Essential Mathematical Formulas and Symbols

1 Important Greek Letters used in Math

α alpha	β beta	γ gamma	Δ δ delta	ϵ epsilon	ζ zeta	η eta	Θ θ theta	Λ λ lambda	μ mu
ν nu	ξ xi	Π π pi	ρ rho	Σ σ sigma	τ tau	Φ ϕ φ phi	χ chi	Ψ ψ psi	Ω ω omega

Note: The pronunciation of ϕ (phi) is similar to π (pi).

2 Algebra Formulas and Equations

Distance Formula: If $P_1 = (x_1, y_1)$ and $P_2 = (x_2, y_2)$, the distance from P_1 to P_2 is

$$d(P_1, P_2) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

Midpoint Formula: If $P_1 = (x_1, y_1)$ and $P_2 = (x_2, y_2)$, the midpoint of $\overline{P_1P_2}$ is

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right).$$

Standard Equation of a Circle: The standard equation of a circle of radius r and center point (h, k) is

$$(x - h)^2 + (y - k)^2 = r^2.$$

Point-Slope Equation of a Line: The equation of a line with slope m incident (touching) the point (x_1, y_1) is

$$y - y_1 = m(x - x_1).$$

Slope-Intercept Equation of a Line: The equation of a line with slope m and y-intercept b is

$$y = mx + b.$$

Note: You read graphs from left to right, just like you read words on paper.

Slope Formula: The formula for computing the slope of a line is

$$m = \frac{y_2 - y_1}{x_2 - x_1}.$$

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Quadratic Formula: The solutions of the equation $ax^2 + bx + c = 0$ with $a \neq 0$ are

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

If $b^2 - 4ac > 0$, there are two unequal real solutions.
If $b^2 - 4ac = 0$, there is a repeated real solutions.
If $b^2 - 4ac < 0$, there are two complex solutions.

Arithmetic Operations

$$a(b+c) = ab + ac \qquad \frac{a}{b} + \frac{c}{d} = \frac{ad+bc}{bd}$$

$$\frac{a+c}{b} = \frac{a}{b} + \frac{c}{b} \qquad \frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c} = \frac{ad}{bc}.$$

Exponents and Radicals

$$x^m x^n = x^{m+n} \qquad \frac{x^m}{x^n} = x^{m-n}$$

$$(x^m)^n = x^{mn} \qquad x^{-n} = \frac{1}{x^n}$$

$$(xy)^n = x^n y^n \qquad \left(\frac{x}{y}\right)^n = \frac{x^n}{y^n}$$

$$x^{\frac{1}{n}} = \sqrt[n]{x} \qquad x^{\frac{m}{n}} = \sqrt[n]{x^m} = \left(\sqrt[n]{x}\right)^m$$

$$\sqrt[n]{xy} = \sqrt[n]{x}\sqrt[n]{y} \qquad \sqrt[n]{\frac{x}{y}} = \frac{\sqrt[n]{x}}{\sqrt[n]{y}}$$

Inequalities and Absolute Values

If $a < b$ and $b < c$, then $a < c$.

If $a < b$, then $a + c < b + c$.

If $a < b$ and $c > 0$, then $ca < cb$.

If $a < b$ and $c < 0$, then $ca > cb$.

If $a > 0$, then

$|x| = a$ means $x = a$ or $x = -a$
$|x| < a$ means $-a < x < a$
$|x| > a$ means $x > a$ or $x < -a$.

Essential Mathematical Formulas and Symbols

Multiplying Binomials and Trinomials (FOIL)

$$(x+y)^2 = (x+y)(x+y) = x^2 + 2xy + y^2$$

$$(x-y)^2 = (x-y)(x-y) = x^2 - 2xy + y^2$$

$$(x+y)^3 = (x+y)(x+y)(x+y) = (x+y)(x^2+2xy+y^2) = x^3 + 3x^2y + 3xy^2 + y^3$$

$$(x-y)^3 = (x-y)(x-y)(x-y) = (x-y)(x^2-2xy+y^2) = x^3 - 3x^2y + 3xy^2 - y^3$$

Factoring Special Polynomials

$$x^2 - y^2 = (x+y)(x-y) \quad \text{(Difference of Squares)}$$

$$x^3 + y^3 = (x+y)(x^2 - xy + y^2) \quad \text{(Sum of Cubes)}$$

$$x^3 - y^3 = (x-y)(x^2 + xy + y^2) \quad \text{(Difference of Cubes)}$$

Formulas for Sums of Sequences

$$\sum_{k=1}^{n} c = c + c + \ldots + c = cn \qquad c \text{ is a constant, not a variable.}$$

$$\sum_{k=1}^{n} k = 1 + 2 + 3 + \ldots + n = \frac{n(n+1)}{2}$$

$$\sum_{k=1}^{n} k^2 = 1^2 + 2^2 + 3^2 + \ldots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{k=1}^{n} k^3 = 1^3 + 2^3 + 3^3 + \ldots + n^3 = \left[\frac{n(n+1)}{2}\right]^2$$

Exponential and Logarithmic Functions

$$\log_b x = y \iff b^y = x$$

$$\ln x = \log_e x, \quad \text{where} \quad \ln e = 1$$

$$\ln x = y \iff e^y = x$$

$$\log_b(b^x) = x \qquad b^{\log_b x} = x$$

$$\ln(e)^x = x \qquad e^{\ln x} = x$$

Essential Mathematical Formulas and Symbols

Properties of Logarithms

$$\log_a(MN) = \log_a(M) + \log_a(N)$$
$$\log_a\left(\frac{M}{N}\right) = \log_a(M) - \log_a(N)$$
$$\log_a(M^r) = r\log_a(M)$$
$$\log_a(M) = \frac{\log(M)}{\log(a)} = \frac{\ln(M)}{\ln(a)} \quad \text{(Change of Base Formula)}$$
$$a^x = e^{x\ln(a)}$$

Permutations & Combinations

$$0! = 1 \quad 1! = 1$$
$$n! = n(n-1)\cdot \ldots \cdot (3)(2)(1)$$
$$P(n,r) = \frac{n!}{(n-r)!}$$
$$C(n,r) = \frac{n!}{(n-r)!r!}$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \cdots + \binom{n}{n-1}ab^{n-1} + b^n, \quad \text{where} \quad \binom{n}{k} = \frac{n(n-1)\cdot \ldots \cdot (n-k+1)}{1\cdot 2\cdot 3\cdot \ldots \cdot k}.$$

Sum of the First n Terms of an Arithmetic Sequence

$$a_1 + (a_1+d) + (a_1+2d) + \cdots + [a_1 + (n-1)d] = \frac{n}{2}[2a_1 + (n-1)d]$$
$$= \frac{n}{2}[a_1 + a_n]$$

Sum of the First n Terms of a Geometric Sequence

$$a_1 + a_1 r + a_1 r^2 + \cdots + a_1 r^{n-1} = a_1 \cdot \tfrac{1-r^n}{1-r}$$

Geometric Series

If $|r| < 1, a_1 + a_1 r + a_1 r^2 + \cdots = \sum_{k=1}^{\infty} a_1 r^{k-1}$

$$= \frac{a_1}{1-r}.$$

3 Geometric Formulas

Circle

For a radius r, area A, and circumference C

$$A = \pi r^2, \qquad C = 2\pi r.$$

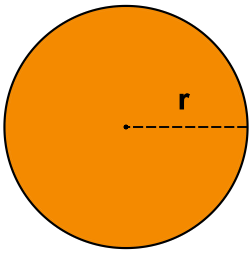


Figure 1: Circle

Triangle

For a base b, altitude (height) h, and area A

$$A = \frac{1}{2}bh.$$

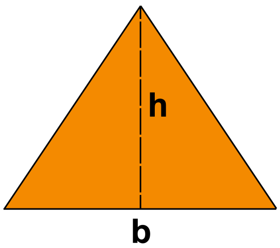


Figure 2: Triangle

Rectangle

For a length l, width w, area A, and perimeter P

$$A = lw, \qquad P = 2l + 2w.$$

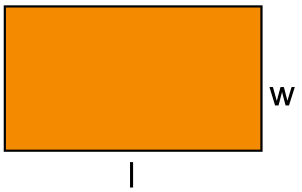


Figure 3: Rectangle

Note: A square is a special case of a rectangle.

Rectangular Box

For a length l, width w, height h, volume V, and surface area S

$$V = lwh, \qquad S = 2lw + 2lh + 2wh.$$

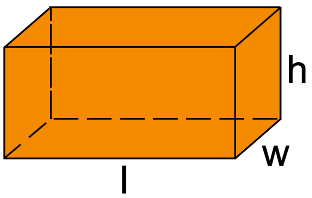


Figure 4: Rectangular Box

Sphere

For a radius r, volume V, and surface area S

$$V = \frac{4}{3}\pi r^3, \qquad S = 4\pi r^2.$$

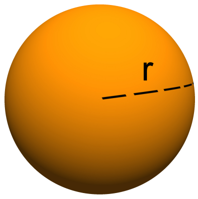


Figure 5: Sphere

Right Circular Cylinder

For a radius r, height h, volume V, and surface area S

$$V = \pi r^2 h, \qquad S = 2\pi r^2 + 2\pi r h.$$

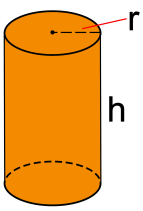


Figure 6: Right Circular Cylinder

Cone

For a radius r, height h, volume V, and area A

$$V = \frac{1}{3}\pi r^2 h, \qquad A = \pi r\sqrt{r^2 + h^2}.$$

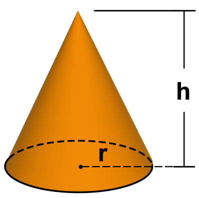


Figure 7: Cone

4 Trigonometric Functions

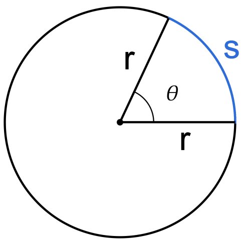
Note: All trigonometry functions require angles to be in radians when computations are carried out by hand or when using a calculator that is set to "rad."

4.1 Trigonometric Functions of Important Angles

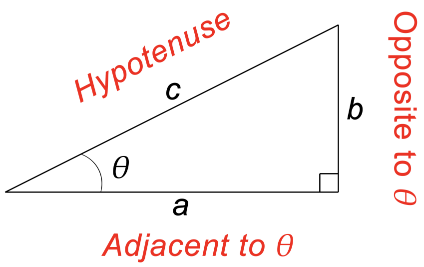
θ	radians	$\sin\theta$	$\cos\theta$	$\tan\theta$
$0°$	0	0	1	0
$30°$	$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$
$45°$	$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
$60°$	$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
$90°$	$\frac{\pi}{2}$	1	0	–

4.2 Angle Measure

π radians $= 180°$ $\quad 1° = \frac{\pi}{180}$ rad $\quad 1$ rad $= \frac{180°}{\pi}$ $\quad s = r\theta \quad (\theta$ in radians$)$



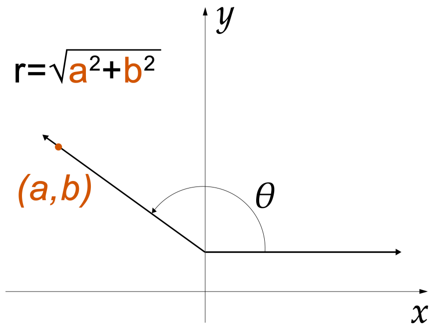
4.3 (SohCahToa) - For Acute Angle θ; i.e., $\theta < 90°$



$\sin(\theta) = \frac{b}{c} = \frac{\text{Opposite}}{\text{Hypotenuse}}$ $\quad \cos(\theta) = \frac{a}{c} = \frac{\text{Adjacent}}{\text{Hypotenuse}}$ $\quad \tan(\theta) = \frac{b}{a} = \frac{\text{Opposite}}{\text{Adjacent}}$

$\csc(\theta) = \frac{c}{b} = \frac{\text{Hypotenuse}}{\text{Opposite}}$ $\quad \sec(\theta) = \frac{c}{a} = \frac{\text{Hypotenuse}}{\text{Adjacent}}$ $\quad \cot(\theta) = \frac{a}{b} = \frac{\text{Adjacent}}{\text{Opposite}}$

4.4 For a General Angle θ



$$\sin(\theta) = \tfrac{b}{r} \qquad \cos(\theta) = \tfrac{a}{r} \qquad \tan(\theta) = \tfrac{b}{a}, \quad a \neq 0$$

$$\csc(\theta) = \tfrac{r}{b}, \quad b \neq 0 \qquad \sec(\theta) = \tfrac{r}{a}, \quad a \neq 0 \qquad \cot(\theta) = \tfrac{a}{b}, \quad b \neq 0$$

4.5 Pythagorean Theorem

For a right triangle with legs a and b and hypotenuse c, the Pythagorean Theorem is $a^2 + b^2 = c^2$.

5 Trigonometric Identities

5.1 Fundamental Identities (All Trig. Functions Can Be Built Using $\sin(\theta)$ and $\cos(\theta)$.

$$\tan(\theta) = \tfrac{\sin(\theta)}{\cos(\theta)} \qquad \cot(\theta) = \tfrac{\cos(\theta)}{\sin(\theta)}$$

$$\csc(\theta) = \tfrac{1}{\sin(\theta)} \qquad \sec(\theta) = \tfrac{1}{\cos(\theta)} \qquad \cot(\theta) = \tfrac{1}{\tan(\theta)}$$

$$\sin^2(\theta) + \cos^2(\theta) = 1 \quad \text{(Essential for Proving Trig. Identities and Integration in Calc. 2)}$$

$$\tan^2(\theta) + 1 = \sec^2(\theta) \iff \frac{\sin^2(\theta)}{\cos^2(\theta)} + \frac{\cos^2(\theta)}{\cos^2(\theta)} = \frac{1}{\cos^2(\theta)}$$

$$\cot^2(\theta) + 1 = \csc^2(\theta) \iff \frac{\cos^2(\theta)}{\sin^2(\theta)} + \frac{\sin^2(\theta)}{\sin^2(\theta)} = \frac{1}{\sin^2(\theta)}$$

$$\sin\left(\tfrac{\pi}{2} - \theta\right) = \cos(\theta) \qquad \cos\left(\tfrac{\pi}{2} - \theta\right) = \sin(\theta)$$

$$\tan\left(\tfrac{\pi}{2} - \theta\right) = \cot(\theta)$$

5.2 Half-Angle Formulas

$$\sin\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1-\cos(\theta)}{2}}$$

$$\cos\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1+\cos(\theta)}{2}}$$

$$\tan\left(\frac{\theta}{2}\right) = \frac{1-\cos(\theta)}{\sin(\theta)}$$

5.3 Double-Angle Formulas

$$\sin(2\theta) = 2\sin(\theta)\cos(\theta) \quad \text{(Frequently Used With Trig. Substitution In Calc. 2)}$$

$$\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta)$$

$$\cos(2\theta) = 2\cos^2(\theta) - 1 \quad \text{(Frequently Used With Trig. Substitution In Calc. 2)}$$

$$\cos(2\theta) = 1 - 2\sin^2(\theta) \quad \text{(Frequently Used With Trig. Substitution In Calc. 2)}$$

$$\tan(2\theta) = \frac{2\tan(\theta)}{1-\tan^2(\theta)}$$

5.4 Even-Odd Identities

$$\sin(-\theta) = -\sin(\theta) \qquad \csc(-\theta) = -\csc(\theta)$$

$$\cos(-\theta) = \cos(\theta) \qquad \sec(-\theta) = \sec(\theta)$$

$$\tan(-\theta) = -\tan(\theta) \qquad \cot(-\theta) = -\cot(\theta)$$

5.5 Product-to-Sum Formulas

$$\sin(\alpha)\sin(\beta) = \frac{1}{2}[\cos(\alpha-\beta) - \cos(\alpha+\beta)]$$

$$\cos(\alpha)\cos(\beta) = \frac{1}{2}[\cos(\alpha-\beta) + \cos(\alpha+\beta)]$$

$$\sin(\alpha)\cos(\beta) = \frac{1}{2}[\sin(\alpha+\beta) + \sin(\alpha-\beta)]$$

$$\cos(\alpha)\sin(\beta) = \frac{1}{2}[\sin(\alpha+\beta) - \sin(\alpha-\beta)]$$

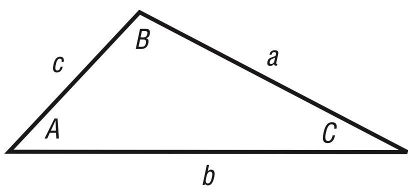
5.6 Sum and Difference Formulas

$$\sin(\alpha+\beta) = \sin(\alpha)\cos(\beta) + \cos(\alpha)\sin(\beta)$$
$$\sin(\alpha-\beta) = \sin(\alpha)\cos(\beta) - \cos(\alpha)\sin(\beta)$$
$$\cos(\alpha+\beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta)$$
$$\cos(\alpha-\beta) = \cos(\alpha)\cos(\beta) + \sin(\alpha)\sin(\beta)$$
$$\tan(\alpha+\beta) = \frac{\tan(\alpha)+\tan(\beta)}{1-\tan(\alpha)\tan(\beta)}$$
$$\tan(\alpha-\beta) = \frac{\tan(\alpha)-\tan(\beta)}{1+\tan(\alpha)\tan(\beta)}$$

5.7 Sum-to-Product Formulas

$$\sin(\alpha)+\sin(\beta) = 2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$$
$$\sin(\alpha)-\sin(\beta) = 2\sin\left(\frac{\alpha-\beta}{2}\right)\cos\left(\frac{\alpha+\beta}{2}\right)$$
$$\cos(\alpha)+\cos(\beta) = 2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$$
$$\cos(\alpha)-\cos(\beta) = -2\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$$

6 Solving Triangles



Law of Sines

$$\frac{\sin(A)}{a} = \frac{\sin(B)}{b} = \frac{\sin(C)}{c}$$

Law of Cosines

$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$b^2 = a^2 + c^2 - 2ac\cos B$$
$$c^2 = a^2 + b^2 - 2ab\cos C$$

7 Conics

7.1 Parabola

Parabola Opening Up: $x^2 = 4ay$

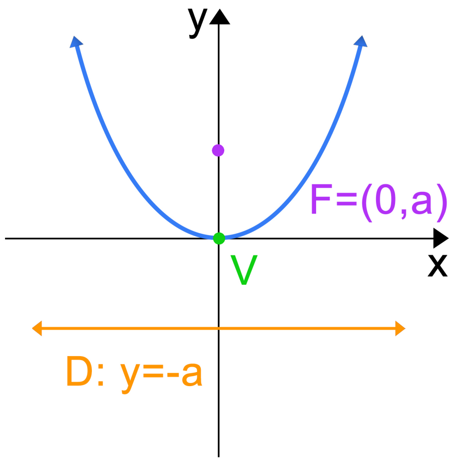


Figure 8: Parabola Opening Up

Parabola Opening Down: $x^2 = -4ay$

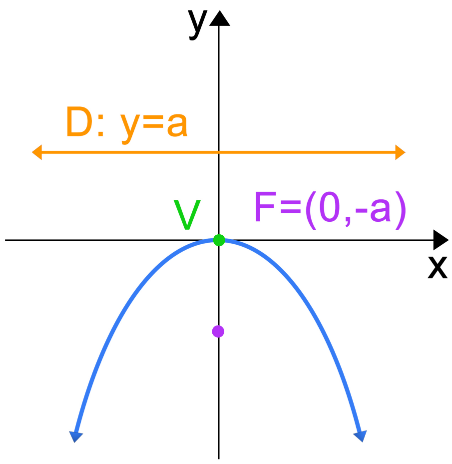


Figure 9: Parabola Opening Down

Parabola Opening Right: $y^2 = 4ax$

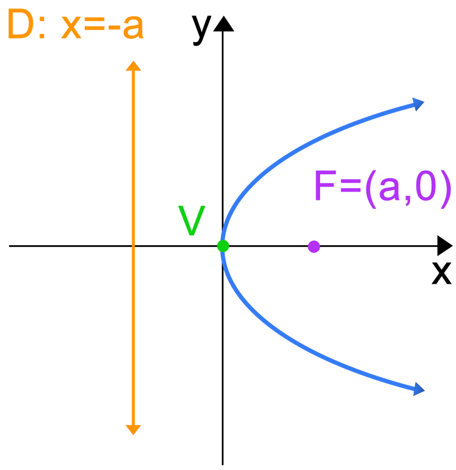


Figure 10: Parabola Opening Right

Parabola Opening Left: $y^2 = -4ax$

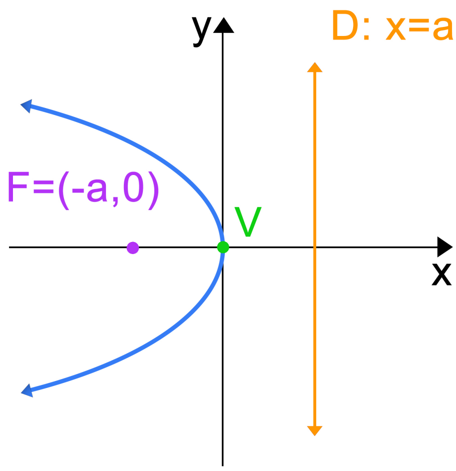


Figure 11: Parabola Opening Left

7.2 Ellipse

Horizontal Ellipse: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, **a > b**, $c^2 = a^2 - b^2$

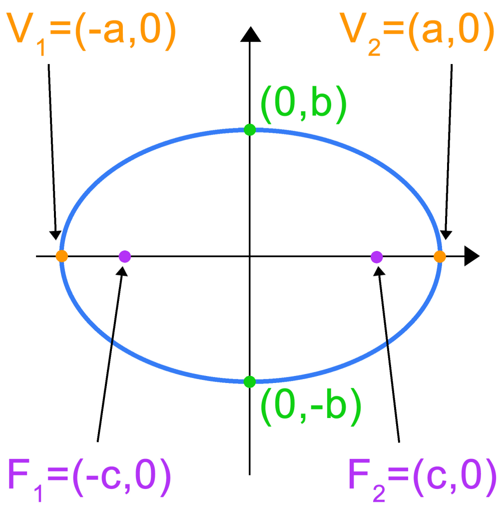


Figure 12: Horizontal Ellipse

Vertical Ellipse: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, **b > a**, $c^2 = b^2 - a^2$

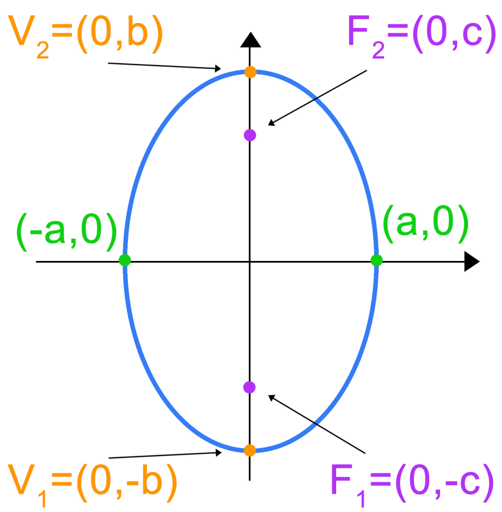


Figure 13: Vertical Ellipse

7.3 Hyperbola

Horizontal Hyperbola: $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, $c^2 = a^2 + b^2$, **Asymptotes:** $y = \frac{b}{a}x$, $y = -\frac{b}{a}x$

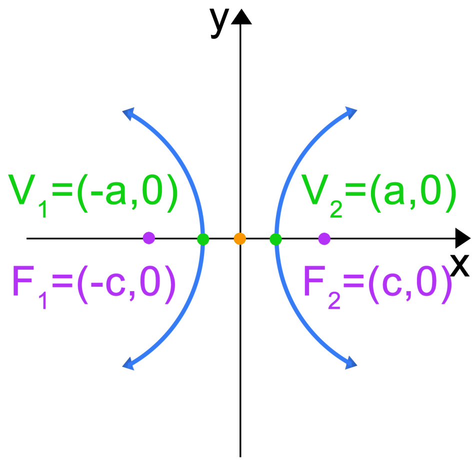


Figure 14: Vertical Hyperbola

Vertical Hyperbola: $\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$, $c^2 = a^2 + b^2$, **Asymptotes:** $y = \frac{b}{a}x$, $y = -\frac{b}{a}x$

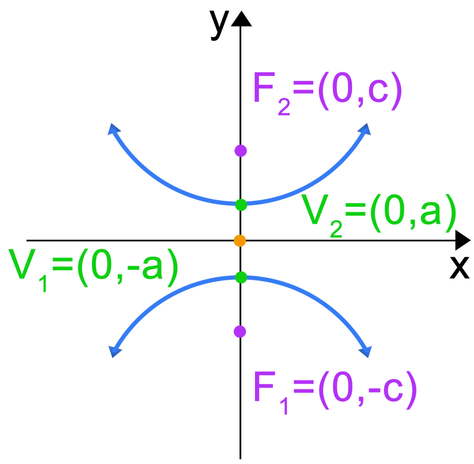


Figure 15: Vertical Hyperbola

8 Differentiation Rules

8.1 General Formulas

Let c be a constant and f and g be functions.

$$\frac{d}{dx}(c) = 0 \quad \text{(Derivative of Constant)}$$

$$\frac{d}{dx}[cf(x)] = cf'(x)$$

$$\frac{d}{dx}[f(x) + g(x)] = f'(x) + g'(x)$$

$$\frac{d}{dx}[f(x) - g(x)] = f'(x) - g'(x)$$

$$\frac{d}{dx}(x^n) = nx^{n-1} \quad \text{(Power Rule)}$$

$$\frac{d}{dx}[f(x)g(x)] = f'(x)g(x) + f(x)g'(x) \quad \text{(Product Rule)}$$

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2} \quad \text{(Quotient Rule)}$$

$$\frac{d}{dx}f(g(x)) = f'(g(x))g'(x) \quad \text{(Chain Rule)}$$

8.2 Exponential and Logarithmic Functions

$$\frac{d}{dx}(e^x) = e^x$$

$$\frac{d}{dx}(b^x) = b^x \ln b$$

$$\frac{d}{dx}\ln|x| = \frac{1}{x}$$

$$\frac{d}{dx}(\log_b x) = \frac{1}{x \ln b}$$

8.3 Trigonometric Functions

$$\frac{d}{dx}(\sin x) = \cos x$$

$$\frac{d}{dx}(\cos x) = -\sin x$$

$$\frac{d}{dx}(\tan x) = \sec^2 x$$

$$\frac{d}{dx}(\csc x) = -\csc x \cot x$$

$$\frac{d}{dx}(\sec x) = \sec x \tan x$$

$$\frac{d}{dx}(\cot x) = -\csc^2 x$$

8.4 Inverse Trigonometric Functions

$$\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\cos^{-1} x) = -\frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$$

$$\frac{d}{dx}(\csc^{-1} x) = -\frac{1}{x\sqrt{x^2-1}}$$

$$\frac{d}{dx}(\sec^{-1} x) = \frac{1}{x\sqrt{x^2-1}}$$

$$\frac{d}{dx}(\cot^{-1} x) = -\frac{1}{1+x^2}$$

9 Table of Integral

9.1 Basic Forms

$$\int u^n \, du = \frac{u^{n+1}}{n+1} + C, \quad n \neq -1$$

$$\int u \, dv = uv - \int v \, du \qquad \text{(Integration by Parts)}$$

$$\int \frac{du}{u} = \ln|u| + C$$

$$\int e^u \, du = e^u + C$$

$$\int b^u \, du = \frac{b^u}{\ln b} + C$$

9.2 Basic Trigonometric Integrals

$$\int \sin u \, du = -\cos u + C$$

$$\int \cos u \, du = \sin u + C$$

$$\int \sec^2 u \, du = \tan u + C$$

$$\int \csc^2 u \, du = -\cot u + C$$

$$\int \sec u \tan u \, du = \sec u + C$$

$$\int \csc u \cot u \, du = -\csc u + C$$

$$\int \tan u \, du = \ln|\sec u| + C$$

$$\int \cot u \, du = \ln|\sin u| + C$$

$$\int \sec u \, du = \ln|\sec u + \tan u| + C$$

$$\int \csc u \, du = \ln|\csc u - \cot u| + C$$

$$\int \frac{du}{\sqrt{a^2 - u^2}} = \sin^{-1}\frac{u}{a} + C$$

$$\int \frac{du}{a^2 + u^2} = \frac{1}{a}\tan^{-1}\frac{u}{a} + C$$

$$\int \frac{du}{u\sqrt{u^2 - a^2}} = \frac{1}{a}\sec^{-1}\frac{u}{a} + C$$

$$\int \frac{du}{a^2 - u^2} = \frac{1}{2a}\ln\left|\frac{u+a}{u-a}\right| + C$$

$$\int \frac{du}{u^2 - a^2} = \frac{1}{2a}\ln\left|\frac{u-a}{u+a}\right| + C$$