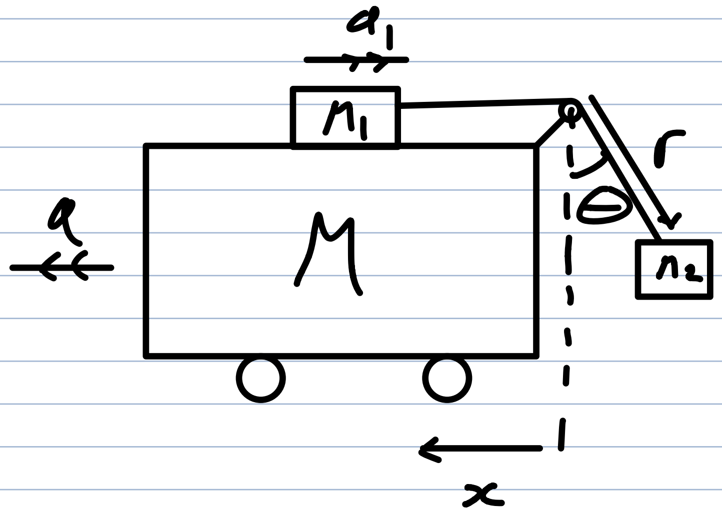


Initial conditions:

$t = 0$;

$\theta = 0$

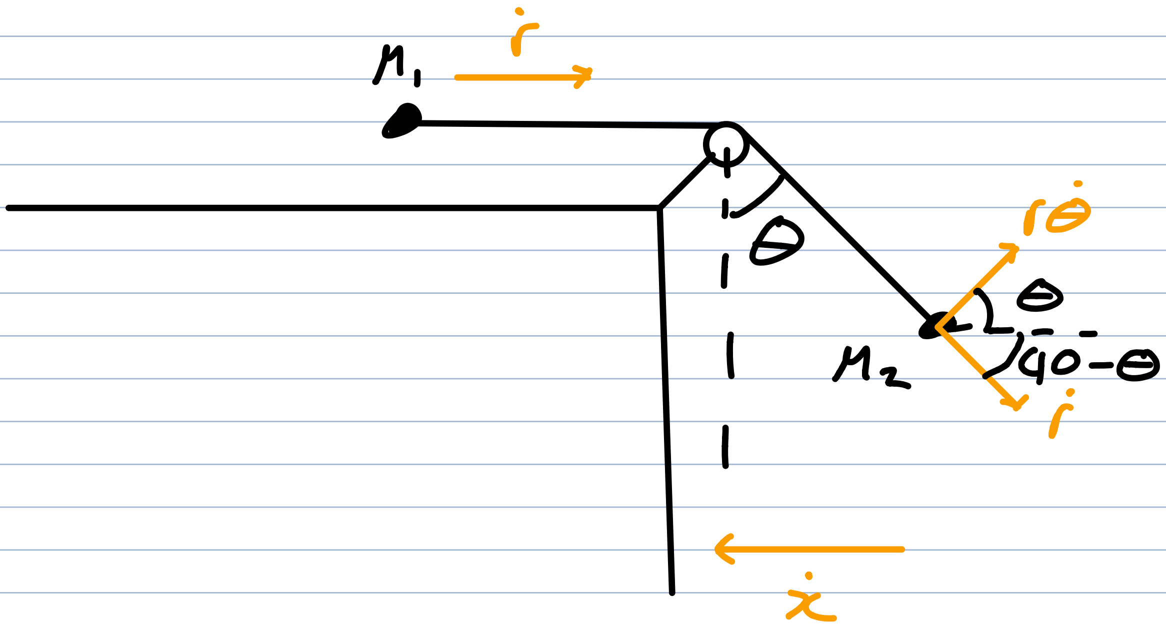
$r = r_0$

$x = 0$

$\dot{\theta} = 0$

$\dot{r} = 0$

$\dot{x} = 0$



$$\mathcal{L} = T - U$$

$$T = \frac{1}{2} m_1 (\dot{r} - \dot{x})^2 + \frac{1}{2} m_2 \Big[(r\dot{\theta}\cos\theta + \dot{r}\sin\theta - \dot{x})^2 + (r\dot{\theta}\sin\theta - \dot{r}\cos\theta)^2 \Big] + \frac{1}{2} M \dot{x}^2$$

$$U = -m_2 g r \cos\theta$$

$$\mathcal{L} = \frac{1}{2}m_1(\dot{r}-\dot{x})^2 + \frac{1}{2}m_2\left[(r\dot{\theta}\cos\theta + \dot{r}\sin\theta - \dot{x})^2\right.$$

$$\left. + (r\dot{\theta}\sin\theta - \dot{r}\cos\theta)^2\right] + \frac{1}{2}M\dot{x}^2 + m_2 g r\cos\theta$$

Euler-Lagrange Eqns:

$$\frac{\partial \mathcal{L}}{\partial q} = \frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}}\right)$$

where $q = x, r, \theta$

Work to first order in θ and $\dot{\theta}$

Neglect terms like $\theta^2, \dot{\theta}^2, \theta\dot{\theta}$

[Probably ok if $M \gg m_1, m_2$]

$$\mathcal{L} \simeq \frac{1}{2}m_1(\dot{r}-\dot{x})^2 + \frac{1}{2}m_2\left[(r\dot{\theta} + \dot{r}\theta - \dot{x})^2 + \dot{r}^2\right]$$

$$+ \frac{1}{2}M\dot{x}^2 + m_2 g r$$

$\underline{q = x}$

$$0 = \frac{d}{dt}\left(-m_1(\dot r - \dot x) - m_2(\dot r\dot\theta + \dot r\theta - \dot x) + M\dot x\right)$$

$$\Rightarrow M\ddot x - m_1\ddot x - m_2\ddot r\dot\theta - m_2\dot r\ddot\theta - m_2\ddot r\theta - m_2\dot r\dot\theta - m_2\ddot x = 0$$

$$\boxed{\begin{array}{l} \ddot x(M - m_1 - m_2) - m_2(\ddot r\dot\theta + \dot r\ddot\theta + \ddot r\theta + \dot r\dot\theta) - m_1\ddot r = 0 \\ \text{OR } -m_1(\dot r - \dot x) - m_2(\dot r\dot\theta + \dot r\theta - \dot x) + M\dot x = D \end{array}}$$

$\underline{q = r}$

$$m_2\dot\theta(r\dot\theta + \dot r\theta - \dot x) + m_2 g = \frac{d}{dt}\left[m_1(\dot r - \dot x) + m_2\theta(r\dot\theta + \dot r\theta - \dot x) + m_2\dot r\right]$$

$$-\cancel{m_2\dot\theta\dot x} + m_2 g = m_1(\ddot r - \ddot x) - \cancel{m_2\dot\theta\dot x} - m_2\theta\ddot x + m_2\ddot r$$

$$\boxed{(m_1 + m_2)\ddot r - (m_1 + m_2\theta)\ddot x = m_2 g}$$

$q = \theta$

$$\cancel{m_2 \dot{r}(r\dot{\theta} + \dot{r}\theta - \dot{x})} = \frac{d}{dt}\left[m_2 r(r\dot{\theta} + \dot{r}\theta - \dot{x})\right]$$

$$= \cancel{m_2 \dot{r}(r\dot{\theta} + \dot{r}\theta - \dot{x})}$$

$$+ m_2 r(\dot{r}\dot{\theta} + r\ddot{\theta} + \ddot{r}\theta + \dot{r}\dot{\theta} - \ddot{x})$$

$$\frac{d}{dt}(r\dot{\theta} + \dot{r}\theta - \dot{x}) = 0$$

$$\Rightarrow r\dot{\theta} + \dot{r}\theta - \dot{x} = \overset{\text{constant}}{K}$$

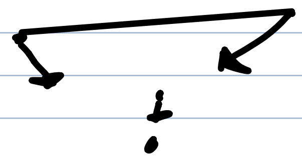
$$\frac{d}{dt}(r\theta - x) = K$$

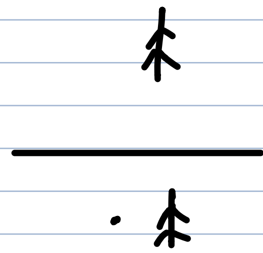
$$\boxed{r\theta - x = Kt + \overset{\text{constant}}{C}}$$

Can use to eliminate
x from previous
eqn's

→ Solve for r and θ numerically.

other less careless approximations possible...

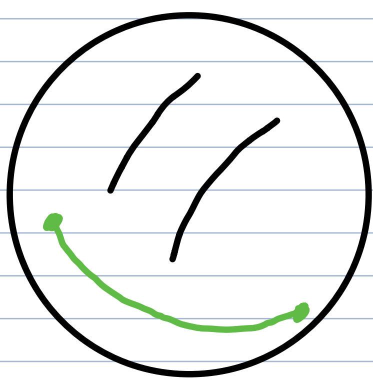




gravity ~ paths through space
(geodesics)

paths through space
(geodesics) ~ curvature

Gravity ~ curvature of
spacetime



Gravity ~ Matter

Matter ~ curvature
of spacetime

Matter $= T_{\mu\nu}$ ← rank-2

Need a rank-2 curvature tensor

$R_{\mu\nu}$ ← Ricci tensor

Try $R_{\mu\nu} \sim T_{\mu\nu}$

SR: $\partial^{\mu} T_{\mu\nu} = 0$

GR: $\nabla^{\mu} T_{\mu\nu} = 0$

$\nabla^{\mu} R_{\mu\nu} \neq 0$ in general

$g^{\mu\nu} R_{\mu\nu} = R_{\mu}{}^{\mu}$

$$\nabla^{\mu}\Big(\underbrace{R_{\mu\nu}}_{\text{Ricci tensor}} - \frac{1}{2}\underbrace{R}_{\text{Ricci scalar}}\underbrace{g_{\mu\nu}}_{\text{Metric tensor}}\Big) \equiv 0$$

$\nabla^{\mu} g_{\mu\nu} = 0$

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} \sim T_{\mu\nu}$$

$$\underbrace{R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}}_{G_{\mu\nu}} = K T_{\mu\nu}$$

Einstein tensor

$$G_{\mu\nu} = K T_{\mu\nu}$$

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

$$R_{\mu\nu} = \partial_\mu \Gamma^\alpha_{\alpha\nu} - \partial_\alpha \Gamma^\alpha_{\mu\nu} + \Gamma^{\cdot}_{\cdot\cdot}\Gamma^{\cdot}_{\cdot\cdot} - \Gamma^{\cdot}_{\cdot\cdot}\Gamma^{\cdot}_{\cdot\cdot}$$

$$\Gamma^\mu_{\alpha\beta} = \frac{1}{2} g^{\mu\varepsilon}\left[\partial_\alpha g_{\varepsilon\beta} + \partial_\beta g_{\varepsilon\alpha} - \partial_\varepsilon g_{\alpha\beta}\right]$$

find $K = \frac{8\pi G}{c^4}$

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

$$\kappa S = \int d^4x \sqrt{|\det g|} \left[R + R^2 + R^{\mu\nu} R_{\mu\nu} \right]$$

$$\frac{\delta S}{\delta g^{\mu\nu}} = 0$$