Maxwell's Equations:

(i) $\underline{\nabla} \cdot \underline{E} = \frac{\rho}{\varepsilon_0}$

(ii) $\underline{\nabla} \cdot \underline{B} = 0 \rightarrow \underline{B} = \underline{\nabla} \times \underline{A}$

(iii) $\underline{\nabla} \times \underline{E} = - \frac{\partial \underline{B}}{\partial t}$

(iv) $\underline{\nabla} \times \underline{B} = \mu_0 \varepsilon_0 \frac{\partial \underline{E}}{\partial t} + \mu_0 \underline{J}$

$$\underline{\nabla} \times \underline{E} = - \frac{\partial \underline{B}}{\partial t}$$

$$\underline{\nabla} \times \underline{E} = - \underline{\nabla} \times \frac{\partial \underline{A}}{\partial t} \qquad \underline{\nabla} \times (\underline{\nabla} \phi) \equiv 0$$

$$\underline{\nabla} \times \left(\underline{E} + \frac{\partial \underline{A}}{\partial t} \right) = 0$$

$$\underline{E} + \frac{\partial \underline{A}}{\partial t} = - \underline{\nabla} \phi$$

(iii) $\boxed{\underline{E} = - \underline{\nabla} \phi - \frac{\partial \underline{A}}{\partial t}}$

(i) $\boxed{\underline{B} = \underline{\nabla} \times \underline{A}}$

$$\underline{\nabla} \times \underline{B} = \mu_0 \varepsilon_0 \frac{\partial \underline{E}}{\partial t} + \mu_0 \underline{J}$$

$$\underbrace{\underline{\nabla} \times (\underline{\nabla} \times \underline{A})}_{-\underline{\nabla}^2 \underline{A} + \underline{\nabla}(\underline{\nabla} \cdot \underline{A})} = -\mu_0 \varepsilon_0 \frac{\partial}{\partial t}\left(\underline{\nabla}\phi + \frac{\partial A}{\partial t}\right) + \mu_0 \underline{J}$$

$$-\underline{\nabla}^2 \underline{A} + \underline{\nabla}(\underline{\nabla} \cdot \underline{A}) = -\mu_0 \varepsilon_0 \underline{\nabla}\left(\frac{\partial \phi}{\partial t}\right) - \mu_0 \varepsilon_0 \frac{\partial^2 A}{\partial t^2} + \mu_0 \underline{J}$$

$$-\mu_0 \varepsilon_0 \frac{\partial^2 \underline{A}}{\partial t^2} + \underline{\nabla}^2 \underline{A} - \mu_0 \varepsilon_0 \underline{\nabla}\underbrace{\left(\frac{\partial \phi}{\partial t} + \frac{1}{\varepsilon_0 \mu_0} \underline{\nabla} \cdot \underline{A}\right)}_{\text{free to set to zero}} = -\mu_0 \underline{J}$$

$$\varepsilon_0 \mu_0 \frac{\partial \phi}{\partial t} + \underline{\nabla} \cdot \underline{A} = 0$$

$$c^2 = \frac{1}{\varepsilon_0 \mu_0}$$

$$\frac{1}{c^2} \frac{\partial \phi}{\partial t} + \underline{\nabla} \cdot \underline{A} = 0$$

'Lorenz guage'

$$\boxed{-\frac{1}{c^2} \frac{\partial^2 \underline{A}}{\partial t^2} + \underline{\nabla}^2 \underline{A} = -\mu_0 \underline{J}}$$

(i) $$\underline{\nabla} \cdot \left(-\underline{\nabla}\phi - \frac{\partial A}{\partial t} \right) = \frac{\rho}{\varepsilon_0}$$

$$-\underline{\nabla}^2 \phi - \frac{\partial}{\partial t}\left(\underline{\nabla} \cdot \underline{A}\right) = \frac{\rho}{\varepsilon_0}$$

$$-\underline{\nabla}^2 \phi + \frac{\partial}{\partial t}\left(\frac{1}{c^2} \frac{\partial \phi}{\partial t} \right) = \frac{\rho}{\varepsilon_0}$$

$$\boxed{-\frac{1}{c^2} \frac{\partial^2 \phi}{\partial t^2} + \underline{\nabla}^2 \phi = -\frac{\rho}{\varepsilon_0}}$$

$$\boxed{-\frac{1}{c^2} \frac{\partial^2 \underline{A}}{\partial t^2} + \underline{\nabla}^2 \underline{A} = -\mu_0 \underline{J}}$$

$$\underbrace{\left(-\frac{1}{c^2} \frac{\partial^2}{\partial t^2} + \underline{\nabla}^2 \right)}_{\partial_\mu \partial^\mu} \phi = -\frac{\rho}{\varepsilon_0}$$

$$\boxed{A^\mu = \begin{pmatrix} \phi/c \\ \underline{A} \end{pmatrix} \qquad J^\mu = \begin{pmatrix} \rho c \\ \underline{J} \end{pmatrix}}$$

$$\boxed{\partial_\mu \partial^\mu A^\nu = -\mu_0 J^\nu}$$

$$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$$

$$\& \quad A^\mu = \begin{pmatrix} \frac{\phi}{c} \\ \underline{A} \end{pmatrix}$$

$$\partial^\mu = \frac{\partial}{\partial x_\mu}$$

$$x^\mu = \begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix}$$

$$x_\mu = \begin{pmatrix} -ct \\ x \\ y \\ z \end{pmatrix}$$

$$F^{\mu\nu} = \begin{pmatrix} 0 & \frac{E_x}{c} & \frac{E_y}{c} & \frac{E_z}{c} \\ -\frac{E_x}{c} & 0 & B_z & -B_y \\ -\frac{E_y}{c} & -B_z & 0 & B_x \\ -\frac{E_z}{c} & B_y & -B_x & 0 \end{pmatrix}$$

rewrite Maxwell's equ's using $F^{\mu\nu}$, J^{μ}, ∂_μ

(i) $\underline{\nabla} \cdot \underline{E} = \dfrac{\rho}{\varepsilon_0}$

(ii) $\underline{\nabla} \cdot \underline{B} = 0 \rightarrow \underline{B} = \underline{\nabla} \times \underline{A}$

(iii) $\underline{\nabla} \times \underline{E} = -\dfrac{\partial \underline{B}}{\partial t}$

(iv) $\underline{\nabla} \times \underline{B} = \mu_0 \varepsilon_0 \dfrac{\partial \underline{E}}{\partial t} + \mu_0 \underline{J}$

(i) $\underline{\nabla} \cdot \underline{E} = \dfrac{\rho}{\varepsilon_0}$

$$\frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} = \frac{\rho}{\varepsilon_0}$$

$$\frac{\partial}{\partial x}(cF^{01}) + \frac{\partial}{\partial y}(cF^{02}) + \frac{\partial}{\partial z}(cF^{03}) = \frac{J^0/c}{\varepsilon_0}$$

$$x^\mu = \begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix} \rightarrow \begin{array}{l} x^0 = ct \\ x^1 = x \\ x^2 = y \\ x^3 = z \end{array}$$

$$\partial_1 F^{01} + \partial_2 F^{02} + \partial_3 F^{03} = \frac{J^0}{\varepsilon_0 c^2}$$

$$\underline{\nabla} \cdot \underline{E} = \frac{\rho}{\varepsilon_0} \qquad c^2 = \frac{1}{\varepsilon_0 \mu_0}$$

(i) $$\boxed{\partial_1 F^{01} + \partial_2 F^{02} + \partial_3 F^{03} = \mu_0 J^0}$$

(iv) $$\underline{\nabla} \times \underline{B} = \mu_0 \varepsilon_0 \frac{\partial \underline{E}}{\partial t} + \mu_0 \underline{J}$$

x comp: $$\frac{\partial B_z}{\partial y} - \frac{\partial B_y}{\partial z} = \mu_0 \varepsilon_0 \frac{\partial E_x}{\partial t} + \mu_0 \underline{J_x}$$

$$\frac{\partial F^{12}}{\partial y} - \frac{\partial}{\partial z}\left(-F^{13}\right) = \mu_0 \varepsilon_0 \frac{\partial}{\partial t}\left(cF^{01}\right) + \mu_0 J^1$$

$$\partial_0 = \frac{1}{c}\frac{\partial}{\partial t}$$

$$\partial_2 F^{12} + \partial_3 F^{13} = \partial_0 F^{01} + \mu_0 J^1$$

$$\partial_2 F^{12} + \partial_3 F^{13} - \partial_0 F^{01} = \mu_0 J^1$$

$$F^{\mu\nu} = -F^{\nu\mu}$$

$$\boxed{\partial_2 F^{12} + \partial_3 F^{13} + \partial_0 F^{10} = \mu_0 J^1}$$ x comp

J comp

$$\partial_1 F^{21} + \partial_3 F^{23} + \partial_0 F^{20} = \mu_0 J^2$$

$$\boxed{\partial_1 F^{01} + \partial_2 F^{02} + \partial_3 F^{03} = \mu_0 J^0}$$ $\nabla \cdot E$

$$\boxed{\partial_2 F^{12} + \partial_3 F^{13} + \partial_0 F^{10} = \mu_0 J^1}$$ $\nabla \times B$

$\nabla \cdot E \uparrow (;)$, $\nabla \times B \uparrow (;\nu)$

$$\boxed{\boxed{\partial_\nu F^{\mu\nu} = \mu_0 J^\nu}}$$

$$F^{\mu\nu} = \begin{pmatrix} 0 & E_x & E_y & E_z \\ & 0 & -B_y & B_z \\ & & 0 & \\ & & & 0 \end{pmatrix}$$

$$\tilde{F}^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\alpha\beta} F_{\alpha\beta}$$

Dual tensor

SWAPS $\underline{B} \leftrightarrow -\frac{\underline{E}}{c}$

$$\tilde{F}^{\mu\nu} = \begin{pmatrix} 0 & -B_x & -B_y & -B_z \\ B_x & 0 & -\frac{E_z}{c} & \frac{E_y}{c} \\ B_y & & 0 & -\frac{E_x}{c} \\ B_z & & & 0 \end{pmatrix}$$

$\nabla\cdot\underline{B}$ $\quad$ $\nabla\times\underline{E}$

$(\vdots), (\vdots)$

$$\boxed{\boxed{\partial_\mu \tilde{F}^{\mu\nu} = 0}}$$

$$\partial_\mu\left(\frac{1}{2}\epsilon^{\mu\nu\alpha\beta} F_{\alpha\beta}\right) = 0$$

$$\rightarrow \boxed{\boxed{\partial_\mu F_{\alpha\beta} + \partial_\alpha F_{\beta\mu} + \partial_\beta F_{\mu\alpha} = 0}}$$