Black Holes and Beyond

The Science of Space:

‘A Physicists Guide to the Galaxy’

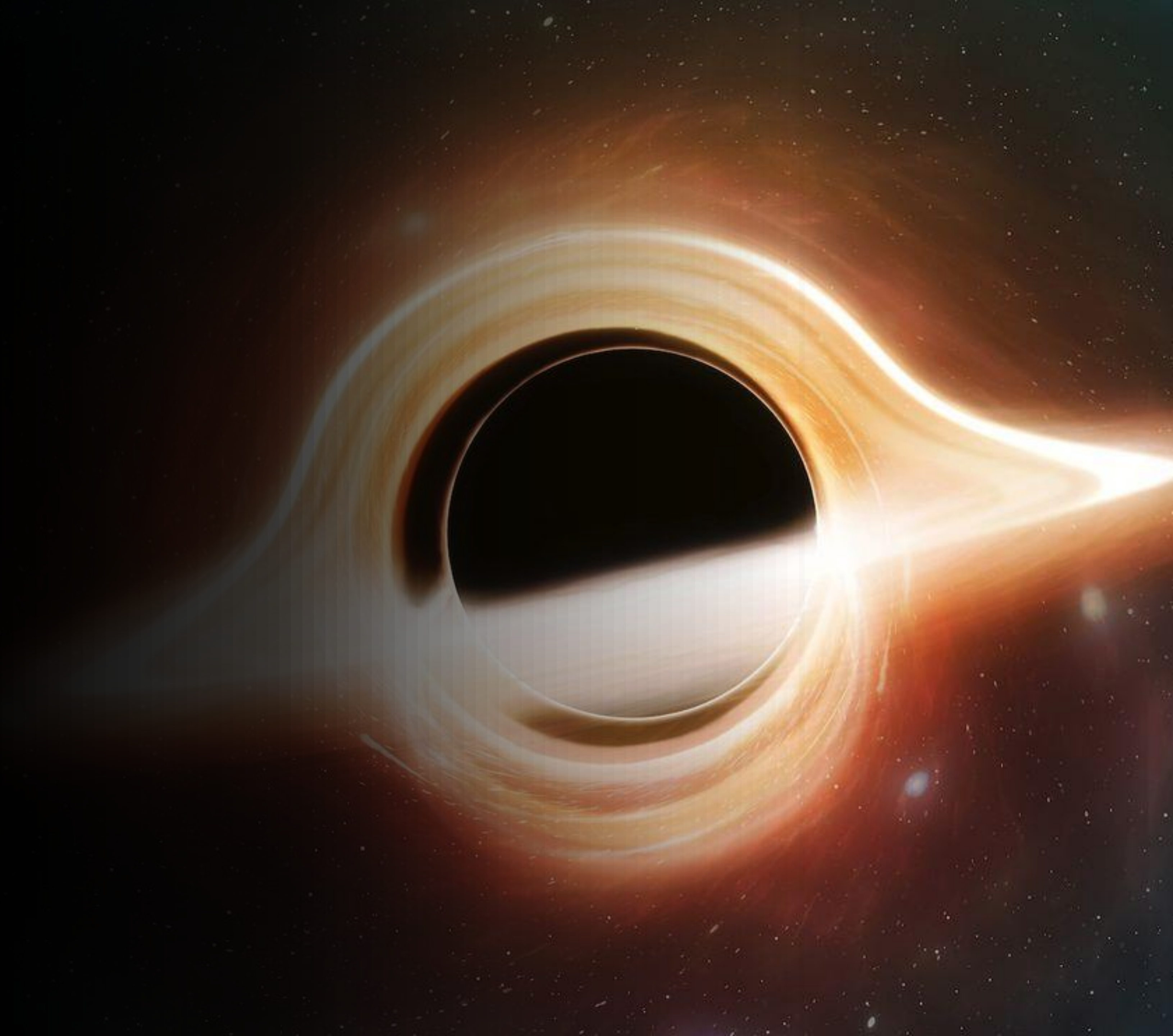
Robert Clemenson - robert.clemenson@rhul.ac.uk

(Sussex U & Royal Holloway UoL)

Southend Planetarium – 25.05.2025

Lecture Overview

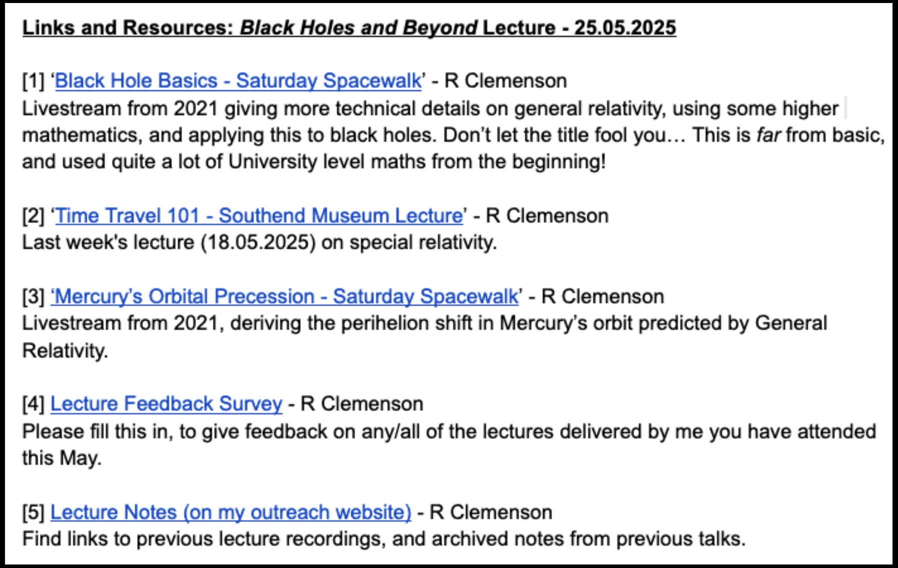
- The History of Gravity
- General Relativity
- Singularities & Quantum Gravity
- Observing Black Holes
- Black Holes as Time Machines
- Evaporation and Hawking Radiation
- Q&A (**Questions welcome throughout the talk!**)



Lecture Live Links (LLL)

Throughout the lecture, I will make a couple of references to previous talks, livestreams, and other online materials.

If you would like to check these out after the talk (or view recordings of previous lectures), please feel free to scan the QR code shown here.



Links and Resources: *Black Holes and Beyond* Lecture - 25.05.2025

[1] 'Black Hole Basics - Saturday Spacewalk' - R Clemenson
Livestream from 2021 giving more technical details on general relativity, using some higher mathematics, and applying this to black holes. Don't let the title fool you… This is *far* from basic, and used quite a lot of University level maths from the beginning!

[2] 'Time Travel 101 - Southend Museum Lecture' - R Clemenson
Last week's lecture (18.05.2025) on special relativity.

[3] 'Mercury's Orbital Precession - Saturday Spacewalk' - R Clemenson
Livestream from 2021, deriving the perihelion shift in Mercury's orbit predicted by General Relativity.

[4] Lecture Feedback Survey - R Clemenson
Please fill this in, to give feedback on any/all of the lectures delivered by me you have attended this May.

[5] Lecture Notes (on my outreach website) - R Clemenson
Find links to previous lecture recordings, and archived notes from previous talks.

Scan the QR code above, or simply click the QR code in the PDF of the lecture slides.

‘*Gravity*’ and ‘*Levity*’

In the ancient world, an objects desire to ‘fall’ or ‘float’ is described as an innate property of that object.

Aristotle tells us that Air and Fire posses levity (they want to move further from the ground), and Water and Earth possess gravity (they want to move closer to the ground).

Water has Gravity, Fire has Levity (so says Aristotle!).

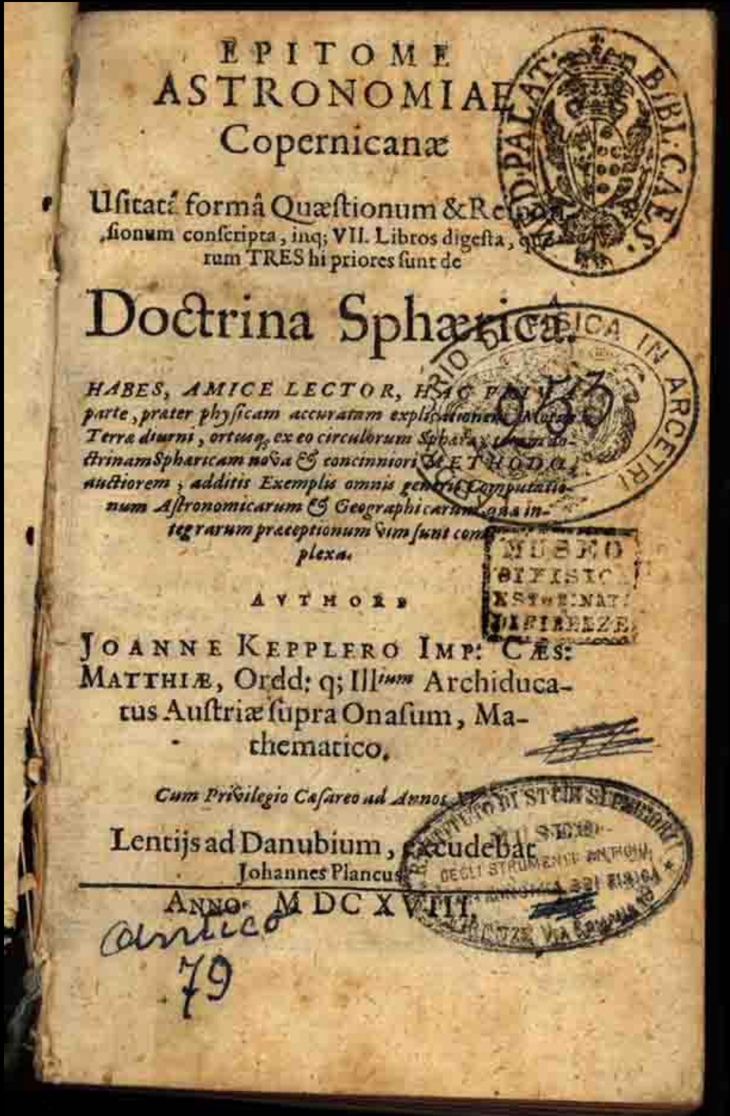
Aristotle, 330 BC.

Kepler’s Laws of Planetary Motion

Kepler published the third of his three laws of planetary motion in 1619.

These three laws describe the motions of the planets, and build upon Copernicus’ 1543 Heliocentric Model.

These laws come directly from observation, as opposed to theory. Kepler used the observations of Tycho Brahe (a 16th century Danish astronomer).



EPITOME
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Uſitatâ formâ Quæſtionum & Re[illegible]ſionum conſcripta, inq; VII. Libros digeſta, q[illegible]rum TRES hi priores ſunt de

Doctrina Sphærica.

HABES, AMICE LECTOR, HAC P[illegible] parte, præter phyſicam accuratam explicationem Motûs Terræ diurni, ortuq; ex eo circulorum Sphæræ totam doctrinam Sphæricam nova & concinniori METHODO auctiorem, additis Exemplis omnis generis Computationum Aſtronomicarum & Geographicarum, quæ integrarum præceptionum vim ſunt complexa.

AVTHORE

JOANNE KEPPLERO IMP: CÆS: MATTHIÆ, Ordd: q; Ill^ium Archiducatus Auſtriæ ſupra Onaſum, Mathematico.

Cum Privilegio Cæſareo ad Annos XV.

Lentijs ad Danubium, excudebat
Johannes Plancus.
ANNO MDCXVIII.

antico
79

Epitome Astronomiae Copernicanae, where Kepler first published his three laws together.

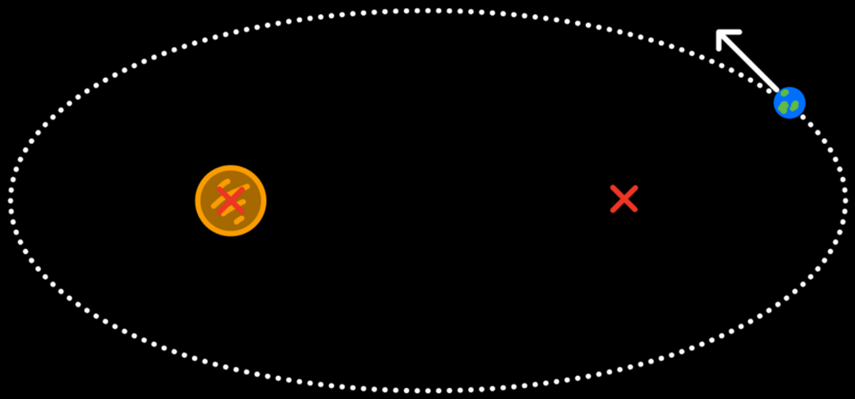
Johannes Kepler, 1610.

Kepler’s Laws of Planetary Motion

Kepler’s First Law

Planets move along elliptical paths, with the sun centered at one focus.

An ellipse is a kind of squashed circle, with geometric properties that have been studied by mathematicians since the 4th century BC.

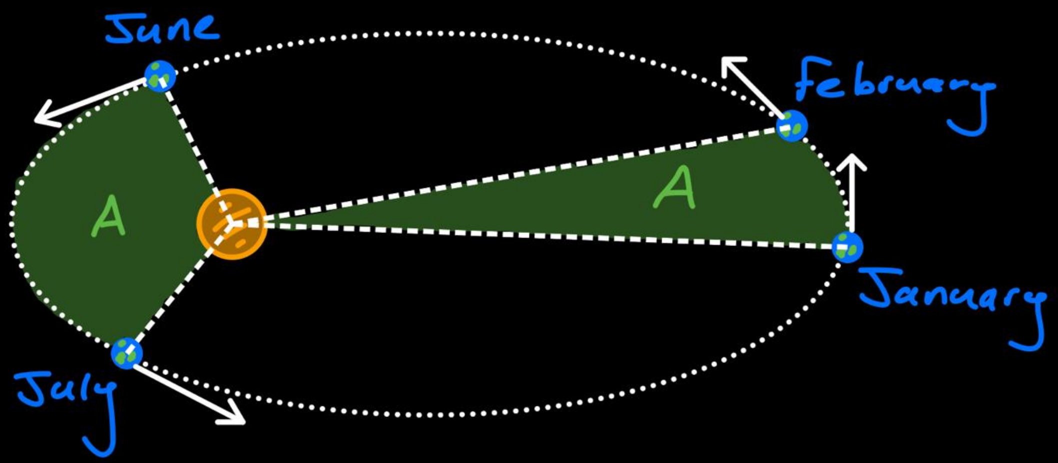


Kepler's Laws of Planetary Motion

Kepler's Second Law

The area *swept out* by a planet in orbit is unchanged, given a fixed duration.

The origin of this law, is the conservation of angular momentum. As a planet moves closer to its star, it speeds up.



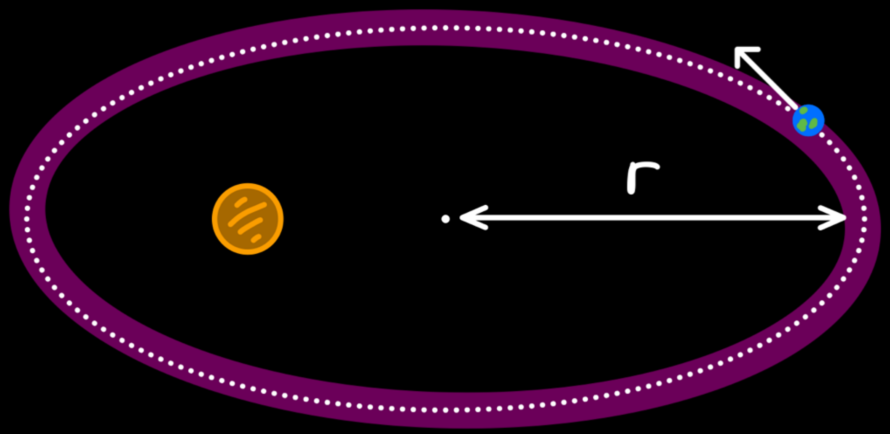
Kepler’s Laws of Planetary Motion

Kepler’s Third Law

The square of the orbital period is directly proportional to the cube of the orbital radius cubed.

This law was the key for Newton to work out his equation of gravity.

Technically an ellipse doesn’t have a radius… The length shown is in fact called the *semi-major axis* of the ellipse.



$$T^2 \propto r^3$$

Newton’s Law of Universal Gravitation

In 1687, Isaac Newtons published his breakthrough text, *Philosophiæ Naturalis Principia Mathematica*.

The Principia outlines his law of Universal Gravitation.

Universal – Gravity affects all objects with mass. Not only the heavenly bodies.

Newton’s apple tree in the grounds of Woolsthorpe Manor in Lincolnshire.

Isaac Newton, 1702.

Newton’s Law of Universal Gravitation

“After dinner, the weather being warm, we went into the garden and drank thea, under the shade of some apple trees…he told me, he was just in the same situation, as when formerly, the notion of gravitation came into his mind. It was occasion’d by the fall of an apple, as he sat in contemplative mood. Why should that apple always descend perpendicularly to the ground, thought he to himself.”

Taken from an early biography of Newtons, written by William Stukeley

Newton’s apple tree in the grounds of Woolsthorpe Manor in Lincolnshire.

Newton's Law of Universal Gravitation

At a more technical level, it was the insights from Kepler's laws that allowed Newton to figure out his equation of gravity.

In order for the planets to move in elliptical paths, the force of gravity between two masses must go like the inverse square of the distance between the masses.

To come to this realization, Isaac Newtons had to invent the core mathematical field of *calculus* (**all before turning 23!**).

$$F = \frac{GMm}{d^2}$$

$$G = 6.67 \times 10^{-11} = 0.0000000000667$$

Newton’s Law of Universal Gravitation

The Law instructs us on how to calculate the force of gravitational attraction between two masses.

The ‘big G’ constant at the front bears Newton’s name (Newton’s Gravitational Constant).

Note the small size of this constant, surprising the strength of the gravitational force.

The relative weakness of gravity, compared to the other fundamental forces remains an open problem in particle physics.

$$F = \frac{GMm}{d^2}$$

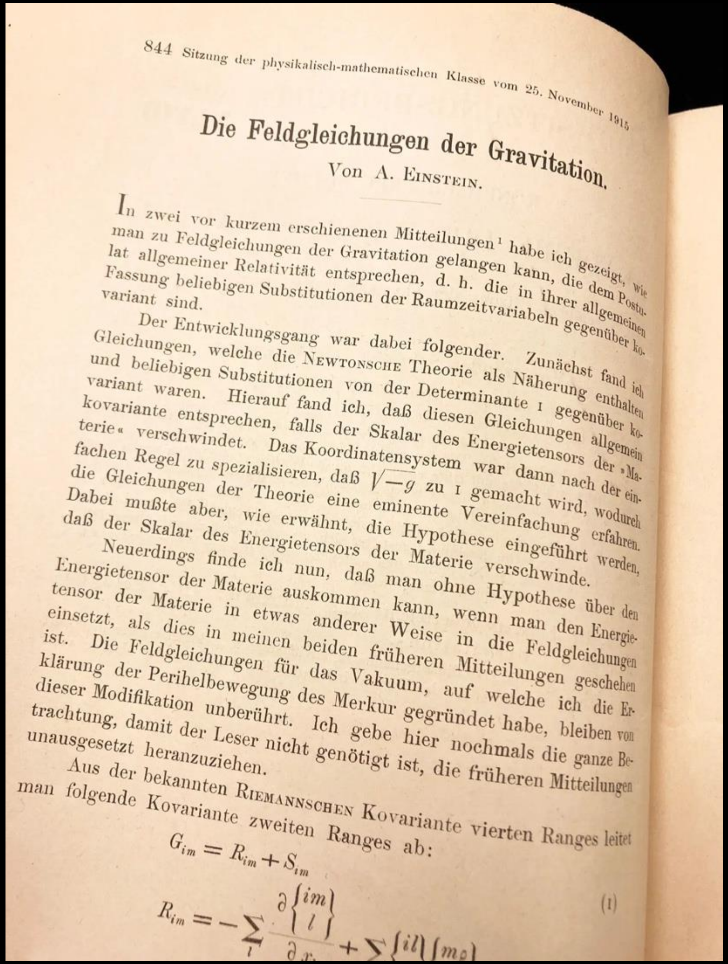
$$G = 6.67 \times 10^{-11} = 0.00000000000667$$

General Relativity

In November 1915, Albert Einstein publishes four papers, on four successive Thursdays.

The fourth of these papers, *The Field Equations of Gravitation*, sets out Einstein's new mathematical description of gravity.

The details of Einstein's theory are highly complex, but we will discuss some of the core ideas.



844 Sitzung der physikalisch-mathematischen Klasse vom 25. November 1915

Die Feldgleichungen der Gravitation.

Von A. Einstein.

In zwei vor kurzem erschienenen Mitteilungen[1] habe ich gezeigt, wie man zu Feldgleichungen der Gravitation gelangen kann, die dem Postulat allgemeiner Relativität entsprechen, d. h. die in ihrer allgemeinen Fassung beliebigen Substitutionen der Raumzeitvariabeln gegenüber kovariant sind.

Der Entwicklungsgang war dabei folgender. Zunächst fand ich Gleichungen, welche die Newtonsche Theorie als Näherung enthalten und beliebigen Substitutionen von der Determinante 1 gegenüber kovariant waren. Hierauf fand ich, daß diesen Gleichungen allgemein kovariante entsprechen, falls der Skalar des Energietensors der »Materie« verschwindet. Das Koordinatensystem war dann nach der einfachen Regel zu spezialisieren, daß $\sqrt{-g}$ zu 1 gemacht wird, wodurch die Gleichungen der Theorie eine eminente Vereinfachung erfahren. Dabei mußte aber, wie erwähnt, die Hypothese eingeführt werden, daß der Skalar des Energietensors der Materie verschwinde.

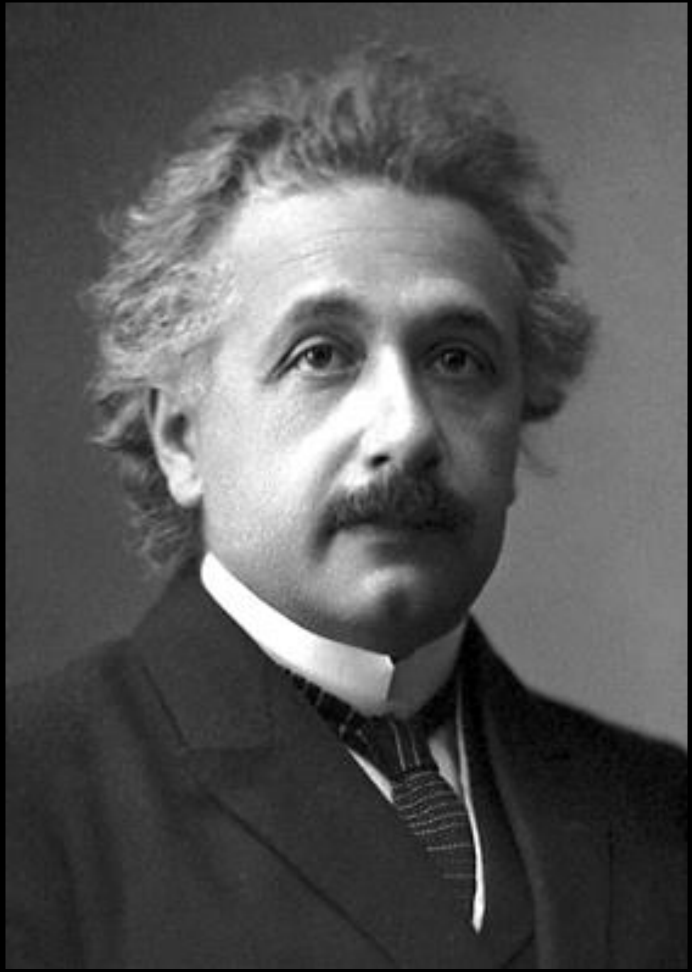
Neuerdings finde ich nun, daß man ohne Hypothese über den Energietensor der Materie auskommen kann, wenn man den Energietensor der Materie in etwas anderer Weise in die Feldgleichungen einsetzt, als dies in meinen beiden früheren Mitteilungen geschehen ist. Die Feldgleichungen für das Vakuum, auf welche ich die Erklärung der Perihelbewegung des Merkur gegründet habe, bleiben von dieser Modifikation unberührt. Ich gebe hier nochmals die ganze Betrachtung, damit der Leser nicht genötigt ist, die früheren Mitteilungen unausgesetzt heranzuziehen.

Aus der bekannten Riemannschen Kovariante vierten Ranges leitet man folgende Kovariante zweiten Ranges ab:

$$G_{im} = R_{im} + S_{im} \qquad (1)$$

$$R_{im} = -\sum_{l} \frac{\partial \begin{Bmatrix} im \\ l \end{Bmatrix}}{\partial x_l} + \sum \begin{Bmatrix} il \\ \end{Bmatrix} \begin{Bmatrix} m\rho \\ \end{Bmatrix}$$

The Field Equations of Gravitation, published Nov 25th 1915.



Albert Einstein, 1921.

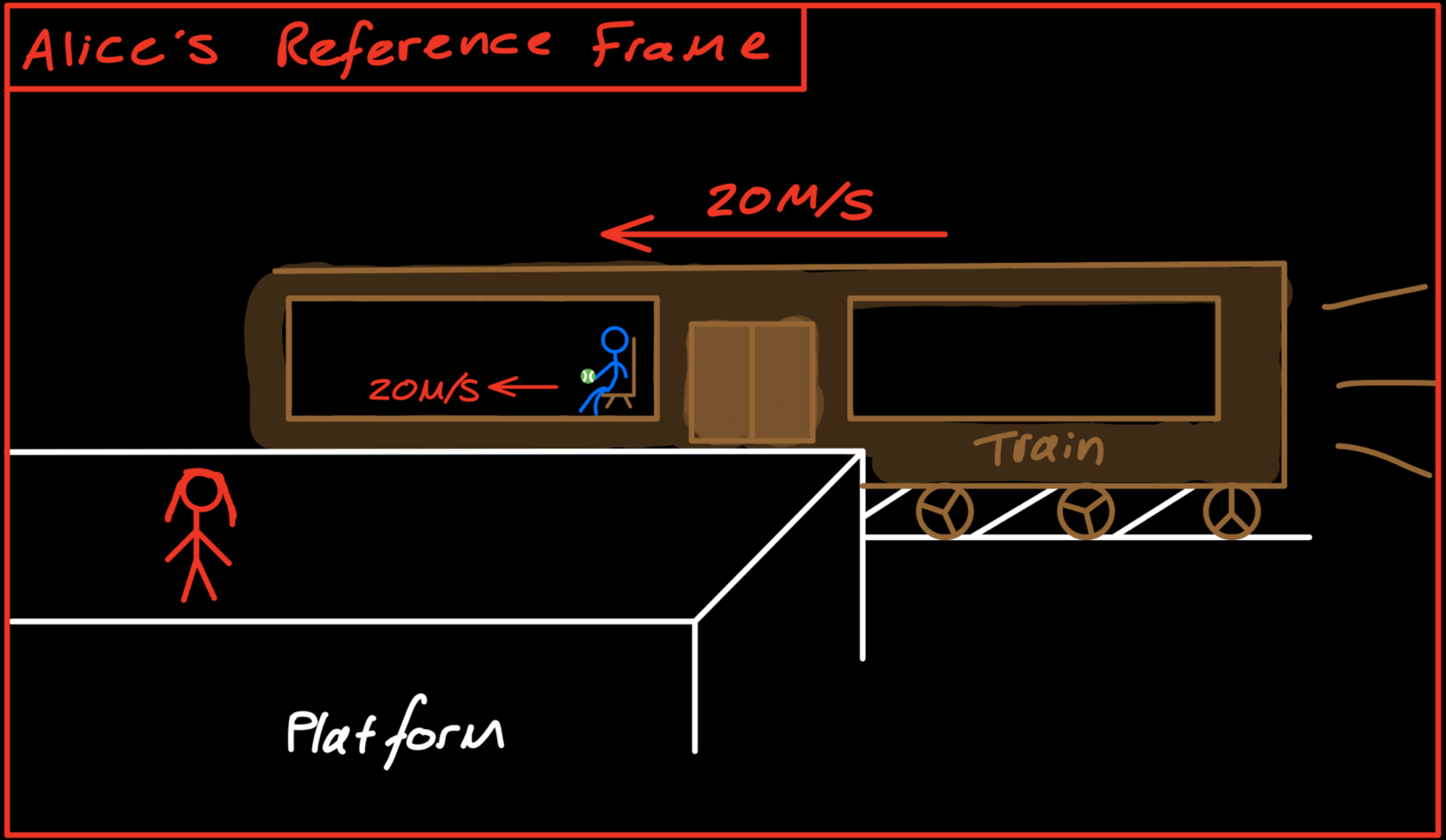
General Relativity

LLL [1]

Recall our discussion of trains and reference frames last week.

Suppose Bob rides a train, which passes Alice on a train platform at a constant speed of 20 meters per second.

Bob throws a tennis ball up in the air.



General Relativity

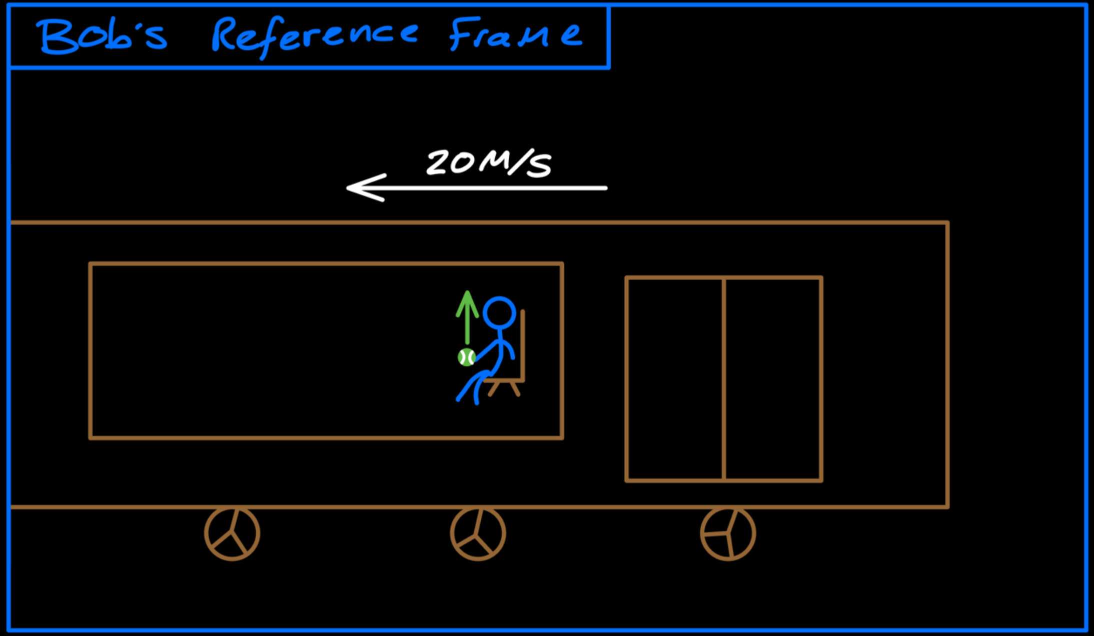
Recap: Inertial (Non-Accelerating) Reference Frames

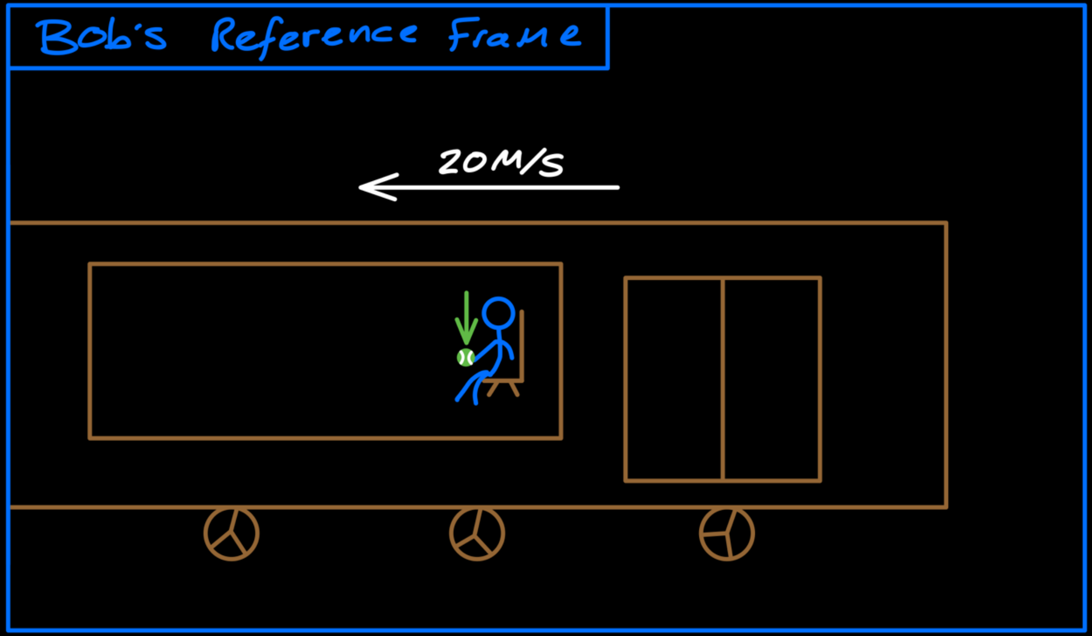
We call reference frames moving at a constant speed an **Inertial Reference Frame**.

In Bob's reference frame, the ball moves upwards in a straight line and falls back into his lap.

This is exactly what Bob would observe when the train is stationary at a station.

Bob *cannot* tell whether his train is at rest in the station, or moving at a constant speed away from the station.





General Relativity

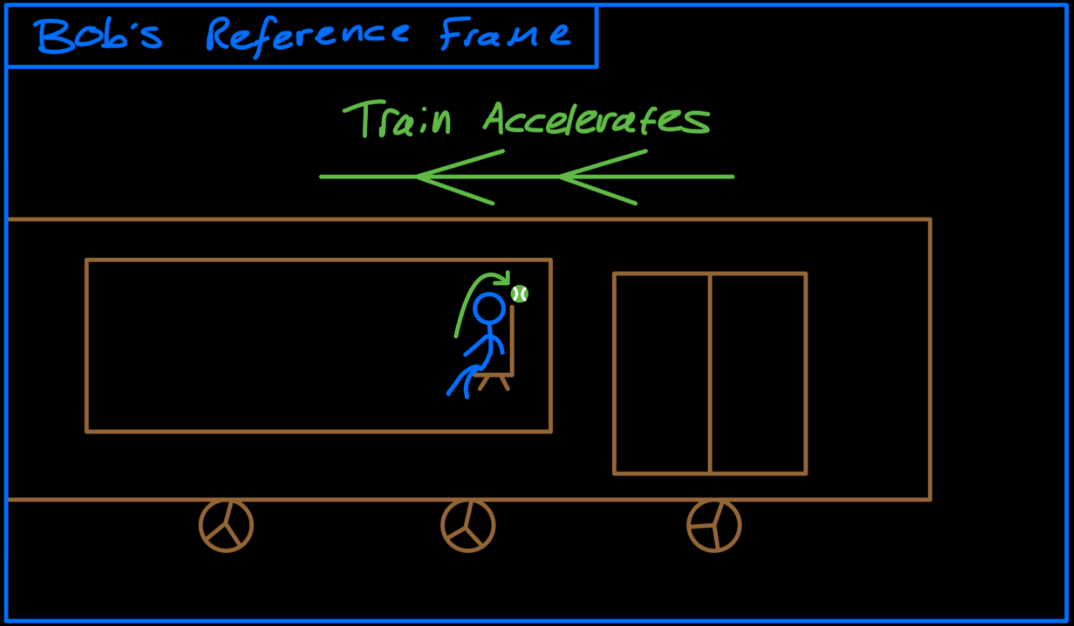
Recap: Accelerating Reference Frames

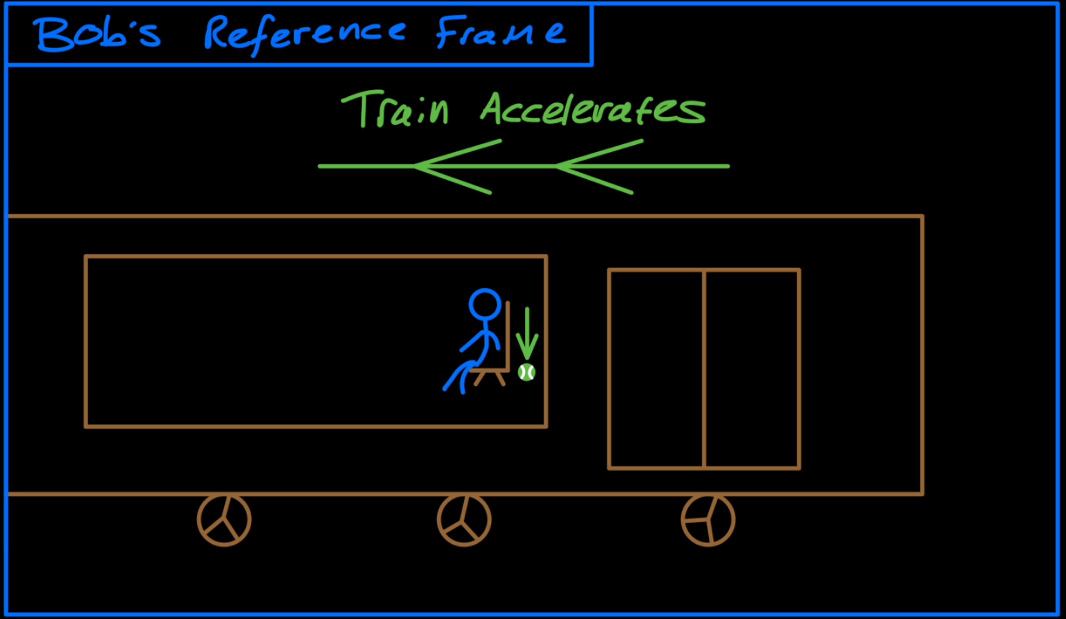
If the train accelerates (speeds up), Bob will observe something different.

He throws the ball upwards, but the ball does not end up on his lap. It shoots over his head, and lands behind him.

This is different to what he would observer on a stationary train.

Bob *can* tell that he is on an accelerating train, distinct from a stationary, or non-accelerating train.



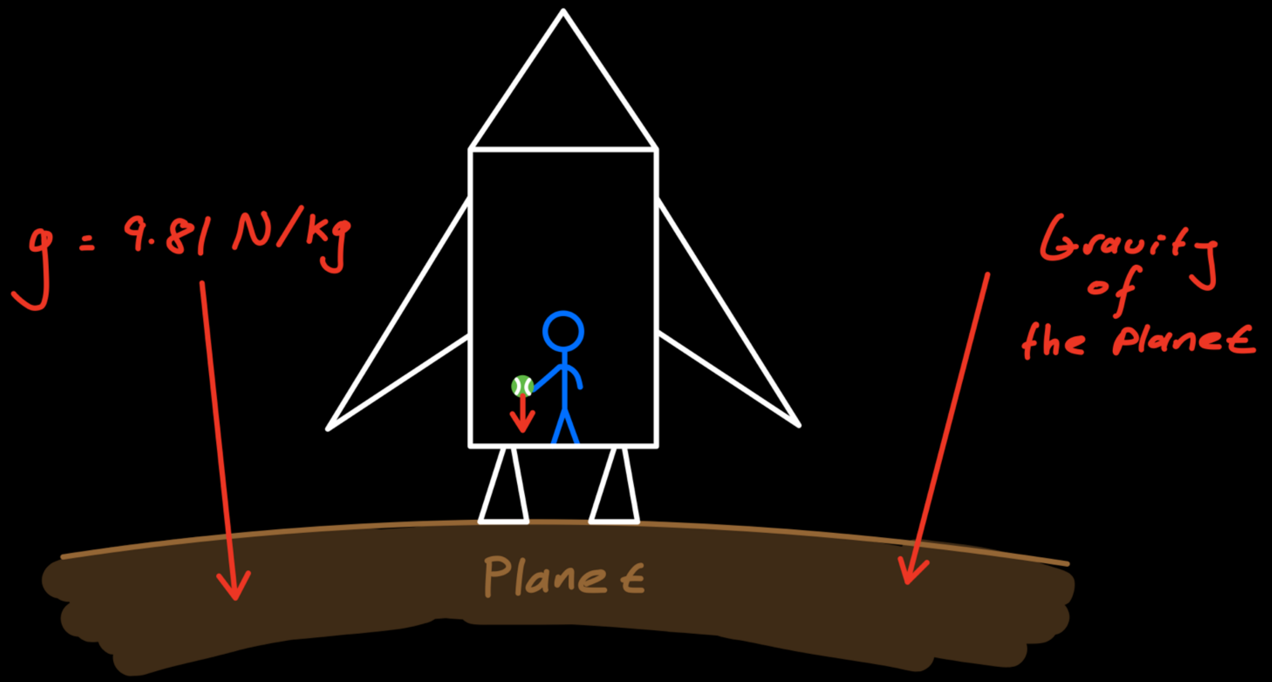


General Relativity

The Equivalence Principle

Suppose Bob is in a rocket, parked at rest on the surface of the Earth. The Earths surface gravity exerts a force of 9.81 Newton's per kilogram of mass.

If Bob drops the tennis ball, it will accelerate downwards at a rate of 9.81 meters per second per second, falling under the influence of gravity.



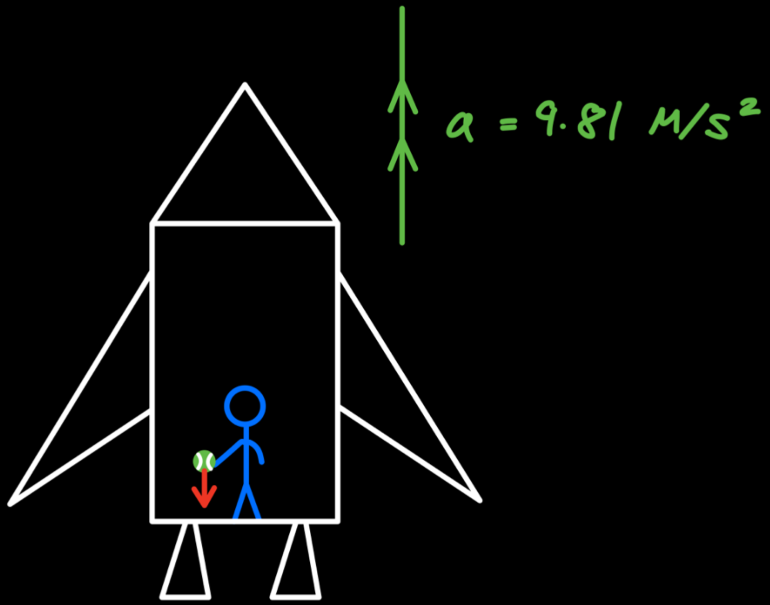
General Relativity

The Equivalence Principle

Suppose instead that Bob's rocket, accelerates upwards at precisely 9.81 meters per second per second.

Bob's feet would feel the floor of the rocket pushing up on him.

If Bob drops the tennis ball, it will appear to him to accelerate downwards at a rate of 9.81 meters per second per second.



General Relativity

Could Bob tell which situation he finds himself in? Both scenarios lead to the tennis ball accelerating down to the ground at a rate of 9.81 meters per second per second.

The indistinguishability of these situations is encapsulated in **Einstein's Equivalence Principle**.

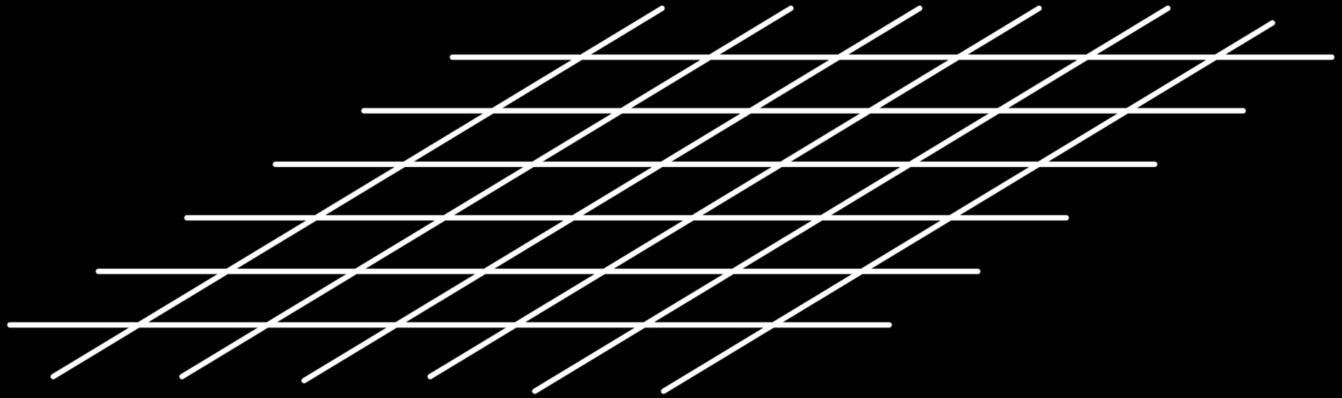
General Relativity

Einstein’s Equivalence Principle:

Gravity is indistinguishable from acceleration.

This is the insight that allows Einstein to conclude that gravity is a *geometric phenomena*.

Einstein’s earlier work on the theory of special relativity (1905) puts space and time into a single united framework, *four-dimensional space time*.



General Relativity

Einstein's Equivalence Principle:

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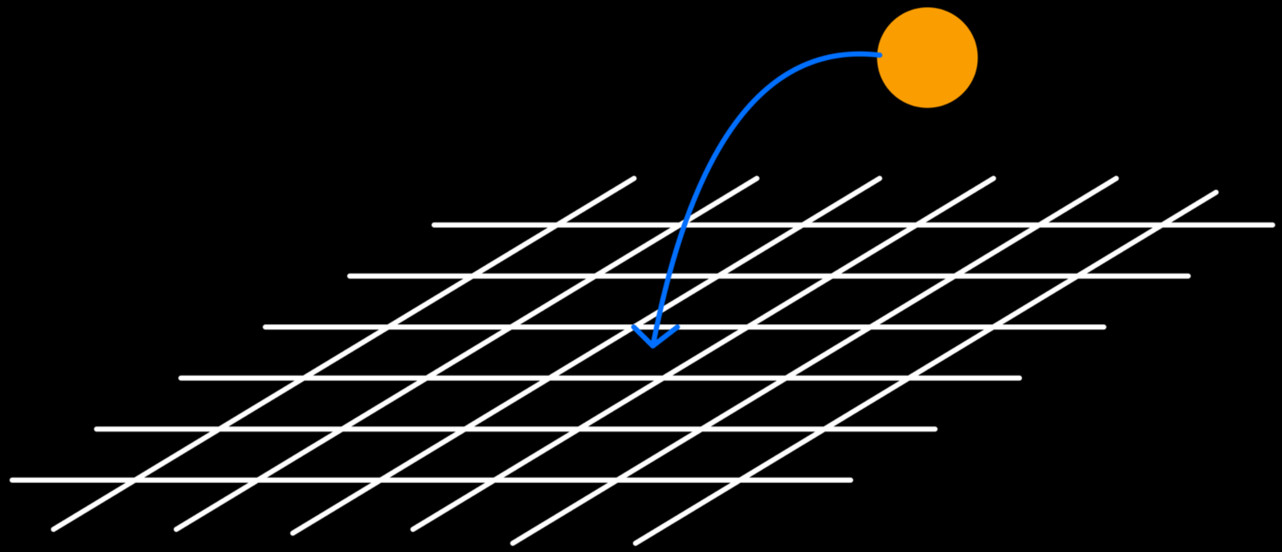
Space and time are more than just the stage on which physics happens, they are players.



Let's add a piece of matter into this flat region of space.

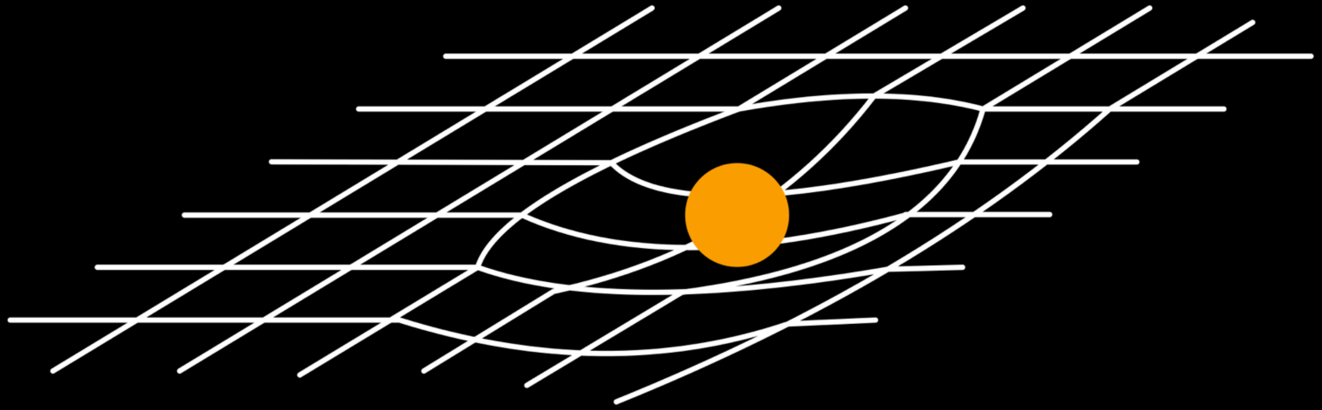
General Relativity

The presence of matter, causes space-time to curves.

The curvature of space-time cases matter to experience the illusion of a gravitational force.

'Spacetime tells matter how to move; matter tells spacetime how to curve.' – John Wheeler

Space-time is more than just the stage on which physics happens, they are players too.



$$\underset{\text{Space-time Curvature}}{G_{\mu\nu}} = \frac{8\pi G}{\underset{\text{Speed of Light}}{c^4}} \underset{\text{Matter}}{T_{\mu\nu}}$$

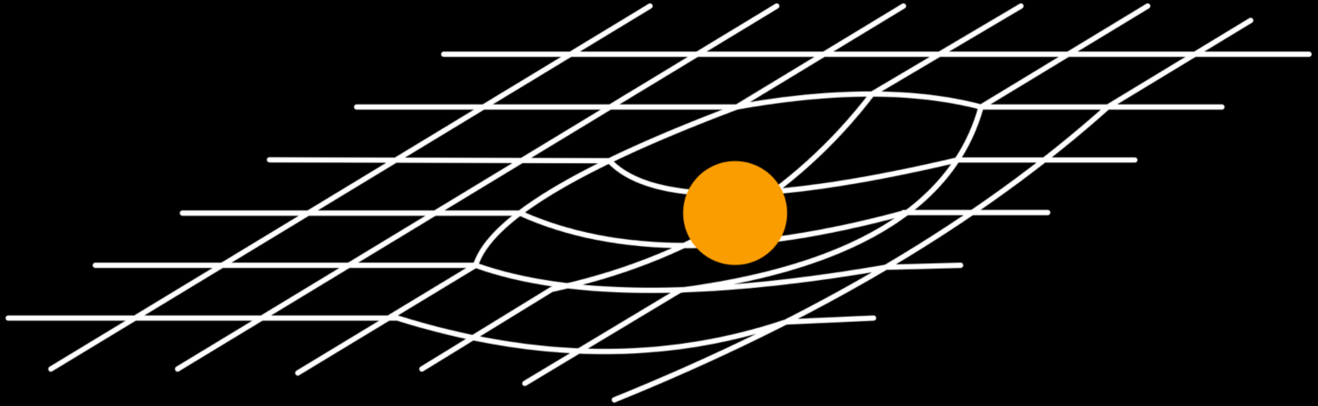
General Relativity

LLL [2]

More theory on General Relativity:

Black Hole Basics - Saturday Spacewalk

Not for the faint of heart!...



$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

Space-time Curvature

Speed of Light

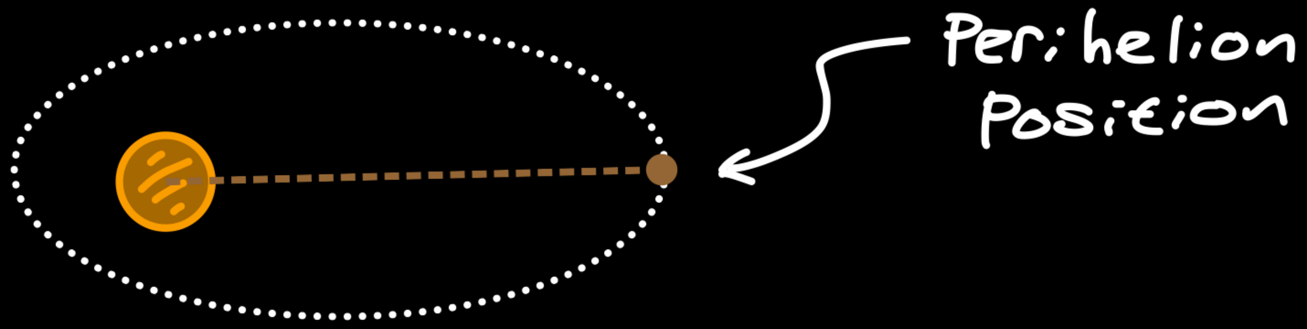
Matter

General Relativity

How do we know Einstein was right?

The abnormal precession of Mercury’s perihelion was a longstanding problem in celestial mechanics (first pointed out in 1859 by Urbain La Verrier).

When physicists used Newton’s Equation of Gravity to calculate the shape of Mercury’s orbit, the calculated rate of advance of Mercury’s perihelion is very far from the rate observed by astronomers.



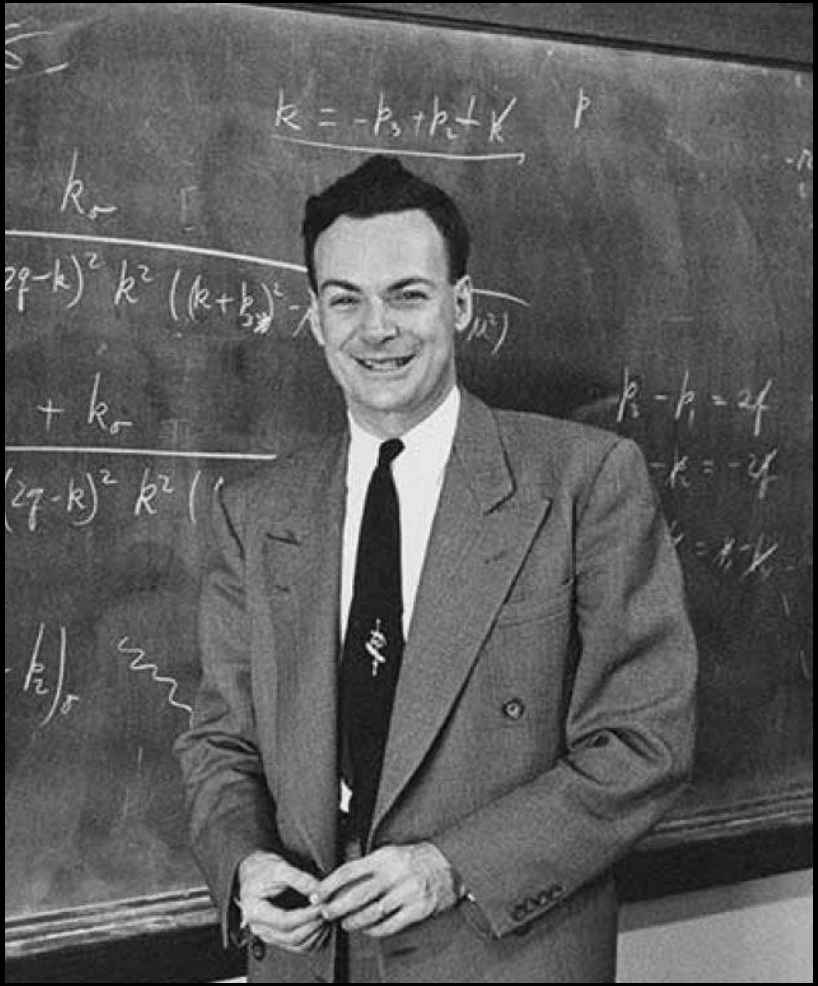
Perihelion – Point of greatest distance from the sun, in a planets elliptical orbit.

Feynman’s Golden Rule

“In general, we look for a new law by the following process: First we guess it; then we compute the consequences of the guess to see what would be implied if this law that we guessed is right; then we compare the result of the computation to nature, with experiment or experience, compare it directly with observation, to see if it works.

If it disagrees with experiment, it is wrong. In that simple statement is the key to science. It does not make any difference how beautiful your guess is, it does not make any difference how smart you are, who made the guess, or what his name is — ***if it disagrees with experiment, it is wrong****.”*

Richard Feynman

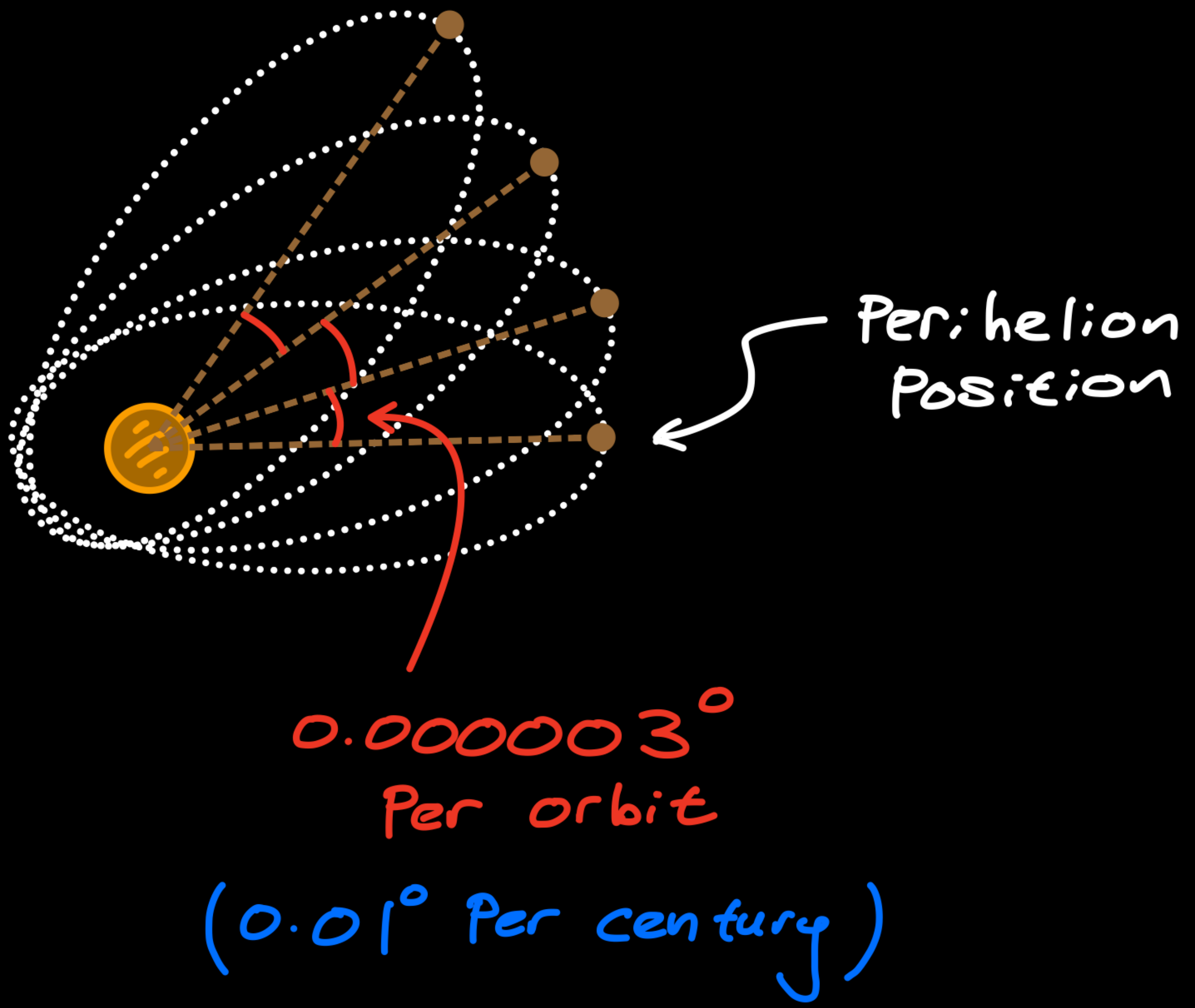


General Relativity

How do we know Einstein was right?

When we use Einstein's Equation of Gravity to perform the same calculation, the answer matches the observation made by astronomers to a very high degree of accuracy.

This is one feather in General Relativity's cap!



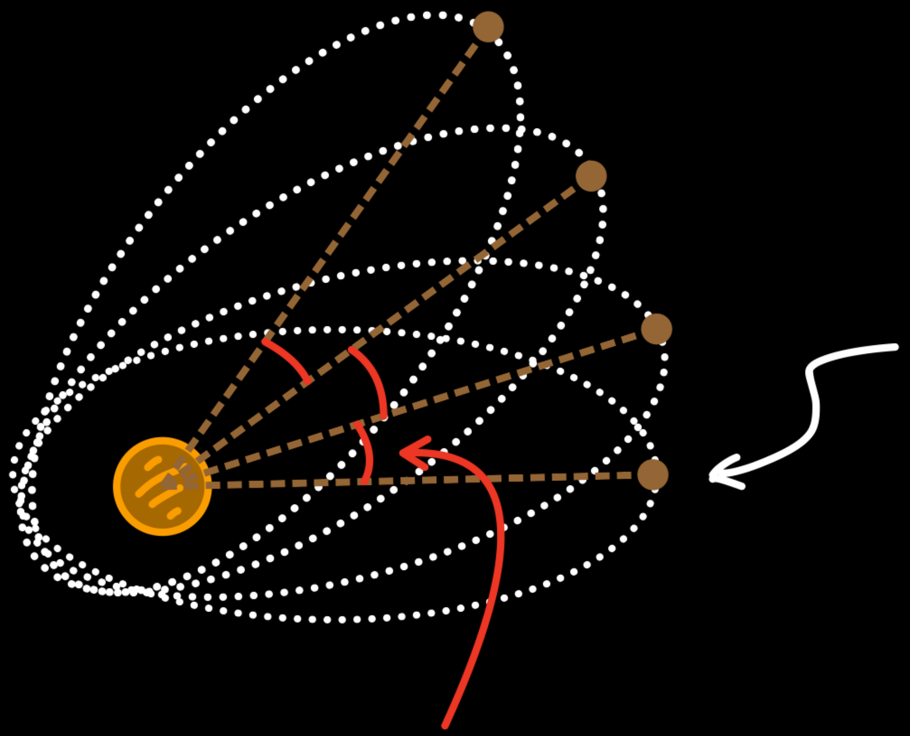
General Relativity

If you have a spare 1hr37min, you can watch me do this calculation in real time!

Mercury's Orbital Precession - Saturday Spacewalk Jan 24th

This is rough...

Don't say I didn't warn you!



Perihelion Position

0.000003° Per orbit

(0.01° Per century)

LLL [3]

Newton vs Einstein

***"If it disagrees with experiment, it is wrong."** – R.P. Feynman*

However... **Wrongness is a spectrum!**

"Absolute truth" is a challenging concept, and not one we grapple with in the sciences.

Science deals with models:

A description of a physical system that allows us to understand *some* aspects of the phenomena and make testable predictions. This does not mean that the model has to work in *all circumstances*.

Newton vs Einstein. The battle for the laws of gravity

Note: This is an AI generated image.

Newton vs Einstein

Einstein vs Newton is a good example.

Newton's theory of gravity allows us to (mostly) explain the orbits of planets in our solar system, and the dynamics of our galaxy. Newton's theory allowed us to get to the moon.

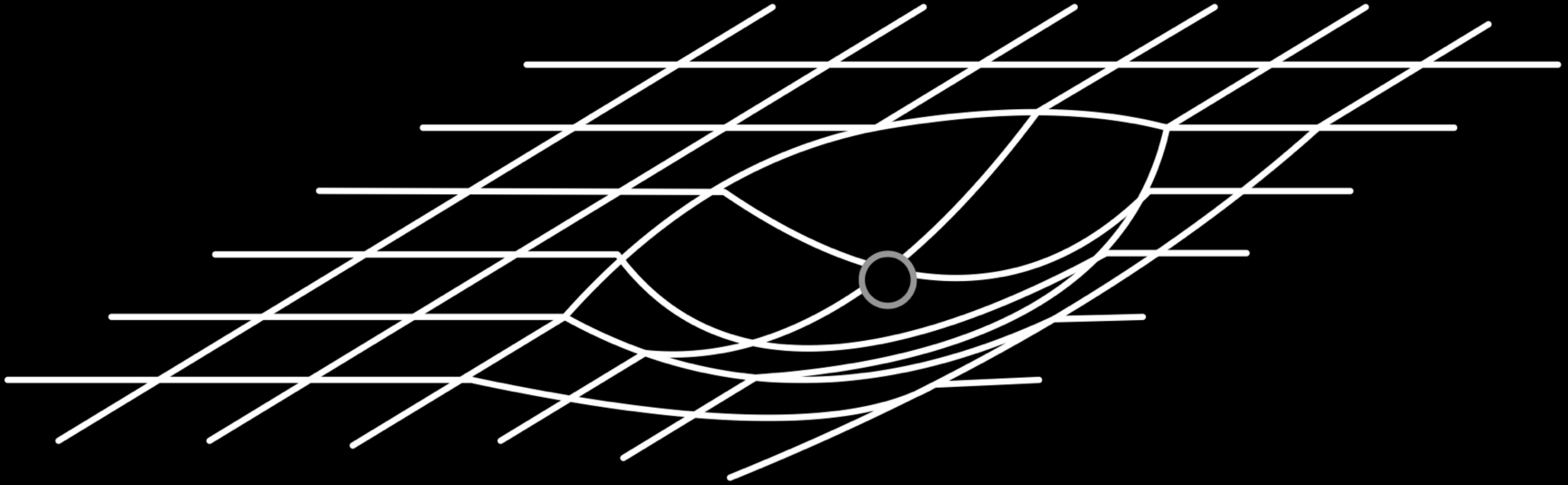
Einstein's theory of gravity (General Relativity) predicts the existence of black holes, and solves subtle problems related to the orbit of the planets that Newton's theory cannot explain.

Newton vs Einstein. The battle for the laws of gravity

Note: This is an AI generated image.

Newton's theory works for (relatively) small masses. Einstein's theory picks up where Newton's left off, and tells us more about larger masses and subtle (harder to observe) effects.

The Monster Hidden in the Equations

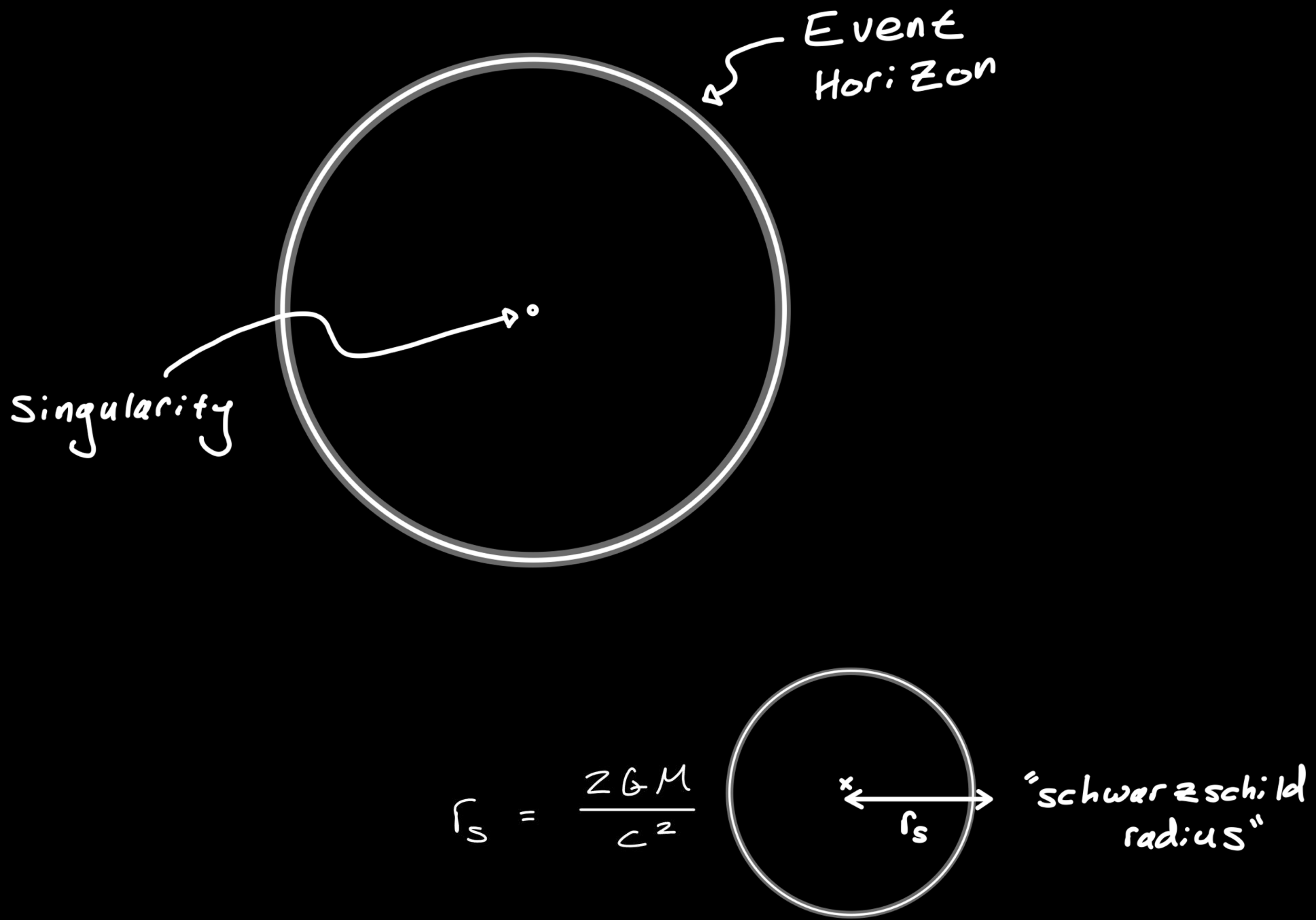


$$ds^2 = -\left(1-\frac{2GM}{rc^2}\right)c^2dt^2+\left(1-\frac{2GM}{rc^2}\right)^{-1}dr^2+r^2d\Omega^2$$

Karl Schwarzschild.

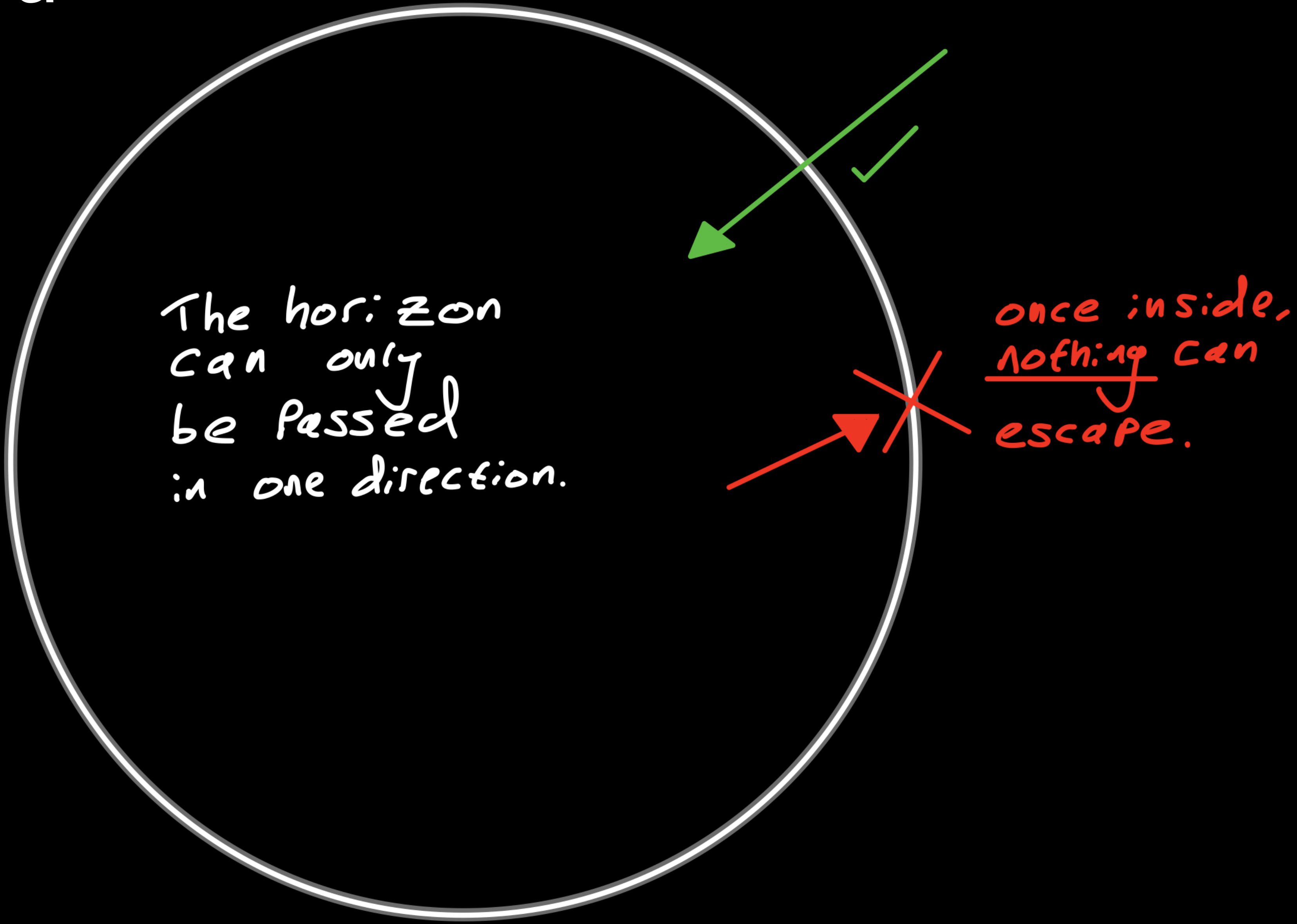
Anatomy of a Black Hole

Describing a black hole is simple enough, as they have very few identifying features.



$$r_s = \frac{2GM}{c^2}$$

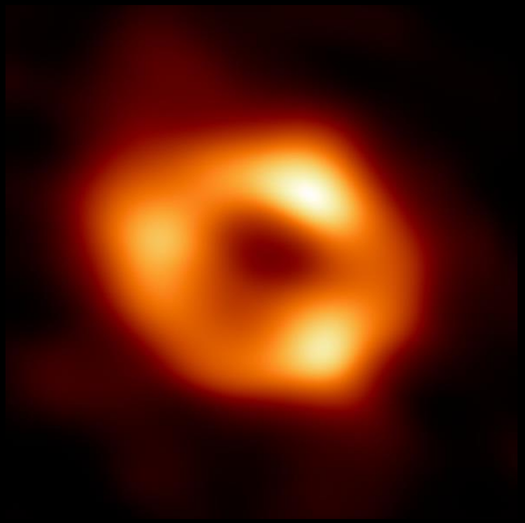
Anatomy of a Black Hole



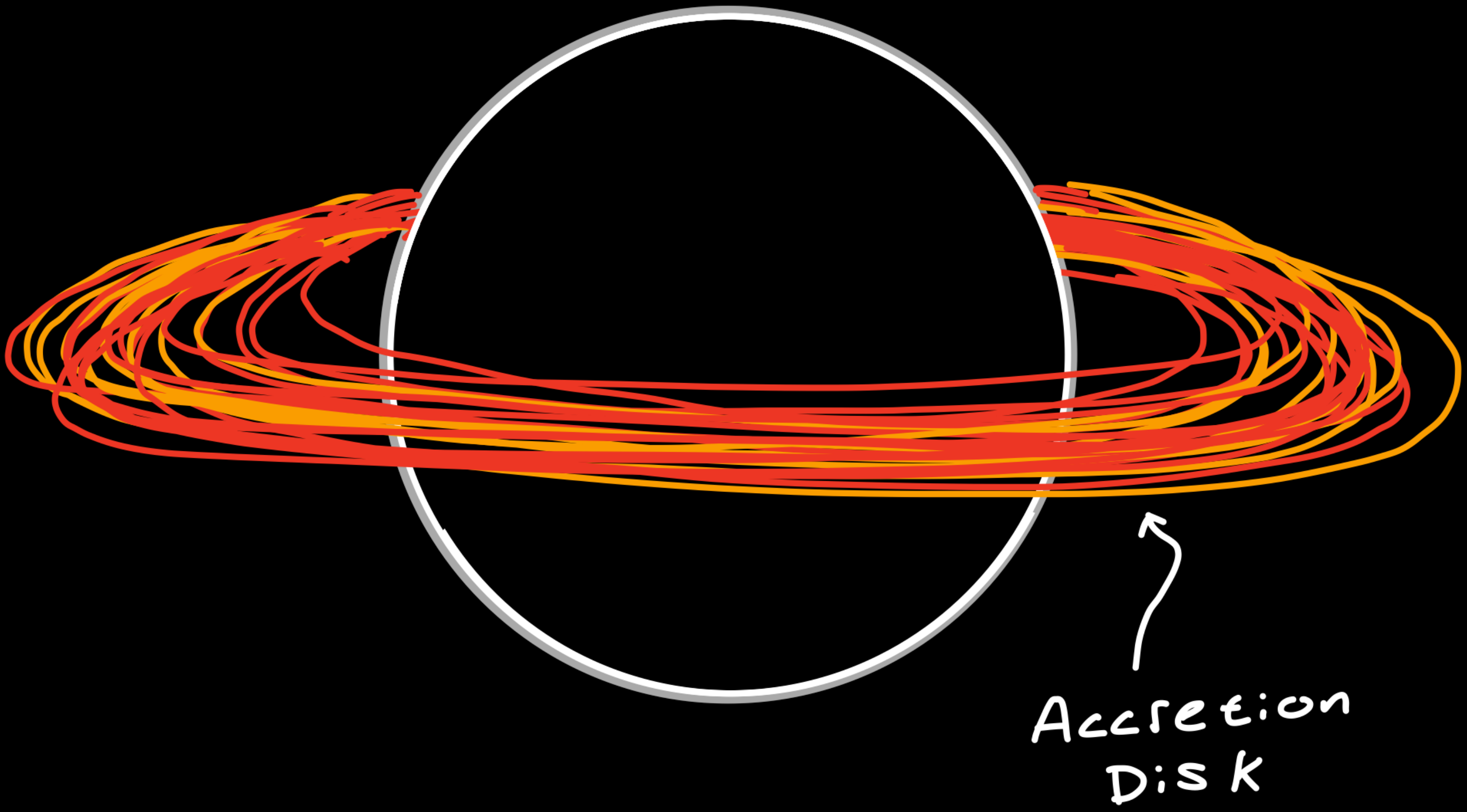
The horizon can only be passed in one direction.

once inside, nothing can escape.

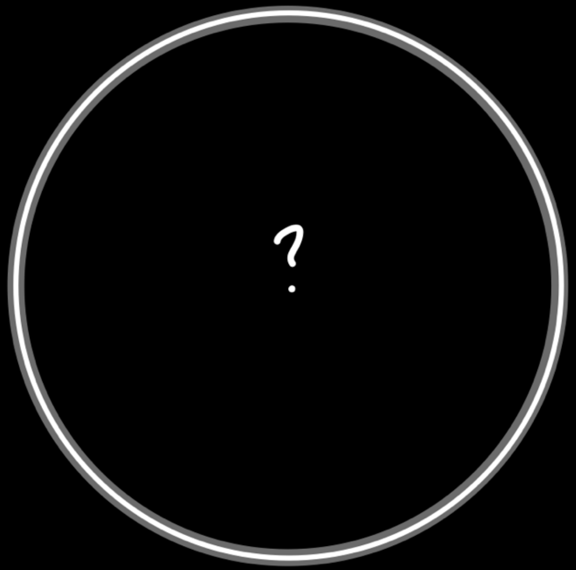
Anatomy of a Black Hole



Event Horizon Telescope image of Sagittarius A*.



Anatomy of a Black Hole

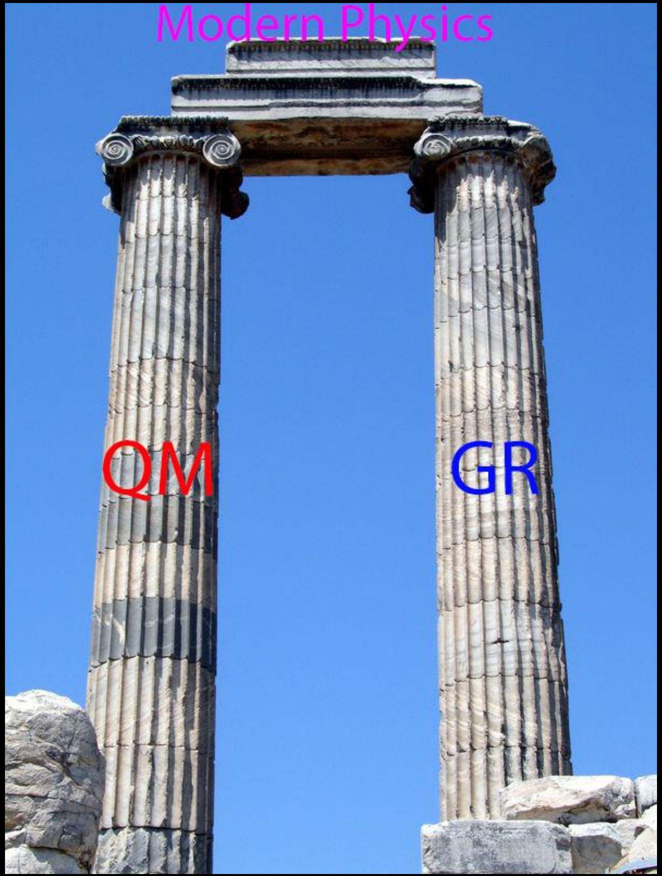


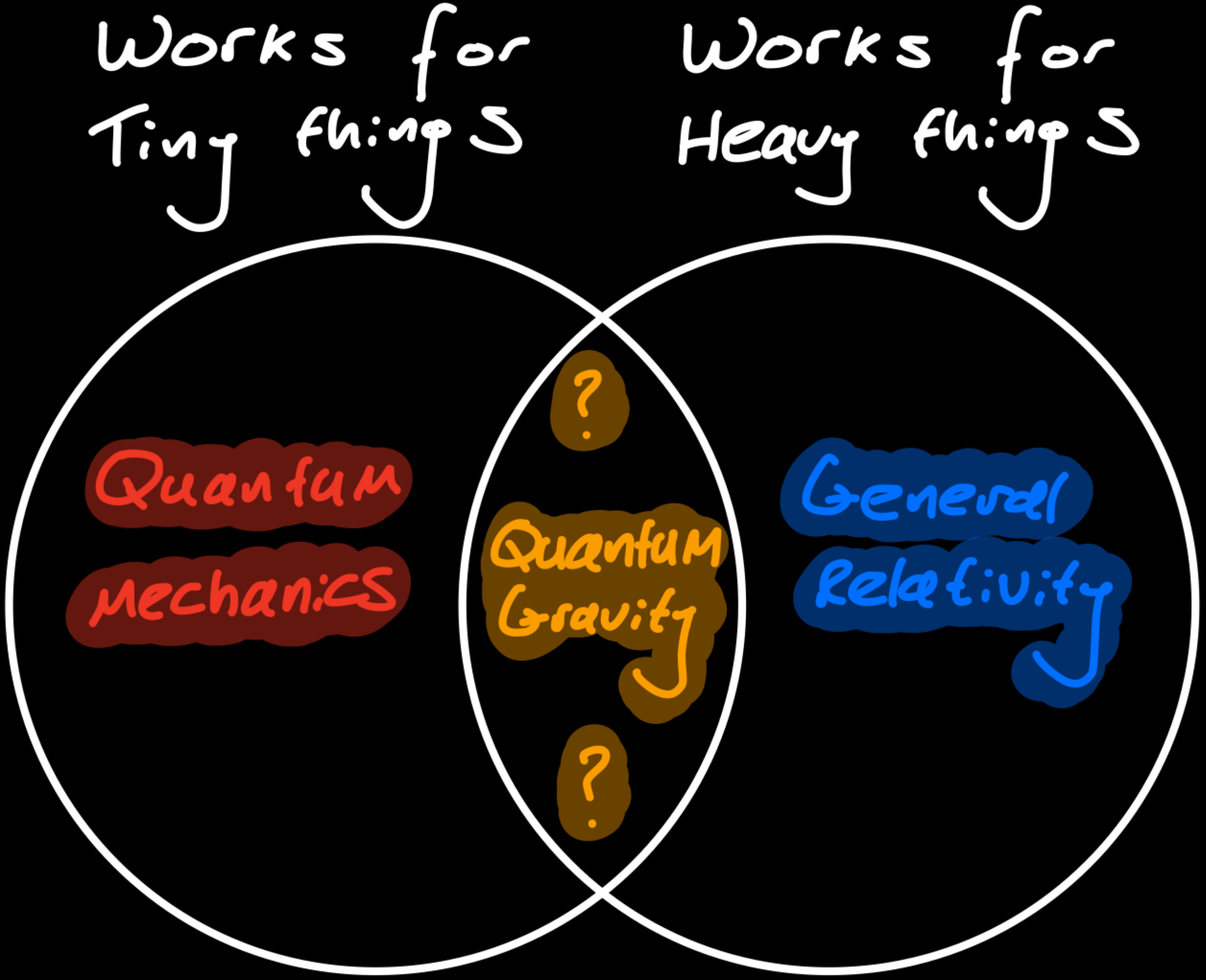
The Singularity:

- The Point at the centre of a black hole.
- 'Infinitely Dense'.
- The Laws of Physics break down.

The Singularity

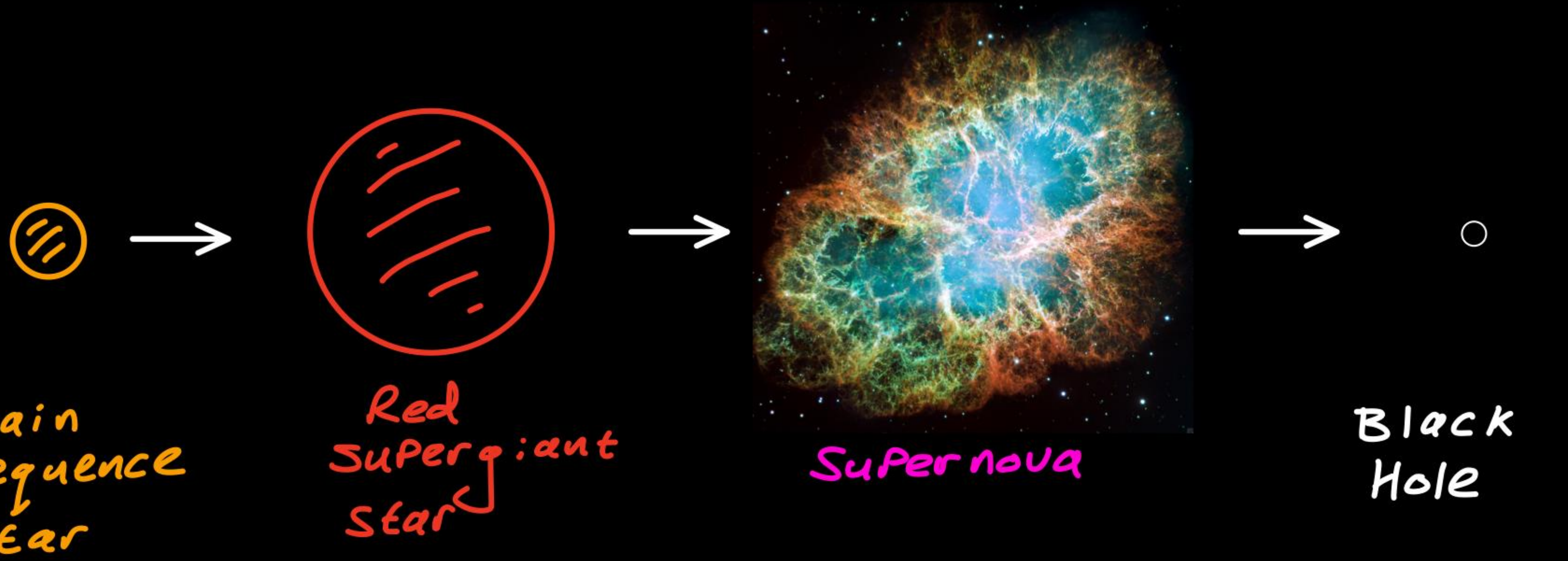
Why is the singularity so hard to understand?





Black Hole Formation

Black holes form when giant stars run out of fuel, and collapse under their own weight.

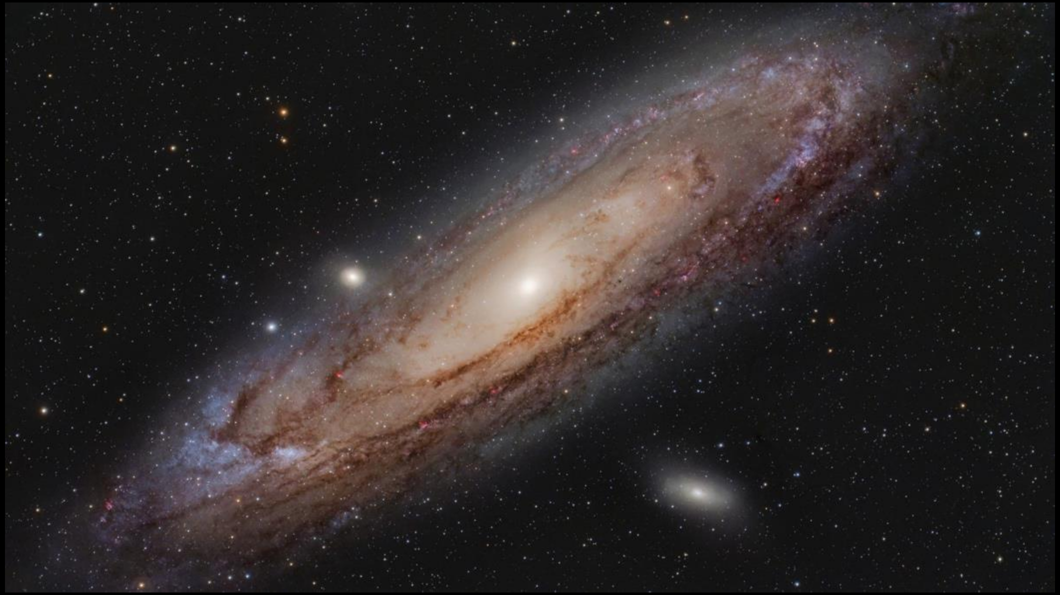


Roger Penrose and Stephen Hawking bridged the gap from Schwarzschild’s theoretical solution of Einstein’s equations, to show that black holes can form in real life (imperfect) conditions.

Supermassive Black Holes

Every large galaxy has a Supermassive Black Hole at its center.

An artist's impression of a Quasar, a particularly energetic kind of galactic nuclei.



The Andromeda Galaxy. The nearest neighboring galaxy to our own.

Black Holes are thought to be crucial to the formation of galaxies, in ways we don’t yet fully understand.

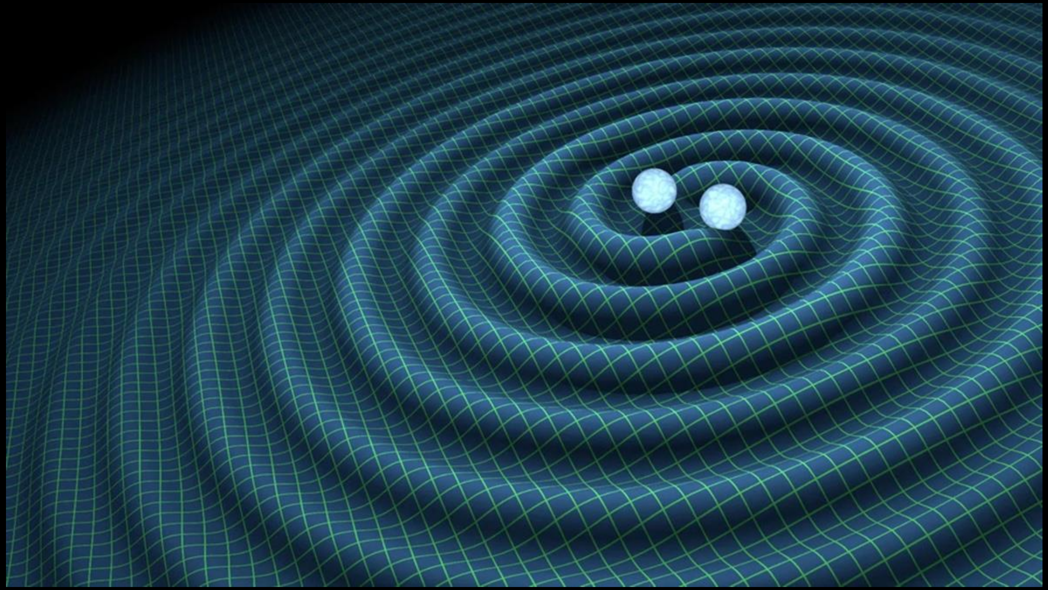
Note: Supermassive = Between one hundred thousand, and 1 billion times the mass of the sun.

Observing Black Holes

We now have numerous sources of observational evidence for the existence of black holes.

The first discovery of Gravitational Waves was announced in early 2016.

A prediction of General Relativity, these ripples in space-time could only have originated from the collision of two large black holes.



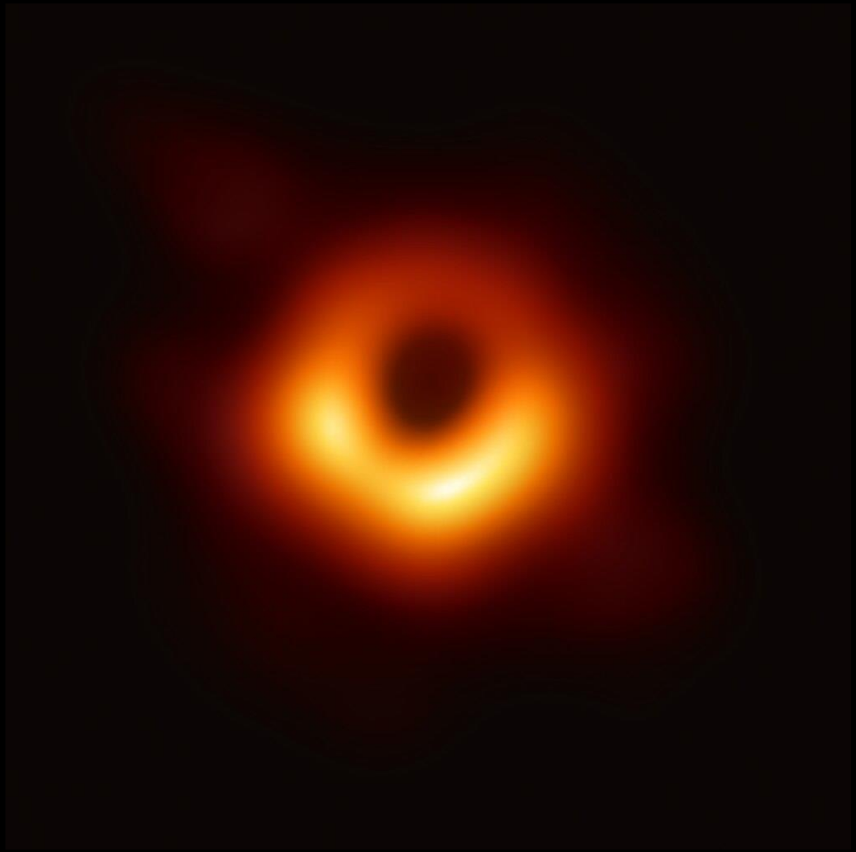
The ripples in space time, created by two colliding black holes.

The LIGO Hanford facility in Washington State, US.

Observing Black Holes

In the last several years, we have even been able to observe black holes directly using the event horizon telescope array (EHT).

The EHT combines data from numerous radio telescopes around the world.

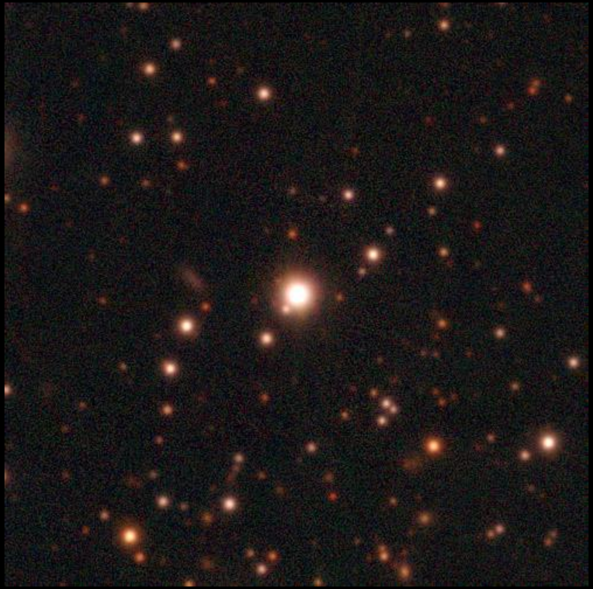


The first image of M87* (the supermassive black hole at the center of the Messier 87 galaxy.

Our Nearest Neighbor

1560 light-years away, a star very similar to our own sun orbits around a black hole.

This black hole has a mass nearly ten times greater than our sun.



Pan-STARRS image of the Gaia BH1 system.

Gaia BH-1

$$M = 9.62\, M_{\odot} = 9.62 \times 1.99 \times 10^{30}$$

$$= 1.91 \times 10^{31}\ \text{kg}$$

$$r_s = \frac{2GM}{c^2} = \frac{2 \times 6.67 \times 10^{-11} \times 1.91 \times 10^{31}}{(3 \times 10^{8})^2}$$

$$= 28310.4\ \text{M}$$

$$= 28.3\ \text{KM}$$

Our Nearest Neighbor

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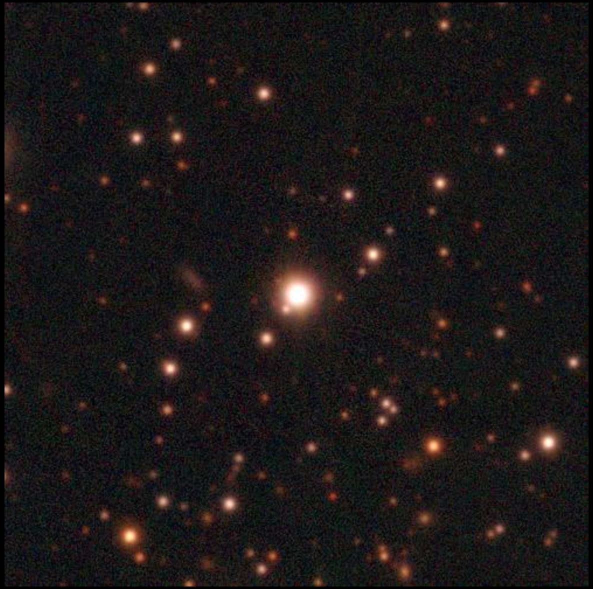


Pan-STARRS image of the Gaia BH1 system.

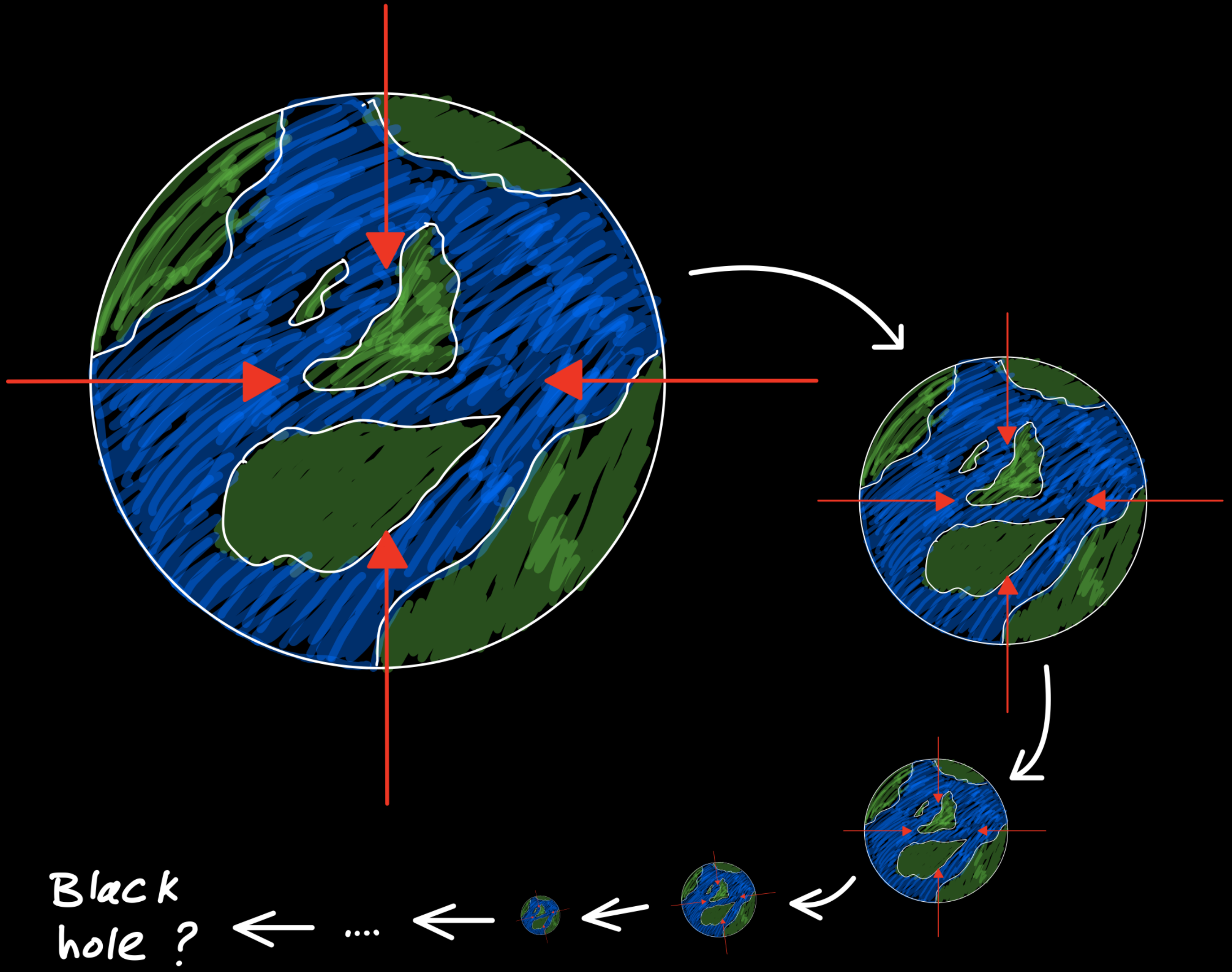
This is the size of our nearest black hole (Gaia BH1), compared to Essex.

The radius stretches roughly from the Beecroft Gallery to the Dartford Bridge.

A Sense of Scale

Let's squash the Earth down until it turns into a black hole.

How far would we need to squash it?



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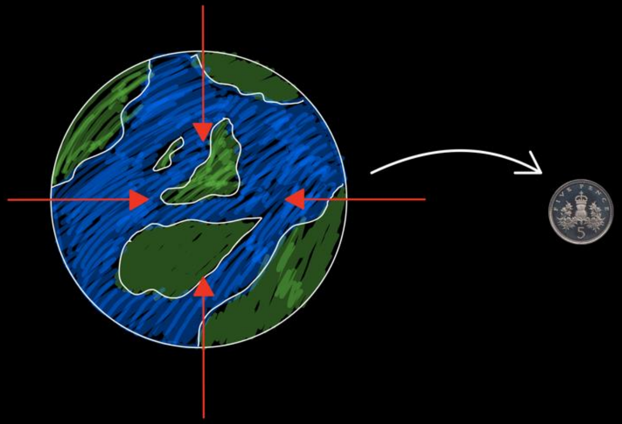


Earth Mass Black Hole

$$M_{Earth} = 5.97 \times 10^{24} \text{ kg}$$

$$r_s = \frac{2GM}{c^2} = \frac{2 \times 6.67 \times 10^{-11} \times 5.97 \times 10^{24}}{(3 \times 10^8)^2}$$

$$= 8.85 \times 10^{-3} \text{ m}$$

$$= 8.85 \text{ mm}$$

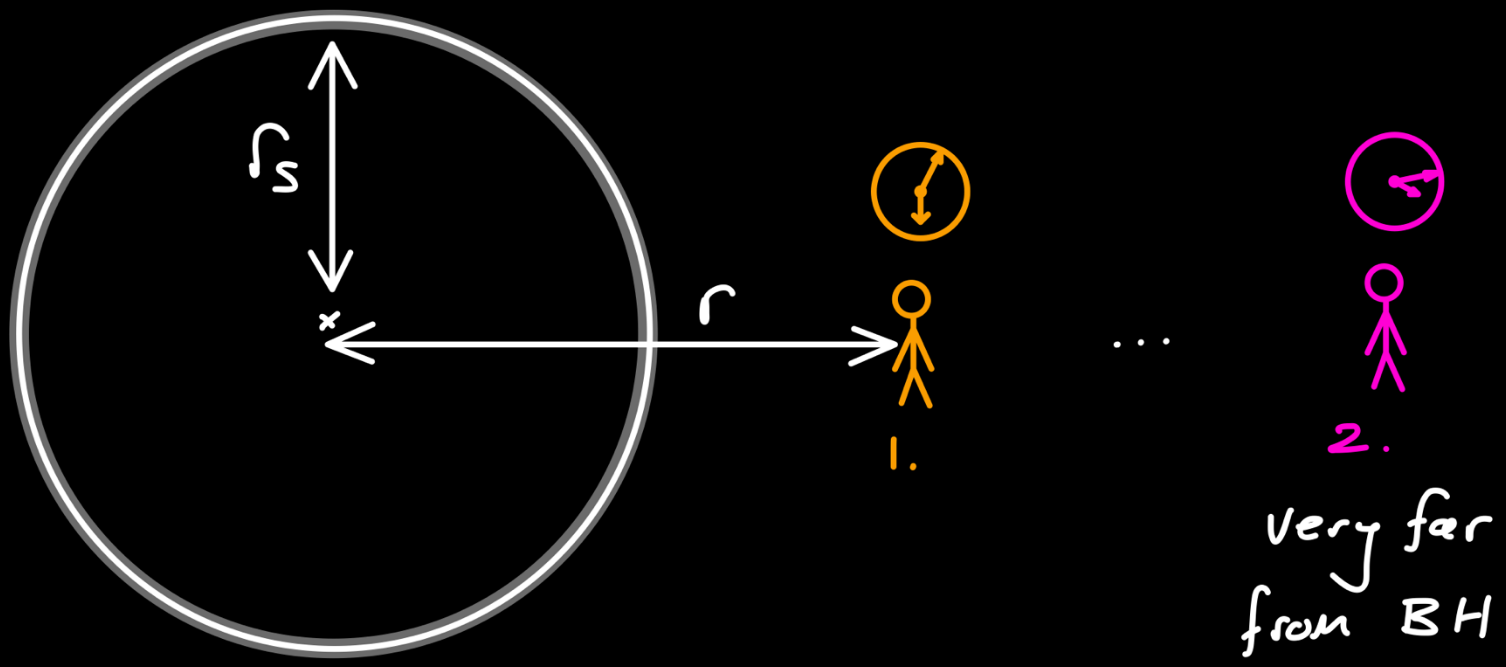
$$\Rightarrow \text{Diameter} \simeq 18\text{mm}$$

About the same as a 5p coin.

Natural Time Machines

Perhaps the strangest property of black holes, is how the bend and stretch time for those nearby...

$$\boxed{\Delta t_{near} = \Delta t_{far}\sqrt{1 - \frac{r_s}{r}}}$$



Person 2's clock will tick much faster than Person 1's.

Let's put this into perspective using Gaia BH1, the nearest known black hole to the Earth.

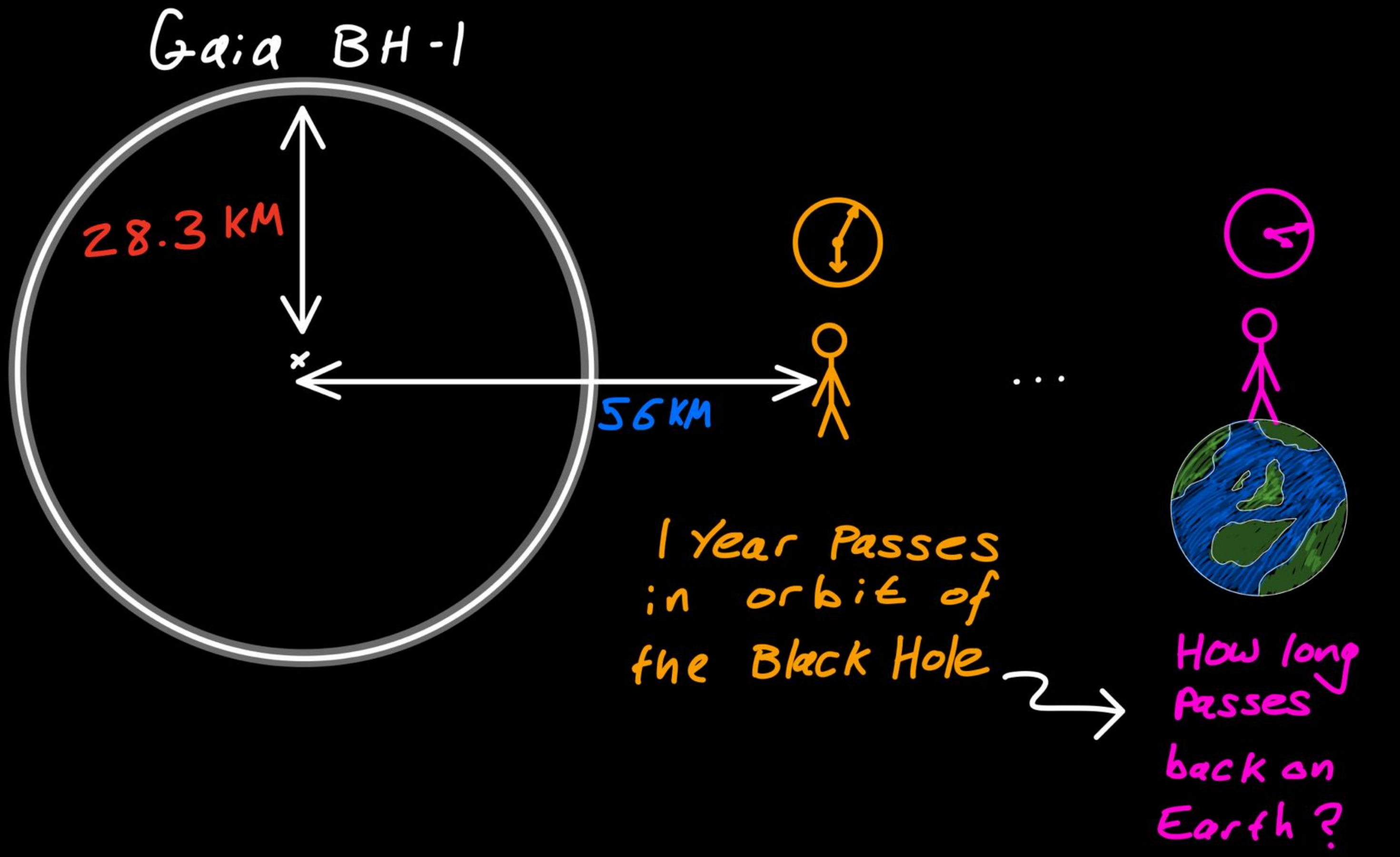
Natural Time Machines

Recall, Gaia BH1's radius stretched out to Dartford.

Suppose we get into an orbit that would stretch out to central London.

Natural Time Machines

Spend 1 year orbiting our nearest known black hole, and you will have travelled forwards in time by 5 months compared to everyone home on Earth.

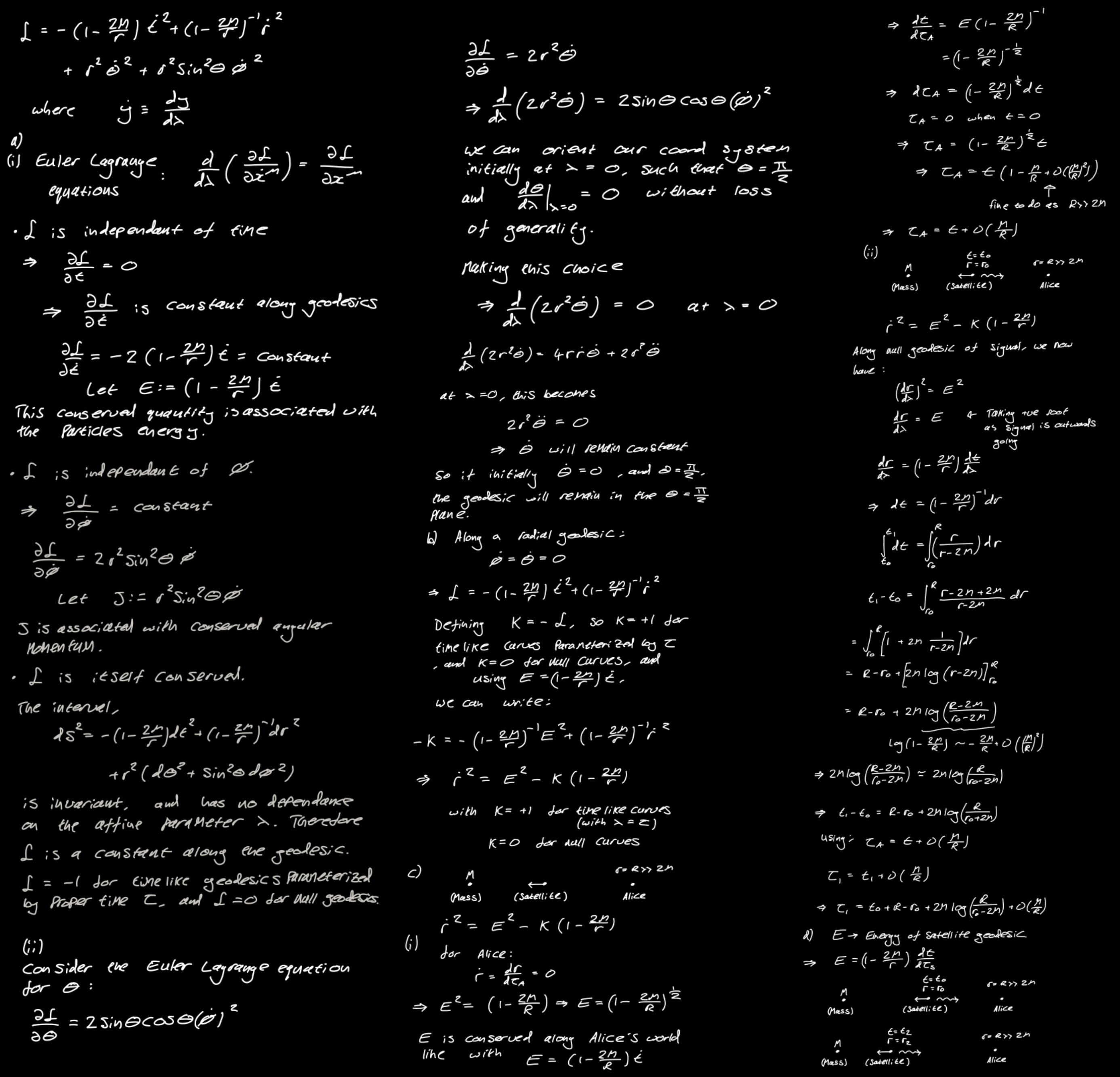


$$\Delta t_{Earth} = \frac{1\,yr}{\sqrt{1 - \frac{28.3\,km}{56\,km}}} = 1\,yr + 5\,months$$

How do we know this?

“How can you possibly know this is correct? Given we’ve never been to a black hole!”

We (physicists) did the maths!



$$\mathcal{L} = -\left(1-\frac{2M}{r}\right)\dot{t}^2 + \left(1-\frac{2M}{r}\right)^{-1}\dot{r}^2 + r^2\dot{\theta}^2 + r^2\sin^2\theta\,\dot{\phi}^2$$

where $\dot{y} \equiv \frac{dy}{d\lambda}$

a)

(i) Euler Lagrange equations: $\frac{d}{d\lambda}\left(\frac{\partial\mathcal{L}}{\partial\dot{x}^\mu}\right) = \frac{\partial\mathcal{L}}{\partial x^\mu}$

- $\mathcal{L}$ is independant of time

$\Rightarrow \frac{\partial\mathcal{L}}{\partial t} = 0$

$\Rightarrow \frac{\partial\mathcal{L}}{\partial\dot{t}}$ is constant along geodesics

$$\frac{\partial\mathcal{L}}{\partial\dot{t}} = -2\left(1-\frac{2M}{r}\right)\dot{t} = \text{constant}$$

Let $E := \left(1-\frac{2M}{r}\right)\dot{t}$

This conserved quantity is associated with the Particles energy.

- $\mathcal{L}$ is independant of ϕ.

$\Rightarrow \frac{\partial\mathcal{L}}{\partial\dot{\phi}} = \text{constant}$

$$\frac{\partial\mathcal{L}}{\partial\dot{\phi}} = 2r^2\sin^2\theta\,\dot{\phi}$$

Let $J := r^2\sin^2\theta\,\dot{\phi}$

J is associated with conserved angular momentum.

- $\mathcal{L}$ is itself conserved.

The interval,

$$ds^2 = -\left(1-\frac{2M}{r}\right)dt^2 + \left(1-\frac{2M}{r}\right)^{-1}dr^2 + r^2(d\theta^2 + \sin^2\theta\,d\phi^2)$$

is invariant, and has no dependance on the affine parameter λ. Therefore $\mathcal{L}$ is a constant along the geodesic.

$\mathcal{L} = -1$ for timelike geodesics parameterized by proper time τ, and $\mathcal{L} = 0$ for null geodesics.

(ii)

Consider the Euler Lagrange equation for θ:

$$\frac{\partial\mathcal{L}}{\partial\theta} = 2\sin\theta\cos\theta(\dot{\phi})^2$$

$$\frac{\partial\mathcal{L}}{\partial\dot{\theta}} = 2r^2\dot{\theta}$$

$$\Rightarrow \frac{d}{d\lambda}(2r^2\dot{\theta}) = 2\sin\theta\cos\theta(\dot{\phi})^2$$

we can orient our coord system initially at $\lambda = 0$, such that $\theta = \frac{\pi}{2}$ and $\left.\frac{d\theta}{d\lambda}\right|_{\lambda=0} = 0$ without loss of generality.

Making this choice

$\Rightarrow \frac{d}{d\lambda}(2r^2\dot{\theta}) = 0$ at $\lambda = 0$

$$\frac{d}{d\lambda}(2r^2\dot{\theta}) = 4r\dot{r}\dot{\theta} + 2r^2\ddot{\theta}$$

at $\lambda = 0$, this becomes

$$2r^2\ddot{\theta} = 0$$

$\Rightarrow \dot{\theta}$ will remain constant

So if initially $\dot{\theta} = 0$, and $\theta = \frac{\pi}{2}$, the geodesic will remain in the $\theta = \frac{\pi}{2}$ plane.

b) Along a radial geodesic:

$$\dot{\phi} = \dot{\theta} = 0$$

$$\Rightarrow \mathcal{L} = -\left(1-\frac{2M}{r}\right)\dot{t}^2 + \left(1-\frac{2M}{r}\right)^{-1}\dot{r}^2$$

Defining $K = -\mathcal{L}$, so $K = +1$ for timelike curves Parameterized by τ, and $K = 0$ for null curves, and using $E = \left(1-\frac{2M}{r}\right)\dot{t}$,

we can write:

$$-K = -\left(1-\frac{2M}{r}\right)^{-1}E^2 + \left(1-\frac{2M}{r}\right)^{-1}\dot{r}^2$$

$$\Rightarrow \dot{r}^2 = E^2 - K\left(1-\frac{2M}{r}\right)$$

with $K = +1$ for timelike curves (with $\lambda = \tau$)

$K = 0$ for null curves

c)

M (Mass) $\longleftrightarrow$ (Satellite) $r = R \gg 2M$ Alice

(i)

$$\dot{r}^2 = E^2 - K\left(1-\frac{2M}{r}\right)$$

for Alice:

$$\dot{r} = \frac{dr}{d\tau_A} = 0$$

$$\Rightarrow E^2 = \left(1-\frac{2M}{R}\right) \Rightarrow E = \left(1-\frac{2M}{R}\right)^{\frac{1}{2}}$$

E is conserved along Alice's world line with $E = \left(1-\frac{2M}{R}\right)\dot{t}$

$$\Rightarrow \frac{dt}{d\tau_A} = E\left(1-\frac{2M}{R}\right)^{-1} = \left(1-\frac{2M}{R}\right)^{-\frac{1}{2}}$$

$$\Rightarrow d\tau_A = \left(1-\frac{2M}{R}\right)^{\frac{1}{2}}dt$$

$\tau_A = 0$ when $t = 0$

$$\Rightarrow \tau_A = \left(1-\frac{2M}{R}\right)^{\frac{1}{2}}t$$

$$\Rightarrow \tau_A = t\left(1-\frac{M}{R}+O\left(\left(\frac{M}{R}\right)^2\right)\right)$$

$\uparrow$ fine to do as $R \gg 2M$

$$\Rightarrow \tau_A = t + O\left(\frac{M}{R}\right)$$

(ii)

M (Mass) $\longleftrightarrow$ $t = t_0$, $r = r_0$ (Satellite) $\rightsquigarrow$ $r = R \gg 2M$ Alice

$$\dot{r}^2 = E^2 - K\left(1-\frac{2M}{r}\right)$$

Along null geodesic of signal, we now have:

$$\left(\frac{dr}{d\lambda}\right)^2 = E^2$$

$\frac{dr}{d\lambda} = E$ ← Taking +ve root as signal is outwards going

$$\frac{dr}{d\lambda} = \left(1-\frac{2M}{r}\right)\frac{dt}{d\lambda}$$

$$\Rightarrow dt = \left(1-\frac{2M}{r}\right)^{-1}dr$$

$$\int_{t_0}^{t_1}dt = \int_{r_0}^{R}\left(\frac{r}{r-2M}\right)dr$$

$$t_1 - t_0 = \int_{r_0}^{R}\frac{r-2M+2M}{r-2M}\,dr$$

$$= \int_{r_0}^{R}\left[1 + 2M\frac{1}{r-2M}\right]dr$$

$$= R - r_0 + \Big[2M\log(r-2M)\Big]_{r_0}^{R}$$

$$= R - r_0 + 2M\underbrace{\log\left(\frac{R-2M}{r_0-2M}\right)}_{}$$

$$\log\left(1-\frac{2M}{R}\right) \sim -\frac{2M}{R} + O\left(\left(\frac{M}{R}\right)^2\right)$$

$$\Rightarrow 2M\log\left(\frac{R-2M}{r_0-2M}\right) \simeq 2M\log\left(\frac{R}{r_0-2M}\right)$$

$$\Rightarrow t_1 - t_0 = R - r_0 + 2M\log\left(\frac{R}{r_0+2M}\right)$$

using: $\tau_A = t + O\left(\frac{M}{R}\right)$

$$\tau_1 = t_1 + O\left(\frac{M}{R}\right)$$

$$\Rightarrow \tau_1 = t_0 + R - r_0 + 2M\log\left(\frac{R}{r_0-2M}\right) + O\left(\frac{M}{R}\right)$$

d) $E \rightarrow$ Energy of satellite geodesic

$$\Rightarrow E = \left(1-\frac{2M}{r}\right)\frac{dt}{d\tau_S}$$

M (Mass) $\longleftrightarrow$ $t = t_0$, $r = r_0$ (Satellite) $\rightsquigarrow$ $r = R \gg 2M$ Alice

M (Mass) $\longleftrightarrow$ $t = t_2$, $r = r_2$ (Satellite) $\rightsquigarrow$ $r = R \gg 2M$ Alice

How do we know this?

“How can you possibly know this is correct? Given we’ve never been to a black hole!”

We (physicists) did the maths!

“But how do you know the maths works? You can’t test this in a lab.

There are many predictions we can test in the real world.

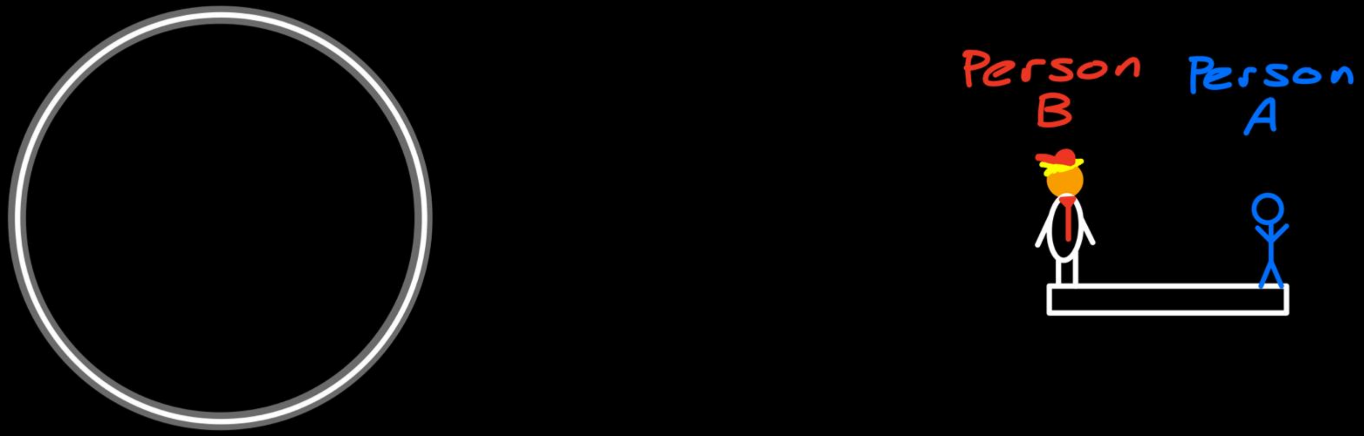
E.g. GPS

GPS satellites calculate location by pinging a signal between themselves and devices on the Earth. Extremely accurate clocks are needed to do this.

General relativity has to be accounted for in order for GPS to work.

Death by Black Hole

Person A pushes Person B into a black hole.



Death by Black Hole

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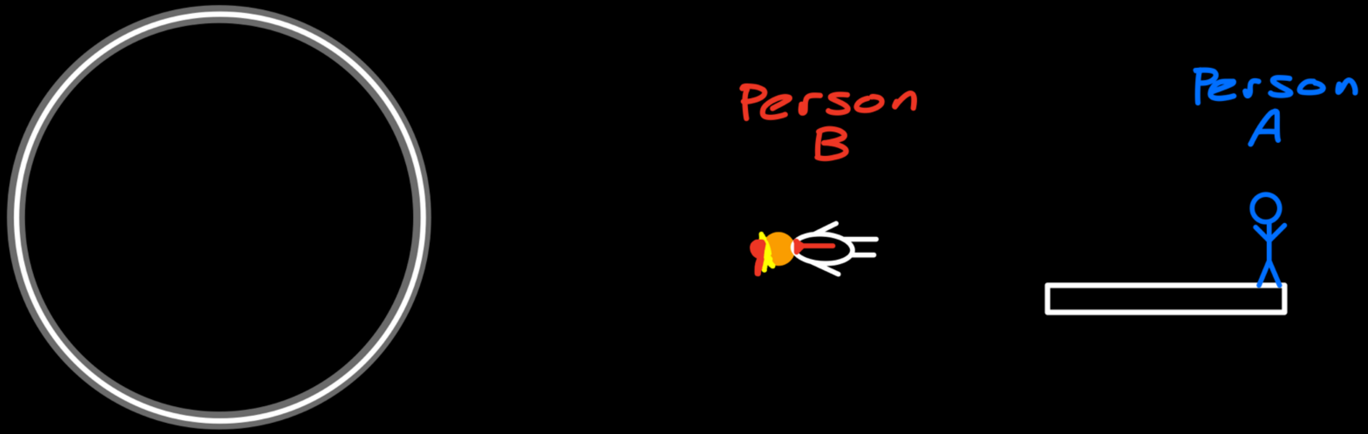


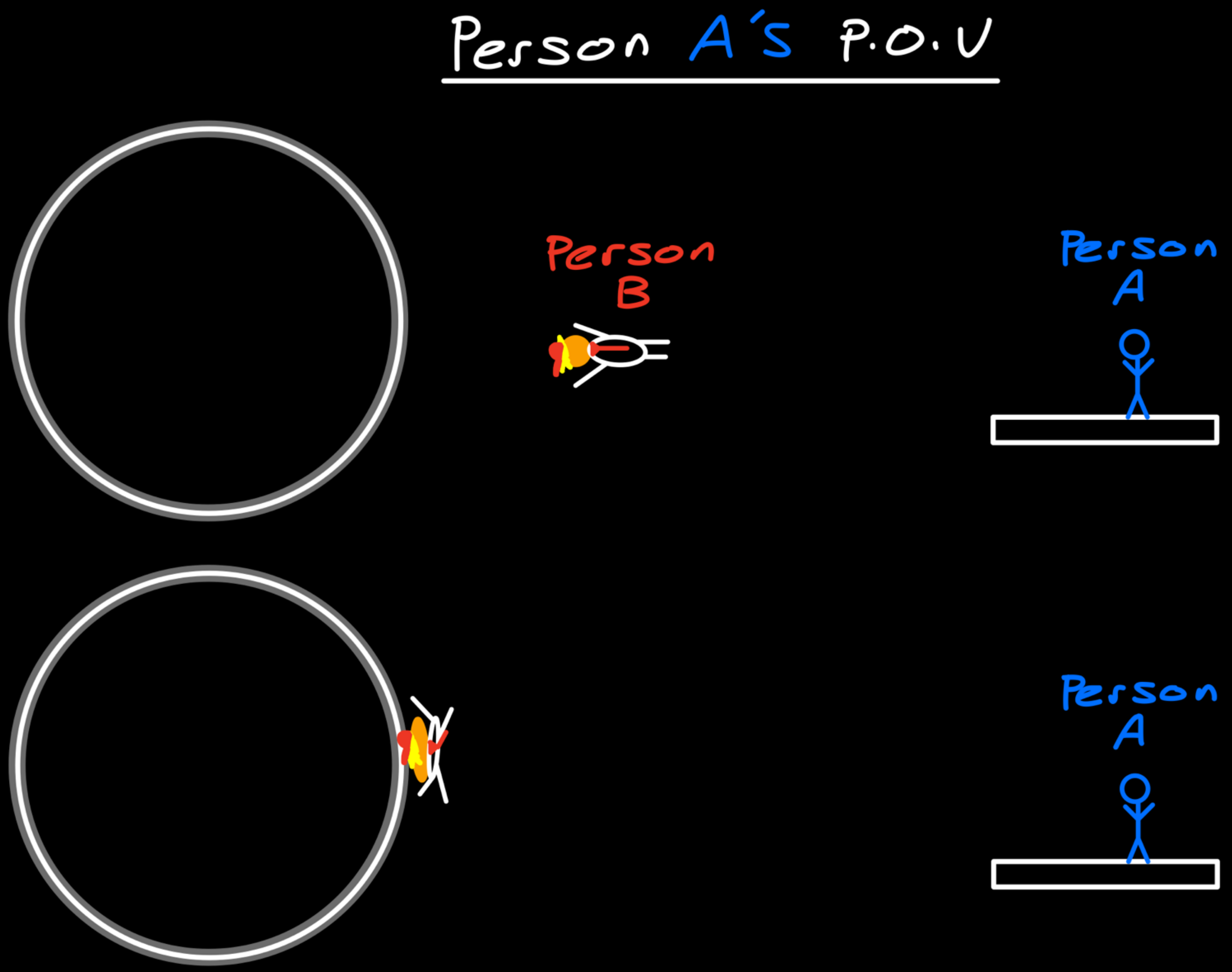
Let's consider how this looks to each person involved…

Death by Black Hole

Person A's POV

From Person A's POV, person B never crosses the event horizon.

Instead, Person B smears along the horizon, become flatter and flatter over billions of years.



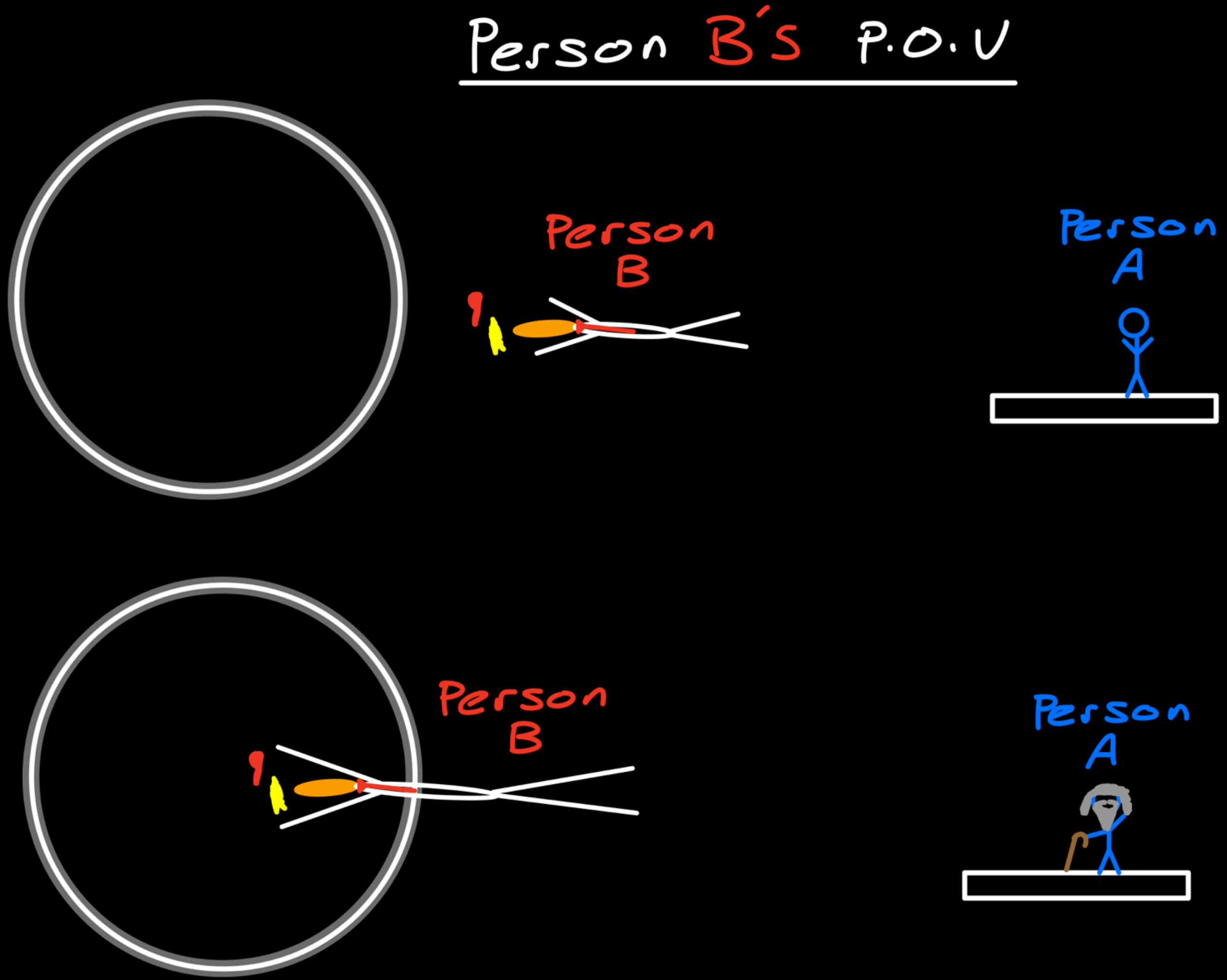
Death by Black Hole

Person B's POV

From Person B's POV, they pass through the horizon without much difficulty.

If the black hole is very small, they will be 'spaghettified' by the fall.

As Person B looks back at Person A, A begins to age rapidly.



Death ***of a*** Black Hole

LLL [4]

In 1974, Stephen Hawking showed that Black Holes can in fact radiate particles, and hence reduce their size.

The radiation they release is called Hawking Radiation.

This process is **extremely** slow.

A black hole with the mass of our sun (impossibly tiny for a naturally occurring black hole) would take 10^{67} years to fully evaporate.

Stephen Hawking with a group of young physicists (including a 19-year-old me!), 2016.

Hawking Radiation

To understand Hawking radiation, we need to combine two key ideas in 20th century physics. Quantum Mechanics & Special Relativity.

'Fundamental Uncertainty' is an unavoidable aspect of Quantum Theory.

This uncertainty is described by Heisenberg's Uncertainty Principle(s).

The more accurately we measure *how much* energy there is, the less sure we can be about *how long* that energy is present.

One version of Heisenberg's Uncertainty Principle:

$$\boxed{\Delta E \, \Delta t \gtrsim \frac{\hbar}{2}}$$

$\Rightarrow$ We can 'borrow' energy from the vacuum, just so long as we return it quickly...

Hawking Radiation

During Einstein's *annus mirabilis* of 1905, in which he published four ground-breaking papers, he discovered his famous relation between mass and energy.

This relationship laid the foundation for the nuclear age, and our understanding of how the stars shine.

Einstein's famous formula, from his special theory of relativity:

$$\boxed{\Delta E = \Delta M c^2}$$

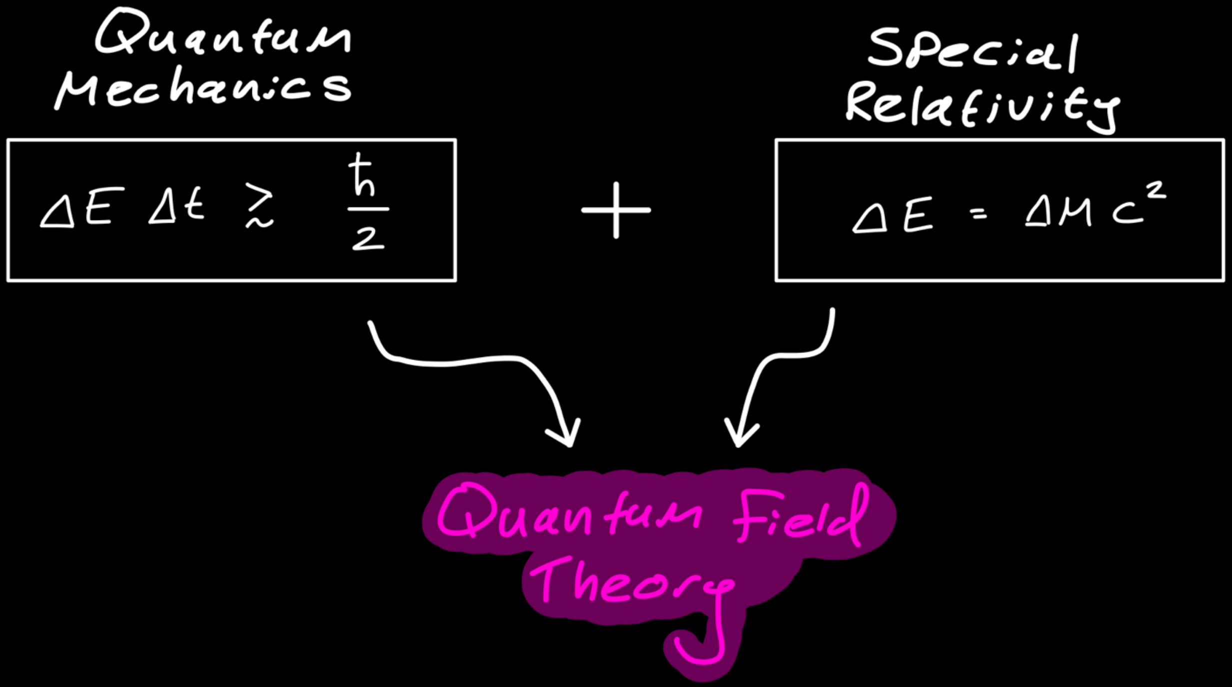
⇒ Matter and Energy are interchangeable. We can 'buy' matter by 'spending' energy.

Hawking Radiation

LLL [5]

The fusion of Quantum Mechanics and Special Relativity leads to our most complete theory of nature yet, Quantum Field Theory.

Schrodinger's Cat in the Particle Zoo - Southend Museum Lecture



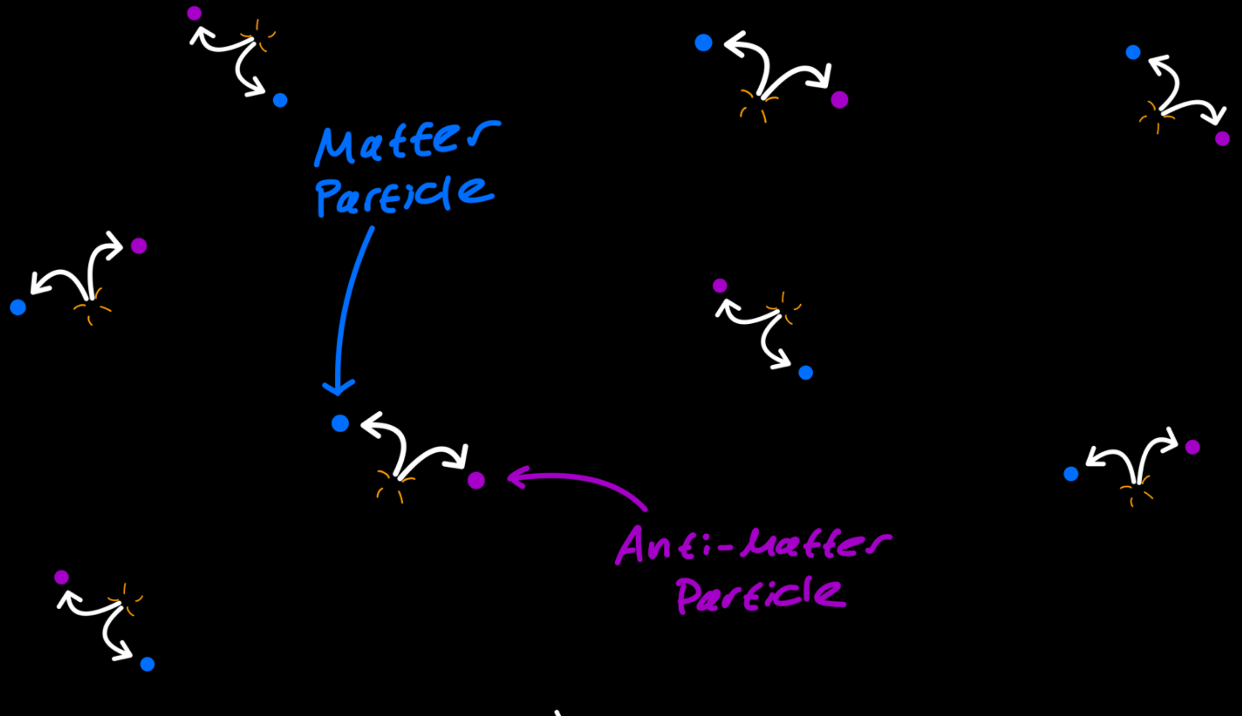
$$\boxed{\Delta E \, \Delta t \gtrsim \frac{\hbar}{2}} + \boxed{\Delta E = \Delta M c^2}$$

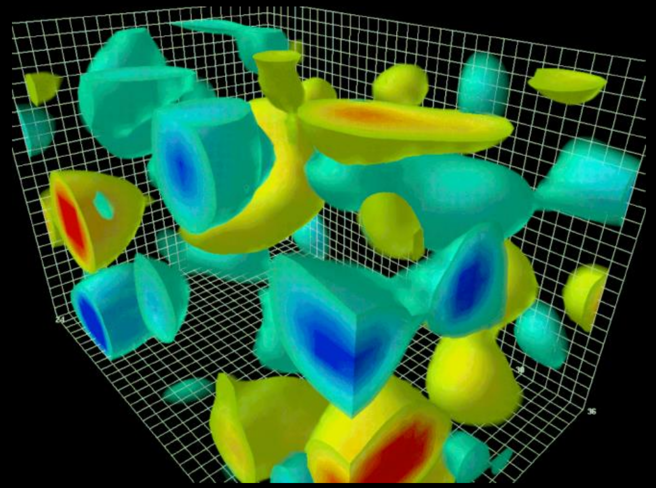
Hawking Radiation

Quantum Field Theory describes matter particles, at the most fundamental level, as excitations of some undulating underlying ‘*Quantum Fields*’.

Particles pop into existence and dissolve back into the vacuum a short time later.

This *fizz* of of creating and annihilating particles fills the entire Universe.





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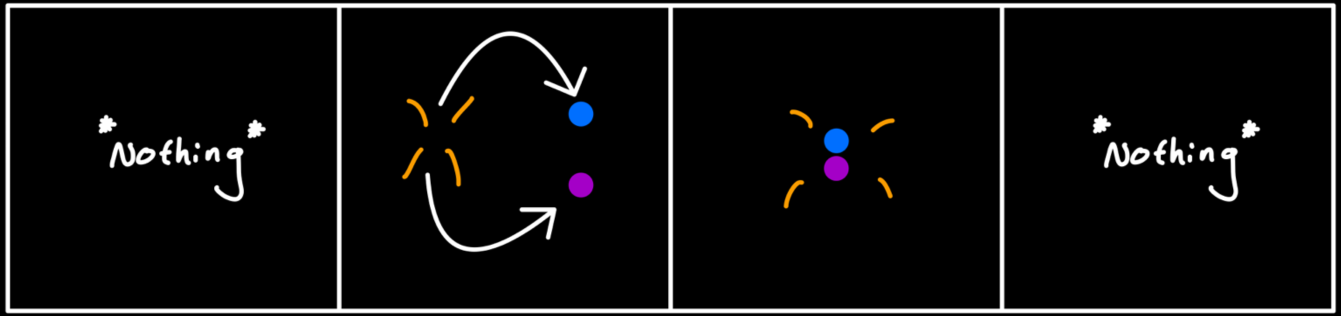
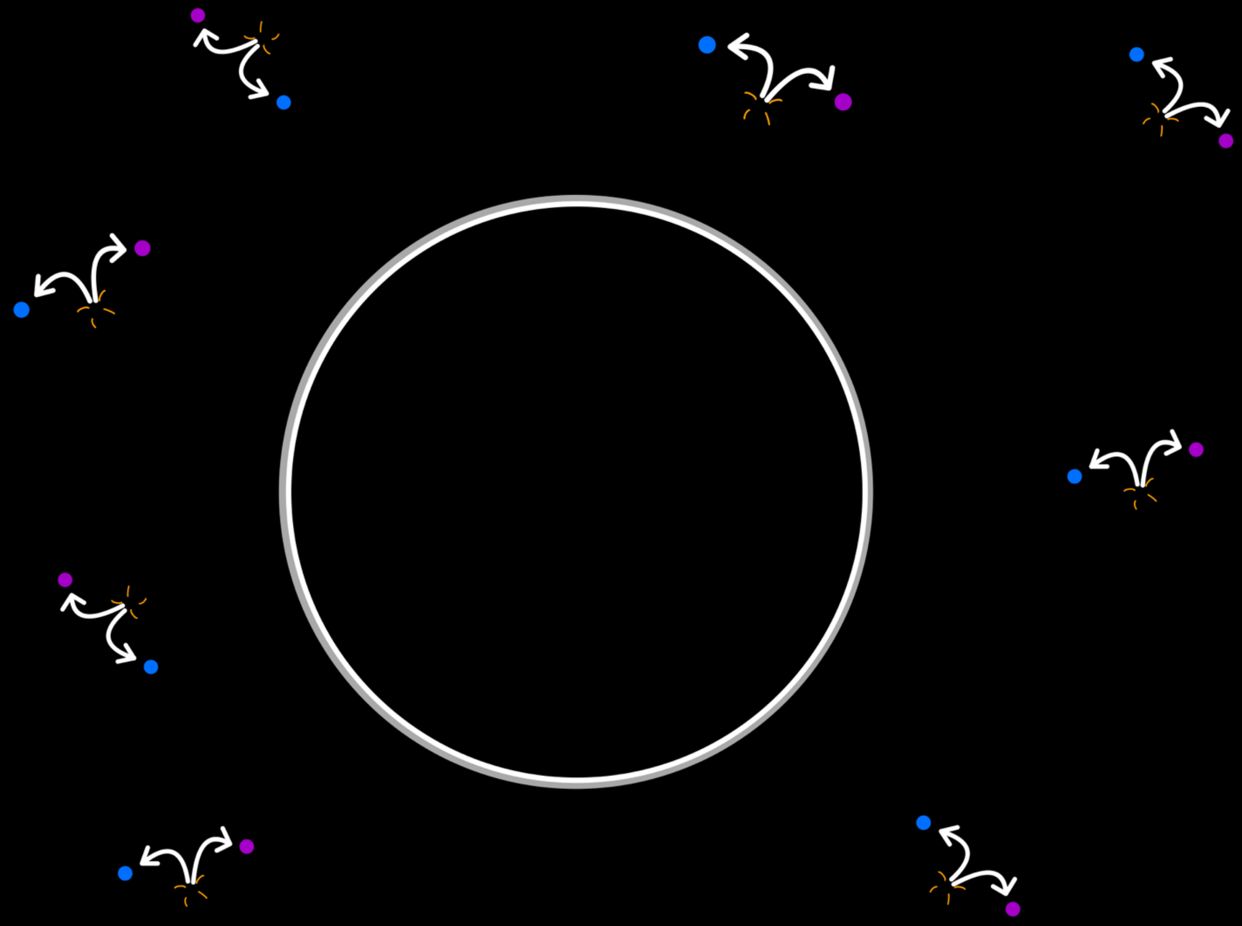


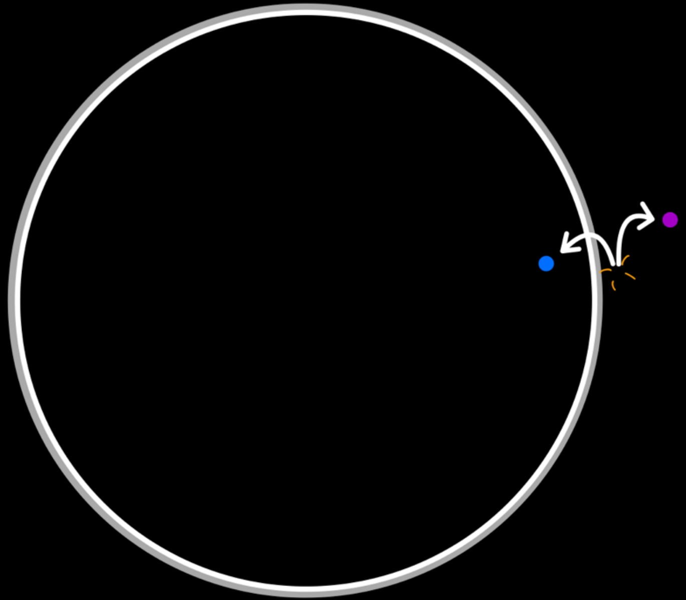
Thanks to Heisenberg's Uncertainty Principle, we can break the law of energy/mass conservation – so long as we don't break it for very long!

Hawking Radiation

These ‘*Quantum Fluctuations*’, of particles popping in and out of existence occur around black holes too.



Hawking Radiation



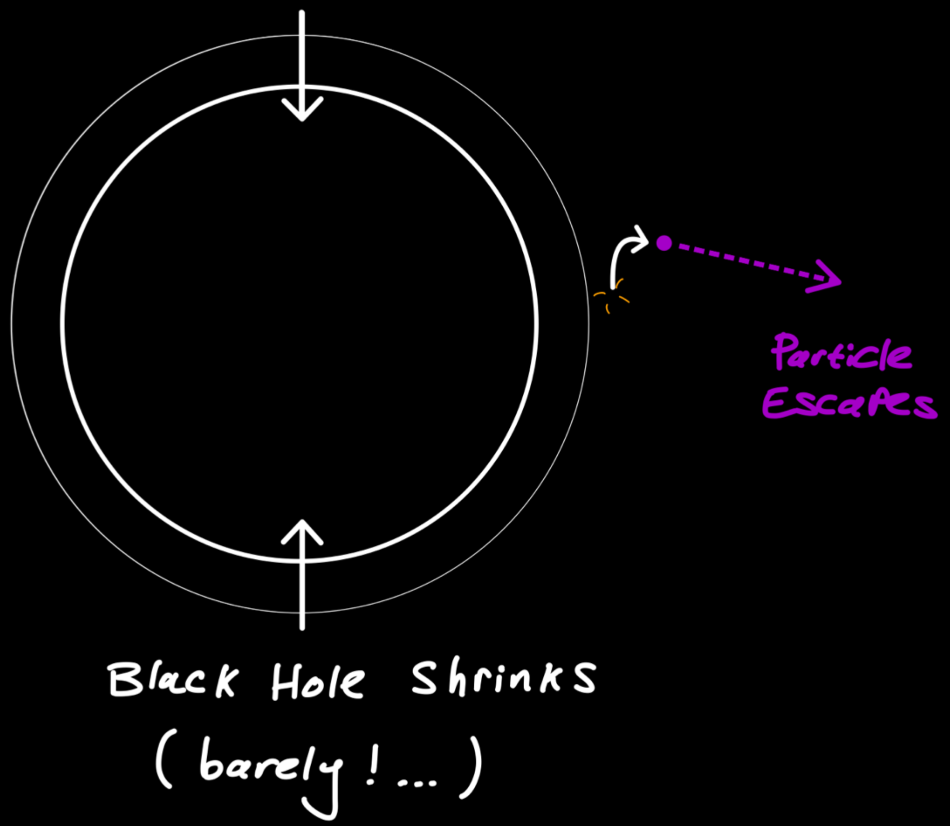
What happens if one of the particles produced in these quantum fluctuations falls into the black hole?

Hawking Radiation

The black hole cheats nature, and prevents the particles from annihilating and dissolving back into the vacuum.

But nature cannot be fooled… The escaping particle, who's mas was borrowed from the vacuum carries a debt that must be paid.

The Black Hole pays this debt, and reduced in mass by a miniscule amount (equal to the mass of the escaping particle).



Questions!

Coming Up...

Watch this space!

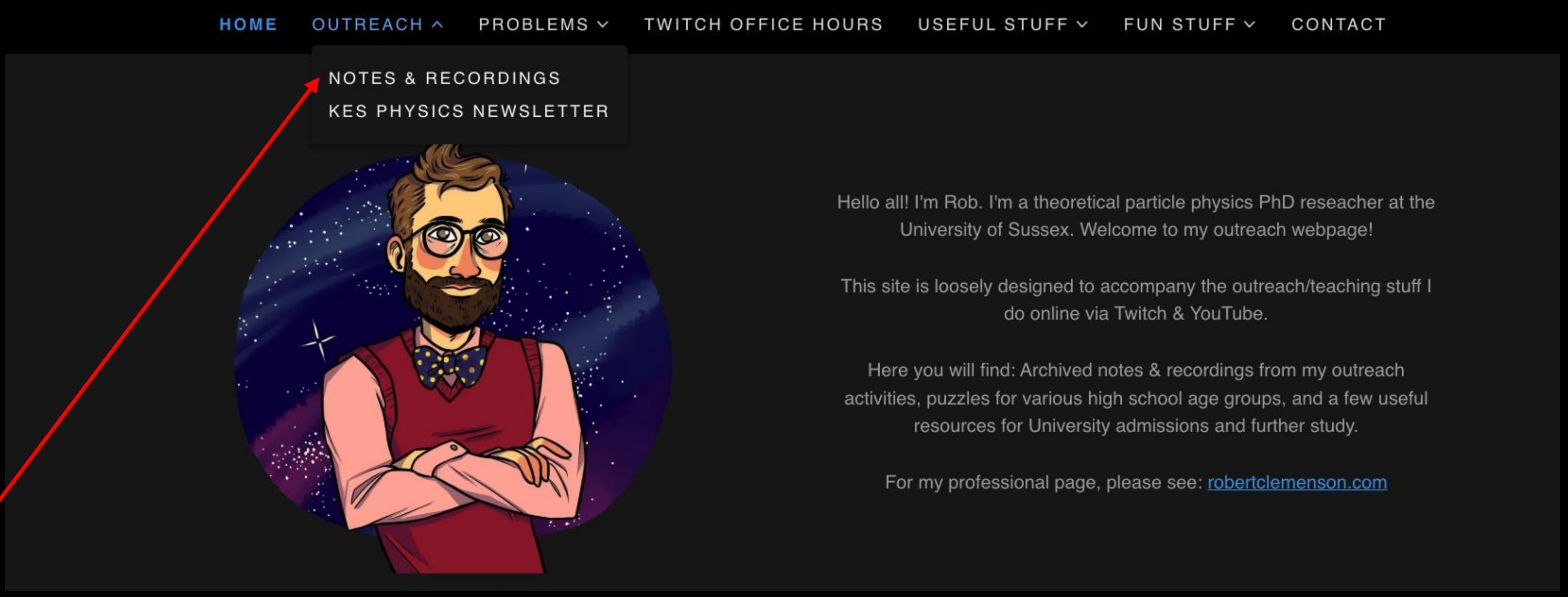
In the meantime...

Please fill in the anonymous survey (4 tick-box questions + 2 optional written response questions).

There is a space to recommend future topics for talks!

LLL [6]

Lecture Slides



Hello all! I'm Rob. I'm a theoretical particle physics PhD reseacher at the University of Sussex. Welcome to my outreach webpage!

This site is loosely designed to accompany the outreach/teaching stuff I do online via Twitch & YouTube.

Here you will find: Archived notes & recordings from my outreach activities, puzzles for various high school age groups, and a few useful resources for University admissions and further study.

For my professional page, please see: robertclemenson.com

These lecture slides are available on my outreach website:

CosmicConundra.com

LLL [7]