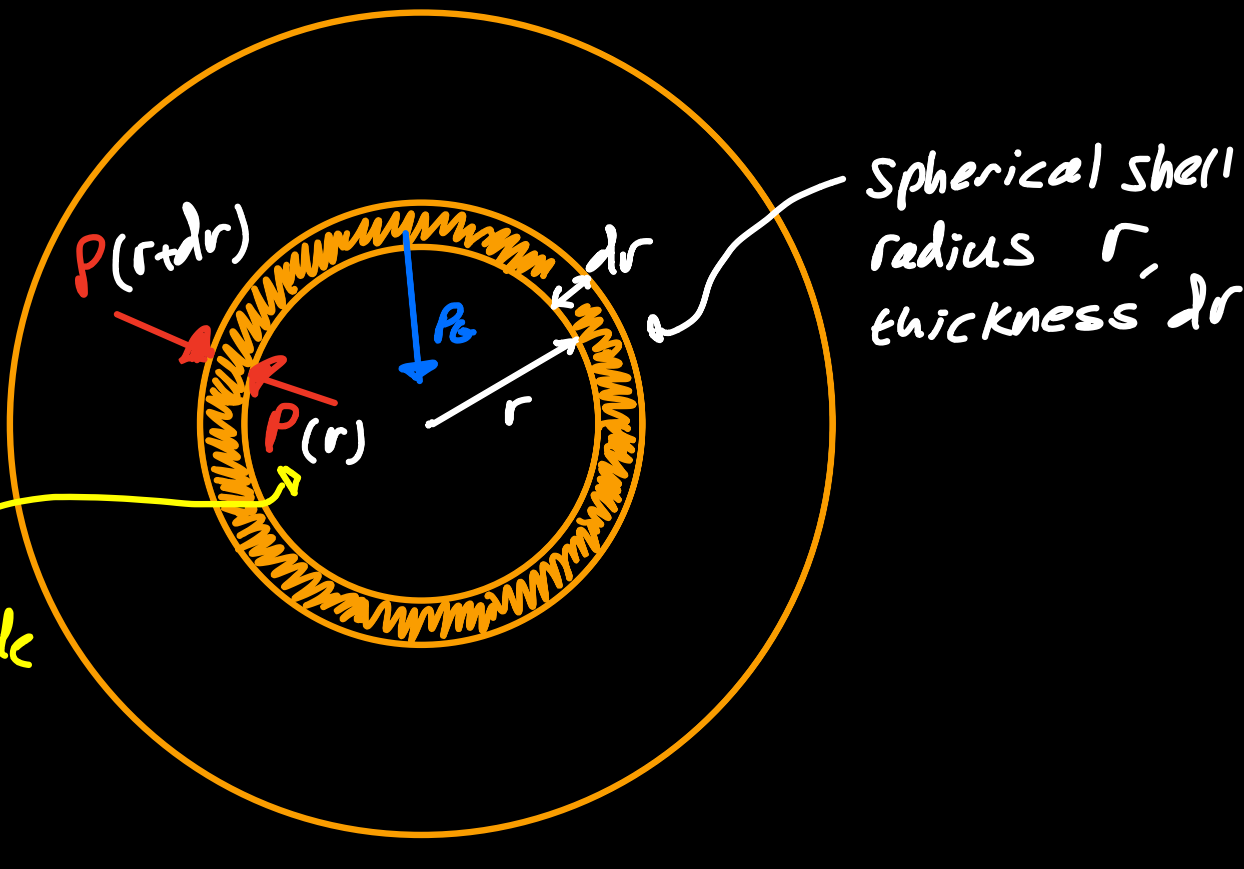
Gravitational Pressure



Most of Stars life is spent in 'hydrostatic' equillibrium.

Inward Pressure = outward Pressure

$$P_G = P$$



- force Pulling spherical shell inwards:

$$F_G(r) = \frac{G\,M(r) \times \text{Mass of Shell}}{r^2}$$

Mass of shell $= \rho(r)\, 4\pi r^2 dr$

$$F_G(r) = 4\pi G M(r) \rho(r)\, dr$$

$$P_G(r) = \frac{F_G(r)}{\text{Area}} = \frac{F_G(r)}{4\pi r^2}$$

$$P_G(r) = \frac{G M(r) \rho(r)}{r^2}\, dr$$

- Net outward Thermal Pressure:

$$P(r) - P(r+dr)$$

$\Rightarrow$ In Hydrostatic equillibrium:

$$P(r) - P(r+dr) = \frac{G M(r) \rho(r)}{r^2}\, dr$$

$$\Rightarrow \quad \frac{P(r+dr) - P(r)}{dr} = -\frac{GM(r)\rho(r)}{r^2}$$

$$\boxed{\frac{dP}{dr} = -\frac{GM(r)\rho(r)}{r^2}}$$

Equation of hydrostatic equillibrium

- Mass continuity equation:

$$M(r) = \int_0^r \rho(\bar{r})4\pi\bar{r}^2 d\bar{r}$$

$$\Rightarrow \quad \boxed{\rho(r) = \frac{1}{4\pi r^2}\frac{dM}{dr}}$$

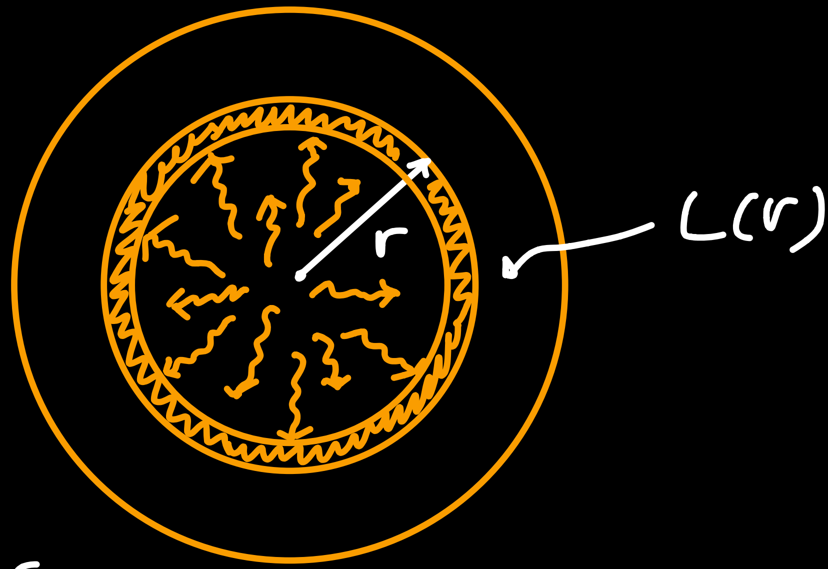
- Energy Transport:

Three mechanisms of energy transport in stars:

1) Radiation ← Dominates for smaller temp gradients
2) Convection ← Dominates for large temp gradients
3) Conduction ← Dominates compact stars (WD's, NS's)

Introduce $\varepsilon(r)$ ← Energy generation per unit mass per unit time

$L(r)$ ← Luminosity produced inside radius r (Total power produced within radius r)



$$L(r) = \int_0^r \varepsilon(\bar{r}) \times 4\pi \bar{r}^2 \rho(\bar{r})\, d\bar{r}$$

$$\Rightarrow \boxed{\frac{dL}{dr} = 4\pi r^2 \rho(r)\, \varepsilon(r)}$$

Energy continuity equation

– Radiation dominated energy transport :

Consider the radiation momentum flux (radiation pressure) on a layer:

Radiation pressure from blackbody source: $P_{rad} = \frac{4}{3} \frac{\sigma T^4}{c}$

$\frac{L(r)}{c} \chi \rho(r) dr$ ← momentum flux absorbed by layer

$-\frac{dP_{rad}}{dr} 4\pi r^2 dr$ ← momentum flux deposited to layer

$$\frac{L(r)}{c} \chi \rho(r) \cancel{dr} = -\frac{dP_{rad}}{dr} 4\pi r^2 \cancel{dr}$$

$$\Rightarrow L(r) = -\frac{4\pi c r^2}{\kappa \rho(r)} \frac{dP_{rad}}{dr}$$

$$= -\frac{4\pi c r^2}{\kappa \rho(r)} \frac{d}{dr}\left(\frac{4}{3}\frac{\sigma T^4}{c}\right)$$

$$\Rightarrow \boxed{\frac{dT}{dr} = -\frac{3\kappa \rho(r) L(r)}{64 \pi r^2 \sigma T^3}}$$

Radiative Energy transport Equation.

- <u>Scaling Relations:</u>

Equations so far:

(1) $$\frac{dP}{dr} = -\frac{GM(r)\rho(r)}{r^2}$$

(2) $$\rho(r) = \frac{1}{4\pi r^2}\frac{dM}{dr}$$

(3) $$\frac{dL}{dr} = 4\pi r^2 \rho(r)\varepsilon(r)$$

(4) $$\frac{dT}{dr} = -\frac{3\kappa\rho(r)L(r)}{64\pi r^2 \sigma T^3}$$

<u>Ideal gas equation</u> $PV = Nk_BT$

$$\Rightarrow P \propto M\frac{T}{R^3}$$

(1) $\Rightarrow \dfrac{P}{R} \propto \dfrac{M^2}{R^3}$

$$\Rightarrow P \propto \frac{M^2}{R^4}$$

for both to be consistent

$$\Rightarrow M \frac{T}{R^3} \propto \frac{M^2}{R^4}$$

$$\Rightarrow \boxed{T \propto \frac{M}{R}}$$

(4) $\Rightarrow \dfrac{T}{R} \propto \dfrac{\frac{M}{R^3} \times L}{R^2 T^3}$

$$\Rightarrow L \propto \frac{R^4 T^4}{M} \propto \frac{R^4 \left(\frac{M}{R}\right)^4}{M}$$

$$\Rightarrow \quad \boxed{L \propto M^3}$$

- **Main Sequence Lifetime:**

$$\tau_{MS} \propto \frac{M}{L} \propto M^{-2}$$

Main sequence lifetime

$\Rightarrow$ More massive stars die younger

• Estimating Core Temperature:

$$\frac{dP}{dr} = -\frac{GM(r)\rho(r)}{r^2}$$

$$\frac{dM}{dr} = 4\pi r^2 \rho(r)$$

$$\Rightarrow \frac{dP}{dM} = -\frac{GM(r)}{4\pi r^4}$$

$$\Rightarrow \int_{P_c}^{P_s} dP = -\int_0^{M_\odot} \frac{GM(r)}{4\pi r^4}\, dM$$

$$\Rightarrow P_c = -\int_0^{M_s} \frac{GM(r)}{4\pi r^4}\, dM > \int_0^{M_s} \frac{GM}{4\pi R_s^4}$$

$$P_c \gtrsim \frac{G M_s^2}{8\pi R_s^4}$$

Ideal gas equation

$$P = \frac{\rho}{\bar{m}} k_B T$$

Mean mass of gas constituents

$\bar{m} = \frac{1}{2} m_H$ for fully ionized Hydrogen plasma (assuming negligable mass for electrons)

$$\rho = \frac{M_s}{\frac{4}{3}\pi R_s^3}$$

(Heavy approximations...)

$$T_c \gtrsim \frac{G M_s^2}{8\pi R_s^4} \frac{\bar{m} \frac{4}{3}\pi R_s^3}{M_s k_B}$$

$$T_C \gtrsim \frac{G M_S \bar{m}}{6 k_B R_S}$$

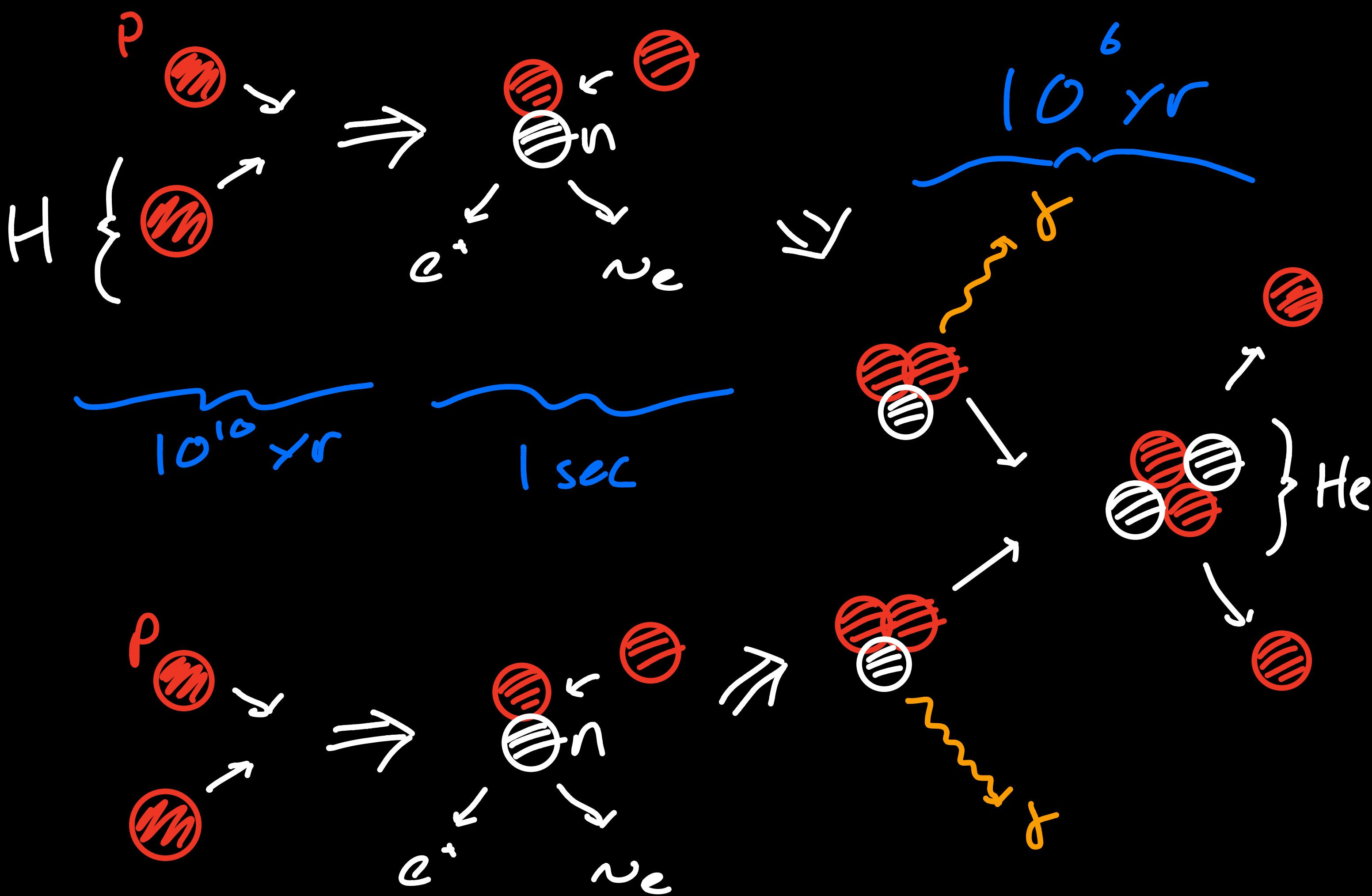
$$= \frac{(6.67\times10^{-11})\times(1.99\times10^{30})\times(\frac{1}{2}\times1.67\times10^{-27})}{6\times(1.38\times10^{-23})\times(6.96\times10^{8})}$$

$$\Rightarrow T_C \gtrsim 2\times10^{6}\ K$$

More sophisticated estimates give around 15 million Kelvin. So not bad for 'back of the envelope'!

• Energy Generation in the Sun:

Energy generated by Nuclear fusion in the stars core. for our Sun this is Mostly Hydrogen fusion via the PP-1 chain (PP-1 accounts for ~85%) energy generation in the Sun.



Net reaction:

$$4H \longrightarrow He + 2e^{+} + 2\nu_e + 2\gamma$$

Nuclear Binding Energy $\Delta E = \Delta m c^2$

$$\Delta M = 4M_H - M_{He} \simeq 4.77 \times 10^{-29} \text{kg}$$

$$\Rightarrow \Delta E = 4.3 \times 10^{-12} \text{J} = 27 \text{MeV}$$

- Practicalities of solar fusion:

Protons must be brought close enough so that the strong force can take over. ($\sim 2 \text{fm} = 2 \times 10^{-15} \text{m}$)

$$\text{Coulomb repulsion} \sim \frac{e^2}{4\pi\varepsilon_0 (2 \times 10^{-15})}$$

$$\simeq 1.15 \times 10^{-13} \text{J} = 0.72 \text{MeV}$$

Recall, $T_c \simeq 2 \times 10^6$ K

$\Rightarrow$ Kinetic energy of Nuclei $= \frac{3}{2} k_B T_c$

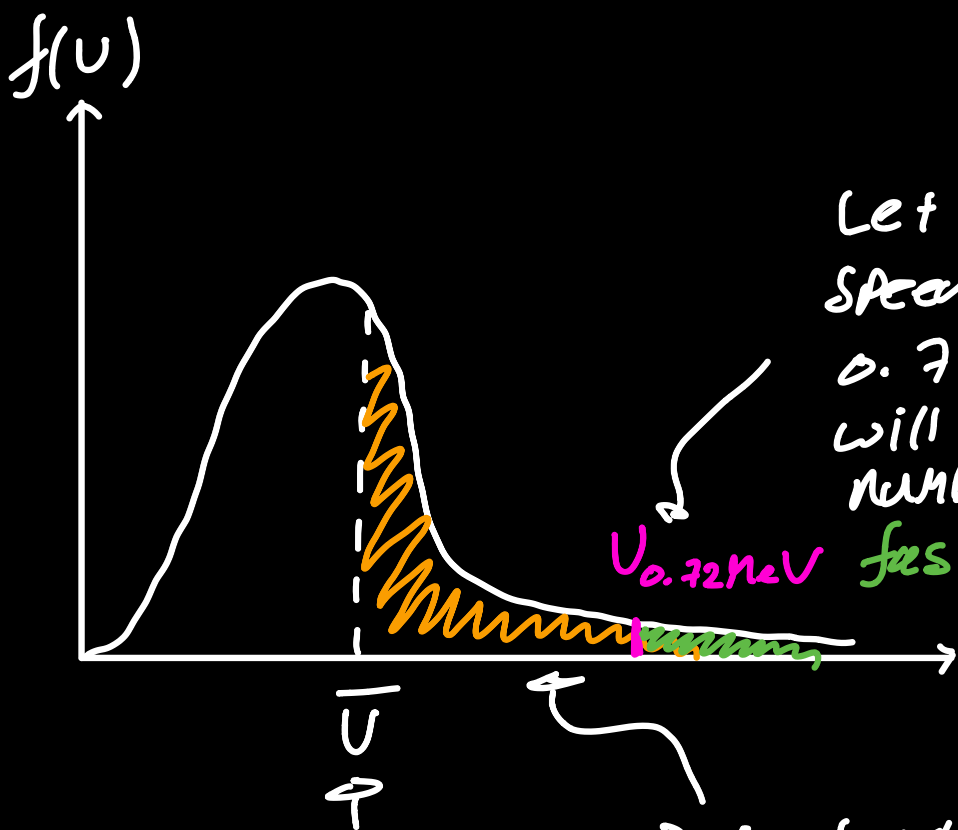
$\simeq 4.14 \times 10^{-17}$ J $= 0.00026$ MeV

Energy needed to overcome coulomb force	Energy of Nuclei in core
~ 0.72 MeV	~ 0.00026 MeV

So why does fusion happen;

1. Boltzmann tail.
2. Quantum tunneling.

1.

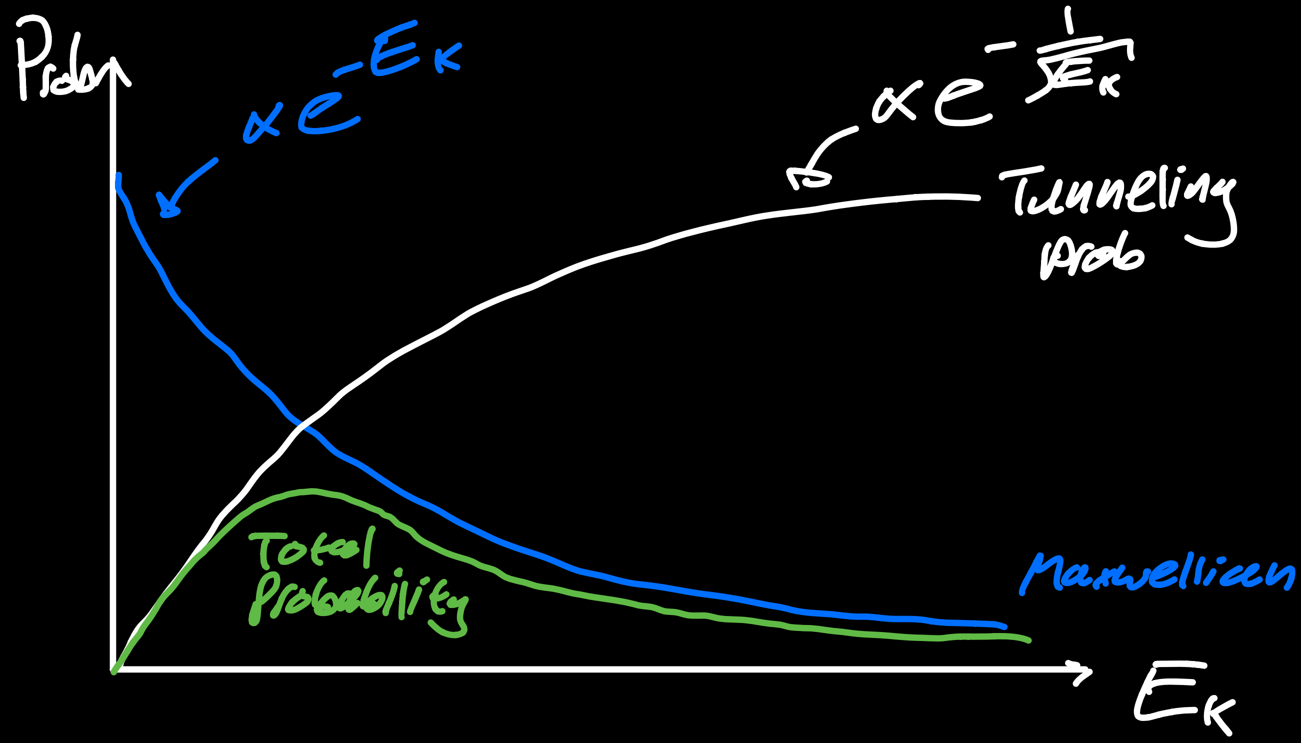


Let this be the speed with KE 0.72 MeV. There will be a small number of particle faster than this speed. which will fuse

This is what we calculated above

But clearly a number of nuclei have velocities higher than this.

2. Nuclei can tunnel through the coulomb barrier quantum mechanically, with some tunnelling probability $P_{tunn} \sim e^{-\frac{1}{\sqrt{E_k}}}$



Quantum mechanical tunneling dominates.

The sun wouldn't shine without quantum mechanics.