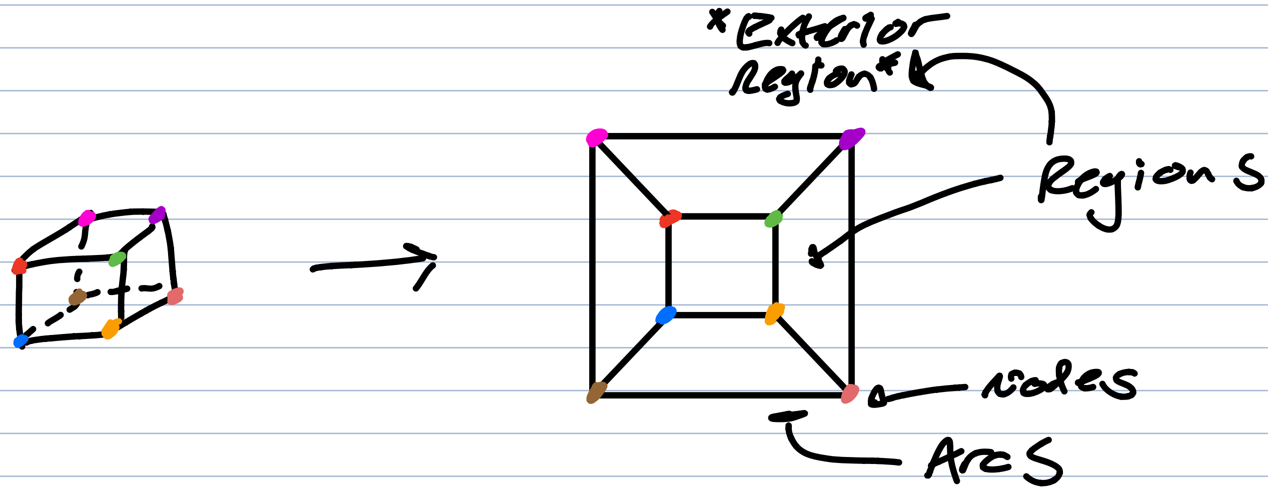
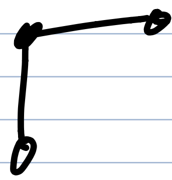


$N + R - 2 = A$ → This holds for all planar graphs



1. Cut Arcs that form loops.
   → Turn graph into a tree

   $A \rightarrow A-1$
   $R \rightarrow R-1$ } formula stays balanced

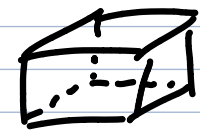
2. Cut off loose nodes / Arcs

   $N \rightarrow N-1$
   $A \rightarrow A-1$

$$N + R - 2 = A$$

$$V + f - 2 = E$$

Vertices faces Edges



S → # sides on a face

M → # faces which meet at a vertex

$$E = \frac{Vm}{2} \Rightarrow V = \frac{2E}{m}$$

$$E = \frac{fs}{2} \Rightarrow f = \frac{2E}{s}$$

$$\frac{2E}{m} + \frac{2E}{S} - 2 = E$$

$$\div 2E$$

$$\frac{1}{m} + \frac{1}{S} - \frac{1}{E} = \frac{1}{2}$$

$$\frac{1}{m} + \frac{1}{S} - \frac{1}{2} = \underbrace{\frac{1}{E}}_{>0}$$

$$\frac{1}{m} + \frac{1}{s} - \frac{1}{2} > 0$$

$$\frac{1}{m} + \frac{1}{s} > \frac{1}{2}$$

$$m, s > 2$$

$s = 3,\ m = 3 \rightarrow \frac{2}{3} > \frac{1}{2}$ Δ ✓ Tetrahedron

$s = 3,\ m = 4 \rightarrow \frac{7}{12} > \frac{1}{2}$ ✓ octahedron

$s = 3,\ m = 5 \rightarrow \frac{8}{15} > \frac{1}{2}$ ✓ Icosahedron

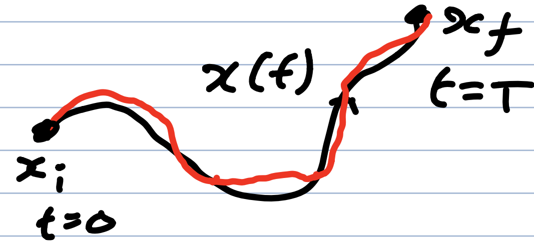
$s = 3,\ m = 6 \rightarrow \frac{9}{18} = \frac{1}{2} \not> \frac{1}{2}$ ✗

$s = 4,\ m = 3 \rightarrow \frac{7}{12} > \frac{1}{2}$ ✓ Cube

$s = 5,\ m = 3 \rightarrow \frac{8}{15} > \frac{1}{2}$ ✓ Dodecahedron

$s = 5,\ m = 4 \rightarrow \frac{9}{20} \not> \frac{1}{2}$ ✗

$$M\ddot{x} = -U'(x)$$



$$\tilde{U}[x] = \frac{1}{T}\int_0^T U(x(t))\,dt \qquad e^{iS[x(t)]}$$

$$\tilde{T}[x] = \frac{1}{T}\int_0^T \frac{1}{2}m\dot{x}(t)^2\,dt$$

$$x(t) + \delta x(t)$$

$$\delta\tilde{U}[x] = \frac{1}{T}\int_0^T \Big(U[x(t)+\delta x(t)] - U[x(t)]\Big)dt$$

$$= \frac{1}{T}\int_0^T \Big(\cancel{U(x(t))} + U'\delta x(t) - \cancel{U[x(t)]}\Big)dt$$

$$= \frac{1}{T}\int_0^T U'\,\delta x(t)\,dt$$

$$\delta\tilde{T}[x] = \frac{1}{T}\int_0^T \Big[\frac{1}{2}m\big(\dot{x}(t)+\delta\dot{x}(t)\big)^2 - \frac{1}{2}m\dot{x}(t)^2\Big]dt$$

$$= \frac{1}{T}\int_0^T M\dot{x}(t)\,\delta\dot{x}(t)\,dt$$

$$\int u \frac{dv}{dx}dx = uv - \int v \frac{du}{dx}dx$$

$$= \frac{1}{T}\int_0^T M\dot{x}(t)\,\frac{d}{dt}\left[\delta x(t)\right]dt$$

$$= \frac{1}{T}\left[M\dot{x}(t)\,\cancel{\delta x(t)}\Big|_0^T - \int_0^T M\ddot{x}(t)\,\delta x\,dt\right]$$

$$\delta\tilde{T}[x] = -\frac{1}{T}\int_0^T M\ddot{x}(t)\,\delta x\,dx$$

$$\delta\tilde{U}[x] = \frac{1}{T}\int_0^T U'\,\delta x(t)\,dt$$

$$\text{NII:}\quad \underbrace{M\ddot{x} = -U'}_{}$$

$$M\ddot{x} + U' = 0$$

$$\delta \tilde{T}[x] - \delta \tilde{U}[x] = -\frac{1}{T}\int_0^T \underbrace{(m\ddot{x} + U')}_{=0} \delta x \, dt$$

$$\delta\left[T \times \left(\tilde{T}[x] - \tilde{U}[x]\right)\right] = 0$$

$$\tilde{U}[x] = \frac{1}{T}\int_0^T U(x(t))\, dt$$

$$\tilde{T}[x] = \frac{1}{T}\int_0^T \frac{1}{2} m \dot{x}(t)^2 \, dt$$

$$S[x(t)] = \int_0^t dt \left[T - U\right]$$

$$\delta S = 0$$

Hamiltons Principle

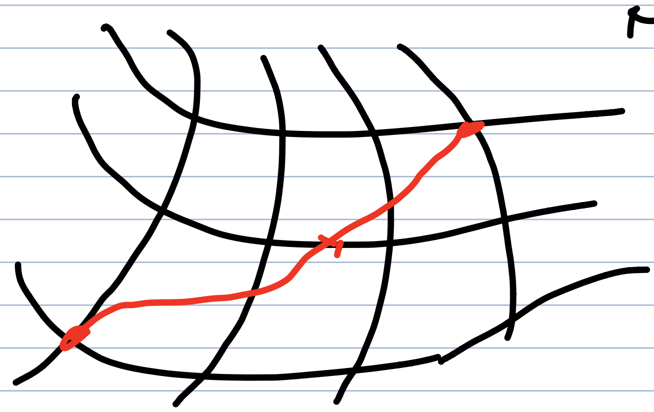
$$\frac{\partial \mathcal{L}}{\partial x} - \frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{x}}\right)$$

$$ds^2 = -dt^2 + dx^2 + dy^2 + dz^2$$

$$ds^2 = -\left(1 - \frac{r_s}{r}\right)dt^2 + \left(1 - \frac{r_s}{r}\right)^{-1}dr^2 + r^2 d\Omega^2$$

$$= -d\tau^2$$

L



$$S = \int d\tau$$

$$\delta S = 0$$