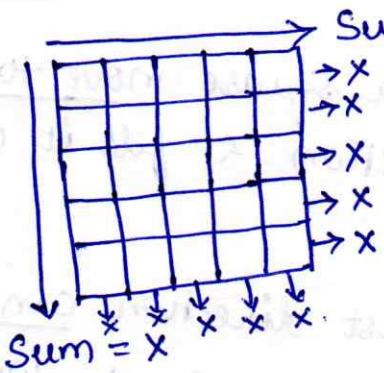
01. Magic Squares

Magic Square is a term given to Squares which are with consecutive Integers in such a way that the total of rows & total of columns are same.



Condition: Odd NO. of Rows & columns.

Sample : 1

3×3 Square
Max NO. → 9.

4	3	8	= 15
9	5	1	= 15
2	7	6	= 15
= 15	= 15	= 15	

Sample : 2

11	10	4	23	17
18	12	6	5	24
25	19	13	7	1
2	21	20	14	8
9	3	22	16	15

5×5 Square
Max NO. → 25
Total → rows → 65
Columns → 65.

A. Forming these Squares:

Rules:

(1) Always put the Number '1' in the centre-most Square of the last column.

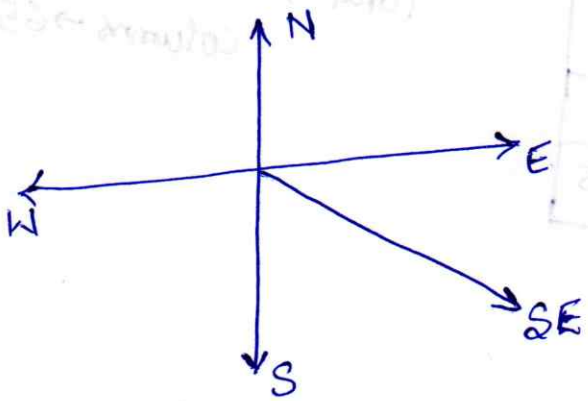
(2) After Inserting a number in a square move to the square in the south east direction & fill it with the next number.

(3) If the square is the south-east direction cannot be filled, then move to the square in the west and fill it with the next number.

(4) When you have filled a number in the last square of the grid, fill the next number in the square to its west.

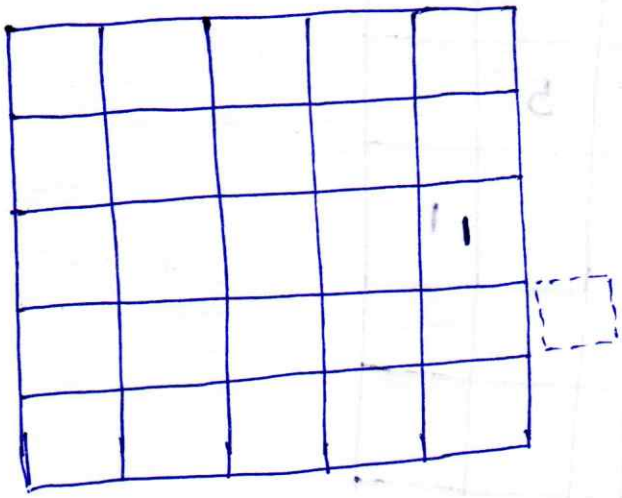
Eg: Five by five grid (5×5 Squares)
: Max NO. → 25

Key of directions:



(a) First we follow rule 1 :

place in centremost - last column.

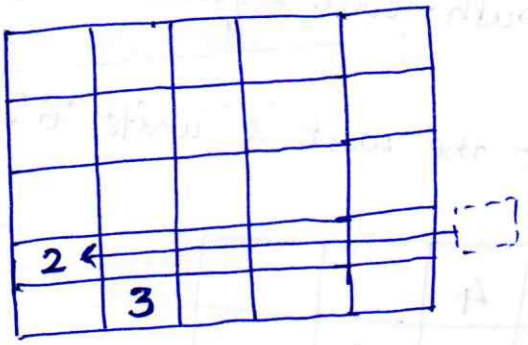


(b) Next we move to the south-east direction from this square.

→ However, there is nothing here.

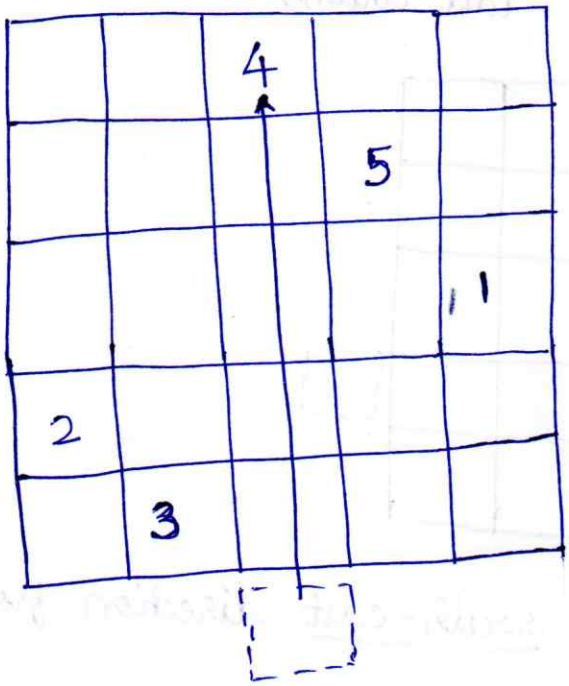
→ Create imaginary square. (No. square hence '2' cannot be put here).

→ Hence number '2' will be written in a square farthest from it.



(c) Move to South-east and write '3'

(d) From 3, move to South-east & create imaginary square.



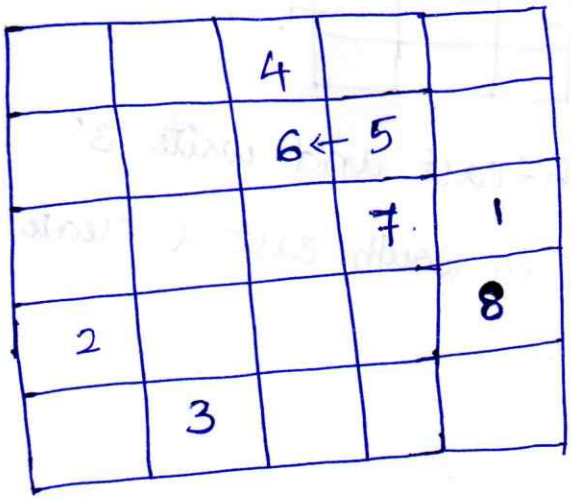
(e). From 4, we move to South-east and fill the number 5.

(f). Next from 5, we have to move to the South-east to fill number 6.

In the South-east square we already have '1'.

Follow Rule 3;

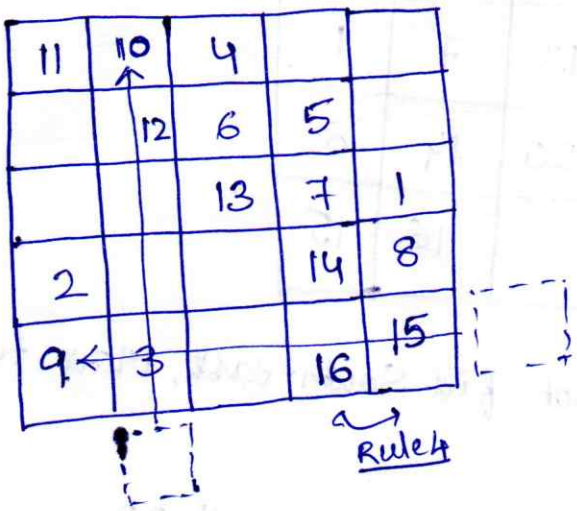
We move to the west & write '6'.



(g). From 6, Move to South-east and write number 7.

From 7, Move South-east and write 8.

(h). From 8, Move to South-east & create imaginary square, we will fill the number '9' in the square farthest from it. (Same direction).



(i) From 9, Move to South-east & make imaginary square & fill the number 10 in the square farthest in the same direction.

(j). From 10, Move to South-east, we cannot fill-in. Hence move to west. and fill '11'.

(k). From 11, Move South-east and fill 12.
From 12, Move South-east & fill the Numbers till 15.

(l). From 15, we follow Rule 4
It states - after filling last square, write the next number to its west.

(m) From 16, create an imaginary square and fill 17.
From 17, we create imaginary square and fill 18.
From 18, move South-east & fill 19 to 20.

11	10	4		17
18	12	6	5	
	19	13	7	1
2		20	14	8
9	3		16	15

(n). From 20, we cannot fill South-east, Move to west & fill 21.

From 21, Move South-east & fill 22.

From 22, Move South-east & create imaginary square & move direction above & fill 23.

From 23, Move South-east & fill 24.

From 24, Move South-east & Move last cell & fill 25.

11	10	4	23	17
18	12	6	5	24
25	19	13	7	
2	21	20	14	8
9	3	22	16	15

B. Properties of Magic Squares-

a. No. of Rows = No. of columns = ODD

So, we can have 3×3, 5×5, 7×7 etc.

b. First Number & Last Number lies in the same row & exactly opposite to each other.

c. Total of any side = Centremost Square × No. of Squares in one Row/column.

Eg: 5 × 3 = 15

d. Centre most Square Number:

1. Take max. No. (3×3) grid = 9
2. ÷ by 2 = $\frac{9}{2}$ = 4.5
3. Round it off to next Number = 5

III^ly, 5×5 Grid.

1. Max No. = 25.
2. $25/2$ = 12.5
3. Next No. = 13 (Rounded).

e. Total of any side of magic square

7×7 grid.

1. 49 Squares – Highest No. 49.

$\frac{49}{2}$ = 24.5

Centre Square - 25

Total of any side - 25×7 = 175.

Ex: 9 x 9 Grid.

Max. NO. 81.

Centre NO. - $\frac{81}{2} = 40.5$

$= 41.$

Then $41 \times 9 = 369$ (Sum).

(f.). There are many possible ways by which a magic square can be made out of a certain grid.

4	3	8
9	5	1
2	7	6

8	1	6
3	5	7
4	9	2

6	7	2
1	5	9
8	3	4

2	9	4
7	5	3
6	1	8

Imp: Tilting the Square in four directions.
(The Sq. we had formed earlier).

Grid - 7×7

Max. NO. - 49.

Centre Most - 25

Total - 25 × 7 = 175

22	21	13	5	46	38	30
31	23	15	14	6	47	39
40	32	24	16	8	7	48
49	41	33	25	17	9	
2	43	42	34	26	18	10
11	3	44	36	35	27	19
20	12	4	45	37	29	28

Grid - 9×9

Max NO. - 81.

Centre Most - 41

Total - 41 × 9 = 369

37	36	26	16	6	77	67	57	47
48	38	28	27	17	7	78	68	58
59	49	39	29	19	18	8	79	69
70	60	50	40	30	20	10	9	80
81	71	61	51	41	31	21	11	1
2	73	72	62	52	42	32	22	12
13	3	74	64	63	53	43	33	23
24	14	4	75	65	55	54	44	34
35	25	15	5	76	66	56	46	45

EXERCISE

PART A :

(a) Make a 3×3 grid using the first 9 even numbers.
(2, 4, 6, 8, .. 18)

Max NO. – 18.
Centre – 9 ⇒ 10. ✓

(b) Will the total of each side of such grid equal to? – YES ✓

Total → 10×3 = 30. ✓

(c) Centre Square NO.?

(d) Total of each side of the grid?

PART B:

(a) Make 5×5 grid using multiples of 3 (3, 6, 9, ...)

(b) Make, if you start the grid mentioned above with FN = 3 the number 15 (instead of NO. 3).

C1: Max – 75
CN – 37.5 → 39.
Total – 39×5
= 195

Will the sides-equal totals. (Yes)

Research

(c) What will be the difference in the value of the centremost square as compared in case (a) & Case (b)

+4 units (12) → FN = 15
Max – 87
CN – 43.5 ⇒ 45 (51)

(d) What will be the difference in the total as computed in case (a) & case (b) mentioned above?

4
30

Total – 45×5 = 225

PART C :

(a) ~~Make a 5×5 grid~~ Using the multiples of 5, make a 3×3 grid & represent it in four different ways.

(b) A man has 49 cows. The first cow gives one litre of milk, second cow gives 2 lts, & so on. upto 49th cow. giving 49 lts.

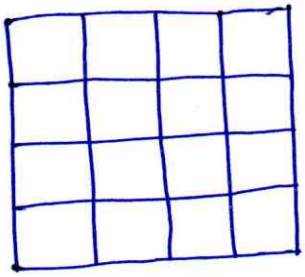
7×7 grid.

This man has seven sons. He wants to divide the 49 cows such that each one recieves the same NO. of cows & same quantity of milk.

provide solution.

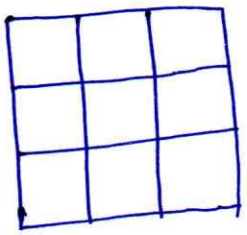
C. Counting of Squares: (4×4)

Eg:



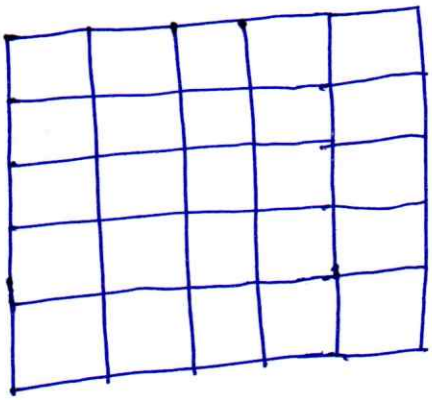
- $1 \times 1 = 1$
- $2 \times 2 = 4$
- $3 \times 3 = 9$
- $4 \times 4 = 16$

30 Squares



- $1 \times 1 = 1$
- $2 \times 2 = 4$
- $3 \times 3 = 9$

14 Squares



- $1 \times 1 = 1$
- $2 \times 2 = 4$
- $3 \times 3 = 9$
- $4 \times 4 = 16$
- $5 \times 5 = 25$

55

1. Consecutive & un-consecutive Numbers

* No.'s following each other one after the other are called Consecutive Numbers.
* No.'s having a skip in between are called unconse-cutive Numbers

Eg: Consecutive Numbers.

1, 2, 3, 4, 5, 6, 7.....

94, 95, 96, 97, 98.....

unconsecutive Numbers

43, 44, 47, 49, 50, 52, 55.....

136, 142, 151, 153, 157.....

A. Summation of Consecutive Numbers:

Eg: 1 Find the Sum of Nos. in the below Series (10 Numbers).

43, 44, 45, 46, 47, 48, 49, 50, 51, 52.

↑

Step 1:

Identify the middle No. in this Series.

* Total 10 Numbers.
* $10/2$ - 5th Number.

  i.e. 47

* Add '5' to the end (append).

  i.e. 475

Sum = 475

Eg: 2 Find the Sum of Series. (10).

186, 187, 188, ... 195.

Centre No. - 190.5

= 1905

186 + 84.

2. Sum of even numbers (Starting from 2) - 'n' Nos.

Eg1: 2, 4, 6, 8, 10 28

S1: Count the numbers (n).

$n = 14 \quad (28/2)$

S2: Apply formula $Sum = n(n+1)$

$Sum = 14(14+1)$

$= 14 \times 15$

$= 170$

Eg2: 2, 4, 6, 8, 10 96

S1: $n = 48 \quad (96/2)$.

S2: $n(n+1) = 48(49)$

$= 2352$

50 - 2

50 - 1

2500.02

4 8

4 9 ↓

2352

7

7

36

32

68

3. Sum of odd Nos. (starting from 1)

Eg 1: 1, 3, 5, 7 ..... 13.

S1: Count the numbers (n)

$n = 7$ $(13/2 = 6.5 \Rightarrow 7)$

S2: Apply the formula.

$\text{Sum} = n \times n$.

$= 7 \times 7$

$= 49$.

Eg 2: 1, 3, 5, 7 ..... 373.

S1: $n = 187$ $(373/2 = 186.5 \Rightarrow 187)$.

S2: $\text{Sum} = n \times n$

$= 187 \times 187$.

Square of 187

$(200-13)^2 = 200^2 - 2(200)(13) + 13^2$

$= 40000 - 5200 + 169$.

$= 34800 + 169$

$= \underline{34969}$.

Eg 3: 1, 3, 5, 7, ... 983.

S1: $n = 492$ $(983/2 = 491.5 \Rightarrow 492)$

$(500-8)^2 = 500^2 - 2(500)(8) + 8^2$

$= 250000 - 8000 + 64$

$= 242000 + 64$

$= \underline{242064}$.

BODMAS | PEDMAS Rule

Order of solving an expression.

B - Bracket
O - Order
D - Division
P - Paranthesis.
E - Exponential.
M - Multiplication
A - Addition
S - Subtraction

BODMAS | PEDMAS.
L → R.

Eg 1: $6 \div 2(3)$.

* Resolve the Bracket first.

$6 \div 2 \times 3$

* Solve the expression from left to right.

$6 \div 2 \times 3 = 3 \times 3.$
$= 9.$

Eg 2: $4 \times 2(6) \div 8(2)$.

$\rightarrow 4 \times 2 \times 6 \div 8 \times 2.$
$\downarrow$
$8 \times 6 \div 8 \times 2$
$\downarrow$
$48 \div 8 \times 2$
$\downarrow$
$6 \times 2 = 12$

Four exciting Magic Tricks : (Mental Magic)

A. Predict a person's date of birth :

a. Ask the people to take the number of the month in which they were born (Jan-1, Feb-2 etc.)

b. Ask them to double the number.

c. Add '5' to it.

d. Multiply it by 5

e. put a zero at the end.

f. Add their date of birth (if they were born on 5th Jan); then add 5.

* Ask them to tell the final answer.

By just listening; you can tell their date of birth.

Answer :

* Mentally subtract 50 from the last two digits & - date.

* Subtract '2' from the remaining digits which gives month.

Example : Date of birth is 22-June.

1. Month = 6
2. Double = 6 × 2 = 12
3. Add 5 = 12 + 5 = 17.
4. Mult 5 = 85
5. Add 0 = 850.
6. Add date = 850 + 22 = 872.

8	72
-2	50
6	22

More Examples :

12 - Oct.

1. Month = 10.
2. Double = 2 × 10 = 20.
3. Add 5 = 20 + 5 = 25
4. Mult 5 = 5 × 25 = 125
5. Append 0 = 1250.
6. Add date = 1250 + 12 = 1262.

12	62
-2	50
10	12
(month)	(date

~~4~~. Total. - 765, 1480, 1071.

5 | 15 12 | 30 8 | 21.

B. predict Money in other's pocket :

a. Ask him to take the amount he has in his pocket (just Rs., ignore ps.)

b. Add 5.

c. ~~Double~~. Multiply 5.

d. Double the answer

e. Ask him to add a digit favourite to this.

f. Add 10.

Ask him Answer, then you can easily predict answer.

Steps to find :

1. Ignore the digit in units place.
2. From the remaining NO., Subtract 6 & you will come to know the amount.

Example:

Suppose a person has 20 Rs. in the pocket.

1. Add 5 - 5 + 20 = ~~100~~ 25
2. Mult 5 - 25 x 5 = ~~500~~ 125
3. Double Ans. - ~~125~~ x 2 = ~~1000~~ 250.
4. Add favourite NO. (Say 8) - ~~1008~~ 258.
5. Add 10. - 268.

Steps to find :

Ignore - 8.

From 26

- 6

20 Rs. In the pocket.

Example :

Suppose a person has 37 Rs.

1. Add 5 - 42
2. Mult 5 - 210
3. Double - 420.
4. Add favourite NO. (3) - 423.
5. Add 10. - 433.

Igore 3.

43 - 6 = 37

Check - 1062, 63 & 170. → 100, 0, 11.

C. Find brothers / sisters for a person

a. Ask him to take number of brothers (0 if no brother).

b. Add 3

c. Multiply by 5

d. Add 20.

e. Double the answer

f. Ask him to add NO. of sisters (0 if no sister).

g. Ask him to add 1.

Ask the Answer & predict brothers/sisters.

Steps:

① final person - 71

② last digit - NO. of sisters.
Remaining digits - NO. of brothers.

Example:

person has 1 brother & 1 sister.

1. Add 3 - 1+3 = 4 (brother)
2. Mult 5 - 20.
3. Add 20 - 40.
4. Double - 80.
5. Add Sis (1) - 80+1 = 81.
6. Add 1 - 82.

•

8 2
7 1
---
1 1
(B) (S)

Example : 3 brothers - 2 sisters.

a. Add 3 - 3+3 = 6.

b. Mult 5 - 6×5 = 30.

c. Add 20 - 20*30 = 50.

d. Double - 100.

e. Add (sis)(2) - 102.

f. Add 1. - 103.

```
103
 71
---
 3 2
(b) (s).
```

III[ly] 71, 93, 102 → 0,0 (b s), 2,2 (b s), 3,1 (b s)

D. Find Answer without knowing the question.

Ask Member to give a 3 digit.

Suppose - 801

Write 801; leave four lines & write answer 2799

(S1)

```
801
354
645
.
.
---
2799
```

- Ask another number (3 digit)
  354
- Next my turn,
  I give 645

(S2)

```
801
354
645
800
199
---
2799
```

- Next again (Aud) - (3 digit)
  800
- Next my turn. - 199.

Steps :

First Number - 801.

Ans. Final. → ① 2 first digit

② 801 - 2 = 799 (Rest digits)

2799

Eg: 2 ⓐ 673. ⓑ 567

2671 2565

Step 2 : My first NO. - previous given NO. by audience. and Sub. All digits by 9.

Eg: 354 (Subs. from 9's).

645

Again for 800, (subs from 9's).

199

Example 2 :

600 (A)
286 (A)
713 (My).
924 (A)
75 (My)

2598.

Tips for competetive exams.

CAT, SAT, GMAT, GRE, UPSC, Railways, Defense, Bank PO etc.

- quick calculation
- Accurate estimation of numerical problems.

(In many cases, average time per question - 60 secs. or so)

* Extremely important to develop the skill to quickly tackle questions.

A. Mental calculation of numbers

In school, we are taught to do all calculations from Right to left. (i.e. units place first).

Suppose - 4639 + 1235.

$$\begin{array}{r} 4\ 6\ 3\ 9 \\ 1\ 2\ 3\ 5 \\ \hline \end{array}$$

First you start adding 9/5 & then proceed behind ways. with carry to 3+3 etc.

ON paper, this works fine.

However, It is quicker to start from left to Right.

Keep 4639 in mind.

1235 = 1000 + 200 + 30 + 5

4639 + 1000 = 5639

5639 + 200 = 5839

5839 + 30 = 5869

5869 + 5 = 5874

Q. Subtract 4142 from 7580.

$7580 - 4142 (4000 + 100 + 40 + 2)$

$7580 - 4000 = 3580.$

$3580 - 100 = 3480.$

$3480 - 40 = 3440.$

$3440 - 2 = 3438$

Q. Multiplication - 76×7

$(70+6) \times 7$

$490 + 42$

$490 + (10 + 32)$

$= 532$

83×8

$(80+3) \times 8$

$640 + 24.$

$= 664.$

B. Estimation of Square Roots (Imperfect)

This gives a rough estimated value (an idea).

a. Find the square root of 70.

S1: Find a perfect Sq. Root $< 70.$

i.e. $64. = 8.$
(Sq root)

S2: Divide 70 by 8

i.e. $\frac{70}{8} = 8.75.$

S3: Take average 8.75 & 8

$= 8.37.$

8.37 is the closest Sq. Root of 70.

Q. Find the Square root of 150.

S1: $144. = 12.$

S2: $\frac{150}{12} = 12.5$

S3: Avg. $\frac{12+12.5}{2} = 12.25$

Q. Find the Square root of 8200

S1: Square just below is 8100
$= 90.$

S2: $\frac{8200}{90} = 91.11.$

S3: $\frac{90+91.11}{2} = 90.55$

Average : 75, 72 & 70.

Here pass 2 from 75 to 70.

73, 72, 72

Avg. - 72 → 1 left

$1/3 = 0.33.$

⇒ 72.33

JO quickly arrive at answer.

c. Fractions:

a. Addition of Fractions:

Eq:① $\frac{3}{5} + \frac{1}{2}$.

$\frac{3}{5} \overset{N}{\underset{D}{\times}} \frac{1}{2} = \frac{3\times2 + 5\times1}{10} = \frac{11}{10}$.

Eq:② $\frac{5}{8}$ & $\frac{2}{3}$.

$\frac{5\times3 + 8\times2}{8\times3} = \frac{15+16}{24} = \frac{31}{24}$

Eq:③ $\frac{11}{15} + \frac{3}{5}$

$\frac{55+45}{75} = \frac{100}{75} = \frac{4}{3}$.

$\frac{3}{5} \times \frac{3}{3} = \frac{9}{15}$

$\frac{11+9}{15} = \frac{20}{15} = \frac{4}{3}$.

Eq:④ $\frac{1}{2} + \frac{1}{4} + \frac{3}{5}$.

$\frac{4+2}{8} = \frac{6}{8} = \frac{3}{4}$.

$\frac{3}{4} + \frac{3}{5} \rightarrow \frac{15+12}{20} = \frac{27}{20}$

b. Subtraction of fractions

Q: $\frac{8}{7} - \frac{8}{2}$.

$= \frac{16-56}{14} = \frac{-40}{14} = \frac{-20}{7}$.

2 Numbers - cross method easy.
More than 3 Numbers LCM may be easy.

Simple Interest & Compound Interest

A. Simple Interest:

principal – P.

Time – T

Rate of Interest – R.

Total Amount – A

$A = P + SI$

$$\text{Simple Interest (SI)} = \frac{PTR}{100}$$

Q1: What is the Simple Interest on Rs. 1000 for 5 years at the rate of 8% per annum? Also find the final value payable back.

$P = 1000$ Rs.

$T = 5$ years

$R = 8\%$ p.a.

$$SI = \frac{PTR}{100} = \frac{1000 \times 5 \times 8}{100} = 400 \text{ Rs.}$$

Amount = Principal + Interest (SI)

$= 1000 + 400$

$= 1400$.

Q2: A certain amount of money doubles itself every 4 yrs at a certain simple Interest. In how much time will it become six times itself.

Instant Method.:

* Substract '1' from the number of times that you want (Here six times)

  i.e. $6 - 1 = 5$

* Multiply itself by the time at which it becomes double. (4 yrs).

  i.e. $5 \times 4 = 20$

Hence the amount becomes '6' times in 20 yrs

Q3. A certain Amount of money becomes 3540 Rs. in 3 yrs. at 6% Interest rate. In how many years will it become Rs. 4260 at 7% Interest rate.

Case 1: $A = 3540$, $P = ?$, $T = 3$ yrs, $R = 6\%$.

$$A = P + SI$$

$$= P + \frac{PTR}{100}$$

$$A = \frac{100P + PTR}{100}$$

$$100 \times A = P(100 + TR)$$

$$\boxed{P = \frac{100 \times A}{100 + TR}} = \frac{100 \times 3540}{100 + 3 \times 6}$$

$$= \frac{100 \times 3540}{118}$$

$$= 3000$$

$P = 3000$

Case 2:

$A = 4260$

$P = 3000$ (from case 1)

$SI = 7\%$

$T = ?$

$$P = \frac{100 \times A}{100 + TR}$$

$$3000 = \frac{100 \times 4260}{100 + T \times 7}$$

$$100 + 7 \times T = \frac{4260\cancel{00}}{30\cancel{00}} = 142$$

$$7T = 42$$

$$T = \frac{42}{7} = 6 \text{ yrs}$$

Q4. An amount was invested at a simple Interest rate for 2 yrs. Had it been put at 4% higher rate, it would have fetched Rs. 400 more. Find the invested amount.

$$SI = \frac{P \times 2 \times R}{100} \quad \text{— (1)}$$

$$(SI + 400) = \frac{P \times 2 \times (R+4)}{100} \quad \text{— (2)}$$

$$\frac{SI}{(SI+400)} = \frac{\frac{P \times 2 \times R}{100}}{\frac{P \times 2 \times (R+4)}{100}}$$

$$SI \times P \times 2 \times (R+4) = P \times 2 \times R \times (SI + 400)$$

$$SI \times R + 4 \times SI$$

$$SI = \frac{2PR}{100}$$

$$SI + 400 = \frac{2P(R+4)}{100}$$

$$400 = \frac{2P(R+4)}{100} - \frac{2PR}{100}$$

$$= \frac{2PR}{100} + \frac{8P}{100} - \frac{2PR}{100}$$

$$P = \frac{400 \times 100}{8}$$

$$= 5000$$

Use direct formula:

$$\text{Principal} = \frac{\text{Additional Interest} \times 100}{\text{Additional rate} \times \text{time}}$$

$$= \frac{400 \times 100}{4 \times 2} = 5000$$

Q5. Find the simple Interest on Rs. 4000 at 3% per annum for 2 years. Also find the amount.

$P = 4000$ Rs.

$T = 2$ yrs.

$R = 3\%$ p.a.

$$SI = \frac{PTR}{100} = \frac{4000 \times 2 \times 3}{100} = \underline{240 \text{ Rs}}$$

$$A = P + SI = 4000 + 240 = \underline{4240}.$$

Q6. Saniya Invested Rs. 10,000 in a savings bank account at a rate of interest 2% per annum. Find simple interest & amount earned by Saniya if the amount was kept in bank for 4 years.

$P = 10,000$ Rs.

$T = 4$ yrs.

$R = 2\%$ p.a.

$$SI = \frac{10,000 \times 4 \times 2}{100} = 800 \text{ Rs.}$$

$$A = 10,000 + 800 = 10,800 \text{ Rs.}$$

Q7. Calculate the simple Interest on Rs. 9200 at 8% p.a. for 8 Months.

$8M = 8/12$ Yrs.

$$SI = \frac{9200 \times 8 \times 8/12}{100}$$

$$= \frac{1472}{3} = \underline{490.67}.$$

Q8: Calculate the Simple Interest on 7400 Rs. at 10% p.a. for 9 months

$$SI = \frac{7400 \times 10 \times 9/12}{100} = 740 \text{ Rs.}$$

$$= 185 \text{ Rs.}$$

Q9 Seema borrowed Rs. 3400 at the rate of 8% p.a. for 225 days. Find the Simple Interest.

$$SI = \frac{3400 \times 8 \times \frac{225}{365}}{100}$$

$$= \frac{272 \times 45}{73}$$

$$= \frac{12240}{73}$$

$$= 167\frac{49}{73}$$

$$= 167.67$$

Q10 Narayan Invested 4500 at the rate of 6% p.a. for 300 days. Find the amount he got backs.

$$SI = \frac{4500 \times 6 \times \frac{300}{365}}{100}$$

$$= \frac{270 \times 60}{73}$$

$$= \frac{16200}{73}$$

$$= 221\frac{16}{27} = 221.92$$

$$A = 4721.92$$

Q 11. If the simple Interest is Rs. 390 at 3% p.a. for 2 years. Find the principal & total amount?

$$SI = \frac{PTR}{100}$$

$$390 = \frac{P \times 6}{100}$$

$$P = 6500$$

$$A = 6500 + 390 = 6890$$

Q12: If the simple Interest is Rs. 500 at 4% p.a. for 2 years. Find the principal & amount.

$$500 = \frac{P \times 4 \times 2}{100}$$

$$\frac{500 \times 100}{8} = P$$

$$P = 6250$$

$$A = 6750$$

Q13: Kiran payed an amount of Rs. 2300 to a bank with a simple interest of 300 Rs. for 3 years. Find the principal & Rate of Interest.

$$P = 2300 - 300 = 2000$$

$$300 = \frac{2000 \times 3 \times R}{100} \quad = 5\%$$

Q14: Sanjay invested an amount of Rs. 4500 to bank with a simple Interest of 500 Rs. for 5 yrs. Find the principal & Rate of Interest.

$$\therefore P = A - SI$$

$$= 4500 - 500 = 4000$$

$$500 = \frac{4000 \times 5 \times R}{100} = 2.5\% \text{ p.a.}$$

Q 15. Find time when, Principal = 1500 Rs., Simple Interest = 450 Rs.
Rate = 5% p.a.

$$450 = \frac{1500 \times 5 \times T}{100} = 6 \text{ yrs}$$

Q 16. Find time when Principal = 3500 Rs, Simple Interest = 700 Rs.
Rate = 4% p.a.

$$700 = \frac{3500 \times 4 \times T}{100} \qquad T = 5 \text{ years}$$

EXERCISE:

I. Find the Amount & Simple Interest of the following

a. P = Rs. 4500, R = 3% p.a.
T = 2 years, A = ? S.I. = ?

b. P = Rs. 6800, R = 4% p.a.
T = 3 years, A = ? S.I. = ?

c. P = Rs. 2500, R = 3% p.a.
T = 8 months, A = ? S.I. = ?

d. P = Rs. 3000, R = 1% p.a.
T = 6 months, S.I. = ?

e. P = 5500 Rs., R = 3% p.a.
T = 150 days, S.I. = ?

f. P = 2000 Rs., R = 5% p.a.
T = 350 days, S.I. = ?

g. S.I. = Rs. 600, R = 4% p.a.
T = 4 years, P = ? A = ?

h. S.I. = Rs. 400, R = 2% p.a.
T = 5 years, P = ? A = ?

i. S.I. = Rs. 340, P = 4000 Rs.
T = 5 years, R = ? A = ?

j. S.I. = Rs. 280, P = Rs. 2800
T = 3 years, R = ? A = ?

k. S.I. = Rs. 460, P = Rs. 7000.
R = 2% p.a., T = ? A = ?

l. S.I. = Rs. 520, P = Rs. 3500
R = 4% p.a., T = ? A = ?

II. Solve the Following

a. A loan of Rs. 10,000 has been issued for 6 years. Calculate the amount to be repaid to the lender, if simple Interest is charged at 5% per year.

b. A woman has deposited Rs. 6000 in a saving account. Bank pays an interest at a rate of 9% per year. Find the amount of Interest that will be earned over 12 years.

c. Calculate the simple Interest on Rs. 8000 at 7% p.a. for 6 months.

d. Sneha Invested Rs. 1500 at the rate of 6% p.a. for 7 years & 3 months. Find the simple Interest.

e. Krishna Invested Rs. 6000 at the rate of 4% p.a. for 200 days. Find the amount he got back.

f. Chaitra borrowed Rs. 5000 at the rate of 2% p.a. for 120 days. Find the amount she has to pay after 120 days.

g. Find the principal & Amount when, time is 4 years, Interest is Rs. 400 & rate is 5% p.a.

h. Find the principal & Amount which gives an Interest of Rs. 516 at the rate of 8% for 5 years.

i. Find the rate of interest & amount when, principal is Rs. 6000, Interest is 900 & time is 7 years.

j. Find the rate of interest & amount when, Principal is Rs. 3500, Interest is Rs. 300 & time is 5 years.

k. Find the time & amount when principal = Rs. 500, Rate = 7.5% p.a., S.I. = 150 Rs.

l. Find the time & amount when principal = 7000 Rs., Rate = 4% p.a., S.I. = Rs. 400.

B. Compound Interest

Eg①: Find the compound Interest on 10,000 Rs. for 3 years at 11%.

$$A = P\left[1+\frac{IR}{100}\right]^t$$

$P = 10,000$.

$T = 3$ yrs.

$IR = 11\%$

Simple Method:

$A_1 = P \times 1\cdot(IR)$ (1 year)

$A_2 = P \times 1\cdot(IR) \times 1\cdot(IR)$ (2 years)

$A_3 = P \times 1\cdot(IR) \times 1\cdot(IR) \times 1\cdot(IR)$ (3 years)

$= P \times 1\cdot(11) \times 1\cdot11 \times 1\cdot11.$

$= 10,000 \times 1\cdot367631$

$A = 13676\cdot31$

10,000 will become 13676.31.

Compound Interest = 3676.31

$a=11$ $b=1$

111

$100+11$

$11^3 = {}_2121 \times 11$

$1331.$

$1331 \times \frac{1}{11} = 121$

$121 \times \frac{1}{11} = 11.$

$1.$

1331 121 11 1

242 22

1331000

36300

330

1

1367631

Eg 2: If Rs. 5000 is compounded at 18% for 2 yrs, Calculate the Amount after 2 years.

$$A = 5000 \times 1.18 \times 1.18$$

$$= \frac{10000}{2} \times 162.42$$

$$= 8121$$

$$A = 6962 \text{ Rs.}$$

Eg 3: Find the compound Interest & Amount on the principal Rs. 20,000 borrowed at 6% compounded annually for 3 years.

$$A = 20{,}000 \times 1.06 \times 1.06 \times 1.06$$

$$= 20{,}000 \times 1.19$$

$$= 2380000$$

$$A = 23800$$

$$CI = 3800$$

Eg 4: Find the compound Interest & amount on the principal Rs. 7000 borrowed at 4% compounded annually for 2 yrs.

$$A = 7000 \times 1.04 \times 1.04$$

$$= 7000 \times 1.08$$

$$= 7.56 \times 1000$$

$$A = 7560$$

$$CI = 560$$

$11^3 = 1331$

$1331 \times \frac{8}{11} = 968$

$968 \times \frac{8}{11} = 88$

$8^3 = 216$

1331 968 88 216

1936 176

2904 264

1331000

2904

2640

1331000

290400

2640

216

1624256

106^3

$10^3 = 1000$

$1000 \times \frac{6}{10} = 600$

$600 \times \frac{6}{10} = 360$

$6^3 = 216$

1000	600	360	216
	1200	720	

1080

1000000

180000

10800

216

1191016

104^2

$(100)^2 + 2(100)(4) + 4^2$

$10000 + 800 + 16$

10816

Eg 5: Find the compound Interest on Rs. 2500 invested at 6% p.a., compound semi-annually for 8 years? 1.03?

-4011.73
-1511.73.

Eg 6: Find compound interest on Rs. 5000 invested at 8% per. annually, compound semi-annually for 2 yrs?

Ans 5:

$$A = P\left[1 + \frac{R/2}{100}\right]^{2T}$$

$$A = P\left[1 + \frac{6/2}{100}\right]^{2\times 8}$$

$$A = 2500\left[1 + \frac{3}{100}\right]^{16}$$

$$= 2500[1.03]^{16}$$

$$= 2500[1.6047]$$

$$= 4011.73$$

$$CI = 1511.73$$

Ans 6: $A = P\left[1 + \frac{R/2}{100}\right]^{2T}$

$$= 5000\left[1 + \frac{8/2}{100}\right]^{2\times 2}$$

$$= 5000[1.04]^{4}$$

$$= 5000[1.1698]$$

$$A = 5849.29$$

$$CI = 849.29.$$

Q7. Find the compound amount which would be obtained from the interest of Rs. 2000 at 4% compounded quarterly for 5 yrs.

$$A = 2000\left[1+\frac{R/4}{100}\right]^{4\times T}$$

$$= 2000\left[1+\frac{4/4}{100}\right]^{4\times 5}$$

$$= 2000[1.01]^{20}$$

$$A = \underline{2440.38}$$

$$CI = \underline{440.38}$$

Q8. Find the compound amount which would be obtained from the interest of Rs. 6000 at 8% compounded quarterly for 3 yrs.

$$A = 6000\left[1+\frac{R/4}{100}\right]^{4T}$$

$$= 6000\left[1+\frac{8/4}{100}\right]^{4\times 3}$$

$$= 6000[1.02]^{12}$$

$$= \underline{7609.45}$$

$$CI = 1609.45$$

Q9. If the rate of compound interest for the first & the second year be 4% & 3% respectively. Find the amount & the compound Interest of Rs. 12,000 in 2 years.

$$A = P\left[1+\frac{R_1}{100}\right]\left[1+\frac{R_2}{100}\right]$$

$$= P\left[1+\frac{4}{100}\right]\left[1+\frac{3}{100}\right]$$

$$= 12,000(1.04)(1.03)$$

$$= \underline{12854.4}$$

$$\underline{CI = 854.4}$$

$$A = 12,000 \times 1.04 \times 1.03$$

$$= 12,000 \times 1.0712$$

$$= 12854.4$$

$$\begin{array}{r} 104 \\ 103 \\ \hline 107012 \end{array}$$

$$1.0712$$

Q. 10 If the rate of compound Interest for the first & second year be 5% & 2% respectively. Find the amount & the compound interest on Rs. 8000 in 2 years.

$$A = 8000 \times 1.05 \times 1.02$$
$$= 8000 \times 1.071$$
$$= 8568$$

$$CI = 568$$

$$\begin{array}{r} 105 \\ \times\ 102 \\ \hline 10710 \end{array}$$

Q. 11: Find the compound Interest on Rs. 5000 for 3 years at 8% p.a. compounded annually (year wise soln.)

Y1:
$P = 5000$ Rs.
$T = 1$ yr.
$R = 8\%$ p.a.

$$SI = \frac{5000 \times 1 \times 8}{100} = 400$$

$$A = 5400$$

Y2:
$P = 5400$ Rs.
$T = 1$ yr.
$R = 8\%$

$$SI = \frac{5400 \times 1 \times 8}{100} = 432$$

$$A = 5400 + 432 = 5832$$

Y3:
$P = 5832$
$T = 1$ yr.
$R = 8\%$

$$SI = \frac{5832 \times 1 \times 8}{100} = 466.56$$

$$A = 5832 + 466.56 = 6298.56$$
$$CI = 6298.56 - 5000 = 1298.56$$

Q12: Find the compound Interest on Rs. 4000 for 2 yrs. at 4% P.A., compounded annually (year wise soln.).

I. Find the compound Amount & compound Interest of the following.

a. P = Rs. 7500, R = 6%, T = 2 years
A = ? C.I. = ?

b. P = Rs. 10,000, R = 8%, T = 2 years
A = ? C.I. = ?

c. P = Rs. 70,000, R = 4%, T = 2 years
A = ? C.I. = ?

d. P = Rs. 3000, R = 2%, T = 3 years
A = ? C.I. = ?

e. P = Rs. 2300, R = 12%, T = 2 years
A = ? CI = ?

f. P = Rs. 4200, R = 8%, T = 1 year
A = ? CI = ?

g. P = Rs. 6200, R_1 = 8%, R_2 = 4%, T = 2 years
A = ? CI = ?

h. P = Rs. 3200, R_1 = 4%, R_2 = 2%, T = 2 years
A = ? CI = ?

i. P_1 = Rs. 6200, P_2 = ?, R = 3%, T = 2 years
A = ? CI = ?

j. P_1 = Rs. 2000, P_2 = ?, R = 7%, T = 2 years
A = ? CI = ?

II. Solve the following.

1. Find the compound amount and compound interest on the principal Rs. 8000 borrowed at 2% compounded annually for 4 years.
2. Find the compound amount and compound interest on the principal Rs. 12000 borrowed at 3% compounded annually for 2 years.
3. Find the compound interest on Rs. 6500 invested at 8% annually, Compound semi-annually for 2 years.
4. Find compound interest on Rs. 5000 invested at 4% annually, compound semi-annually for 3 years.
5. Find the compound amount which would be obtained from the interest of Rs. 9000 at 12% compounded quarterly for 2 years.
6. Find the compound amount which would be obtained from the Interest of 11000 at 4% compounded quarterly for 1 year.
7. If the rate of compound interest for the first & second year be 5% & 4% respectively, find the amount & the compound interest in 2 years.
8. If the rate of compound interest for the first & second year be 2% and 6% respectively, find the amount and the compound Interest on Rs. 10,000 in 2 years.
9. Find the amount and the compound interest on Rs. 16000 for 3 years at 5% per annum compounded annually.
10. Find the amount & the compound interest on Rs. 2500 for 2 years at 10% per annum, compounded annually.

Speed Distance Formulas

Speed: A measure of how quickly an object moves from one place to another.

It is equal to the distance travelled divided by time.

Speed = distance / time.

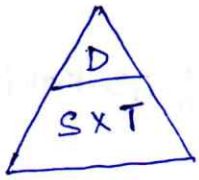
i.e. The distance travelled by a body or vehicle in a unit time is known as its speed.

Distance: Distance is a path covered by a moving object (person or vehicle).

distance = speed × time.

Time: Time = distance / speed.

Note: The unit of time in speed should be the same as that of the given time.



S - Speed → (mts/sec.).
d - distance → mts
t - time → sec.

* In some cases, we need to convert mts/sec. to km/hr. & vice-versa.

Eg:

Speed related problems:

1. Shreya travelled a distance of 234 Km. by car in 6 hrs. Find the speed of the car.

distance = 234 kms.
time = 6 hrs.
speed = $d/t = \frac{234}{6} = 39$ kms./hr.

2. A car travels a distance of 495 kms. in 8½ hrs. What is its speed.

distance = 495 kms.
time = 8½ hrs.
speed = $d/t = \frac{495}{8.5} = \frac{4950}{85} = \frac{990}{17} = 58.23$ kms/hr.

Distance related problems:

1. How much distance will be covered in 6 hrs. at a speed of 45 km/hr.

speed (s) = 45 km./hr.
time (t) = 6 hrs.
distance (d) = s x t
= 45 x 6 = 270 kms.

2. How much distance will be covered in 4 hrs. at a speed of 72 km/hr.

time (t) = 4 hrs.
speed (s) = 72 kms./hr.
distance (d) = s x t
= 72 x 4
= 288 kms.

3. A bus travels at a speed of 45 km./hr. How far will it travel in 36 minutes.

speed (s) = 45 km./hr.
time (t) = 36 mins. = 36/60 hrs.
distance (d) = s x t
= 45/3 x 36/60 = 27 kms.

Time related problems:

1. How much time will be taken to cover a distance of 550 km. at a speed of 50 km./hr.

distance (d) = 550 km.
speed (s) = 50 km./hr.
time (t) = d/s = 550/50 = 11 hrs.

2. A motorized rickshaw covers a distance of 120 Km at a speed of 30 Km./hr. Find the time taken to cover this distance.

distance (d) = 120 Km.
speed (s) = 30 Km./hr.
time (t) = $d/s = \frac{120}{30} = 4$ hrs.

I. Find the Speed, distance, and time of the following

a. Distance = 320 Km.
Time = 3 hrs.
speed = ?

b. Distance = 428 Km.
Time = 4 hrs.
speed = ?

c. Speed = 45 Km./hr.
Time = 5 hrs.
Distance = ?

d. Speed = 30 Km/hr
Time = 4 hrs.
Distance = ?

e. Speed = 35 Km./hr
Time = 120 mins.
Distance = ?

f. Speed = 55 Km./hr
Time = 180 mins.
Distance = ?

g. Distance = 440 Km.
Speed = 20 Km./hr.
Time = ?

h. Distance = 690 km.
Speed = 30 km./hr.
Time = ?

II. Solve the following.

a. Surya travelled a distance of 755 km. by car in 5 hrs. Find the Speed.

b. A car travels a distance of 330 km. in $3\frac{1}{2}$ hrs. what is its speed.

c. How much distance will be covered in 4 hrs. at a speed of 30 km per hour

d. How much distance will be covered in 7 hrs. at a speed of 82 km./hr

e. A car travels at a speed of 35 km/hr. How far will it travel in 420 mins.

f. A bus travels at a speed of 40 km./hr. How far will it travel in 90 mins.

g. How much time will be taken to cover a distance of 390 km at a speed of 50 km./hr.

h. How much time will be taken to cover a distance of 840 km. at a speed of 40 km./hr.

i. How much time will be taken to cover 20 m at a speed of 20 cms/sec.

j. A man runs at a speed of 15 km./hr. How much time will he take to cover 750 mts.

Fractions & Decimals.

a. Introduction to Fractions.

* An object divided into NO. of equal parts - FRACTION

Eg: Two fifth of an object.
- 2/5.

Types of fraction:

1. Proper fraction
   num < denom

Eg: 2/5, 1/5, 7/16 etc.

2. Improper fraction
   num ≥ denom

Eg: 7/4, 6/6 etc.

3. Mixed fraction
   Whole Number + fraction.

Eg: $3\frac{4}{5}$, $7\frac{9}{2}$ etc.

Improper fractions & mixed fractions conversion.

Improper fraction → can be expressed as mixed fraction & Vice-Versa.

1. Eg: 13/5 as mixed fraction.

   S1: 13/5 Q→2 R→3.

   S2: $2\frac{3}{5}$

2. Eg: 38/9 Q→4 R→2

   S2: $4\frac{2}{9}$.

3. Express $4\frac{3}{7}$ as improper fraction

   S1: $4 \overset{+}{\underset{\times}{}} \frac{3}{7}$

   S2: 4×7+3 = 31/7.

4. $5\frac{5}{12}$.

S1: $5 \times 12 + 5 = 65$

S2: $\frac{65}{12}$

b. Decimals

→ Decimal number contains a decimal point.
→ contains both whole part & decimal part.

$$\underset{\leftarrow x}{abc.}\underset{y \rightarrow}{def}$$

$x \rightarrow$ powers of 10.

$a \times 10^2 + b \times 10^1 + c \times 10^0$.

$y \rightarrow$ value decreases in reciprocals of power of 10.

$d - \frac{1}{10}$ $e - \frac{1}{100}$ $f - \frac{1}{1000}$. etc.

1. Eg: 25.8

25 → whole part
8 → decimal part.

c. Types of Decimal Numbers.

1. Terminating decimals.

- The decimal numbers which terminate / contain finite digits in the decimal part are called Terminating decimals.

Eg: 15.222345, 9856.784321, 0.147935862.

2. Non-Terminating decimals

- The decimal numbers which do not terminate / contain in-finite digits in the decimal part are called Non-Terminating decimals.

There are 2-types of non-terminating decimals.

a. Recurring decimals

- decimal digits repeats itself / recurs endlessly (infinitely) is called Recurring decimals.

digits → single / multigroup.

Eg: 0.3333... , 28.454545... , 147.25792579... , 7.1333...

The recurring decimals are represented as $0.\dot{3}$, $28.\dot{4}\dot{5}$, $147.\dot{2}5\dot{7}\dot{9}$, $7.1\dot{3}$, $52.3\dot{8}\dot{4}$.

$52.3\dot{8}\dot{4}$, $7.1\dot{3}$: These kind of decimal are also called recurring decimals with terminating part, since one or more digits are terminating and there is recurring part as well.

b. Non-Recurring decimals

- The decimal numbers in which the decimal part contains infinite digits but do not recur are called Non-Recurring decimals.

Eg: $\pi = 3.141592....$, 57.896542310872...

d. Rules to identify the types of decimals

1. Rules to identify terminating decimals.
   * if the denominator has only 2 & 5, sometimes both as some of the factors then it is terminating decimal.

   Eg: $7/2$, $19/5$, $42/10$, $55/16$, $62/25$ etc.

2. Rules to identify non-terminating decimals
   a). Recurring decimals
   * if the denominator has prime numbers as some of the factors but no 2's & 5's, then it is recurring decimal.

   Eg: $22/3$, $34/7$, $63/27$, $85/11$.

   b). Recurring decimals with terminating part
   * if the denominator has prime numbers (i.e. 2,3,5,7) as some of the factors then it is recurring decimal

with terminating part.

Eg: $1/12$, $33/18$, $7/24$ etc.

II Conversions of decimals to fractions

1. For terminating decimals

- Multiply & divide by powers of 10.
- The power of 10 is chosen based on the NO. of digits after the decimal point.
- Suppose there are 3-digits after the decimal point then we multiply & divide by 1000.
(i.e. the NO. of digits after the decimal point & the NO. of ~~digits~~ zeros in the power of 10 must be equal).

a. Eg: 0.24

→ No. of digits after decimal point is 2

→ SO NO. of zeros in power of 10 must be 2 (i.e. 100)

* Multiply & divide by 100 & simplify to get the fraction equivalent to the decimal.

i.e. $0.24 \times \frac{100}{100} = \frac{24}{100} = 6/25$

b. 34.725

$34 + 0.725$

$34 + 725/1000 = 29/40$

$34\ \frac{29}{40}$

c. 96.72

$96 + 72/100 = 96\ 18/25$

(18/25)

d. 1.73

$1 + 73/100 = 1\ 73/100$

2. For non-terminating decimals (recurring).

* Write the recurring digit as numerator & denominator will contain 9's.
* No. of 9's in denom = No. of digits in the recurring part.

a. Eg: $0.\dot{3}$

* digits recur in decimal part is 3.
* So '3' forms Numerator.
* No. of digits recurring is only one

fraction = $3/9 \Rightarrow 1/3$.

Fraction equivalent of $0.\dot{3} = 1/3$.

b. Eg: $12.\dot{4}\dot{6}$

$\dot{4}\dot{6} = \frac{46}{99}$

$12 + \frac{46}{99} = \frac{(12\times99)+46}{99}$

$= \frac{1234}{99}$

1×2
$9 \quad 9$
1098

3. For Recurring decimals with terminating part.

Eg: $21.3\dot{4}\dot{5}$

$= 21.3454545 \rightarrow ①$

Multiply ① by 10

$= 213.4545 \rightarrow ②$

(Multiply until one sequence is onto other part of decimal)

ie. $21345.45 \rightarrow ③$

Eq. ② & ③ have same digits after decimal part.

Subtract these two eqns.

$1000x = 21345.45$
$-\ 10x \quad\quad 213.45$
$990x \quad\quad 21132.0$

$$x = \frac{21132}{990} = \frac{2348}{110}$$

SR of 21132 is 9, Hence divisible by 9.

Eg: $405.8\dot{7}\dot{1}$

$x = 405.8\dot{7}\dot{1}$

$10x = 4058.\dot{7}\dot{1}$

$1000x = 405871.\dot{7}\dot{1}$

$990x = 405871.\dot{7}\dot{1} - 4058.\dot{7}\dot{1}$

401813

$$x = \frac{401813}{990}$$

III. Converting fractions into decimals.

1. When denominator is power of 10.

* Count no. of zeros, place the decimal point in the numerator after the digits equal to no. of zeros.

a. $28/10 = 2.8$

b. $7/100 = 0.07$

2. When denominator is 5.

* Multiply both num & den by 2
* Then place decimal in num (as in step 1).

a. $97/5 = \frac{97 \times 2}{5 \times 2} = \frac{194}{10} = 19.4$

b. $13/5 = \frac{13 \times 2}{5 \times 2} = \frac{26}{10} = 2.6$

3. When denominator is 2.

* Multiply by 5 (both Num. & den.).
* follow Step 1.

a. Eg: $\frac{33}{2} = \frac{33 \times 5}{2 \times 5} = \frac{165}{10} = 16.5$

b. Eg: $\frac{81}{2} = \frac{81 \times 5}{2 \times 5} = \frac{405}{10} = 40.5$

4. When the denominator is 4

* Multiply both Num & den. by 5 twice.
* follow step 1.

a. Eg. $\frac{23}{4} = \frac{23 \times 5}{4 \times 5} = \frac{115 \times 5}{20 \times 5} = \frac{575}{100} = 5.75$

b. Eg. $\frac{75}{4} = \frac{75 \times 25}{4 \times 25} = \frac{1875}{100} = 18.75$

IV Highest common factor (HCF).

It's highest common factor present b/n 2 or more Nos.

a. Find the HCF of [22, 26].

S1: Difference b/n 2 Numbers.

$26 - 22 = 4$.

S2: Difference is not prime No., we have to find the factors of 4. Factors of 4: 2 & 4.

S3: check the given numbers are divisible by factors of 4.
i.e. both Nos. are divisible by 2, but not by 4.

H.C.F. = 2

b. Find the HCF of [3, 5]

S1: $5 - 3 = 2$

S2: '2' is prime.

S3: Check the given Nos. divisible by '2'
- both Nos. are not divisible by '2'
hence HCF = 1.

c. Find HCF of [138, 161].

S1: 161-138 = 23.

S2: '23' - prime.

S3: 23 divides both Nos.

HCF = 23

d. Find HCF of [20,30].

S1: 30-20 = 10.

S2: Factors of 10 = 2, 5, 10.

S3: All three factors can divide given Nos.

HCF = 10.

Points to Remember:

a. If the difference of 2 Numbers is a prime Number, then check if the numbers are divisible by this number, if yes, then HCF = number
else HCF = 1.

b. difference ≠ prime.
Find the factors.
Check if the numbers are divisible by factors, if it does then the difference itself is HCF otherwise 1.

V. Least Common Multiple (LCM)

A common multiple is a number that is a multiple of 2 or more Numbers.
LCM is a smallest Number (not zero) that is a multiple of both.

Method 1: UT (Urdva Tiryagbhyam).
Meaning: Vertically crosswise.

LCM by UT

a. Find LCM of [12, 18]

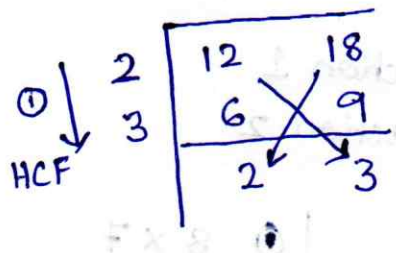
1. General Method. to find LCM

2	12, 18
3	6, 9
	2, 3

$2 \times 3 \times 2 \times 3 = 36$

LCM = 36

2. To find LCM using UT.



HCF = 2 × 3 = 6

LCM = 12 × 3 = 18 × 2
= 36.

Method 2: Anurupyena method.

Meaning: 'Proportionality'

This is a simple method for finding the lowest common multiple of 2 numbers, when the answer is not obvious.

a. Find LCM of [12, 18].

S1: Write HCF = 6

S2: Divide both nos. by $12/6 = 2$ $18/6 = 3$

S3: $6 \overline{)\begin{matrix}12 & 18\\ 2 & 3\end{matrix}}$

LCM = product of 3 Nos.

= 6 × 2 × 3

= 36

VI Basic operations in fractions:

a. Addition & Subtraction of fractions by UT.

UT → vertically & crosswise

* When the denominators are co-prime

① Eg: Add $5/2 + 7/3$

S1: Cross multiply - 'Num' of fraction 1 × 'Den' of fraction 2

$$5/2 \times\!\!\times 7/3 = \frac{5\times3 + 2\times7}{?}$$

S2: We get 'den' by multiplying the denominators.

$$\frac{15+14}{(2\times3)} = \frac{29}{6}$$

2. $\frac{8}{16} - \frac{13}{7}$

S1: Cross multiply - 'Num' of fraction 1 × 'den' of fraction 2.

$\frac{8}{16} \times \frac{13}{7} = \frac{(8\times7)-(13\times16)}{?}$

$= \frac{56-208}{?}$

1×3
$1 \quad 6$
208

S2: Get denominator by multiplying den.

$\frac{56-208}{16\times7} = \frac{-152}{112}$

① 8×7.
$= 1\bar{2} \times 1\bar{3}$
$= 1\ \bar{2}$
$1\ \bar{3}$
$1\,(\bar{5})\,6$
$= 56$

② 13×16
$1\ 3$
$2\ \bar{4}$
$2\ 2\ \bar{12}$
① 88
$2\ 1\ 88$.
208

* When the denominators are not co-prime

1. Eg: $\frac{3}{8} + \frac{5}{12}$

S1: Write down HCF of denominators

$12 - 8 = 4$

Both divisible by '4'

HCF = 4.

S2: Divide the den. by HCF & write below

i.e. $\frac{3}{8} + \frac{5}{12}$

$\downarrow \quad \downarrow$

$4 \quad 2 \quad 3$

S3: 2 & 3 will be new denominator.

Rewrite as below.

$\frac{3}{2} + \frac{5}{3} = \frac{3\times3 + 2\times5}{4\times2\times3} = \frac{19}{24}$

2. $7\frac{5}{9} - 4\frac{7}{15}$

S1: $7 - 4 = 3$.

S2: $15 - 9 = 6 = 2, 3$

H.C.F. = 3.

S3: $\frac{5}{9} \quad \frac{7}{15}$

$\downarrow \quad \downarrow$

$3 \quad 3 \quad 5$

S4: $\frac{5}{3} - \frac{7}{5} = \frac{5\times5 - 7\times3}{3\times3\times5} = \frac{25-21}{45} = \frac{4}{45}$

③

$7\frac{5}{9} - 4\frac{7}{15} = 3\frac{4}{45}$.

* When the denominator is factor of other.

* When the denominators are same

Eg: $\frac{3}{5} + \frac{19}{5}$

Simply add num & place den as-is.

$$\frac{3+19}{5} = \frac{22}{5}$$

* When denominator is factor of other
- Multiply until denom are same.

① Eg: Add $1/4 + 5/16$

S1: 4 is a factor of 16.

$$\frac{1}{4} \times \frac{4}{4} = \frac{4}{16}$$

S2: $\frac{4}{16} + \frac{5}{16} = \frac{4+5}{16} = \frac{9}{16}$.

2. $\frac{4}{5} - \frac{31}{40}$

$$\frac{4}{5} \times \frac{8}{8} - \frac{31}{40} = \frac{32}{40} - \frac{31}{40} = \frac{1}{40}$$

b. Multiplication of fractions -

- Multiply two 'Nums'
- Multiply two 'Dens'

1. Eg: $\frac{3}{\cancel{4}_1} \times \frac{\cancel{12}^3}{} = 9.$

$\frac{\cancel{3}^1}{4} \times \frac{7}{\cancel{8}_2} = \frac{7}{8}$

2. $1\frac{5}{7} \times 1\frac{1}{3} = \frac{\cancel{12}^4}{7} \times \frac{4}{\cancel{3}_1} = \frac{16}{7} = 2\frac{2}{7}$

c. Division of fractions.

Sutra → 'Transpose & Adjust'

→ keep 1st fraction as-is.

→ Turn 2nd fraction upside down.
& change sign ÷ to '×'.

* paravartya Sutra.

1. Eg $1/2 \div 1/9$

$= 1/\cancel{12}_4 \times \frac{\cancel{9}^3}{1} = 3/4$

2. $2\frac{1}{3} \div 1\frac{3}{4}$

$= \cancel{7}/3 \div \cancel{7}/4$

$= \cancel{7}/3 \times \frac{4}{\cancel{7}} = 4/3 = \underline{1\frac{1}{3}}$

EXERCISE

I. convert decimal to fraction.

a. 0·3 b. 0·45 c. 2·3 d. 0·234 e. 0·5

f. 7·65 g. 0·98 h. 5·8 i. 0·123

II. convert fraction to Decimal.

a. $\frac{4}{10}$ b. $8/5$ c. $\frac{6}{10}$ d. $\frac{61}{2}$ e. $\frac{11}{100}$

f. $\frac{123}{100}$ g. $\frac{17}{4}$ h. $\frac{10}{3}$ i. $\frac{21}{5}$

III. convert Improper fraction to mixed fractions

a. $\frac{9}{4}$ b. $7/2$ c. $\frac{11}{3}$ d. $18/5$ e. $\frac{23}{4}$

f. $\frac{49}{8}$ g. $41/5$ h. $29/3$ i. $\frac{54}{10}$

IV convert mixed fractions to Improper fractions.

a. $3\frac{4}{5}$ b. $2\frac{2}{7}$ c. $4\frac{1}{4}$ d. $2\frac{3}{5}$ e. $3\frac{3}{5}$

f. $7\frac{2}{4}$ g. $8\frac{3}{2}$ h. $4\frac{7}{8}$ i. $6\frac{9}{10}$ j. $5\frac{2}{7}$.

V Find the HCF of the following.

a. 3, 4 b. 32, 36 c. 83, 96 d. 66, 44 e. 48, 32

f. 4, 5 g. 2, 4 h. 6, 12

VI Find the HCF of the following

a. 6, 9 b. 9, 8 c. 83, 84.

d. 77, 66 e. 45, 50 f. 4, 7

VII Find the LCM by UT Method -

a. 18, 24 b. 28, 36 c. 6, 12 d. 15, 20

e. 12, 32 f. 48, 52 g. 72, 84 h. 66, 44.

VIII. Find the LCM by UT Method.

a. 6, 8 b. 6, 9 c. 25, 30.

d. 24, 28 e. 8, 18 f. 9, 15.

IX Find the LCM by Anurupyena method.

a. 12, 8 b. 10, 4 c. 15, 10 d. 40, 60.

e. 8, 14 f. 10, 12 g. 12, 14 h. 6, 20.

X. LCM by UT

a. 5, 12 b. 10, 6 c. 4, 5

d. 4, 16 e. 25, 10 f. 5, 7

XI Add the following

a. $\frac{5}{12} + \frac{1}{5}$ b. $\frac{15}{6} + \frac{2}{3}$ c. $6\frac{1}{18} + 4\frac{1}{16}$ d. $\frac{11}{20} + \frac{1}{4}$

e. $\frac{7}{10} + \frac{1}{12}$ f. $\frac{8}{9} + \frac{3}{8}$ g. $\frac{11}{16} + \frac{5}{32}$ h. $11\frac{11}{16} + 2\frac{3}{8}$

XII Solve the following

a. $\frac{4}{15} + \frac{3}{5}$ b. $\frac{1}{17} + \frac{1}{20}$ c. $\frac{15}{16} + \frac{7}{20}$ d. $2\frac{7}{36} + 94\frac{5}{48}$

e. $3\frac{5}{6} + 2\frac{1}{3}$.

XIII Subtract the following

a. $\frac{7}{9} - \frac{23}{36}$ b. $\frac{5}{8} - \frac{5}{11}$ c. $\frac{5}{6} - \frac{12}{25}$ d. $\frac{7}{8} - \frac{5}{24}$

e. $\frac{11}{12} - \frac{4}{15}$ f. $6\frac{12}{25} - 3\frac{3}{40}$ g. $\frac{22}{36} - \frac{5}{24}$ h. $9\frac{9}{6} - 1\frac{1}{28}$

XIV Solve the following

a. $\frac{7}{8} - \frac{55}{64}$ b. $\frac{31}{530} - \frac{1}{3}$ c. $\frac{21}{250} - \frac{7}{100}$

d. $6\frac{3}{4} - 1\frac{3}{14}$ e. $8\frac{9}{10} - 2\frac{7}{15}$.

XV Multiply the following

a. $\frac{3}{7} \times \frac{2}{5}$ b. $\frac{4}{15} \times \frac{7}{12} \times \frac{25}{21}$ c. $3\frac{8}{9} \times 3\frac{6}{7}$

d. $1\frac{1}{2} \times 1\frac{1}{3} \times 1\frac{1}{5}$ e. $\frac{4}{63} \times \frac{21}{32}$ f. $2\frac{1}{4} \times 5\frac{1}{3}$

g. $2\frac{4}{7} \times 4\frac{2}{3}$ h. $2/3 \times 6/7 \times 3/4$

XVI Solve

a. $1/5 \times 8$ b. $2\frac{1}{4} \times 1/3$ c. $3\frac{1}{16} \times \frac{14}{25} \times 2\frac{1}{7}$

d. $\frac{10}{27} \times \frac{9}{35}$ e. $\frac{4}{7} \times \frac{5}{8}$

XVII Divide the following.

a. $\frac{5}{8} \div \frac{2}{3}$ b. $1 \div 1\frac{1}{2}$ c. $3\frac{1}{7} \div 11$ d. $1\frac{11}{45} \div 10\frac{1}{9}$

e. $\frac{2}{9} \div \frac{4}{3}$ f. $1\frac{5}{7} \div 1\frac{1}{17}$ g. $\frac{5}{8} \div \frac{5}{8}$ h. $\frac{28}{48} \div 4\frac{5}{18}$

XVIII Solve the following.

a. $1\frac{1}{3} \div 2/5$ b. $1/12 \div 1/9$ c. $3\frac{3}{14} \div 1\frac{4}{21}$

d. $5 \div 3\frac{1}{3}$ e. $2\frac{1}{3} \div 1\frac{3}{4}$.

Algebraic Expressions

a. Solving equations using Vilokanam Sutra.

Vilokanam Sutra.

Meaning : Observation

An expression is an equation equalling to quantity.

Equal symbol : "="

Eg: $3 \times 4 = 12$

$3 \times \_ = 12$.

OR

$\_ \times 4 = 12$.

* Solve problem by just observation.

Eg: a. $4x \_ = 32$.

$\downarrow$

8

b. $4x \_ = 12$

$\downarrow$

3

c. $x * 5 = 17$

$\downarrow$

12

d. $x + \frac{1}{x} = \frac{5}{2}$

$x + \frac{1}{x} = 2 + \frac{1}{2}$

$x = 2$.

e. $\frac{x}{3} = 12$.

$\frac{x}{3} = \frac{36}{3}$

$x = 36$.

f. $18 - x = 15$

$\downarrow$

3

Solving Equations by 'Transpose & Adjust'

(i) Transpose : to change.

Addition $\xrightarrow{\text{Transpose}}$ Subtraction.

Subtraction $\xrightarrow{\text{Transpose}}$ Addition.

Multiplication $\xrightarrow{\text{Transpose}}$ Division.

Division $\xrightarrow{\text{Transpose}}$ Multiplication.

1. Eg: $2+3x = 20$.

$3x = 20-2$

$3x = 18$

$x = 6$

2. Eg: $5x-11 = 29$

$5x = 29+11$.

$5x = 40$.

$x = 8$

b. Ratios & Proportions.

Ratio : A ratio is a relationship b/n two numbers indicating how many times the first number 'contains' the second.

Anurupyena → 'Proportional'

i. Ratio of 2 Numbers

1. Eg: A school of 150 students & 25 teachers. What is the ratio of students to teachers.

S1: Students : teachers.

150 : 25

S2: Find the HCF (to find ratio).

5	150, 25
5	30, 5
	6, 1

HCF → 25

LCM = 150

S3: $150/25 : 25/25$

$\Rightarrow 6 : 1$

∴ Ratio of students to teachers is 6:1

2. Eg: A class room has 24 girls & 42 boys. What is the ratio of girls to boys in the class room.

S1: Girls : boys.

24 : 42

S2: Find HCF.

2	24, 42
3	12, 21
	4, 7

HCF = 6

S3: $24/6 : 42/6$

4 : 7

∴ Ratio of girls to boys is 4:7

ii. Ratio of 3 Numbers.

1. Eg: In a basket, there are 75 apples, 450 oranges & 125 Gauva. Find the ratio of each fruit in the basket.

S1: Apples : Oranges : Gauva

75 : 450 : 125

S2: Find HCF.

5	75, 450, 125
5	15, 90, 25
	3, 18, 5

HCF = 25

S3: $75/25 : \frac{450}{25} : \frac{125}{25}$

3 : 18 : 5

Ratio of Apples & fruits in basket → 3:18:5

2. Eg: Bring ratio to the simplest form.

12 : 54 : 96.

S1: 12 : 54 : 96.

S2: Find HCF.

$$\begin{array}{r|l} 2 & 12, 54, 96 \\ 3 & 6, 27, 48 \\ \hline & 2, 9, 16 \end{array}$$

HCF = 6.

S3: $12/6 : 54/6 : 96/6$

2 : 9 : 16

Proportions:

proportions means equating two ratios (comparitive) relation b/n things or magnitudes as to size, quantity, number etc.)

Use Madhyamadhyena Adyamantyena sutra to perform calculations.

1. eg: Consider ratio 72:18

Simplest form → 4:1 (ratio).

HCF → 18

$$\begin{array}{r|l} 2 & 72, 18 \\ 9 & 36, 9 \\ \hline & 4, 1 \end{array}$$

72:18 :: 4:1

↓

proportions b/n two ratios.

2. Eg: 15 : 3 & 100 : y are in proportion

determine 'y'

S1: Madyamadhyen Adyamantyena.

means → product of means } = { Product of extremes.

i.e. 15 : 3 = 100 : y

$15y = 300$

S2: Transpose.

$y = \frac{300}{15} = 20.$

3. Eg: $z : 8 :: 14 : 16$

$16z = 14 \times 8$

$z = \frac{14 \times 8}{16} = 7$

$z = 7$.

Percentage:

Percentage means 'per hundred' or 'for every hundred'

* fraction of 100.

Symbol → %

1. Eg: 20%, 12%, 7% etc.

Conversion of percentage to fraction:

% age can be expressed as fraction of 100.

1. Eg: 25%

$\Rightarrow 25/100 = 1/4.$

fraction of 25% = $1/4$.

2. Eg: 64%.

$\Rightarrow \frac{64}{100} = \frac{16}{25}$

fraction of 64% = $16/25$

3. Eg: 33.5%.

$\Rightarrow \frac{33.5}{100} = \frac{335}{1000} = \frac{67}{200}$.

$33\frac{1}{2} = \frac{67}{2}/100 = \frac{67}{200}$.

Conversion of fraction to Percentage

* Given fraction is multiplied by 100 & Simplify to get %. age.

1. Eg: 4/5.

$\Rightarrow 4/5 \times 100^{20} = 80\%$.

2. Eg: 3/8

$\Rightarrow 3/8_2 \times 100^{25} = \frac{75}{2} = 37.5\%$.

Find the Percentage of quantity:

1. Eg: 23% of 45

S1: $\frac{23}{100} \times 45$

S2: 23×45 using UT.

Apply Gunitasamuccayah samuchchayagunitah sutra.

→ Count No. of zeros & place decimal point (divisor) Two zeros, hence put decimal of 2 digits from the right.

$\Rightarrow 10.35$.

$$\begin{array}{r} 2 \overset{②}{\searrow} 3 \\ 4 \quad 5 \\ \hline 8 \quad 22 \quad 15 \\ 8 \quad 23 \quad 5 \\ \hline 1035 \end{array}$$

③ ↓ ①

2. Eg: 22% of 7

S1: $\frac{22}{100} \times 7$.

S2:
$$\begin{array}{r} 2\ 2 \\ 0\ 7 \\ \hline 0\ 14\ 14 \end{array}$$
154.

S3: 1.54.

3. Eg: 12% of 46.

S1: $\frac{12}{100} \times 46$.

S2:
$$\begin{array}{r} 1\ 2 \\ 4\ 6 \\ \hline 4\ 14\ 12 \end{array}$$
4 15 2
552

S3: 5.52

4. Eg: 36% of 850.

S1: $\frac{36}{100} \times 850$.

S2:
$$\begin{array}{r} 0\ 3\ 6 \\ 8\ 5\ 0 \\ \hline 0\ 24\ 63\ 30\ 0 \end{array}$$
24 66 00
30 6 00
= 30600

S3: 306

d. Profit & Loss

Cost price (CP) : The price at which the article has been purchased is called cost price (CP).

Selling price (SP) : The price at which the article ~~has~~ is sold is called Selling price (SP).

Profit : If Selling price (SP) > cost price (CP) then it is said to be profit.

i.e. profit = Selling price (SP) − cost price (CP).

Loss : If the cost price (CP) > Selling price (SP) then it is said to be loss.

i.e. loss = CP − SP.

$$\text{Profit \%} = \frac{\text{Profit}}{CP} \times 100.$$

$$\text{Loss \%} = \frac{\text{Loss}}{CP} \times 100.$$

$$\text{Selling price (SP)} = \frac{100 + \text{profit \%}}{100} \times CP \quad \text{or} \quad \frac{100 - \text{loss \%}}{100} \times CP.$$

$$\text{Cost price (CP)} = \frac{100 \times SP}{100 + \text{profit \%}} \quad \text{or} \quad \frac{100 \times SP}{100 - \text{loss \%}}.$$

1. Eg: An article was purchased for Rs. 850 and sold for Rs. 920. Find the profit percentage.

Cost price (CP) = 850.

Selling price (SP) = 920.

Profit = CP − SP

= 920 − 850 = 70

$$\text{Profit \%} = \frac{70}{\cancel{850}_{17}} \times \cancel{100}^{20} = \frac{140}{17} = \underline{8.23}$$

2. Eg: An article was purchased for Rs. 740 & sold for 680. Find the loss percentage.

Cost Price (CP) = 740 Rs.

Selling Price (SP) = 680 Rs.

Loss = 740 − 680 = 60 Rs.

$$\text{Loss \%} = \frac{\text{Loss}}{CP} \times 100 = \frac{60}{740} \times 100 = \frac{300}{37} = 8.1\%$$

3. Eg: An article was purchased for Rs. 540 and the profit % is 7.5 %. What is the selling price (SP).

Cost price (CP) = 540 Rs.

profit % = 7.5 %

$$\text{Selling price} = 540 + \left(\frac{7.5}{100} \times 540\right)$$

$$= 540 + \left(\frac{75}{100} \times 54\right)$$

$$= 540 + \left(\frac{3}{4} \times 54\right)$$

$$= 540 + \left(\frac{3}{2} \times 27\right)$$

$$= 580.5$$

Alternatively:

$$\text{Profit} = \frac{\text{profit \%} \times CP}{100} = \frac{7.5 \times 540}{100} = 40.50$$

S.P. = C.P + profit

= 540 + 40.5

= 580.5

4. Eg: An article was purchased for Rs. 375 which incurred a loss of 5% on selling. Calculate the Selling price.

C.P. = 375

loss % = 5%

~~Box, 375~~

$$S.P. = CP - \left(\frac{5}{100} \times 375\right)$$

$$= 375 - \frac{75}{4}$$

$$= 375 - 18.75$$

$$= \underline{356.25}$$

5. Eg: A merchant sold a chair & table at Rs. 900. each. He incurred a loss of 6% on the chair and 4% on the table. Calculate the cost price of chair & Table.

S.P. (Table) = 900.

loss = 4%.

C.P = SP − loss.

= 900 − (4% of CP)

$= 900 - \frac{4}{100} \times CP$.

$= 900 - 0.04CP$.

$1.04CP = 900$

$CP = \frac{900}{1.04} = \underline{865.38}$

S.P (chair) = 900

loss = 6%.

C.P. = SP − loss.

= 900 − (6% of CP)

$1.06\,CP = 900$

$CP = \frac{900}{1.06} = 957.44$

$\frac{26 \times 9}{2304}$

$\frac{9}{1.04}$

$= \frac{900}{104}$

$= \frac{450}{52}$

$= \frac{225}{26}$

e. Discount :

Marked price : The price of the article without any rebate is called Marked Price.

If no rebate then marked price = Selling price.

Discount : The reduction of price on the article is called discount. Discount is applied on marked Price.

Discount = Marked Price - Selling price.

$$\text{Discount \%} = \frac{\text{Discount}}{\text{M.P.}} \times 100$$

$$\text{M.P.} = \frac{100 + \text{profit \%}}{(100 - \text{discount \%})} \times \text{cost price}$$

$$\text{C.P.} = \frac{100 - \text{Discount \%}}{100 + \text{profit \%}} \times \text{MP.}$$

1. Eg: A phone case was sold for Rs. 220 but the marked price was 300. calculate the discount & discount %age.

Discount = Marked price - Selling price.

= 300 - 220.

= 80 Rs.

$$\text{Discount \%} = \frac{\text{Discount}}{\text{M.P}} \times 100$$

$$= \frac{80}{300} \times 100 = \underline{26.67}$$

2. Eg: In a book shop a novel series is marked 1999. on wednesdays the shop offers 19% discount on the purchase of the novel series. Calculate the price of the series on wednesdays.

M.P. = 1999.

Discount % = 19.

discount = $\frac{19}{100} \times 1999$ = $\frac{\text{discount \%} \times MP}{100}$

= $\frac{37981}{100}$

= 379.81

S.P. = MP. − discount

= 1999 − 379.81.

= 1619.19

1 9

1 9 9 9

90 81

(20−1) × 1999

39980 − 1999

37981

3. Eg: An agent sold a house for Rs. 7,50,000/- with a discount of 18% yet gained 30%. Find the cost of the house at which the agent has purchased.

Marked price → 7,50,000/-

discount % → 18 %.

profit % → 30.

C.P. = $\frac{100 - \text{discount \%}}{100 + \text{profit \%}} \times MP.$

= $\frac{100 - 18}{100 + 30} \times 7,50,000$

= $\frac{82}{130} \times 7,50,000$ = 47,30,761.54

S.P. = M.P − $\frac{18}{100} \times 75,$

=

4. Eg: A dealer gave discount of 5%. still gained 16%. Find the marked price of the article, if it costs Rs. 950 to the dealer.

$$M.P. = \frac{100 + \text{profit \%}}{100 - \text{discount \%}} \times C.P.$$

$$= \frac{100 + 16}{100 - 5} \times 950.$$

$$= \frac{116}{95} \times 950. = \underline{\underline{1160}}.$$

EXERCISE:

I. Solve the following equations by Vilokanam method.

a. $5x = 45$ b. $x + 9 = 51$ c. $\frac{x}{10} = 23$ d. $\frac{x}{3} = 4$.

e. $x + 24 = 76$ f. $x + 12 = 40$ g. $6 = 10 - x$

II. Solve the following

a. $18 = 3x$ b. $\frac{x}{7} = 1$ c. $5 + x = 85$

b. $\frac{x}{9} = 3$ e. $x + \frac{1}{x} = \frac{10}{3}$ f. $22 = 30 - x$

III Solve the following equations by transpose & Adjust.

a. $1 + 2x = 3$ b. $16 = 4 + 6x$ c. $67 = 12 + 5x$

d. $4x - 2 = 26$ e. $-5 = 7 + 3x$ f. $4 + 9x = 4$

IV Solve the following

a. $3x + 5 = 17$ b. $3x + 3 = 36$ c. $4x - 5 = -13$.

d. $3x + 8 = 2$ e. $7 = 3x - 5$

V Find the ratio of Numbers given below.

a. 85 & 135 b. 40 & 80 c. 75 & 175

d. 30, 60 & 90 e. 125, 100 & 150 f. 24, 12, 48.

VI Find the ratio of below.

a. In an auditorium, there are two levels A & B. Level A has 250 chairs & level B has 150 chairs.
What is the ratio of chairs at level A & level B have.

b. In an office, there are 800 male employees & 400 female empoloyees. What is the ratio of male to female employees working in that office.

c. A Vegetable vendor bought 40 Kgs. of onions, 20 Kgs of tomatoes & 10 Kgs. of potatoes.
What is the ratio of the vegetables respectively he bought.

d. Rahul bought three containers which can hold 150 lts, 250 lts & 500 lts respectively.
What is the ratio of the containers he bought.

VII Solve the following equations using Madyamadhyena Adyamantyena Sutra.

a. $36 : a :: 13 : 39$
b. $y : 25 :: 75 : 15$
c. $100 : 150 :: 480 : z$
d. $h : 4000 :: 800 : 400$.
e. $528 : 3168 :: b : 36$
f. $345 : 69 :: y : 138$.

VIII Convert the following percentages to fractions.

a. 15% b. 54% c. 66% d. 32% e. 8%
f. 79% g. 88% h. 28% i. 46%.

IX Convert the following fractions to percentages

a. $3/4$ b. $6/9$ c. $2/7$ d. $24/25$ e. $1/14$.
f. $33/100$ g. $13/19$ h. $18/50$ i. $15/75$

X Find the percentage of quantity for the following.

a. 17% of 5 b. 25% of 8 c. 49% of 80 d. 32% of 60.
e. 11% of 95 f. 15% of 75 g. 16% of 480 h. 21% of 950
i. 35% of 788

XI Solve the following

a. A boy purchased 10 pens for Rs. 99 from a shopkeeper, who bought them for Rs. 70. Calculate the profit & profit percentage.

b. A merchant incurred a loss of 20% by selling a leather jacket for Rs. 2800. Calculate the price at which the merchant purchased the jacket and the loss.

c. A car sold at 18% profit. Calculate the selling price, if the cost price is Rs. 5,40,000/- & profit.

d. A boy bought 2 shirts at Rs. 400 each & sold them for Rs. 500. Calculate the profit or loss & Also the percentage of profit or loss.

e. A man bought a phone for Rs. 6090 with a profit of 8%. Calculate the Cost Price & profit.

f. A girl purchased 2 bicycles for Rs. 1500 each. She sold one at 10% profit & another at 6% loss. Find the profit or loss in both cases & the price at which she sold the bicycles.

g. A Vendor bought 50 kg onions for Rs. 1000 & sold at Rs. 30 per kg. Calculate the profit or loss percentage.

h. A lady sold books for Rs 5430 with loss of 5%. Calculate at what price the lady should have sold the books to get 12% profit.

i. If cost price is 38% of selling price. Find the profit & profit percentage.

j. A trader uses 900 gm. weight instead of 1 kg weight. Calculate the profit or loss percentage.

XII Solve the following.

a. A quilt was sold for Rs. 1859 which was marked Rs 2999. Calculate the rate of discount.

b. A cake shop has offer of 30% if ordered online. If the cake you bought regularly costs Rs. 350. Calculate how much would you save on discount.

c. On discount of 25%, Shiny bought a sweater for Rs 891. Calculate the actual price of the sweater & discount in cash.

d. A merchant offers 9.5% discount on purchase of goods marked Rs. 3999 but still gains 10% profit. Calculate the price at which the merchants bought the goods.

e. A designer gave 11% discount on the jewellery which costs Rs. 8750 yet gained 15%. Find the marked price of the jewellery.

Geometry

a. Introduction

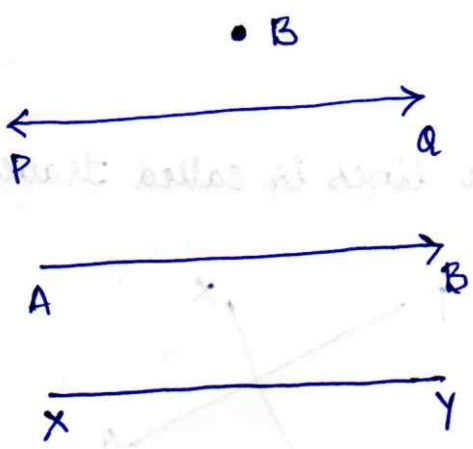
point : A point is a location on the plane or space. It is represented by a dot (•)

Line : A collection of points from a line. It extends on both the directions (i.e. it has no end points)

Ray: A ray is also a collection of points but it extends only in one direction (i.e. it has one end point)

Line Segment : A part of line is called line segment. It has two end points.
A line segment can also be defined as line joining two points.

Representations :



B → point

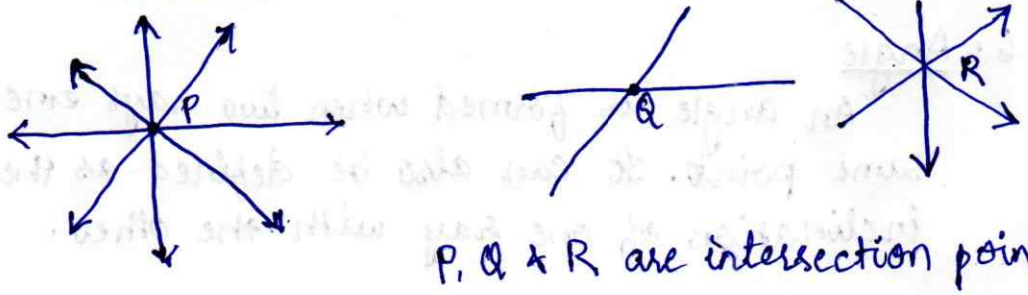
Line → PQ (Here P & Q are not end points.

Ray → AB (A - End point).

Line Segment → XY (X & Y both have end points).

b. classification of lines.

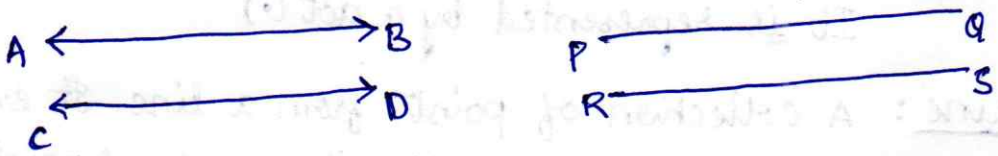
1. **Intersecting Lines** : Two or more lines which meet at a point is called Intersecting lines.
The point where they meet is called the point of Intersection.



P, Q & R are intersection points.

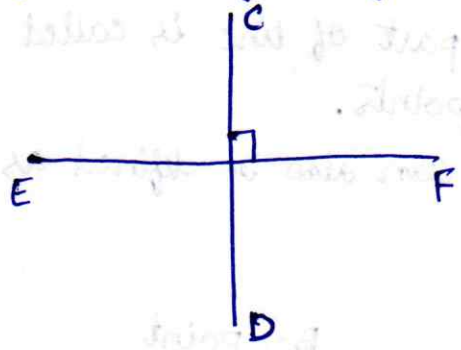
2. Parallel lines

Two lines are called parallel lines, if they never intersect even when extended indefinitely.



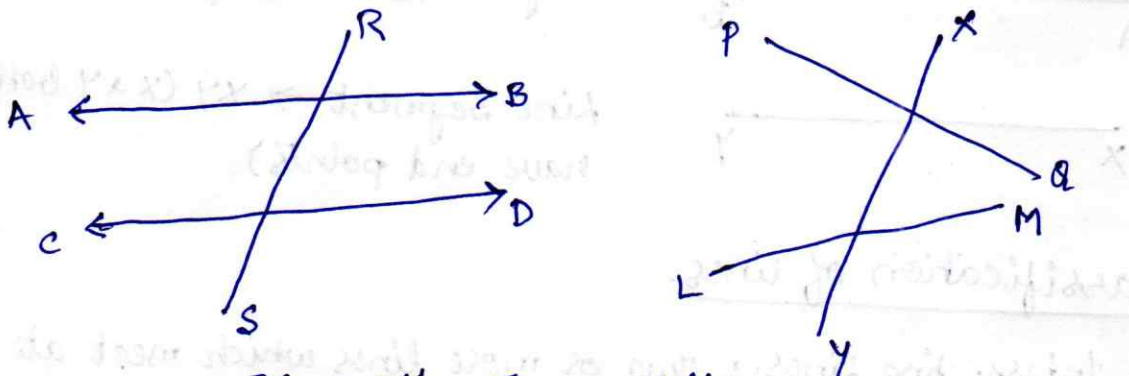
3. Perpendicular lines

Two lines are said to be perpendicular if they intersect at right angle. (i.e. right angle formed) between 2 lines.



4. Transversal lines

A line intersecting two lines is called Traversal line.



RS & XY - Traversal line

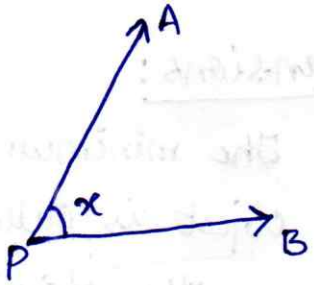
5. Vertex

A point at which two lines meet is called vertex.

6. Angle

An angle is formed when two rays emerge from the same point. It can also be defined as the measure of inclination of one ray with the other.

Here PA & PB are two rays & P is a vertex. 'x' is the angle formed by the rays PA & PB.



7. Surface

A surface is a boundary in a space. It divides the space into 2 regions or it bounds a region in space.

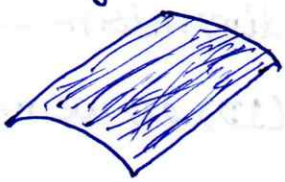


fig 1

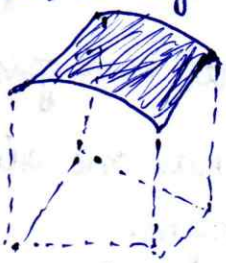


fig 2

Here fig 1 is the surface & fig 2 is the region bounded by the surface

8. Plane :

A plane is a flat surface which extends infinitely. It is a two dimensional surface.

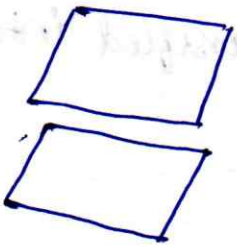


fig 3.

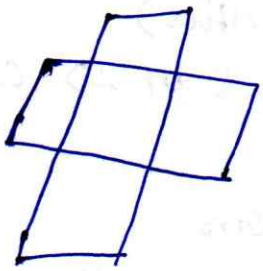


fig 4

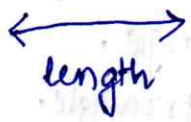
fig 3 : Represents two parallel planes to each other

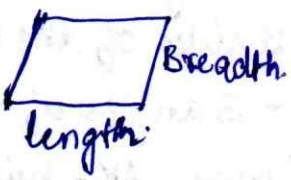
fig 4 : Represents two intersecting planes.

c. Geometric shapes :

A set of points or vertices connected by a set of lines or curves form geomitric shapes.

Eg :





Dimensions:

The minimum No. of co-ordinates required to define the object is called dimensions.

The objects may be points, lines, rays or geometric shapes.

Dimensions of geometric shapes:

1. Point: A point has zero or no-dimension.
2. Line: A line has one dimension (1D) as we measure only length.
3. Plane: A plane has 2 dimensions (2D) as we measure length & breadth.
4. Solid: A solid has 3 dimensions (3D) as we measure length, breadth & width.

2D Geometric shapes:

(Examples with properties).

Geometric shapes in 2D can be classified into 2-main categories.

a. Polygons
b. Curved shapes.

Polygons is derived from the greek word 'Poles' meaning many and 'gon' meaning angle or corner.

Types of Polygons:

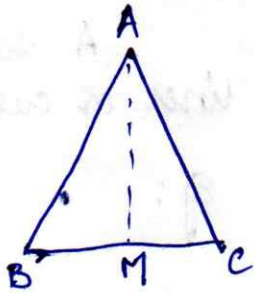
a. Triangle:

The simplest polygon with three sides & three angles is called triangle.

AB, BC, AC - 3 sides of the triangle.

A, B & C are - 3 angles of the triangle.

BC - base of triangle AM - height of triangle.



* Perimeter of triangle:

let a, b & c be the lengths of the triangle

Perimeter = a+b+c.

* Area of triangle.

Area = $\frac{1}{2}$ x base x height.

= $\frac{1}{2}$ x b x h.

b - base length.

h - height length.

b. Quadrilateral

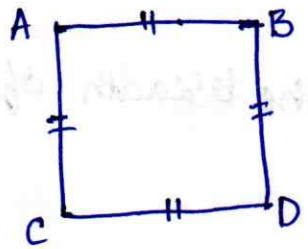
The polygon with four sides & four angles is called. Quadrilateral.

Quadrilateral are further classified based on the length of their sides & angles.

1. Square:

A quadrilateral in which all the sides are equal & interior angles are all right angles is called Square.

- The opposite sides are parallel to each other.



- AB, BD, AC & CD are equal sides.
- AB, CD & AC, BD are pairs of parallel sides.

A, B, C & D are right angles.

Perimeter of Square:

Let the length of the side of the square is 'a'

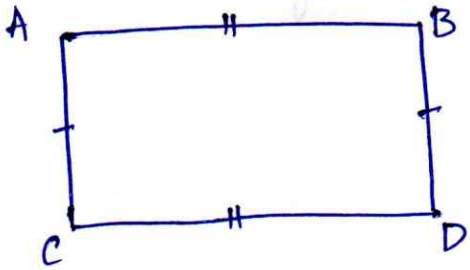
Perimeter = 4 x a = 4a.

Area of the Square = a x a = a^2

2. Rectangle:

A quadrilateral in which only the opposite sides are equal & interior angles are right angles is called Rectangle.

* The opposite sides are parallel to each other.



- AB & CD, AC & BD are parallel sides where AB = CD & AC = BD.
- A, B, C & D are right angles.

Perimeter of Rectangle:

Let the length be 'l' & breadth 'b' of a rectangle.

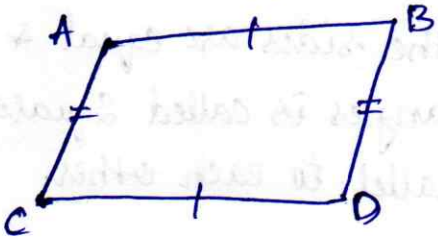
Perimeter = $2 \times (l + b)$.

Area of the rectangle = $l \times b$.

3. Parallelogram:

A quadrilateral in which opposite sides are equal & the interior opposite angles are equal is called parallelogram.

* The opposite sides are parallel to each other.



- CD & AB, AC & BD are pairs of equal & parallel sides.
- A & D, C & B are opposite angles & are equal.

Perimeter of Parallelogram:

let 'l' be the length & 'b' be the breadth of the Parallelogram.

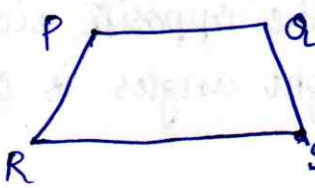
Perimeter = $2 \times (l + b)$.

Area = $l \times b$.

4. Trapezium:

A quadrilateral in which one pair of opposite sides are parallel is called trapezium.

All four sides & four angles are not equal.



- PQ & SR are parallel sides.
- All sides are of different length.
- All the interior angles are not equal.

Perimeter of Trapezium:

let 'a' & 'b' be the length of parallel sides of the trapezium & 'c' & 'd' be the length of the other two sides of the trapezium.

Perimeter = $a+b+c+d$.

Area = $\frac{h}{2}(a+b)$.

d. General formula for area of 2D shapes:

Let us consider the height of each of the shapes as 'h' & 'b' as the base.

Formula in 2-Steps:

S1: Find the average of the parallel sides of 2D shape

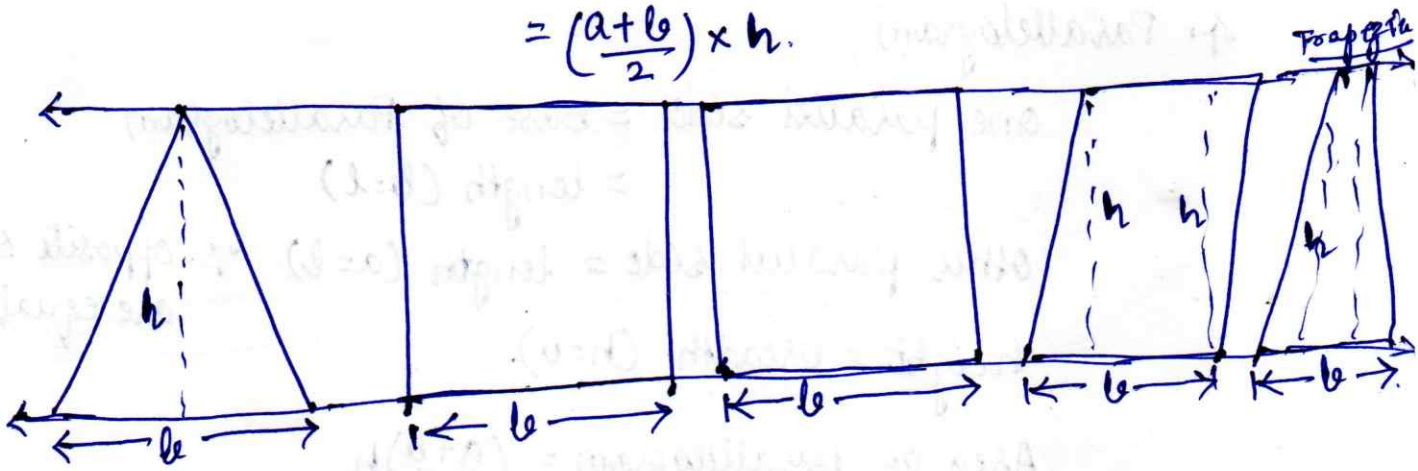
S2: Find the height of the 2D shape.

* Multiply the answers of steps 1 & 2 to get the area.

Let 'a' & 'b' be the parallel sides (where 'b' is the base, & 'h' be the height of the 2D shapes).

Area of 2D shape = Average of parallel sides x height

$= \left(\frac{a+b}{2}\right) \times h.$



Find the area of each geometrical shapes:

1. Triangle

one parallel side = base of triangle (b).
other parallel side = Zero (i.e. a=0).

height of triangle = h.

Area of triangle = $\left(\frac{a+b}{2}\right) \times h = \left(\frac{b+0}{2}\right) \times h$.

$= \frac{1}{2} \times b \times h$.

2. Square

one parallel side = b (base of square)

other parallel side = a ($a = b$).

Height of the Square = h ($h = b$)

Area of Square = $\left(\frac{a+b}{2}\right) \times h = \left(\frac{b+b}{2}\right) \times b$.

$= b \times b = b^2$.

3. Rectangle

one parallel side = base of length (b) = l.

other parallel side = base of ~~length~~ breadth (b) = b.

height of Rectangle = $h = b$.

Area of Rectangle = $\left(\frac{a+b}{2}\right) \times h$

$= \left(\frac{l+b}{2}\right) \times b = \underline{l \times b}$.

4. Parallelogram

one parallel side = Base of Parallelogram

= length ($b = l$)

other parallel side = length ($a = l$) ∵ opposite sides are equal.

height = breadth. ($h = b$).

Area of parallelogram = $\left(\frac{a+b}{2}\right) h$

$= \left(\frac{l+l}{2}\right) \times b$

$= \underline{l \times b}$.

5. Trapezium

One parallel side = base ($b = a$)

Other parallel side = other side ($a = b$).

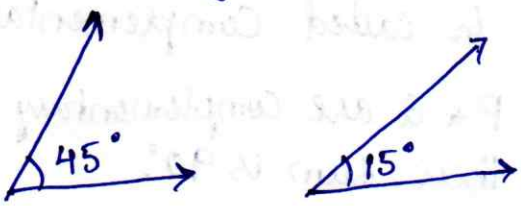
height of trapezium = h.

Area of trapezium = $\left(\frac{a+b}{2}\right) \times h$.

e. Classification of Angles.

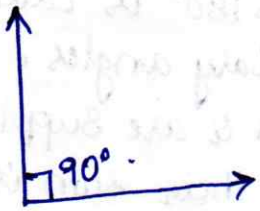
I. Based on measurement of Angles:

1. Acute Angle:



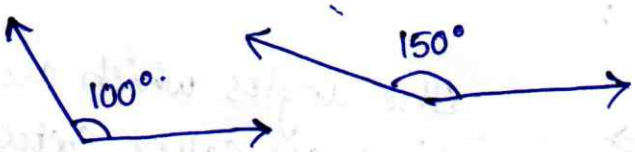
* if the angle measures b/n 0° to 90°.

2. Right Angle.



* If an angle measures exactly 90°.

3. Obtuse Angle

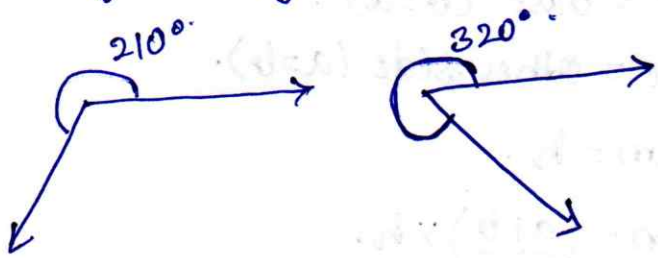


* if an angle measures b/n 90° to 180°.

4. Straight angle.

* if an angle measures exactly - 180°.

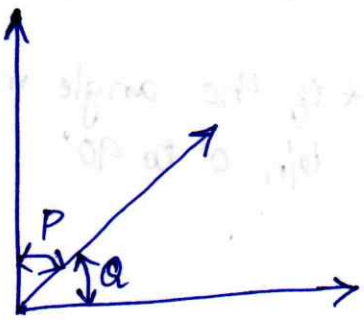
5. Reflex Angle.



* if an angle measures b/n 180° to 360°.

II Based on Sum of Angles

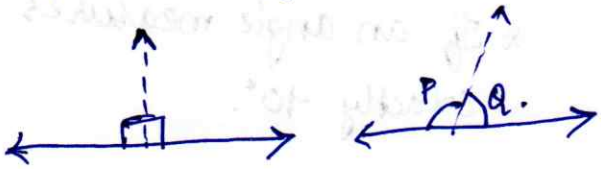
1. Complementary Angles



* pair of angles whose sum is 90° is called Complementary angles.

P & Q are complementary angles as their sum is 90°.

2. Supplementary Angles

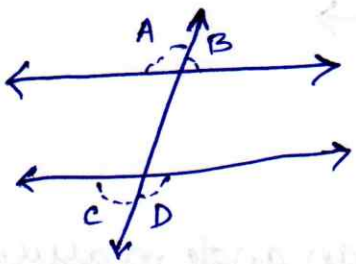


* pair of angles whose sum is 180° is called Supplementary angles.

P & Q are Supplementary angles as their sum is 180°.

III Based on Traversal line :.

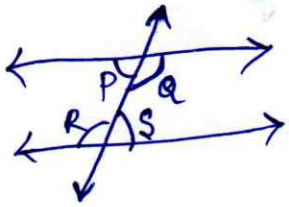
1. Exterior Angles :



The angles which are outside the lines are called Exterior Angles.

A, B, C & D are exterior Angles.

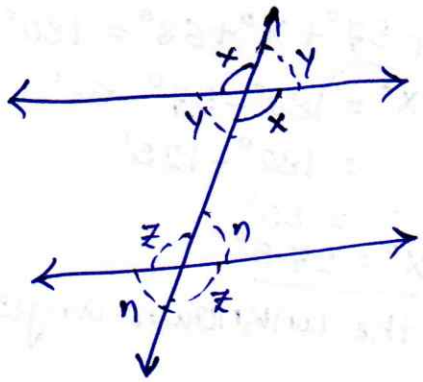
2. Interior Angles.



The angles which are inside the lines are called Interior Angles.

P, Q, R & S are interior Angles.

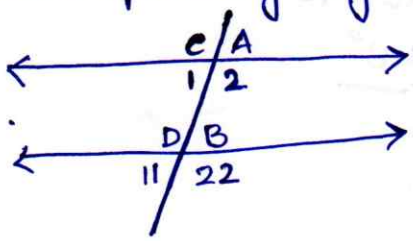
3. Vertically opposite Angles.



A pair of angles which are opposite when two lines intersect are called Vertically opposite angles.

x, y, n, z are opposite angles.

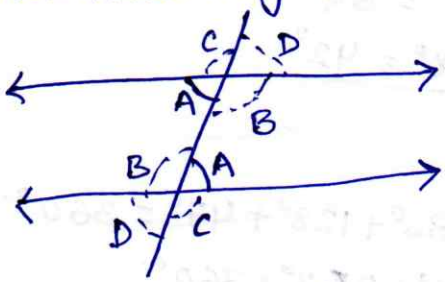
4. corresponding Angles.



Two angles on the same side of the traversal line, among which one is interior & other one is exterior are called corresponding angles.

A&B, C&D, 1&11, 2&22 are corresponding angles.

5. Alternate Angles.



Two angles on the opposite sides of the traversal line, either exterior or interior are called Alternate Angles.

A, B, C & D are Alternate Angles.

6. Finding of Angles.

Sankalana Vyavakalanabhyam Sutra.

1. Missing angle in Straight line

Vedic maths has a sutra for solving these types of questions & i.e. Sankalana Vyavakalanabhyam Sutra. It means "By addn. or Sub.".

Eg 1: Find the answer in ??.

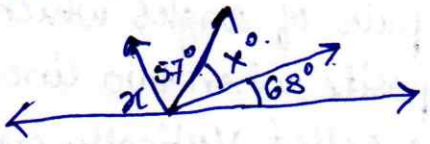
Angle of St. line = 180°.

$97° + x = 180°$

$x = 180° - 97°$

$= 83°$.

Eg: 2 Find what should be the answer in place of x°.



$x^\circ + 57^\circ + x^\circ + 68^\circ = 180^\circ$.

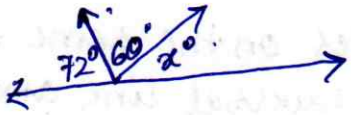
$2x^\circ = 180^\circ - 57^\circ - 68^\circ$.

$= 180^\circ - 125^\circ$.

$= 55^\circ$.

$x^\circ = 27.5^\circ$.

Eg: 3 Find the measurement of the unknown angle which is denoted by x°.



$x^\circ + 72^\circ + 60^\circ = 180^\circ$.

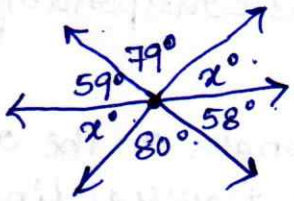
$x^\circ = 180^\circ - 132^\circ$.

$= 48^\circ$.

2. Missing angle at a point

A point has angle of 360°.

Eg 1:



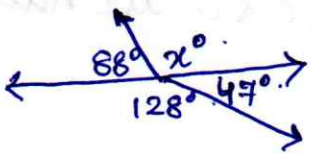
$2x^\circ + 79^\circ + 59^\circ + 58^\circ + 80^\circ = 360^\circ$.

$2x^\circ + 276^\circ = 360^\circ$.

$2x^\circ = 84^\circ$.

$x^\circ = 42^\circ$.

Eg 2: Find the value of x°.

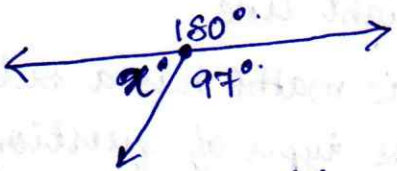


$x^\circ + 88^\circ + 128^\circ + 47^\circ = 360^\circ$.

$x^\circ + 263^\circ = 360^\circ$.

$x^\circ = 97^\circ$.

Eg 3: Find the value of x°.



$x^\circ + 97^\circ + 180^\circ = 360^\circ$.

$x + 277^\circ = 360^\circ$.

$x = 83^\circ$.

3. Missing Angles in Triangle

Triangle has.

a. Three line segments

b. Three angles.

c. Three vertices or Edges.

Triangles are of many types based on two parameters.

a. Based on angles.

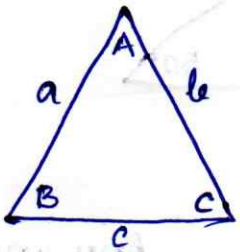
b. Based on sides.

1. Based on sides.

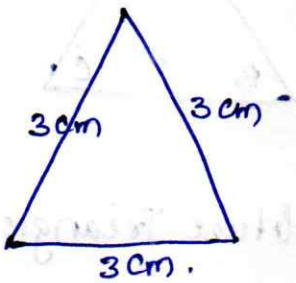
(i) Equilateral Triangle

In this triangle all the three sides are same (measurement).

Eg:



$a = b = c$.

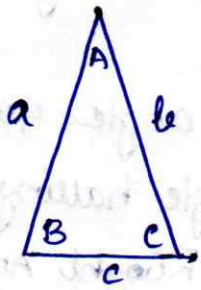


Note: In equilateral triangle, All the 3 sides & angles (= 60°).

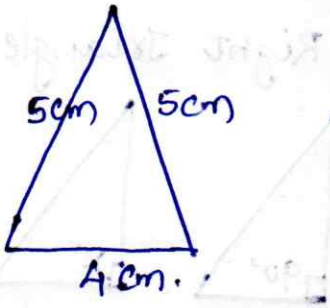
$A = B = C = 60°$.

(ii) Isoceles Triangle

In this triangle any two sides are equal. (measurements).

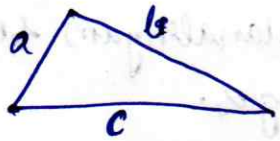


$a = b$

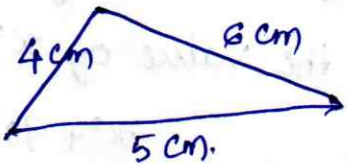


Also, $a = c$ or $b = c$ apply.

(iii) Scalene triangle



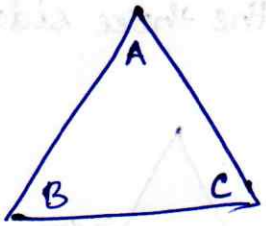
a, b & c are of different sides with different measurements



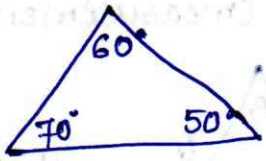
2. Based on Angles.

(i) Acute Triangle

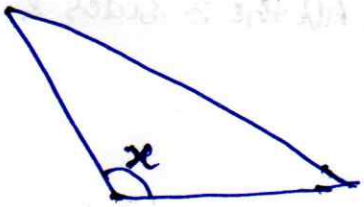
Acute triangle (angle) measure angle b/n 0° & 90°.
So when the angles in a triangle are acute angles then these triangles are known as Acute triangles.

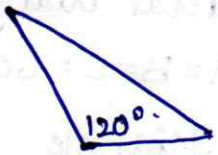


$A < 90°$, $B < 90°$ & $C < 90°$.



(ii) Obtuse Triangle



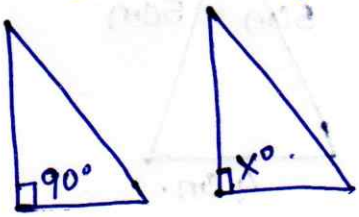


Any angle $> 90°$ is called Obtuse Angle ($< 180°$)

Obtuse Angle $> 90° - x - < 180°$

A triangle having an Obtuse angle is called Obtuse Angle Triangle.

(iii) Right Triangle



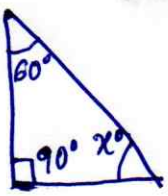
$x = 90°$.

A right angle = 90°.
A triangle having 90° angle is called Right Angle Triangle.

Note: Sum of all angles (3) in a triangle = 180°.

→ Using Sankalana Vyavakalanabhyam sutra to finding the missing angles.

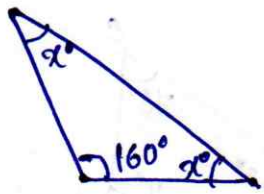
Eg 1: Find the value of $x°$.



$x° + 90° + 60° = 180°$

$x° = 30°$

Eg 2: Find the value of x°.



$x^\circ + 160^\circ + x^\circ = 180^\circ$.

$2x^\circ = 20^\circ$.

$x = 10^\circ$.

9. Finding exterior angle of a triangle

* concept is a combination of both straight line and triangles. We know that straight angle is 180°. and a triangle is also made by straight lines.

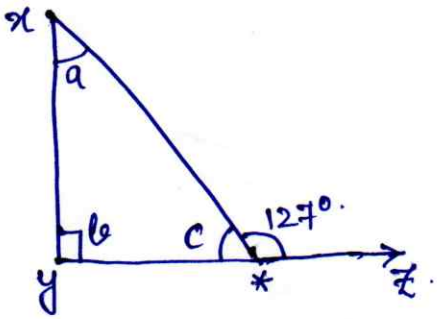


fig a.

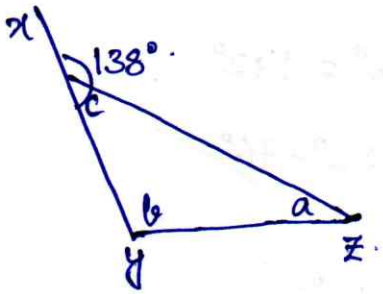


fig b

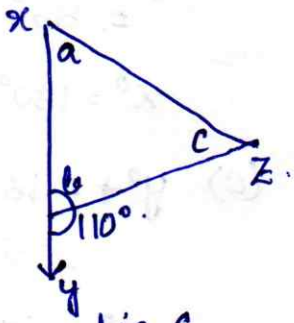


fig c.

Here a, b & c angles are known as interior angles.

127°, 138° & 110° → Exterior angles.

In fig a, a line segment "yz" is extended outside to make exterior angle of 127° but if you see at point (star mark) that point forms a straight line angle is 180°.

$\angle c + 127^\circ = 180^\circ$

$\angle c = 180^\circ - 127^\circ$

$= \underline{53^\circ}$

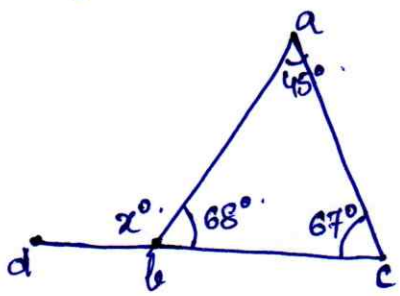
fig b. $\angle c = 180^\circ - 138^\circ$

$\angle c = 42^\circ$.

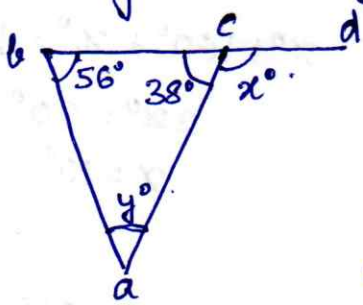
fig c. $\angle c = 180^\circ - 110^\circ$

$\angle c = 70^\circ$.

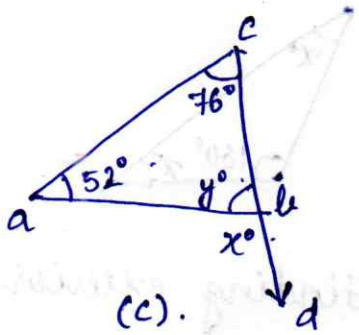
Eg 1: Find the exterior angle in the given triangles.



(a)



(b)



(c).

(a) $x^\circ = 180^\circ - 68^\circ = 112^\circ$.

(b) $y^\circ = 180^\circ - 56^\circ - 38^\circ$.

$= 86^\circ$.

$x^\circ = 180^\circ - 38^\circ = 142^\circ$.

(c) $y^\circ = 180^\circ - 52^\circ - 76^\circ$

$= 52^\circ$

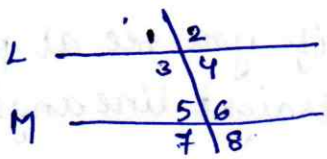
$x^\circ = 180^\circ - 52^\circ$.

$= 128^\circ$.

h. Finding angles using Vilokanam Sutra.

Vilokonam – Observation.

If a traversal line cuts two parallel lines then.

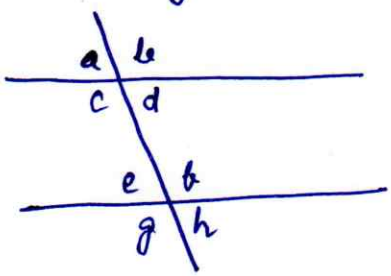


Then $\angle 1 = \angle 8$, $\angle 3 = \angle 6$,

$\angle 4 = \angle 5$, $\angle 2 = \angle 7$.

Interior & Exterior Angles:

Eg 1: using Vilokanam sutra show the vertically opposite angles



$\angle a = \angle d$

$\angle e = \angle h$

$\angle b = \angle g$

$\angle b = \angle c$

2. Eg: Show the Supplementary angles using Vilokanam Sutra

$\angle a + \angle b = 180°$

$\angle c + \angle d = 180°$ etc.

3. Eg: Name Interior & Exterior angles

Interior angles = c, d, e, f.

Exterior angles = a, b, g, h.

Alternate interior angles = c & f, e & d.

Alternate exterior angles = a & h, b & g.

4. Find the corresponding angles

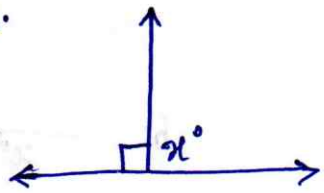
corresponding angles - b & f.

- d & h
- a & e.
- c & g.

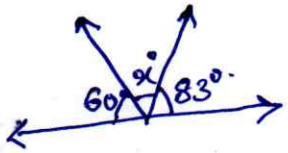
EXERCISE.

I. Find the missing angle in a straight line.

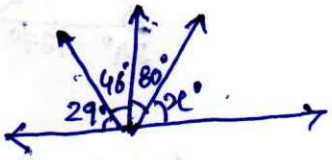
a.



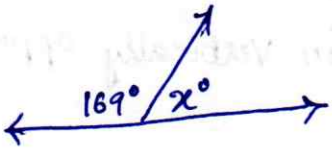
b.



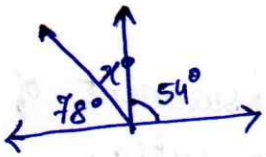
c.



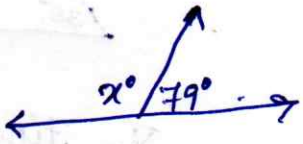
d.



e.

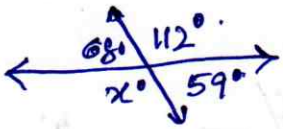


f.

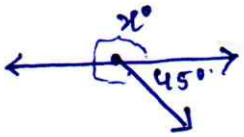


II. Find the missing angle of a point

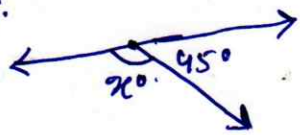
a.



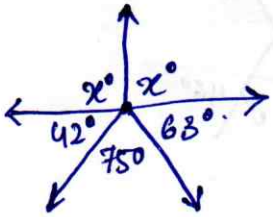
b.



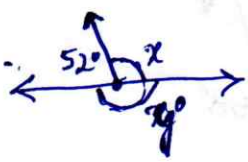
c.



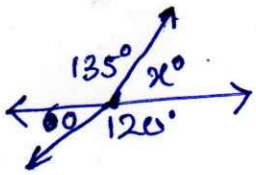
d.



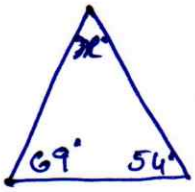
e.

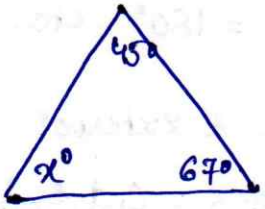


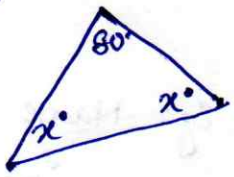
f.

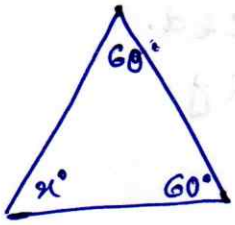


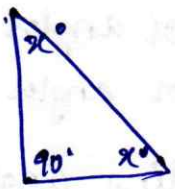
III. Find the Measurement of missing angle in a triangle.

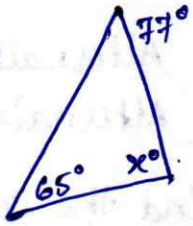
a. 

b. 

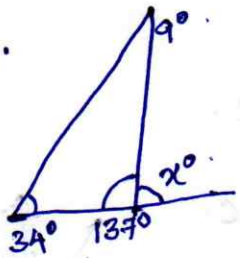
c. 

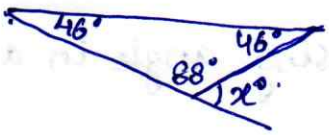
d. 

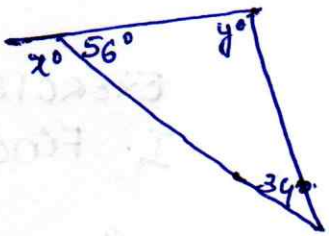
e 

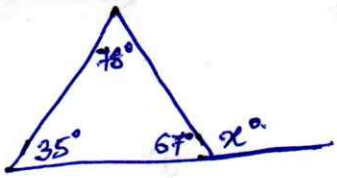
f. 

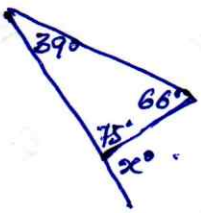
IV Find the measurement of Exterior angles

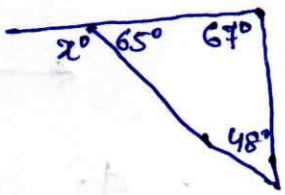
a. 

b. 

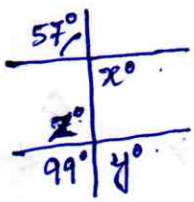
c. 

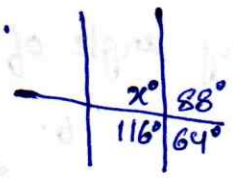
d. 

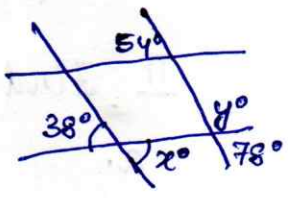
e. 

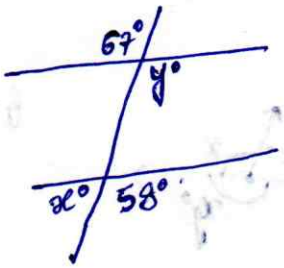
b. 

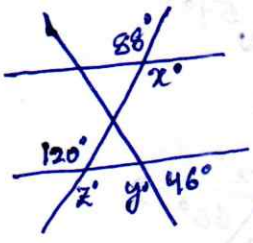
V Find the measurement of unknown Vertically opposite angles by Vikolanam sutra.

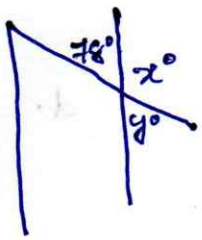
a. 

b. 

c. 

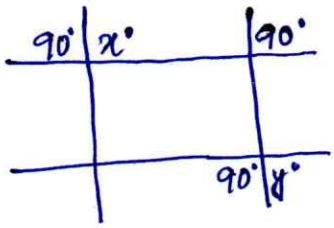
d. 

e. 

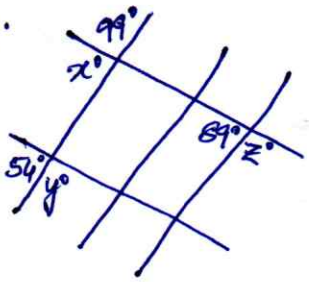
f. 

VI. Find the measurement of unknown supplementary angles by Vilokanam sutra: 10

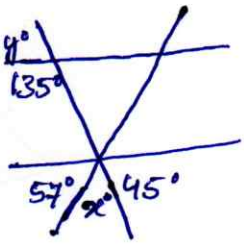
a.



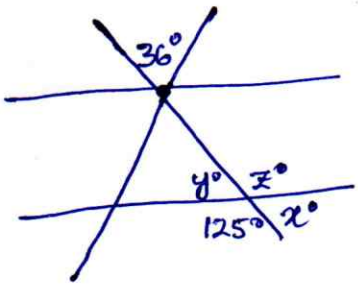
b.



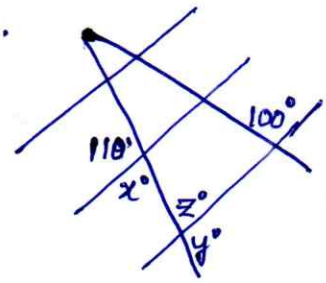
c.



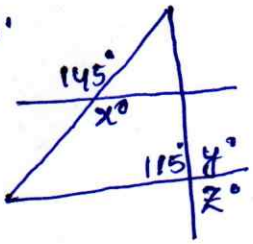
d.



e.



f.



Trignometry

a. Introduction

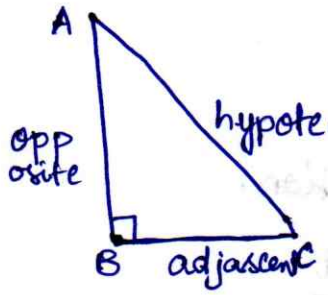
Trignometry → Relationship b/n lengths & angles of a triangle.

Greek word:

Trignometry - 'trigonon' - triangle
& 'metron' - measure.

Hipparchus - Father of Trignometry.

Consider a right angled triangle ABC,



θ (theta) → angle
(0 to 360°)
θ → Greek Alphabet.

In a triangle - Sum of angles = 180°
In a right angled triangle; one angle is 90°. The other two angles are complementary.

b. Trignometric functions

I 1. Sine function (Sin).

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

2. Cosine function (cos).

$$\cos\theta = \frac{\text{Adjascent}}{\text{hypotenuse}}$$

3. Tangent function (tan)

$$\tan\theta = \frac{\text{opposite}}{\text{adjascent}}$$

II Reciprocal of the above functions

1. cosecant function (cosec)
   - Reciprocal of sine function or ratio of hypotenuse to opposite

$$\text{cosec } \theta = \frac{\text{hypotenuse}}{\text{opposite}} = \frac{1}{\sin \theta}$$

2. Secant function (Sec)
   - Reciprocal of cosine function.

$$\sec \theta = \frac{1}{\cos \theta} = \frac{\text{hypotenuse}}{\text{adjascent}}$$

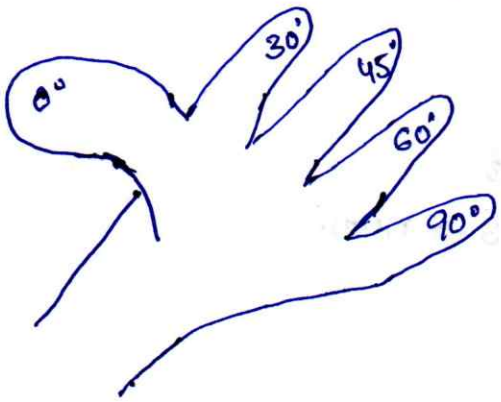
3. cotangent function (cot)
   - Reciprocal of tangent function.

$$\cot \theta = \frac{1}{\tan \theta} = \frac{\text{adjascent}}{\text{opposite}}$$

Trignometric ratios of standard angles.

Ratio \ θ	0°	30°	45°	60°	90°
sin θ	0	1/2	1/√2	√3/2	1
cos θ	1	√3/2	1/√2	1/2	0
tan θ	0	1/√3	1	√3	∞
cosec θ	∞	2	√2	2/√3	1
sec θ	1	2/√3	√2	2	∞
cot θ	∞	√3	1	1/√3	0

c. Trick to remember Trignometric ratio.



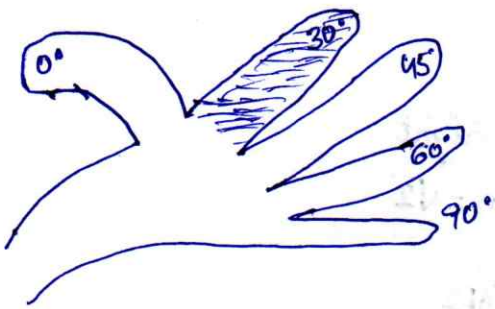
S1: denom 2 common for every calculation

S2: num → depends on 'angle' & the 'trignometric' function.

- First choose the finger based on the angle & fold the finger.
- Based on the trignometric function count the number of fingers beside the folded finger.
- Suppose it is a sine function, then we count the number fingers on the left side of the folded finger
- cosine → right side

- Take the sq. root of the number of fingers folded (beside folded finger) & simplify to get the answer.

Example: Sin 30°



num = 1
den = 2

S1: denom → 2

S2: num → angle & function.

30° → fold finger for sign count.

left of folded finger → Sq. Root of 1 = 1.

= 1/2.

Sin θ = Sin 30° = 1/2.

2. Example - $\sin 60^\circ$

S1: denom = 2.

S2: num = ?

{ left - 3 ; (sin) → left.
{ Sq. root of 3 → $\sqrt{3}$ → num.

$\sin 60^\circ = \sqrt{3}/2$.

3. $\cos 30^\circ$.

S1: den = 2

S2: num = ?

{ Right - 3 ; (cos) → right
{ Sq. root of 3 → num = $\sqrt{3}$.

$\cos 30^\circ = \frac{\sqrt{3}}{2}$.

4. $\cos 90^\circ$.

S1: den = 2

S2: num = ?

{ Right → 0 ; (cos) → right
{ Sq. root of 0 → num = 0.

$\cos 90^\circ = \frac{0}{2} = 0$

5. $\cos 45^\circ$

S1: den = 2

S2: num = ?

{ Right → 2 ; (cos) → right.
{ Sq. root of 2 → num = $\sqrt{2}$

$\cos 45^\circ = \frac{\sqrt{2}}{2} = 1/\sqrt{2}$

d. Heights & distances

Applications of trigonometric ratios / functions & values.

Line of sight:

The line joining the points which represent the observers eye & the object is called 'Line of sight'

Horizontal line:

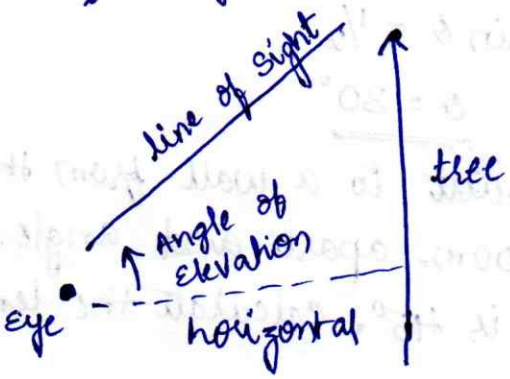
The line which is drawn horizontal from the point representing the observers eye is called 'Horizontal line'

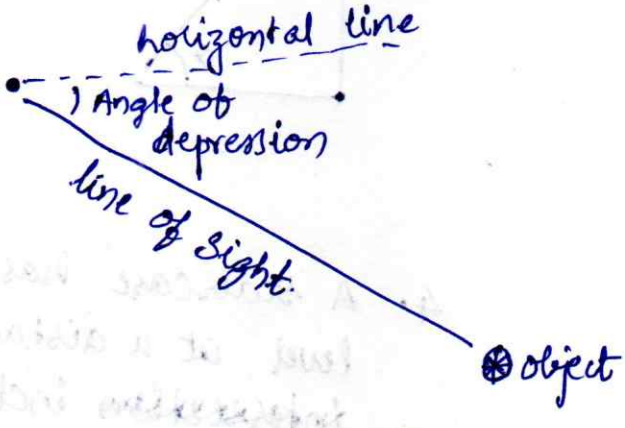
Angle of elevation:

The angle b/n the 'Horizontal' & 'Line of sight' when the object is above the horizontal line is called the 'Angle of elevation'.

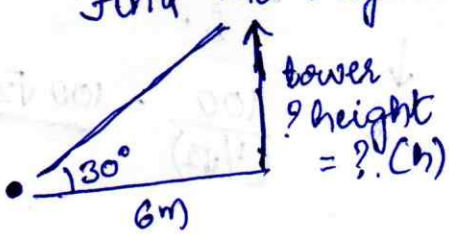
Angle of depression:

The angle b/n the 'horizontal line' & 'Line of sight', When the object is below the horizontal line is called 'Angle of depression'.





1. Eg: Shiny is watching a bird on the top of the tower which is 6m away & the angle of elevation is 30°. Find the height at which the bird is sitting.

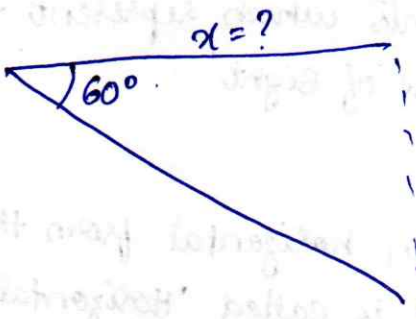


$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}} = 1/\sqrt{3}$$

$$\Rightarrow \frac{h}{6m} = 1/\sqrt{3}$$

$$h = \frac{6}{\sqrt{3}} = \frac{2*3}{\sqrt{3}} = 2\sqrt{3}.$$

2. Example: The angle of depression of an animal from a hilltop is 60°. Calculate how far is the animal from the hill which is $200\sqrt{3}$ ft.



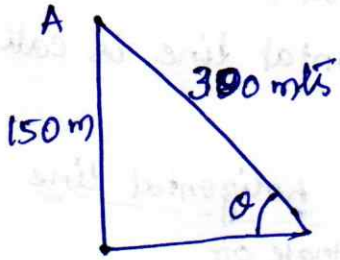
$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

$$= \frac{200\sqrt{3}}{x} = \tan 60°$$

$$x = \frac{200\sqrt{3}}{\tan 60°}$$

$$x = \frac{200\sqrt{3}}{\sqrt{3}} = 200 \text{ mts}$$

3. Example: A glider goes down a ramp of length 150m. & the ramp starts 300m. above the ground. Calculate the angle of inclination of the ramp.



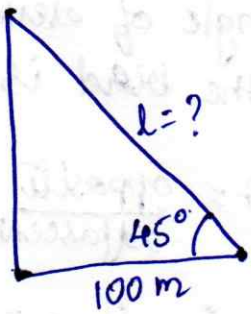
$$\sin\theta = \frac{\text{opposite}}{\text{adjacent}}$$

$$= \frac{150\text{ m}}{300\text{ m}} = 1/2.$$

$$\sin\theta = 1/2$$

$$\theta = 30°$$

4. A staircase has to be built to a wall from the ground level at a distance of 100m. apart and angle of inclination is 45°; calculate the length of staircase.



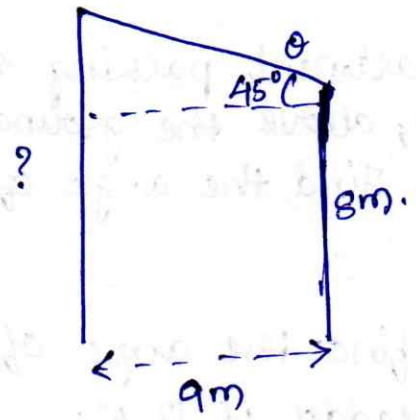
$$\cos\theta = \frac{\text{adjacent}}{\text{opp.}}$$

$$\cos 45° = \frac{100\text{ m}}{l}$$

$$\downarrow$$

$$l = \frac{100}{(1/\sqrt{2})} = 100\sqrt{2}.$$

5. Two poles are 9m apart and angle of elevation from shorter pole is 45°. calculate the length of longer pole if the shorter one is 8m.



$\tan\theta = \frac{\text{opposite}}{\text{adjasecent}}$

$\tan 45° = \frac{\text{opposite}}{9}$

$\downarrow$

$1 = \frac{\text{opposite}}{9}$

opposite = 9.

Total length = 9 + 8 = 17m ans.

EXERCISE

I. Solve the following:

1. The angle of elevation of a tower is 60°, which is 800m tall. Find the distance from where the observer is watching the tower.

2. A sailor can sight the light house on the shore at an angle of elevation 45° and at a distance of 4 km. Calculate the height of the light house.

3. An 80 ft. tall building casts a shadow of length 80 ft. Calculate at which angle the sun rays pass the top of the building.

4. The angle of depression of a car in parking space from an apartment is 30°. If the parking space is 150 m away from the apartment, then find at what height the observer is from the ground. calculate the length of the ramp.

5. A cycle ramp is to be built at an inclination of 45° & one end of the ramp should be 600m high from the ground.

6. A slope has to be built at an apartment parking such that one end of the slope is 45m, above the ground & the length of the slope is 90m. Find the angle of inclination.

7. A ladder is placed to a wall. find the angle of inclination, if the length of the ladder is 12 ft. The distance b/n the wall & foot of the ladder is 12 ft.

8. There are two towers 4m apart and a path-way connects these two towers top which is 800 cm long. Find the angle of elevation from the top of one tower to another.

9. Two buildings are of length 15 ft. & 18 ft. respektively. The angle of depression from the taller building to shorter building is 60°. Calculate the distance b/n the two buildings.

10. Two poles are 30m apart & tied by a rope at the top ends. The angle of elevation from the top of shorter pole is 30°. Calculate the length of the rope used to tie the poles.