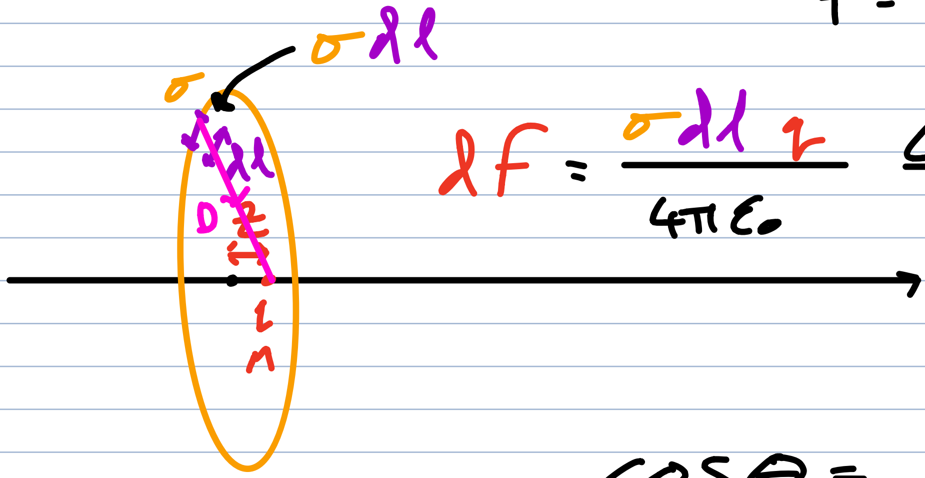
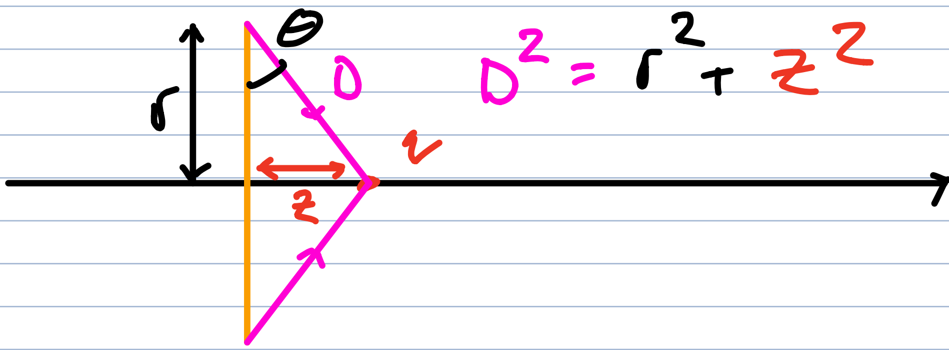
$$f = \frac{\quad}{4\pi\varepsilon_0 r^2}$$



$$df = \frac{\sigma\, dl\, q}{4\pi\varepsilon_0} \frac{\cos\theta}{D^2}$$

$$\cos\theta = \frac{z}{D}$$

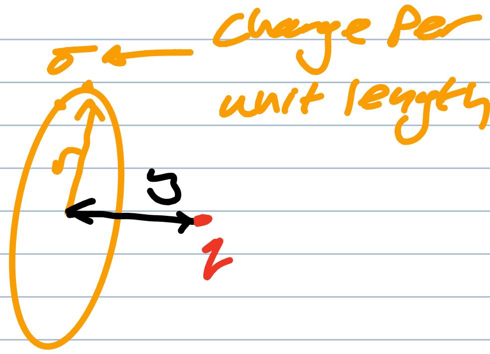


$$D^2 = r^2 + z^2$$

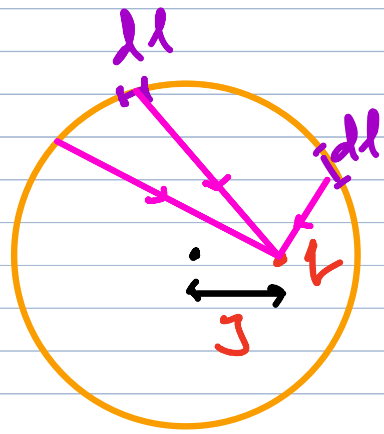
$$df = \frac{\sigma\, dl\, q}{4\pi\varepsilon_0} \frac{z}{D^3}$$

$$f = \sum df \longrightarrow \int df$$

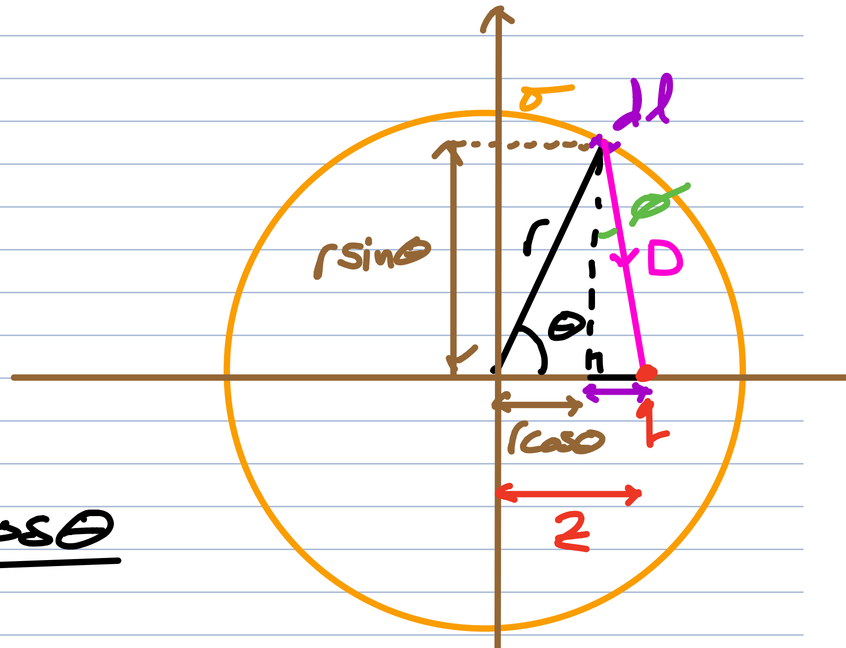
$$f = \frac{\sigma q z}{4\pi\varepsilon_0 D^3} \underbrace{\int dl}_{2\pi r}$$



$$f = \frac{\sigma q z r}{2\varepsilon_0 D^3}$$



$$df = \frac{\sigma \, dl \quad \lambda}{4\pi \varepsilon_0} \frac{\sin \phi}{D^2}$$



$$\sin \phi = \frac{z - r\cos\theta}{D}$$

$$df = \frac{\sigma \lambda}{4\pi \varepsilon_0} \frac{(z - r\cos\theta)}{D^3} \, dl$$

$$D^2 = (z - r\cos\theta)^2 + (r\sin\theta)^2$$
$$= z^2 + r^2 - 2rz\cos\theta$$

$$dl = r \, d\theta$$

$$df = \frac{\sigma \lambda r}{4\pi \varepsilon_0} \frac{(z - r\cos\theta)}{D^3} \, d\theta$$

$$f = \int df = \int \frac{\sigma \lambda r}{4\pi \varepsilon_0} \frac{(z - r\cos\theta)}{D^3} \, d\theta$$

$$f = \int df = \int \frac{\sigma q r}{4\pi \varepsilon_0} \frac{(z - r\cos\theta)}{(z^2 + r^2 - 2rz\cos\theta)^{\frac{3}{2}}} d\theta$$

consider $z << r$

$$\Rightarrow \frac{z}{r} << 1$$

$$f = \frac{\sigma q r}{4\pi \varepsilon_0} \int \frac{(z - r\cos\theta)}{(z^2 + r^2 - 2rz\cos\theta)^{\frac{3}{2}}} d\theta$$

$$= \frac{\sigma q r}{4\pi \varepsilon_0} \frac{1}{r^3} \int \frac{(z - r\cos\theta)}{\left(\frac{z^2}{r^2} + 1 - 2\frac{z}{r}\cos\theta\right)^{\frac{3}{2}}} d\theta$$

$$= \frac{\sigma q}{4\pi \varepsilon_0 r^2} \int \frac{(z - r\cos\theta)}{\left(\frac{z^2}{r^2} + 1 - 2\frac{z}{r}\cos\theta\right)^{\frac{3}{2}}} d\theta$$

$$z/r << 1$$

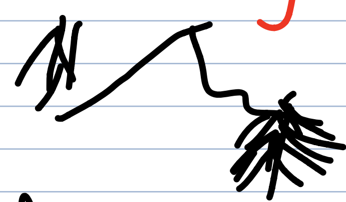
$$(z/r)^2 << (z/r) << 1$$

$$\left(1+\frac{z^2}{r^2}-2\frac{z}{r}\cos\theta\right)^{-\frac{3}{2}}$$

$$\frac{\sigma q}{4\pi\varepsilon_0 r^2}\times\ldots$$

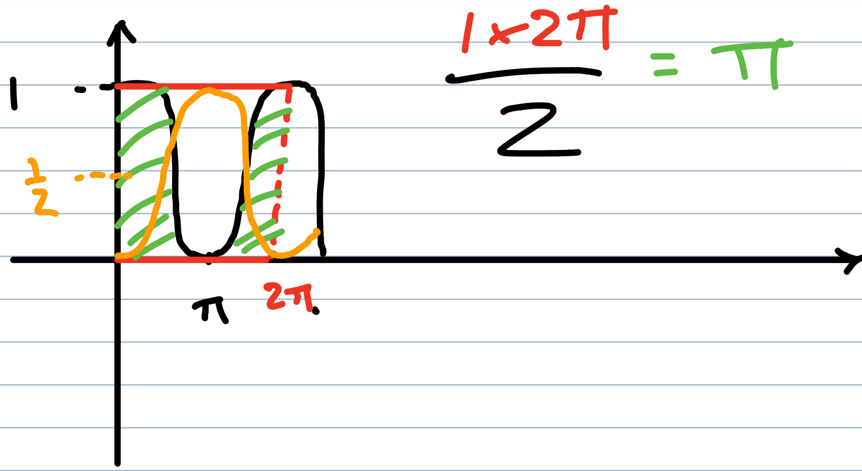
$$\ldots\times\int (z - r\cos\theta)\left[1-\frac{3}{2}\left[\cancel{\frac{z^2}{r^2}} - 2\frac{z}{r}\cos\theta\right]\right]d\theta$$

$$\frac{\sigma q}{4\pi\varepsilon_0 r}\int\left(\frac{z}{r}-\cos\theta\right)\left[\ \ldots\ \right]d\theta$$



$$\frac{\sigma q}{4\pi\varepsilon_0 r}\int_0^{2\pi}\left[\frac{z}{r}-3\frac{z}{r}\cos^2\theta\right]d\theta$$

$$\frac{\sigma q}{4\pi\varepsilon_0 r}\left[\frac{z}{r}2\pi - 3\frac{z}{r}\int_0^{2\pi}\cos^2\theta\, d\theta\right]$$



$$\frac{1\times 2\pi}{2} = \pi$$

$$\frac{\sigma q}{4\pi\varepsilon_0 r}\left[\frac{z}{r}2\pi - 3\frac{z}{r}\pi\right]$$

$$f = -\frac{\sigma q}{4\varepsilon_0 r^2} z$$

γ_1 γ_2 $E_0 = 10^{-3}\,eV$

Before: γ_1: E_0, P_0 γ_2: $E, -P$ $\boxed{E = P}$ Photons

After: e^+: $E_+, -P_+$ e^-: E_-, P_-

$$P^\mu = \begin{pmatrix} E \\ \underline{P} \end{pmatrix} \qquad \boxed{c = 1}$$

$$P^\mu_{before} = \begin{pmatrix} E_0 + E \\ E_0 - E \end{pmatrix}$$

$$P^\mu_{After} = \begin{pmatrix} E_+ + E_- \\ -P_+ + P_- \end{pmatrix}$$

$$P^\mu_{After,\,CM} = \begin{pmatrix} 2M_e \\ 0 \end{pmatrix}$$

$$P^\mu_{before} = P^\mu_{after}$$

$$|P_{before}|^2 = |P_{after}|^2 = |P_{After,\,CM}|^2$$

$$-(E_0+E)^2+(E_0-E)^2 = -4M_e^2$$

$$-2E_0E-2E_0E = -4M_e^2$$

$$E = \frac{M_e^2}{E_0}$$

$$M_e = 511 \text{ keV}$$

$$E_0 = 10^{-3} \text{ eV}$$

$$E = 2.61 \times 10^{14} \text{ eV}$$