Trignometry

a. Introduction

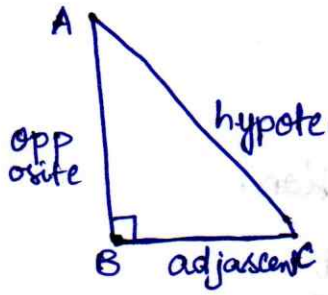
Trignometry → Relationship b/n lengths & angles of a triangle.

Greek word:

Trignometry - 'trigonon' - triangle
& 'metron' - measure.

Hipparchus - Father of Trignometry.

Consider a right angled triangle ABC,



θ (theta) → angle
(0 to 360°)
θ → Greek Alphabet.

In a triangle - Sum of angles = 180°
In a right angled triangle; one angle is 90°. The other two angles are complementary.

b. Trignometric functions

I 1. Sine function (Sin).

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

2. Cosine function (cos).

$$\cos\theta = \frac{\text{Adjascent}}{\text{hypotenuse}}$$

3. Tangent function (tan)

$$\tan\theta = \frac{\text{opposite}}{\text{adjascent}}$$

II Reciprocal of the above functions

1. cosecant function (cosec)
   - Reciprocal of sine function or ratio of hypotenuse to opposite

$$\text{cosec } \theta = \frac{\text{hypotenuse}}{\text{opposite}} = \frac{1}{\sin \theta}$$

2. Secant function (Sec)
   - Reciprocal of cosine function.

$$\sec \theta = \frac{1}{\cos \theta} = \frac{\text{hypotenuse}}{\text{adjascent}}$$

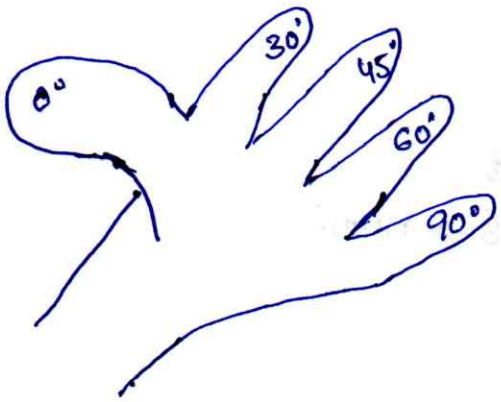
3. cotangent function (cot)
   - Reciprocal of tangent function.

$$\cot \theta = \frac{1}{\tan \theta} = \frac{\text{adjascent}}{\text{opposite}}$$

Trignometric ratios of standard angles.

Ratio \ θ	0°	30°	45°	60°	90°
sin θ	0	1/2	1/√2	√3/2	1
cos θ	1	√3/2	1/√2	1/2	0
tan θ	0	1/√3	1	√3	∞
cosec θ	∞	2	√2	2/√3	1
sec θ	1	2/√3	√2	2	∞
cot θ	∞	√3	1	1/√3	0

c. Trick to remember Trignometric ratio.



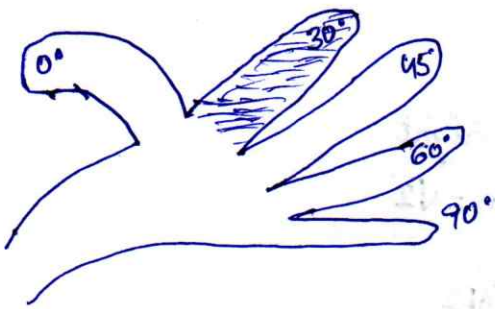
S1: denom 2 common for every calculation

S2: num → depends on 'angle' & the 'trignometric' function.

- First choose the finger based on the angle & fold the finger.
- Based on the trignometric function count the number of fingers beside the folded finger.
- Suppose it is a sine function, then we count the number fingers on the left side of the folded finger
- cosine → right side

- Take the sq. root of the number of fingers folded (beside folded finger) & simplify to get the answer.

Example: Sin 30°



num = 1
den = 2

S1: denom → 2

S2: num → angle & function.

30° → fold finger for sign count.

left of folded finger → Sq. Root of 1 = 1.

= 1/2.

Sin θ = Sin 30° = 1/2.

2. Example - $\sin 60^\circ$

S1: denom = 2.

S2: num = ?

{ left - 3 ; (sin) → left.
{ Sq. root of 3 → $\sqrt{3}$ → num.

$\sin 60^\circ = \sqrt{3}/2$.

3. $\cos 30^\circ$.

S1: den = 2

S2: num = ?

{ Right - 3 ; (cos) → right
{ Sq. root of 3 → num = $\sqrt{3}$.

$\cos 30^\circ = \frac{\sqrt{3}}{2}$.

4. $\cos 90^\circ$.

S1: den = 2

S2: num = ?

{ Right → 0 ; (cos) → right
{ Sq. root of 0 → num = 0.

$\cos 90^\circ = \frac{0}{2} = 0$

5. $\cos 45^\circ$

S1: den = 2

S2: num = ?

{ Right → 2 ; (cos) → right.
{ Sq. root of 2 → num = $\sqrt{2}$

$\cos 45^\circ = \frac{\sqrt{2}}{2} = 1/\sqrt{2}$

d. Heights & distances

Applications of trigonometric ratios / functions & values.

Line of sight:

The line joining the points which represent the observers eye & the object is called 'Line of sight'

Horizontal line:

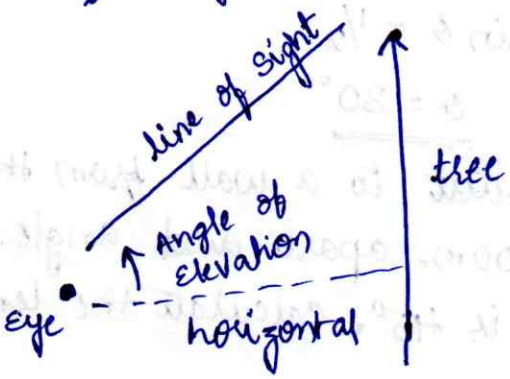
The line which is drawn horizontal from the point representing the observers eye is called 'Horizontal line'

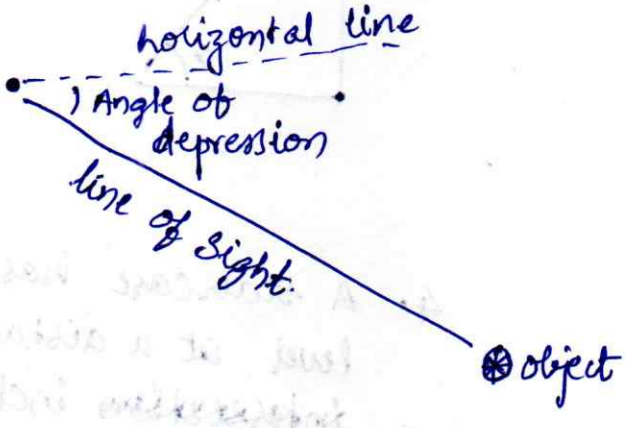
Angle of elevation:

The angle b/n the 'Horizontal' & 'Line of sight' when the object is above the horizontal line is called the 'Angle of elevation'.

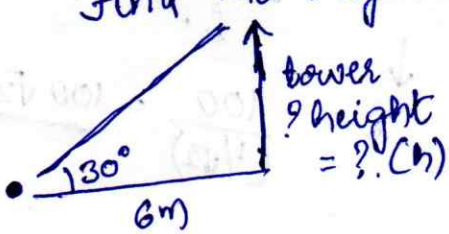
Angle of depression:

The angle b/n the 'horizontal line' & 'Line of sight', When the object is below the horizontal line is called 'Angle of depression'.





1. Eg: Shiny is watching a bird on the top of the tower which is 6m away & the angle of elevation is 30°. Find the height at which the bird is sitting.

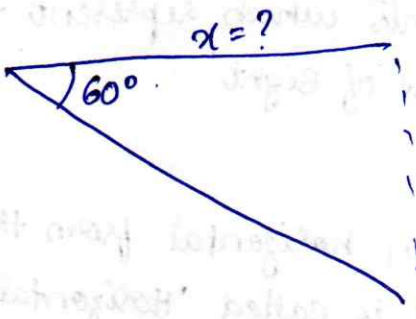


$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}} = 1/\sqrt{3}$$

$$\Rightarrow \frac{h}{6m} = 1/\sqrt{3}$$

$$h = \frac{6}{\sqrt{3}} = \frac{2*3}{\sqrt{3}} = 2\sqrt{3}.$$

2. Example: The angle of depression of an animal from a hilltop is 60°. Calculate how far is the animal from the hill which is $200\sqrt{3}$ ft.



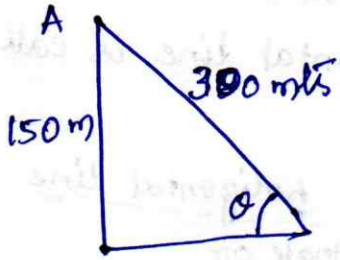
$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

$$= \frac{200\sqrt{3}}{x} = \tan 60°$$

$$x = \frac{200\sqrt{3}}{\tan 60°}$$

$$x = \frac{200\sqrt{3}}{\sqrt{3}} = 200 \text{ mts}$$

3. Example: A glider goes down a ramp of length 150m. & the ramp starts 300m. above the ground. Calculate the angle of inclination of the ramp.



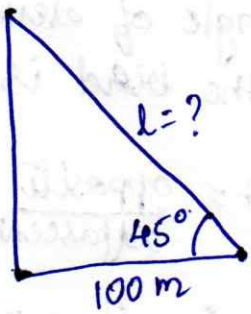
$$\sin\theta = \frac{\text{opposite}}{\text{adjacent}}$$

$$= \frac{150\text{ m}}{300\text{ m}} = 1/2.$$

$$\sin\theta = 1/2$$

$$\theta = 30°$$

4. A staircase has to be built to a wall from the ground level at a distance of 100m. apart and angle of inclination is 45°; calculate the length of staircase.



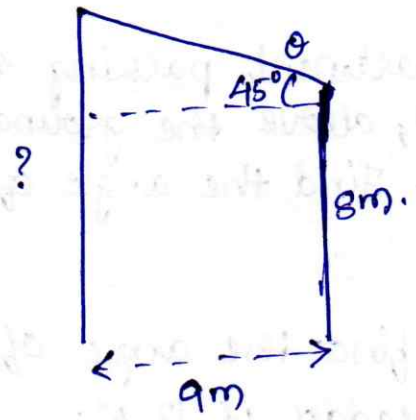
$$\cos\theta = \frac{\text{adjacent}}{\text{opp.}}$$

$$\cos 45° = \frac{100\text{ m}}{l}$$

$$\downarrow$$

$$l = \frac{100}{(1/\sqrt{2})} = 100\sqrt{2}.$$

5. Two poles are 9m apart and angle of elevation from shorter pole is 45°. calculate the length of longer pole if the shorter one is 8m.



$\tan\theta = \frac{\text{opposite}}{\text{adjasecent}}$

$\tan 45° = \frac{\text{opposite}}{9}$

$\downarrow$

$1 = \frac{\text{opposite}}{9}$

opposite = 9.

Total length = 9 + 8 = 17m ans.

EXERCISE

I. Solve the following:

1. The angle of elevation of a tower is 60°, which is 800m tall. Find the distance from where the observer is watching the tower.

2. A sailor can sight the light house on the shore at an angle of elevation 45° and at a distance of 4 km. Calculate the height of the light house.

3. An 80 ft. tall building casts a shadow of length 80 ft. Calculate at which angle the sun rays pass the top of the building.

4. The angle of depression of a car in parking space from an apartment is 30°. If the parking space is 150 m away from the apartment, then find at what height the observer is from the ground. calculate the length of the ramp.

5. A cycle ramp is to be built at an inclination of 45° & one end of the ramp should be 600m high from the ground.

6. A slope has to be built at an apartment parking such that one end of the slope is 45m, above the ground & the length of the slope is 90m. Find the angle of inclination.

7. A ladder is placed to a wall. find the angle of inclination, if the length of the ladder is 12 ft. The distance b/n the wall & foot of the ladder is 12 ft.

8. There are two towers 4m apart and a path-way connects these two towers top which is 800 cm long. Find the angle of elevation from the top of one tower to another.

9. Two buildings are of length 15 ft. & 18 ft. respektively. The angle of depression from the taller building to shorter building is 60°. Calculate the distance b/n the two buildings.

10. Two poles are 30m apart & tied by a rope at the top ends. The angle of elevation from the top of shorter pole is 30°. Calculate the length of the rope used to tie the poles.