1. Numbers:

Number is a mathematical object used to count, label & measure.

a. Types of Numbers:

(i). Natural numbers:

Counting numbers $\{1, 2, 3 \ldots\}$ till infinity

Eg: 1, 15, 18, 386 etc.

(ii) Whole Numbers:

Natural numbers including '0'. $\{0, 1, 2, 3 \ldots\}$.

Eg: 0, 7, 13, 274 etc.

! All Natural numbers are whole numbers.

But, All Whole numbers are NOT natural numbers.

(iii) Integers:

Positive & Negative Counting Numbers as well as '0'.

Eg: ....,-5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5 ....

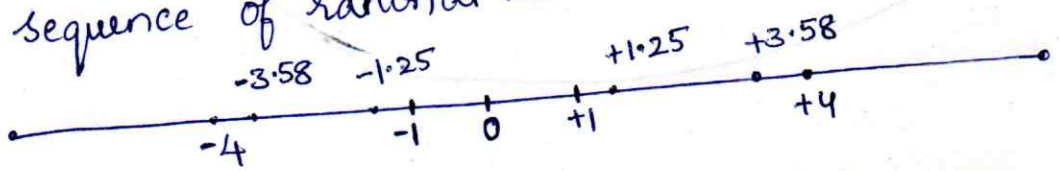
(iv) Rational Numbers:

Numbers that can be expressed as a fraction of an integer & a non-zero integer.

Eg: $\frac{1}{2}$, $\frac{3}{4}$, $\frac{1}{4}$, $-\frac{1}{2}$, $-\frac{1}{3}$ etc.

(v) Real Numbers:

All Numbers that can be expressed as the limit of a sequence of rational Numbers.



Every real Number corresponds to a point on the Number line.

(VI). Irrational Numbers:

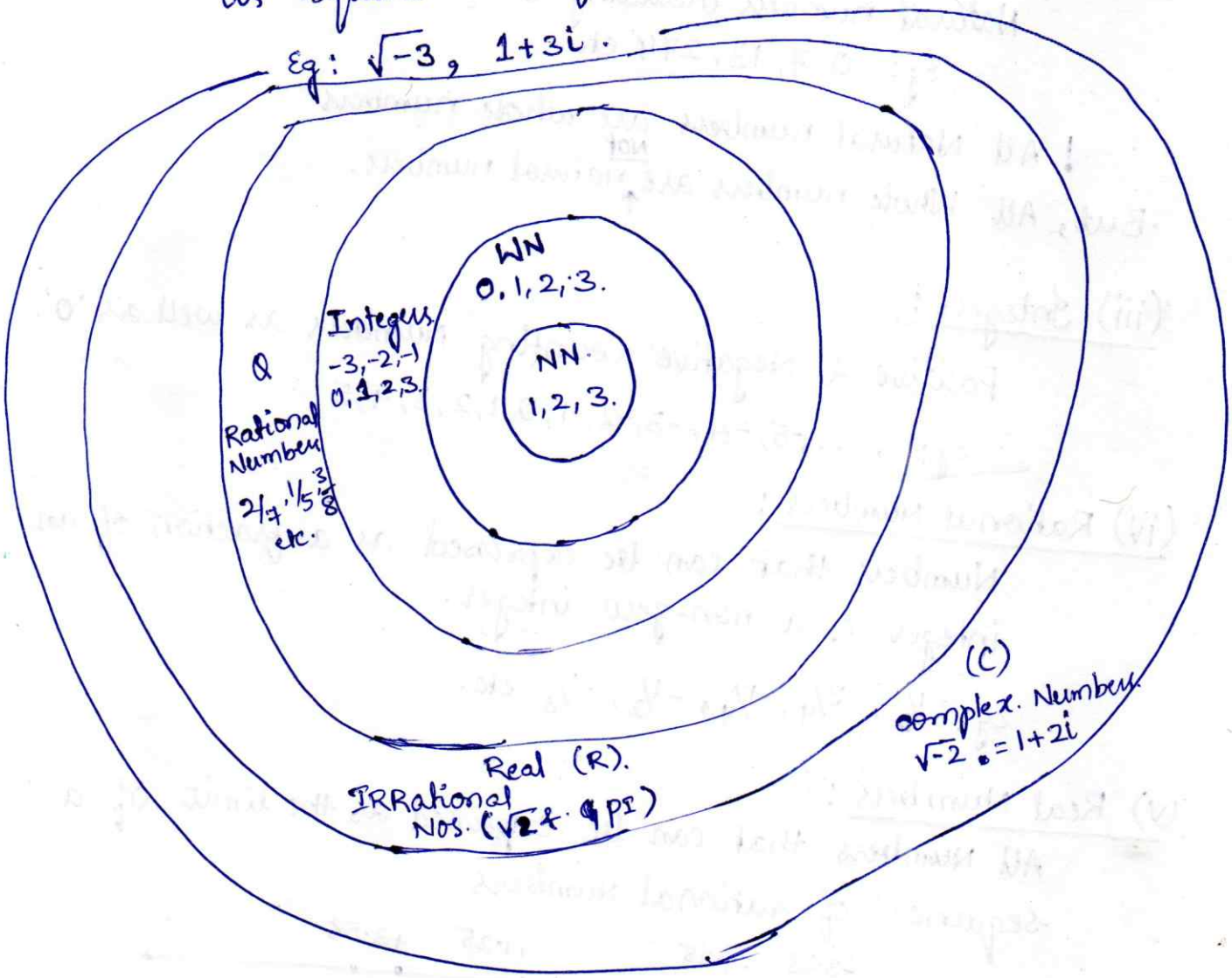
A real no. which is not a rational is called. irrational Number.

Eg: Square root of 2 = 1.41421356.
Golden Ratio (PI) = 1.6180339887 4

(VII) Complex Numbers:

Includes real Numbers & imaginary Numbers, such as square root of -ve number.

Eg: $\sqrt{-3}$, $1+3i$.



b. Prime, Composite, Even & Odd Numbers:

(i) Prime Numbers:

Prime Number is a natural number which are divisible by '1' & 'itself' only.

Eg: 5 is a prime number.
(divisible only '1' & itself).

(ii) Composite Numbers:

Natural Number >1, that is not prime is called Composite Number.

Eg: '6' is a composite number
(∵ can be divisible by '2' & '3', along with '1' & '6').

(iii) Even Numbers:

Any Number (Integer) that can be divisible by '2' (exactly) is called an Even Number.

Eg: 42, -68, 374, -14 etc.

(iv) Odd Numbers:

Any Number (Integer) that cannot be divisible by '2' (exactly) is called an Odd Number.

Eg: 59, 3, 91, 373 etc.

2. Basic Operators:

An operator is a special symbol / phrase that is used to change, check or combine values.

There are '4' basic operators to perform mathematical calculations:

a. Addition
b. Subtraction
c. Multiplication &
d. Division.

a. Addition : (+) - plus sign.

Total Amount of objects together (in a collection)

Eg: $5 + 10 = 15$.

- 5, 10 → Addends
- 15 → Sum/Total.

b. Subtraction : (-) - Minus sign.

Removing the objects (from a collection).

Eg: $24 - 17 = 7$

- 24 → Minuend
- 17 → Subtrahend
- 7 → difference

c. Multiplication : (x) - cross/into

Scaling one number by another

Eg: $8 \times 4 = 32$ → product.

- 8 → Multiplicand (8 times 4)
- 4 → Multiplier

d. Division : (÷ or — or /) - 'by' or 'divided by'.

* $\frac{\text{dividend}}{\text{divisor}}$

Eg: $14 \div 2$ ($\frac{14}{2}$ or $14/2$) $= 7$.

- 14 → dividend
- 2 → divisor
- 7 → Quotient.

3. Sum Root / Digital Root / Digit Sum / add-up.

converting a given Number into a single digit by adding up all the digits in the given Number.

* This method helps to 'check answers' - addition, Subtraction Multiplication, division, Squares, square roots, Cube roots etc.

Eg: (i) Find the digit sum of 2467539.

Ans: $2+4+6+7+5+3+9 = 36$.

$3+6 = 9$.

Hence digit sum = 9.

(ii) Find the digit sum of 56768439

Ans: $5+6+7+6+8+4+3+9 = 48$

$4+8 = 12$

$1+2 = 3$.

Hence digit Sum = 3.

a. Cast of Nines (9):

Any Numbers adding to '9' will result in same Numer

or 'total 9' will be eliminated.

Totalling: 6,3 sums upto 9.
2,7 'these Numbers
1,8 are eliminated while adding up'

Eg: $9+7 = 16$

$1+6 = 7$

Eg: $3+6+8 = 17$

$1+7 = 8$

Eg: (i) Find the digit sum of 12919.

Ans: $1+2+9+1+9 = 4$.

or 12919

$1+2+1 = 4$.

(ii) Find the digit sum of 63727

Ans: $6+3+7+2+7 = 7$.

or $63727 = 7$

b. Answer Verification: (dR, SR, dS, AUP)

(i) Addition:

1. Verify 1827365 + 9988888 + 6300852 + 11111111
= 45674496

digit sum of addends together = digit sum of Sum/Total.

(DS) Sum of Addends) 0 + 4 + 6 + 8 = 9

(SR of Sum/Total) 6+7+4+4+6 = 27 = 9.

2. Verify 978 + 1846 = 2824.

ds(a1) + ds(a2) = ds(s).

(ii) Subtraction:

1. Verify 63 - 21 = 42.

digit sum of Minuend - digit sum of Subtrahend = digit sum of difference

9 - 3 4 2 = 6.

2. Verify. 5798 - 3672 = 2126.

* One limitation is that some cases, Minued (SR) < Subtrahend (SR).
In such cases, Keep Minued at higher value. (Add '9' to Minuend).

3. Verify 67458 - 43756 = 23702

ds(minuend) - ds(Subtrahend) = ds(difference)

(iii) Multiplication:

1. Verify $467532 \times 107777 = 50389196364$

$9 \times 2 = 9$

$= 18 = 9$

$1+8 = 9$

digit sum of Multiplicand x digit sum of Multiplier = digit sum of product

2. Verify $999816 \times 727235 = 727101188760$.

ds(Multiplicand) x ds(Multiplier) = ds(product)

(iv) Division:

1. Verify whether 2308682040 (dividend) divided by 36524 (divisor) equals 63210. (quotient)

ds of dividend = ds of divisor x ds of quotient + ds of Remainder.

$6 = 2 \times 3 + 0$

$= 6 + 0$

$= 6.$

2. Verify $89/3$

$Q = 29 \quad R = 2.$

ds(dividend) = ds(divisor) x ds(quotient) + ds (Remainder).

Hint: (Important).

"The digit-sum method can only tell us whether the answer is wrong or not. It cannot tell us with complete accuracy whether the answer is correct or not."

* digit sums match - Most likely correct.
* Do not match - Wrong.

(V) Squaring / Square rooting :

$ds(\sqrt{\ }) * ds(\sqrt{\ }) = ds(sq.)$

Eg: 23 is square root of 529.
Verify

✓ digit sum of square root = 2+3 = 5.
So. 5 × 5 = 25 = 7.

✓ digit sum of square = 5+2+9 = 7

(VI) Cubing / Cube rooting :

$ds(\sqrt{\ }) * ds(\sqrt{\ }) * ds(\sqrt{\ }) = ds(cube)$.

Eg: Verify whether 2197 is the cube of 13.

$ds(\sqrt{2}) = 1+3 = 4$

$\Rightarrow 4 \times 4 \times 4 = 64 = 1$

$ds(cube) = 2+1+9+7 = 1$.

Restriction Eg:

product of $9993 \times 9997 = 99900021$

$= 3 \times 7 = 2+1$

$21 = 2+1$

$2+1 = 2+1$

$3 = 3$

However;
if we get
99900012.
↓
Here also verification is Right.

Hence Answer verification
- fails 'wrong'.
- tallies 'Most likely correct'

4. Complementary (parama Mitra):

* Two digits are complementary, if their sum is '10'.

Eg: 3 & 7 are complementary.
(∵ 3+7=10).
complementary for 2 is 8 etc.

Complementary to 9

* Two digits are complementary (9), if their sum is '9'.

Eg: 1 is complementary 8.
complementary of 3 is 6 etc.

Exercise:

PART A:

Q. (1). Instantly calculate the digit-sum of the following numbers:

(1) 23456789
(2) 27690815
(3) 7543742
(4) 918273645

PART B:

Q. (1). Verify whether the following answers are correct or incorrect without the actual calculation

(1) $95123 \times 66666 = 6341469918$
(2) $838102050 \div 12345 = 67890$
(3) $88^3 = 681472$
(4) $88^2 = 7444$
(5) $475210 + 936111 + 315270 = 726591$
(6) $9999999 - 6582170 = 3417829$
(7) $6582170 - 9999999 = -3417829$

(8) $\frac{900}{120}$ gives quotient 7 & remainder 60

(9) $0.45632 \times 0.65432 = 0.2985793024$.

PART C :

Q (1). Select the correct answer from the alternatives without doing actual calculation.

(1) $3569 \times 7129 =$ _____

(a) 25443701

(b) 25443421

(c) 25443401

(d) 25445401.

(2) $6524 + 3091 + 8254 + 6324 + 7243 + 5111 + 9902 + 3507 =$ _____

(a) 49952

(b) 49852

(c) 59956

(d) 49956

PART D :

Q 1. Find the complementary 10. for the below numbers.

a. 6 b. 3 c. 9 d. 7 e. 5.

2. Find the complimentary 9. for the below numbers.

a. 3 b. 6 c. 8 d. 2 e. 9.

Tables

Table 1 : Series of counting numbers 1 to 10.

Table 2 :

00	02	04	06	08
10	12	14	16	18
				20

Table 3 :

03	06	09
12	15	18
21	24	27
		30

Table 4 :

04	08
12	16
	20

24	28
32	36
	40

Table 5 :

00	10	20	30	40	50
05	15	25	35	45	55

Table 6 :

03	06	09
12	15	18
21	24	27

30

33	36	39
42	45	48
51	54	57

60

Table 7 :

07	14	21
28	35	42
49	56	63

70

Table 8 :

S1	S2
0	8
1	6
2	4
3	2
4	0
4	8
5	6
6	4
7	2
8	0

S1 ↓ Numbers 0-8 Repeating '4'

↑ S2 Numbers 'even' 0 to 8. ~~Repeating~~ Repeating them.

Table 9 :

S1	S2
0	9
1	8
2	7
3	6
4	5
5	4
6	3
7	2
8	1
9	0

↑ ↓

Table 10 :

S1	S2
1	0
2	0
3	0
4	0
5	0
6	0
7	0
8	0
9	0
10	0

Note: Learn Tables till '5'.

Rest of the tables can be managed with simple multiplication.

01. Method : Table 6

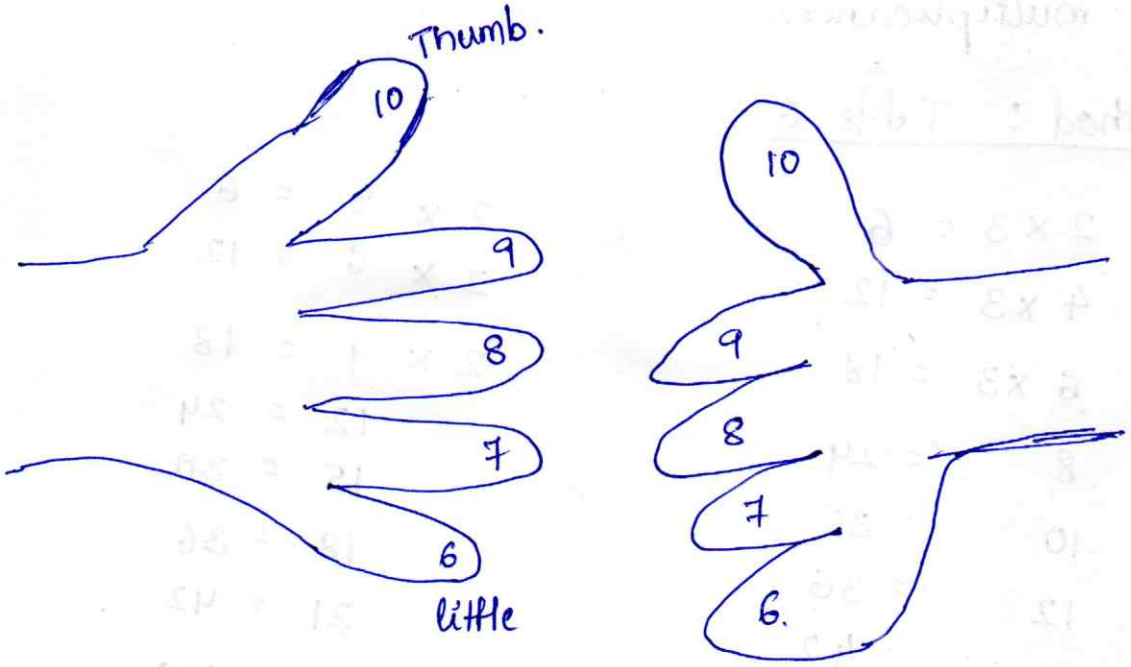
$2 \times 3 = 6$.
$4 \times 3 = 12$
$6 \times 3 = 18$
$8 = 24$.
$10 = 30$
$12 = 36$
$14 = 42$
$16 = 48$
$18 = 54$
$20 = 60$.

$2 \times 3 = 6$
$2 \times 6 = 12$
$2 \times 9 = 18$
$12 = 24$
$15 = 30$
$18 = 36$
$21 = 42$
$24 = 48$
$27 = 54$
$30 = 60$.

Table 8

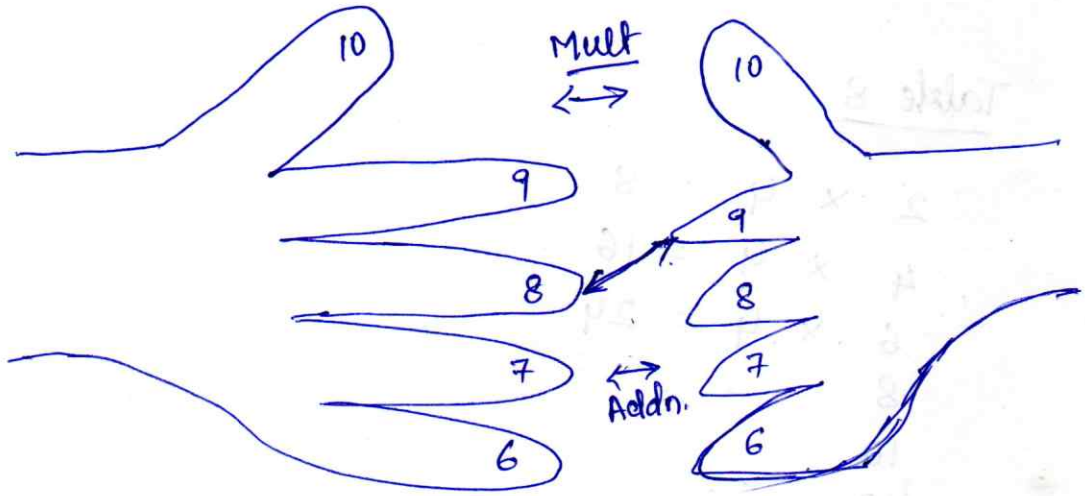
$2 \times 4 = 8$
$4 \times 4 = 16$
$6 \times 4 = 24$
8
10
12
14
16
18
20

02. Method : Using Hands



Eg:

$9 \times 8 =$.



$2 \times 1 = 2$ (Above)

$3 + 4 = 7$. (Below).

Below	Above
7	2

$= 72$

03. Tricks of 2-digit multiplication:

Eg: ① 14×6

$(10+4) \times 6$.

$60+24 = 84$.

② 98×8

$(90+8) \times 8$.

$720+64 = 784$

04. Writing Tables:

Eg: ① 84 Tables

$80 \times 1 + 4 \times 1 = 80+4 = 84$

$80 \times 2 + 4 \times 2 = 160+8 = 168$.

$80 \times 3 + 4 \times 3 = 240+12 = 252$

Eg: ② 96 Tables

9 Table	6 Table.	96 Table.
90	6	= 96
18	← 12	= 192
27	← 18	= 288.

Eg: ③ 976 Table

$900T + 70T + 6T$

	H	T	U.
×1	9	7	6
×2	18	14 ←	12
	18 ←	15	2

1952

Addition & Subtraction

1. Base Method:

Number 'powers of 10' are called 'base numbers'.

Eg: (a) '97' is 3 less than the base Number 100.

Hence Number 97 can be written as '-3'.

(b) '105) is 5 more than the base Number 100.

Hence Number 105 can be written as '+5'

(a). Addition (Base Method)

Condition: Both Numbers are close to 100.

① Eg: 93 + 90.

S1: Base Numbers.

$93 \xrightarrow{(100)} -7$.

$90 \xrightarrow{(100)} -10$.

S2: Add the differences.

$93(100) \rightarrow -7$

$90(100) \rightarrow -10$

-17.

S3: $200 - 17 = \underline{183}$.

② Eg: 82 + 74.

S1:

$82 \xrightarrow{(100)} -18$

$74 \xrightarrow{(100)} -26$

S2: $200 - 44 = \underline{156}$.

2. One more than previous one

a. Addition

While adding numbers, if the sum of digits is more than single digit, we consider that in next digit.

Eg: 263 + 179.

```
  2 6 3
+ 1 7 9
-------
  4 4 2
```

3. Subtraction (Base Method)

(a) Eg: 92 - 89.

S1: Base Numbers

92 (100) → -8 ↓ Sub.
89 (100) → -11 ↓

S2: (100-100) → -8-(-11).

= -8+11 = 3.

(b) Eg: 108 - 97.

S1: 108 (100) → +8 ↓ Sub.
97 (100) → -3. ↓

0 → +8-(-3)

= 8+3 = 11.

4. Subtraction (~~complementary to 10~~ one more than previous one).

Eg: 84 - 36.

Sl: 8 4
*3 6
4 8

'4' is smaller than 6.

① * Find complementary of '6'.
② i.e. 4.
③ Add this 4 to '4' in 84.
④ 8.
⑤ 8* - (3+1) = 4

Eg: 3 9 8
2 4 6
1 5 2

~~① comp of~~
* None of the digits are lesser.

Eg: 4 8 3
*3 *9 6
0 8 7

① complementary of 6 → 4
② Add. 4 to 3. (7).
③ '10' - comp. 0.
④ 8+0 = 8

Multiplication.

01. Multiplication by 5

S1: Multiply by 10.

S2: Divide by 2.

Eg: ① 68×5.

$$\frac{68 \times 10}{2} = 340$$

② 568×5

$$\frac{568 \times 10}{2} = 2840.$$

③ 9846×5

$$\frac{9846 \times 10}{2} = 49230.$$

02. Multiplication by 25

S1: Multiply by 100

S2: Divide by 4.

Eg: ① 9845×25

$$= \frac{9845 \times 100}{4} = \frac{984500}{4} = 246125.$$

② 7643×25

$$= \frac{7643 \times 100}{4} = \frac{764300}{4} = 191075$$

03. Multiplication by 125

S1: Multiplication by 1000.

S2: Divide by 8.

Eg: ① 87×125

$= \frac{87 \times 1000}{8} = \frac{87000}{8} = 10875$

② 255×125

$= \frac{255 \times 1000}{8} = \frac{255000}{8} = 31875$

04. Multiplication by 9

Eg. ① : 56×9

$= 56 \times (10-1)$

$= 560 - 56 = 504$

Eg. ② : 827×9.

$= 827 (10-1)$

$= 8270 - 827 = 7443$

05. Multiplication by 99

Eg. ① : 96×99

$= 96 (100-1)$

$= 9600 - 96$

$= 9504$

-4
-1
$95 \quad 04$

② 7286×999.

$= 7286 (1000-1)$

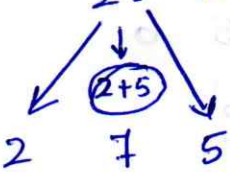
$= 7286000 - 7286$

$= 7278714$

06. Multiplication by 11

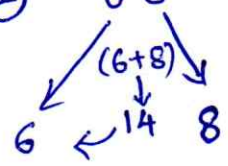
Condition: Two Digit Multiplication

Eg: ① 25 × 11.



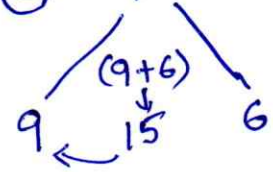
Ans: 275

② 68 × 11



Ans: 748

③ 96 × 11

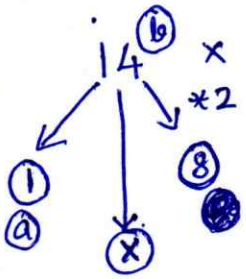


Ans: 1056

07. Multiplication by 12

Cond: Two digit Numbers

Eg: ① 14 × 12.



Ans = 168

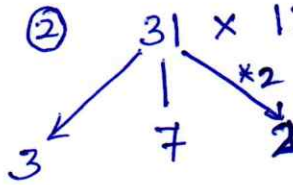
$x = 2 \times a + b$

$= 2 \times 1 + 4$

$= 6.$

2 × Ten's place + Unit's place.

② 31 × 12

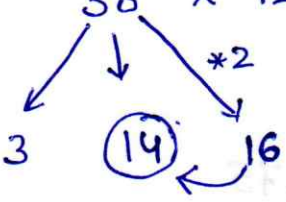


Ans = 372.

$x = 2 \times 3 + 1$

$= 7.$

③ 38 × 12.



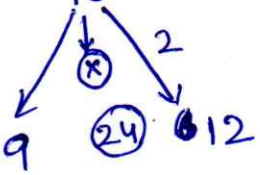
$x = 2 \times 3 + 8$

$= 14$

3 15 6

Ans = 456

④ 96 × 12



$x = 2 \times 9 + 6$

$= 24$

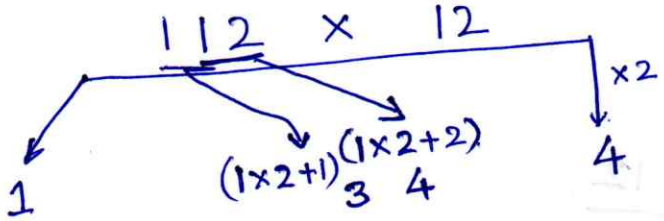
9 25 2

11 5 2

= 1152

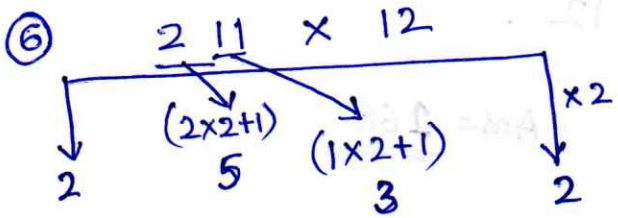
Cond: 3 digit × 12.

⑤ 112 × 12.



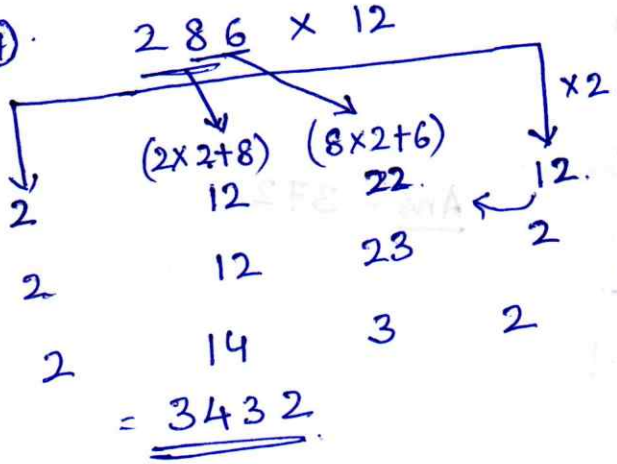
Ans: 1344

⑥ 211 × 12



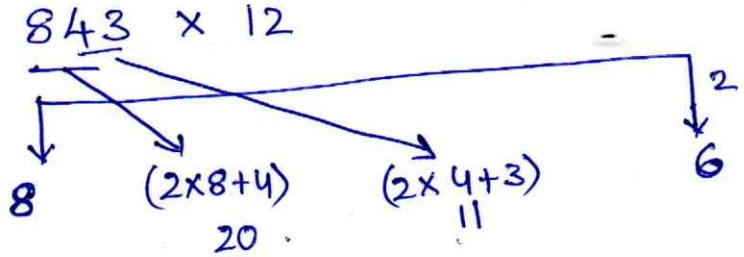
Ans: 2532

⑦ 286 × 12



= 3432

③ 843 × 12



8 21 16.

10116

Specific Simple Methods

1. Squaring of numbers ending with '5'

Q.① Find Square of 65

$6 \times (6+1)$ | 6 5 ← LHS.

6 5

42 25

LHS RHS

RHS = $5 \times 5 = 25$

LHS = $6 \times (6+1)$

$= 6 \times 7 = 42$.

② Find Square of 75

$7(7+1)$ | 7 5

7 5

56 25

③ Find Square of 95

9 5

9 5

90 25

④ Find square of 105

10 5

10 5

110 25

2. Multiplication : Special case

a. Condition :

1. units place digits sum to 10
2. Remaing digits are same (in both the numbers).

Eg: (1)

```
          6 6
6×(6+1)   6 4
         -----
         42 24
```

(2)

```
 10 7
 10 3
------
110 21
```

RHS = 6 × 4 (2 digits)

LHS = 6 × (6+1)

Imp. → units place multiplication two digits (- -)

(3)

```
 9 1
 9 9
-----
9009
```

(4)

```
 5 1
 5 9
-----
30 09
```

(5) 72 × 78 = 5616

(6) 84 × 86 = 7224

(7) 23 × 27 = 621

(8) 89 × 81 = 7209

(9) 106 × 104 = 11024

(10) 1003 × 1007 = 10100 21

b. condition :

* Last two Numbers(digits) from both sum = 100.
* Remaining digits are same

Q.(1): 2048 × 2052.

→ 48 + 52 = 100.

20 48 | 4 8
20 52 ↓ 5 2.

20 × 21 | 2496

4202496

AB = 100
WB = $\frac{100}{2}$ = 50.

48 - 2
52 + 2

2) 50 | -04

(25 × 100) - 04
2500 - 04
2496

Q(2): 23437 × 23463

54990 | 2331

3 7
6 3

✓ 2331

2 3 4
2 3 5

AB = 1000.
WB = $\frac{1000}{4}$ = 250.

234 - 16
235 - 15

4) 219 | 240

1 6
1 5

240

54 3/4 ^ 240.
54 / (3/4 of 1000 + 240)
54 / 750 + 240.
54990

Note: Applicable for units sum = 1000 and rest of digits same.

3. Squaring of numbers b/n 50 & 60

Q. Find the Square of 57

RHS = Multiply unit place digits. (2 places)

LHS = Add 25 + units place digit.

$(5 \times 5 + 7)$

$$\begin{array}{r} 57 \\ \times\ 57 \\ \hline 3249 \end{array}$$

② 50×50

$$\begin{array}{r} 50 \\ \times\ 50 \\ \hline 2500 \end{array}$$

③ 53×53

$$\begin{array}{r} 53 \\ \times\ 53 \\ \hline 2809 \end{array}$$

④ 52×52

$$\begin{array}{r} 52 \\ \times\ 52 \\ \hline 2704 \end{array}$$

$51^2 = 2601$

$52^2 = 2704$

$53^2 = 2809$

$54^2 = 2916$

$55^2 = 3025$

$50^2 = 2500$

$56^2 = 3136$

$57^2 = 3249$

$58^2 = 3364$

$59^2 = 3481$

$60^2 = 3600$

4. Multiplication of numbers with a Series of 9's

Case 1: Multiplying a number with an equal number of 9's

Q. Multiply 654 by 999.

```
  654
x 999
------
653346
```

S1: Sub. '1' from 654 → 653.
S2: Sub. 653 from 999 → 346

Q. Multiply 9994 by 9999

```
 9994
x 9999
-------
99930006
```

Q. Multiply 456789 by 999999

```
 456789
x 999999
---------
456788 543211
```

More examples:

```
  7777
x 9999
-------
77762223
```

```
 65432
x 99999
----------
6543134568
```

```
  447
x 999
-------
446 553
```

```
 90909
x 99999
---------
90908 09191
```

Case 2 : Multiplying a number with a higher number of 9's

Q. Multiply 45 with 999

$$\begin{array}{r} 45 \\ \times\ 999 \\ \hline 044955 \end{array}$$

Q. Multiply 888 with 9999

$$\begin{array}{r} 888 \\ \times\ 9999 \\ \hline 08879112 \end{array}$$

Q. Multiply 123 with 99999

$$\begin{array}{r} 123 \\ \times\ 99999 \\ \hline 00122999877 \end{array}$$

Other Examples :

$$\begin{array}{r} 162 \\ \times\ 9999 \\ \hline 01619838 \end{array}$$

$$\begin{array}{r} 5555 \\ \times\ 99999 \\ \hline 05554\ 94445 \end{array}$$

$$\begin{array}{r} 363 \\ \times\ 999999 \\ \hline 000362999637 \end{array}$$

$$\begin{array}{r} 10101 \\ \times\ 9999999 \\ \hline 0\cancel{10100899}9 \\ \hline 001010049899 \end{array}$$

Case 3 : Multiplying a number with a lower number of 9's

Q. Multiply 654 by 99

$$\begin{array}{r} 654 \\ \times\ 99 \\ \hline \end{array} \quad \rightarrow \quad \begin{array}{r} 65400 \\ -\ 654 \\ \hline 64746 \end{array} \qquad 654 \times (100-1)$$

Q. Multiply 80020 by 999.

$$\begin{array}{r} 80020000 \\ 80020 \\ \hline 79939980 \end{array}$$

5. Multiplication of numbers with a series of 1's

Case 1 : Multiply by 2 1's

Q. Multiply 32 × 11

```
  3 2
x 1 1
-----
3 (3+2) 2

3 5 2
```

Q. Multiply 43 × 11

```
  4 3
x 1 1
-----
4 (4+3) 3

4 7 3
```

Q. Multiply 64 × 11

```
  6 4
x 1 1
-----
6 (6+4) 4

6 10 4

7 0 4
```

Q. Multiply 652 × 11.

```
  6 5 2
x   1 1
-------
  6 11 7 2
    ↘
  7 1 7 2
```

Q. 3102 × 11

```
  3102
×   11
------
 34122
```

Q. 41 × 11 = 451

Q. 303 × 11 = 3333

Q. 1309 × 11 = 14399

Q. 2901265 × 11 = 31913915

Case 2: Multiply by 3 1's

Q. Multiply 203 by 111

```
  203
× 111
-----
22533
```

Q. Multiply 201432 by 111

```
 201432
×   111
--------
22358952
```

More Examples:

111 × 111 = 12321

2035 × 111 = 225885

90321 × 111 = 10025631

6021203 × 111 = 668353533

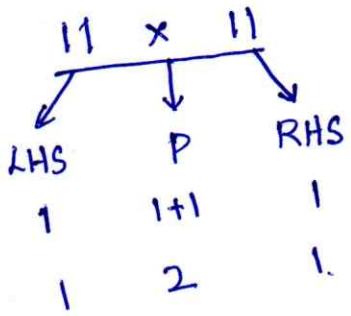
Case 3 : Multiply by 4 1's

Q. Multiply 210432 by 1111.

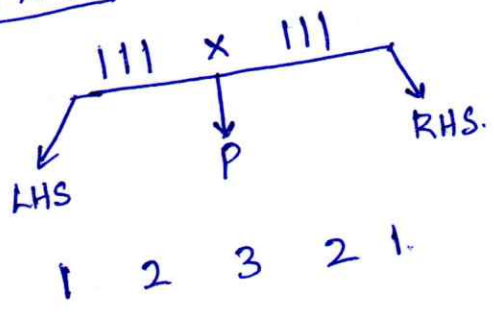
$$\begin{array}{r} 210432 \\ \times \quad 1111 \\ \hline 233789952 \end{array}$$

Case 4 : Multiply Same digits of ones in both numbers.

a. 11 × 11



b. 111 × 111



c. 1111 × 1111

1 2 3 4 3 2 1

d. 11111 × 11111.

1 2 3 4 5 4 3 2 1.

e. 111111 × 111111

1 2 3 4 5 6 5 4 3 2 1

6. Multiplication of numbers with a series of similar digits in multiplier.

Note: This is an extension of series of 1's multiplication.

Q. Multiply 333 by 222

333 × 222

= 333 × 2 × 111.

= 666 × 111.

```
  6 6 6
× 1 1 1
-------
 73926
```

S1: 6

S2: 6+6 = 12 ↓ =

S3: 6+6+6 = 18

18+1 = 19 ↓ =

S4: 6+6 = 12

12+1 = 13 ↓ =

S5: 6+1 = 7 =

Q. 3021 × 333.

= 3021 × 3 × 111.

= 9063 × 111.

= 1005993

S1: 3

S2: 6+3 = 9

S3: 0+6+3 = 9

S4: 9+0+6 = 1 ↓ 5 =

S5: 9+0 = 9+1 = 10 ↓ =

S6: 9+1 = 10

Q. Multiply 1202 by 44

= 1202 × 4 × 11

= 4808 × 11

= 52888

Q. Multiply 2008 by 5555 -

$= 2008 \times 5 \times 1111$

$= 10040 \times 1111$

11154440

Q. Multiply 10503 by 888

$= 10503 \times 8 \times 111$

$= 84024 \times 111$

$= 9326664$

7. Subtraction using the rule 'All from 9 & the last from 10'.

Note: Subtracting a number from powers of 10.

Eg: 10, 100, 1000, 10000 ....

Q. Subtract 54.36 from 100

$$\begin{array}{r} 100.00 \\ -\ 54.36 \\ \hline 045.64 \end{array}$$

Q. Subtract 3478.2281 from 10000.

$$\begin{array}{r} 10000.0000 \\ -\ 3478.2281 \\ \hline 06521.7719 \end{array}$$

More Examples:

a. $\begin{array}{r} 1000.000 \\ -\ 363.633 \\ \hline 636.367 \end{array}$

b. $\begin{array}{r} 10000.00 \\ -\ 9191.09 \\ \hline 0808.91 \end{array}$

c. $\begin{array}{r} 10.000000 \\ -\ 7.142857 \\ \hline 2.857143 \end{array}$

d. $\begin{array}{r} 10000 \\ -\ 23 \\ \hline 9977 \end{array}$

e. $\begin{array}{r} 100000.00 \\ -\ 459.62 \\ \hline 99540.38 \end{array}$

f. $\begin{array}{r} 100.00 \\ -\ 17.10 \\ \hline 82.90 \end{array}$

EXERCISE

Q (1) Find the product of the following numbers ~~between~~ whose last digits add to ten.

a. 45×45

b. 95×95

c. 111×119

d. 107×103.

Q (2). Find the Squares of the following numbers b/n 50 & 60.

a. 56

b. 51

c. 53.

Q (3). Find the product of the following numbers which are multiplied by a series of 9's.

a. 567×999

b. 23249×99999

c. 66×9999

d. 302×99999

Q (4) Find the product of the following numbers which are multiplied by a series of 1's.

a. 32221×11

b. 64609×11

c. 12021×111

d. 80041×111

e. 94362×111

f. 63845×111

Q.(5) Find the product of the following numbers which are multiplied by a series of same numbers.

a. 7005×77

b. 1234×22

c. 2222×222

d. 1203×333.

Q.(6). Subtract the following numbers from a given power of ten.

a. $1000 - 675.43$.

b. $10000 - 7609.98$

c. $10000 - 666$.

d. $1000 - 2.653$

1. Criss Cross Multiplication (UT - Urdhava - Tiryak)

a. Two digits Multiplication

principle 1-2-1

①

S1 4

S2 $(3\times2)+(1\times2) = 8$

S1 $3\times1 = 3$.

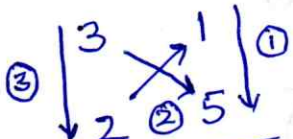
3 2
1 2
3 8 4

a. 

b. 

c. 

② 31 x 25

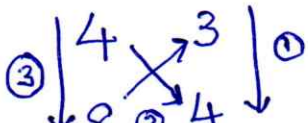


6 (17) 5

6 ← (17) 5

775.

③ 43 x 84.



(32) (40) (12)

32 (41) 2

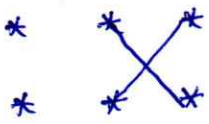
32 ← 41 2

3612.

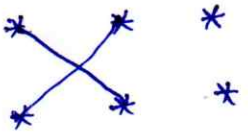
b. Three Digit Multiplication

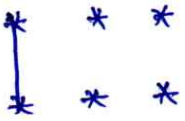
principle 1-2-3-2-1.

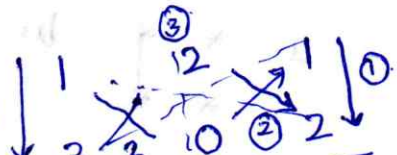
(1) 

(2) 

(3) 

(4) 

(5) 

Eg: ① 

3 15 5 4 2

Eg: ② 

3 11 27 30 20

3 11 27 32 0

3 11 30 2 0

3 14 0 2 0

44020

a. 342
506
173052

b. 412
903
372036

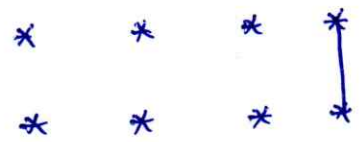
c. 555
222
123210

d. 391
274
107134

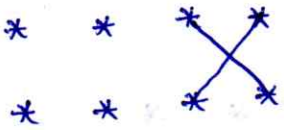
c. Four digit Multiplication

principle 1-2-3-4-3-2-1

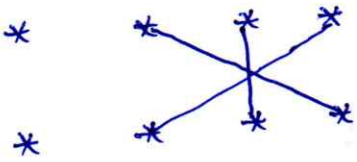
a.



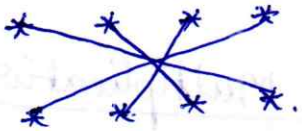
b.



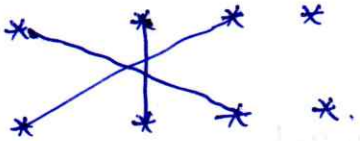
c.



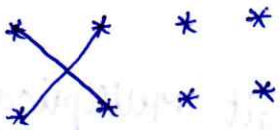
d.



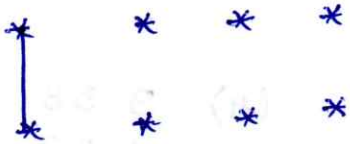
e.



f.



g.

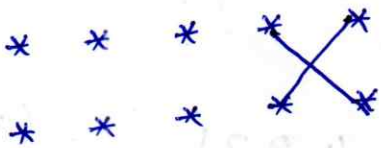


d. Five digit Multiplication

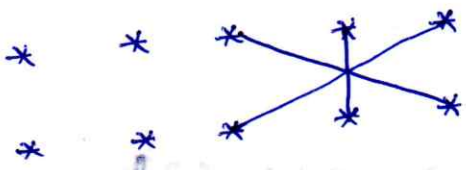
principle. 1-2-3-4-5-4-3-2-1.

a.

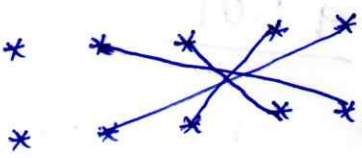
b.



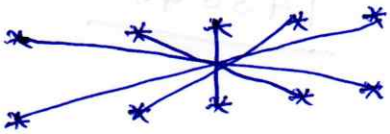
c.



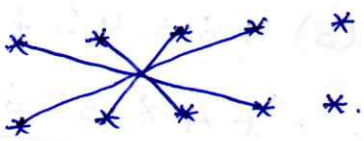
d.

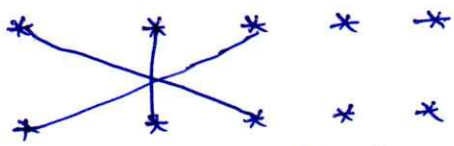


e.

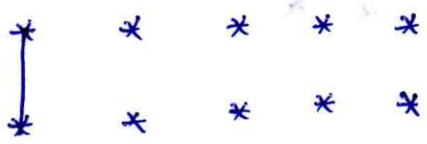


f.



g. 

h. 

i. 

e. Six digit Multiplication

principle 1-2-3-4-5-6-5-4-3-2-1

f. Seven digit Multiplication

principle 1-2-3-4-5-6-7-6-5-4-3-2-1

EXERCISE

(1) 23 × 12

(2) 41 × 13

(3) 423 × 202

(4) 358 × 132

(5) 4321 × 9101

(6) 94386 × 47257

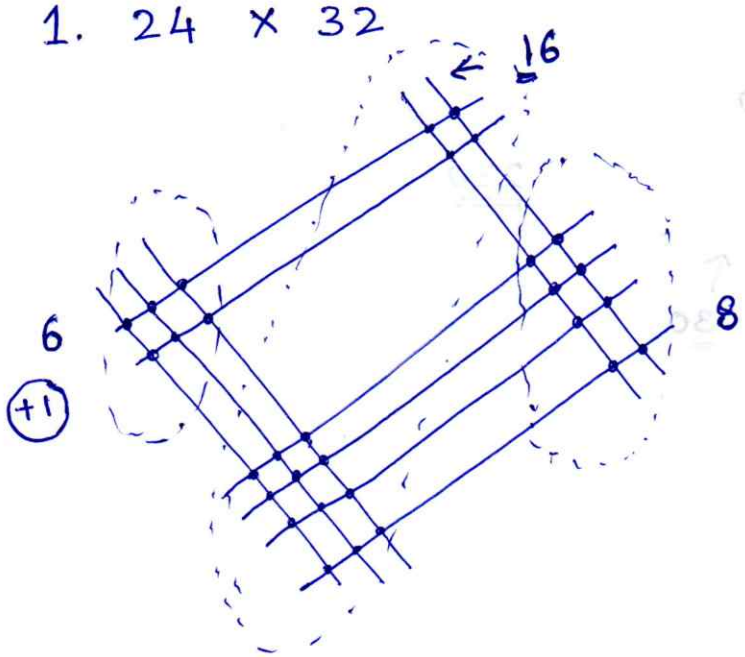
(7) 3867345 × 923687

(8) 1854375 × 6483268

(9) 9483621 × 143648

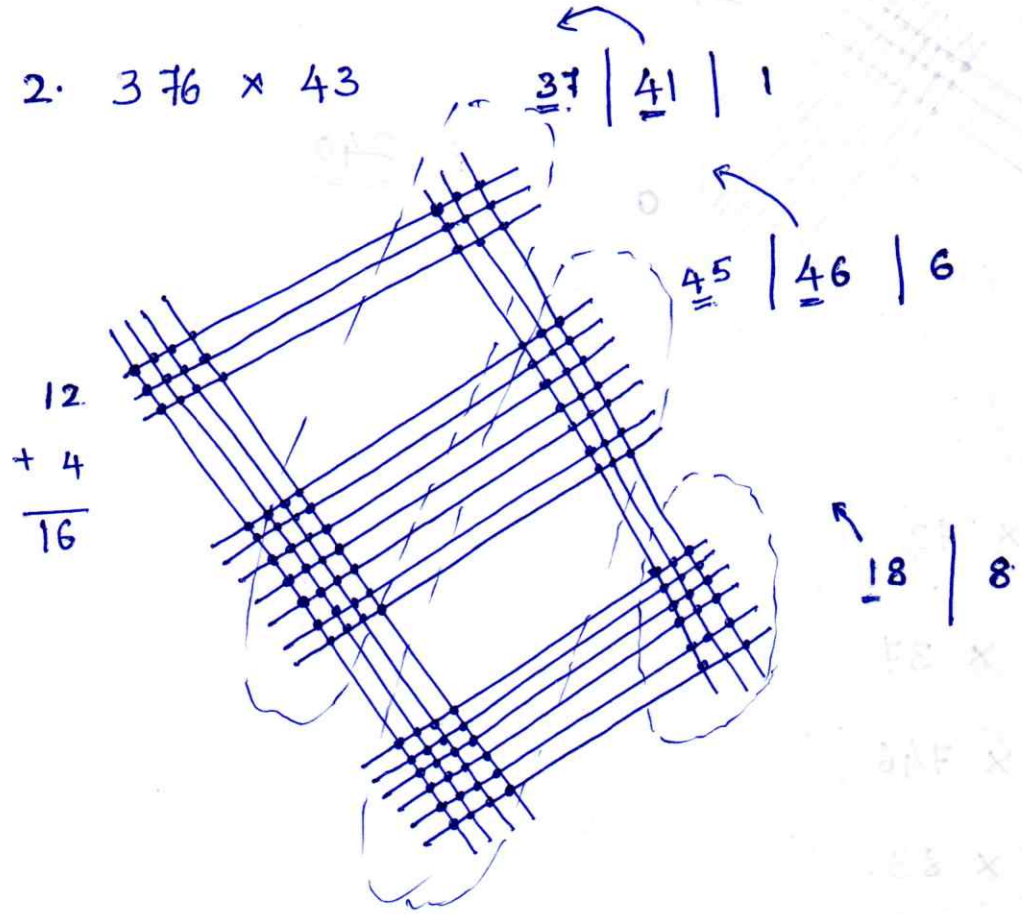
2. Multiplication Technique (Line method)

1. 24 × 32



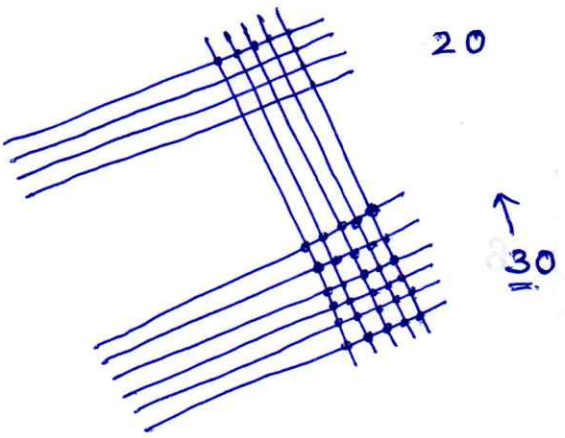
= 768

2. 376 × 43



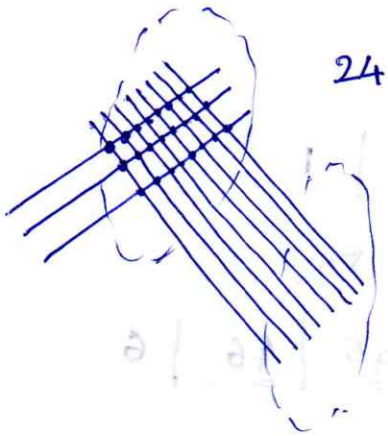
= 16168

3. 46 × 5



= 230

4. 30 × 8



= 240

EXERCISE.

1. 48 × 72

2. 246 × 37

3. 482 × 746

4. 400 × 83.

5. 47 × 4

6. 760 × 32

3. Base Method (Nikhilam)

* powers of 10.

a. condition : Numbers close to base Numbers

Eg:- close to 10, 100, 1000 etc.

We Find the answer in two parts
LHS & RHS.

Suppose 97 & 99 to be multiplied.
These Numbers are close to 100

Diff. 97 → -3 (base)
99 → -1

Steps : ① Find the base & difference
② No. of digits on the RHS = No. of zeros in the base
③ Multiply the differences on the RHS.
④ Put the cross Answer on the LHS.

Eg: ① 97
99

S ① Base = 100.
Diff 97 − 3.
99 − 1

S ②

LHS	RHS.
	●
	− −

S ③ 97 − 3 ×
99 − 1 ↓

LHS	RHS.
	03

S ④ : 97 −3
99 −1

LHS	RHS
(99−3) or (97−1) ↓ 96	03

ANS: 9603

Eg: ② 9989
9995

S1 ① Base - 10000
Diff: 9981 - 11
9995 - 5

S2 ② 9989 - 11
9995 - 5

LHS	RHS.

S3 ③ 9989 - 11
9995 - 5 ↓

LHS	RHS
	0055

S4 ④ (9989-5 or 9995-11)

LHS.	RHS
9984	0055

= 99840055

Eg. ③ 9999999
9999998

Sl ① B - 10000000
Diff - -1
-2 ↓

②

LHS	RHS
	③

③ 9999997 0000002
④

Eg. ④ 9750 × 9998.

→ 10000
→ -250
-2

→

LHS.	RHS.
9748	0500

97480500.

⑤ 1007 by 1010.

→ 10000
→ +7
+10.

→

LHS	RHS
1017	0070.

1007+10
1010+7

10170070

EXERCISE

667 - 333
997 - 3
664 | 999.

9988 - 12
9996 - 4
9984 | 0048.

9500 - 500
9991 - 9
9491 | 4500

808 - 192
999 - 1
807 | 192

997 - 23
980 - 20
957 | 460.

10020 + 20
10020 + 20
10040 | 0400.

1230 + 230
1003 + 3
1233 | 690

123456 + 23456.
100001 + 1
123457 | 23456.

6. Condition:
When the No. of digits exceed in RHS.

Eg: ① 950 × 950

→ 1000.
+50
+50.

→

LHS	RHS
1900	2500.

⇒ 902500

② 1200 × 1020

→ 1000.
+200
+ 20

→

LHS	RHS
1220	4000

= 1224000.

③. 150 × 140.

100
+ 50
+ 40

LHS	RHS
190	2000

= 21000

④ 112 + 12
110 + 10

122	120

12320

⑤ 1300 + 300
1020 + 20

1320	6000

1326000

⑥ 9200 - 800
9200 - 800

8400	64 0000

84640000

⑦ 17 + 7
18 + 8

25	56

306

⑧ 850 - 150
993 - 7

843	1050

844050

⑨ 75 - 25
95 - 5

70	125

7125

c. Multiplying a number above base with below base.

Eg: ① 95 × 115

imp:
LHS × Base

100
95 - 5
115 + 15

RHS	RHS
110	(-75)

= (110 × 100) - 75
= 11000 - 75 = 10925

Eg: ② Multiply 1044 by 998.

1000.
1044 + 44
998 − 2
1042 | −088.

$= (1042 \times 1000) - 088$

$= 1042000 - 088$

$= \underline{1041912}$

Eg: ③ 100032 by 99990.

100000.
+32
−10
(−00320).

100022 |.

$= 100022 \times 100000 - 00320$

$= 10002200000 - 00320$

$= \underline{1000219680}$

④ 800 − 200.
1004 + 4
(−800)

804

$= 804 \times 1000 - 800$.

$= 804000 - 800$.

$= \underline{803200}$

⑤ 120 + 20
97 − 3
117 | (−060)

$= 117 \times 100 - 60$

$= 11700 - 60$

$= \underline{11640}$.

⑥ 14 + 4
9 − 1
13 | −04

$= 13 \times 10 - 04$

$= 130 - 04$

$= \underline{126}$.

d. Multiplying Numbers by different bases

Eg: ① 85 × 995

* Convert the Numbers to same base.
  i.e. 85 to 850.
* In the Result divided by multiplied by (the value) Eg: 10 in this case.

850 - 150.
995 - 5

845 | RHS.
•750

= 845750

Divide by 10 → 84575

Eg: ② ~~995~~ 102 by 999.

* Multiply 102 by 10.

1020 + 20.
999 - 1

1019 | (-020)

= 1019 × 1000 - 020.
= 1019000 - 020
= 1018980

÷ by 10 = 101898

Eg: ③ 9995 × 86.

x - 86 × 100 = 8600.

9995 - 5
8600 - 1400

8595 | 7000.

~~8600000~~. 85957000.

÷ 100 = ~~860000~~ 859570

Eg: ④ 73×997.

× 73 by 10 = 730.

730 − 270.

997 − 3

727 | 810

= 727810

÷ 10 = 72781

Eg: ⑤ 73×990.

⇒ 990/10 = 99.

73 − 27

99 − 1

72. 27.

= 7227 × 10

= 72270

Eg: ⑥ 99×1005

→ 99 × 10 = 990.

990 − 10.

1005 + 5

995 | −050

= 995 × 1000 − 50.

= 995000 − 50

= 994950

÷ by 10. = 99495

e. When Base is not power of 10.

In Base Method we take 10, 100, 1000 etc. as bases. which are called as Actual base.

But, Here we will use 40, 50, 600 etc. as the bases, which are called as Working base

Eg: ① 48×48.

Actual base - 100.

Working base - $100/2 = 50$.

$48 - 2$

$48 - 2$

$2) 46 \mid 04$

2304

Since 50 (working base is obtained by dividing $100/2$)

* We divided LHS by 2 (i.e. $46/2 = 23$).

Alternative :-

Actual base - 10.

Working base - $10 \times 5 = 50$

$48 - 2$

$48 - 2$

$46. \mid 4$

$= 46 \times 5 \mid 4$

230

$= 2304$.

Eg: ② 27×28.

Actual base = 100

Working base = $\frac{100}{5} = 20$.

$27 + 7$

$28 + 8$

$5) 35 \mid 56$

$= 756$.

Eg. ③ 59×58

Actual base = 100.
Working base = 50. = $\frac{100}{2}$

$59 + 9$
$58 + 8$
$67 \mid 72$

Actual base = 10.
working base = $10 \times 6 = 60$.

$59 - 1$
$58 - 2$
$57 \quad 2$
$\times 6$
3402

Eg. ④ 31×32.

AB = 10.
WB = $10 \times 3 = 30$.

$31 + 1$
$32 + 2$
$33 \mid 2$
$\times 3$
992

Eg. ⑤ 57×57

AB - 10, WB - $10 \times 6 = 60$,

$57 - 3$
$57 - 3$
$54 \quad 9$
$\times 6$
3249

Eg: ⑥ 395 × 396.

AB = 100.

WB = 100 × 4 = 400.

395 − 5

396 − 4

394	20
× 4	
1576	20

Eg: ⑦ 228 × 246.

AB = 1000.

WB = 1000/4 = 250.

228 − 22.

246 − 4

224	088

÷ 4

5 6088

Eg: ⑧ 45 × 45

AB = 100. ; WB = 100 × 5 = 50.

45 − 5

45 − 5

40	25
× 5	
200	25.

= 2025

AB = 10 ; WB = 10 × 4 = 40.

45 + 5

45 + 5

50	25.
× 4	
200	25

= 2025.

Eg: ⑨ 58×42.

$AB = 10$

$WB = 10 \times 5 = 50$.

$58 + 8$

$42 - 8$

$50 \mid -6\frac{4}{}$

$\times \quad 5 \mid$

$250 \mid (-64)$

$= 250 \times 10 - 64$

$= 2500 - 64$

$= 2436$

Eg: ⑩ 55×45

$AB = 10$; $WB = 10 \times 5 = 50$.

$55 + 5$

$45 - 5$

$50 \mid -25$

$\times \quad 5 \mid$

$250 \mid -25$

$= 250 \times 10 - 25$

$= 2500 - 25$

$= 2475$

Eg: (11) 45×42

$AB = 100$; $WB = \frac{100}{2} = 50$.

$45 - 5$
$42 - 8$

$2) 37 \cdot | 40$

$18\frac{1}{2} | 40 \cdot$

$= 18/(50+40)$

$= \underline{1890}$

$AB = 10$; $WB = 10 \times 4$

$45 + 5$
$42 + 2$

$47 | 10$
$\times 4 |$

$188 | 10 \cdot$

$= \underline{1890}$

Eg: (12) 245×248

$AB = 100$; $WB = 1000/4 = 250$.

$245 - 5$
$248 - 2$

$4) 243 | 10$
~~4~~ |

$60\frac{3}{4} | 10 \cdot$

$= 60(3/4 \times 1000 + 10)$

$= 60(750 + 10)$

$= \underline{60760}$

EXERCISE

Q 1. Multiply the following Numbers:

a. 990×994

b. 999993×999999

c. 1002×10100

d. 1050×1005

Q. 2. Multiply the following Numbers where the answers in RHS exceeds the No. of zeros in the base.

a. 16×17

b. 1500×1040.

c. 9300×9500.

d. 860×997.

50 100.
4
154 | 200
1560000

Q. 3. Calculate the product of the following (one number is above the base & the other number is below the base).

a. 96×104

b. 890×1004

c. 10080×9960.

d. 970×1010.

Q. 4. Multiply the following numbers using different bases.

a. 73×997

b. 94×990.

c. 82×9995

d. 102×1010.

Q. 5. Multiple the numbers using actual & working bases.

a. 49×48.

b. 22×22

c. 53×49.

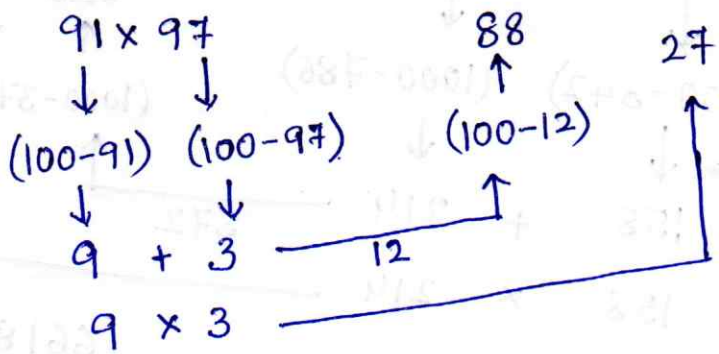
d. 18×17

e. 499×496.

484.
+2
+2
24 4
2
484

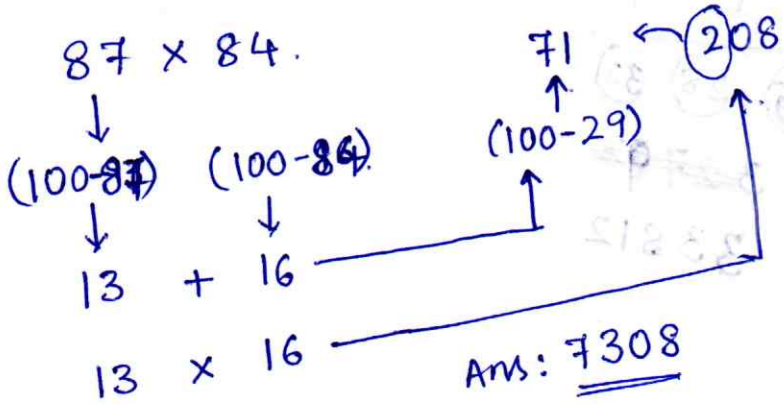
1. Multiplication (using base method).

Eg. ① 91 × 97



Ans: 8827

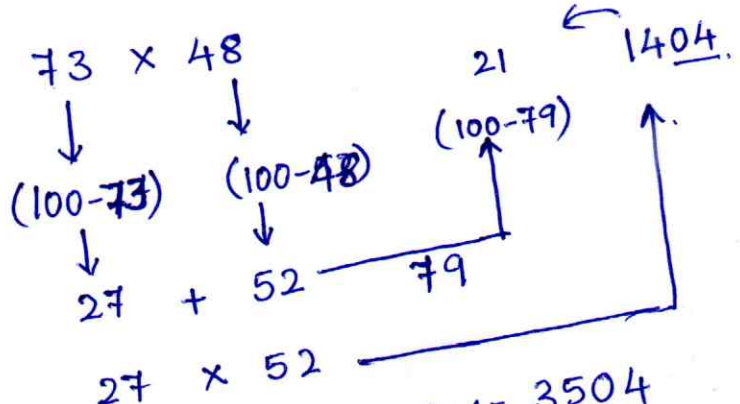
Eg. ② 87 × 84.



Ans: 7308

13 × 16 = 208

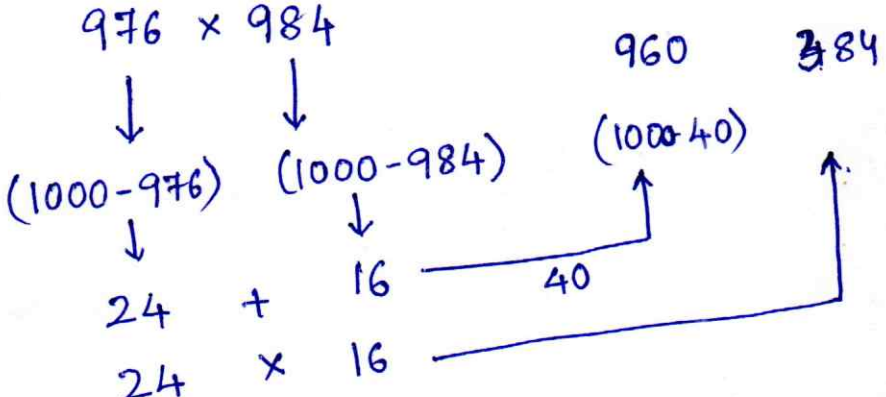
Eg. ③ 73 × 48



Ans = 3504

27 × 52 = 1404

Eg. ④ 976 × 984



Ans = 960384

24 × 16 = 384

Eg: (4)

842 × 786

(1000-842) (1000-786)

158 + 214

158 × 214

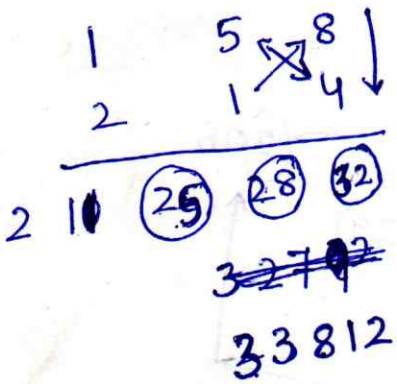
372

(1000-372)

628

33812

661812



01. Multiplication : Special Case

a. Numbers differ by 2

Eg: ① 66×68.

S1: Find No. between the two Numbers.

S2: Then Apply 'Square' & Subtract 1.

$= 67^2 - 1$

$= 4489 - 1$

$= 4488$.

6 7

6^2 $2(6)(7)$ 7^2

36 84 49

36 88 9

4489

② 20×22.

$21^2 - 1 = 441 - 1$

$= 440$

4 2(2)(1) 1

441.

b. Numbers differ by 4

Eg: ① 20×24.

S1: Find the No. between the two Numbers

S2: Then Apply 'Square' & Subtract 4.

$= 22^2 - 4$.

$= 480$

4 2(2)(2) 4

484

Eg: ② 82×86.

$= 84^2 - 4$

$= 7052$.

64 2(8)(4) 16.

64 64 16.

64 65 6.

7056.

Eg. ③ 586×590

$= 588^2 - 4$

Eg ④ 696×700

C. Numbers by 6

S1: Find the NO. between the two Numbers.
S2: Then apply 'Square' & Subtract '9'

Eg: ① 83 × 89.

$\frac{83+89}{2}$

S1: 86

S2: $86^2 - 9$

$7396 - 9$

$= 7387$.

8^2 64 2(8)(6) 36
64 96 36
64 99 6
7396.

② 98 × 104

S1: 101.

S2: $101^2 - 9$.

$= 10201 - 9$

$= 10192$.

101
101
10201

Multiplication Trick

Case: Same digits sequence multiplied by Numbers - 1 to 9

1. $888 \times 2 = 1776$ (① ×, ② +)

2. $9999 \times 3 = 29997$ (① ×, ② +)

3. $777 \times 4 = 2\,10\,10\,8$

$= 21108$

$= 3108$

4. $4444 \times 7 = 2\_\_\_\_8$ (+)

5. $6666 \times 9 = 59994$

DIVISION

1.

Divisibility Rules

Numbers is divisible.

① By 2 : If the number is ~~di~~ Even NO.

② By 3 : Sum Root is divisible by 3. (Digit Sum)

③ By 4 : If the last '2' digits are divisible 4.

④ By 5 : If the Number ends with '0' or '5'

⑤ By 6 : If the Number is divisible by both 2 & 3

⑥ By 7 : Check directly. (there is a solution thou')

⑦ By 8 : If the last '3' digits are divisible by 8.

⑧ By 9 : If Digit Sum is divisible 9.

⑨ By 10 : If the Number ends with '0'

⑩ By 11 : Difference b/n Sum of even digits & Sum of odd digits is multiple of 11. or 0.
~~or difference is~~

⑪ By 12 : Check for divisibility by 3 & 4.

⑫ By 13 : Delete the last digit from given number. Subtract '9' times deleted digit from given rest NO. if result is divisible by 13. Then the NO. is divisible by 13.

2. Specific case : Divisible by 7

S1 : 'x' units place by 2 = Result

S2 : Sub. Result from remaining Number.

S3 : Check the answer in 'S2', divisible by 7. Then the NO. is divisible by 7.

Eg: ① $\frac{595}{2}$ $\frac{5}{1}$

S1 : units place -5.
$5 \times 2 = 10$

S2 : $59 - 10 = 49$.

S3 : $49/7 = 7$.

Hence 595 is divisible by '7'

2. 948

948
×2
94-16 = 78.

'78' Not divisible by 7
Hence 948 Not divisible 7.

3. 1792

179|2
×2
179-4 = 175
×2
17-10 = 7

Divisible by 7.

3. Division by 5

$\frac{\text{DIVIDEND}}{\text{DIVISOR}} = Q + R$

Eg: ① $\frac{45}{5} = \frac{45 \times 2}{5 \times 2} = \frac{90}{10} = 9.$

Q = 9, R = 0.

Eg: ② $\frac{97}{5} = \frac{97 \times 2}{5 \times 2} = \frac{194}{10} = 19.4 = 19\frac{4}{10} = 19\frac{2}{5}$

Q = 19, Remo = 2

Eg: ③ $\frac{582}{5} = \frac{582 \times 2}{5 \times 2} = \frac{1164}{10} = 116.4 = 116\frac{4}{10} = 116\frac{2}{5}$

Q = 116 R = 2

Eg: ④ $\frac{748}{5} = \frac{748 \times 2}{5 \times 2} = \frac{1496}{10} = 149.6 = 149\frac{6}{10} = 149\frac{3}{5}$

Q = 149 R = 3.

Eg: ⑤ $\frac{1483}{5} = \frac{1483 \times 2}{5 \times 2} = \frac{2966}{10} = 296\frac{6}{10} = 296\frac{3}{5}.$

Q = 296 R = 3.

4. Division by 9

S1: First digit is written 'as-is' in answer part.

S2: Add 2nd digit to the answer part written in 1st step (S1)

S3: Repeat until last digit (right most digit) in the answer part is the remainder. Rest of the digits form the quotient.

① Eg: $53 \div 9$.

	9	Q	R
S1:		5	3.
		↓ 5	↗
S2:			5+3
		↓	
S3:		5	8

$Q = 5 \quad R = 8.$

10 | 5 | 3.
9 | | 5
1 | 5 | 8.

Verification: SR(Dividend) = SR(Q) × SR(divisor) + SR(R)

$8 = 5 \times 9 + 8$
$= 45 + 8$
$= 9 + 8 = 8$

10.	11	2
9	↓ 1	2
1	← 1 2	4.

Eg: 2 112×9.

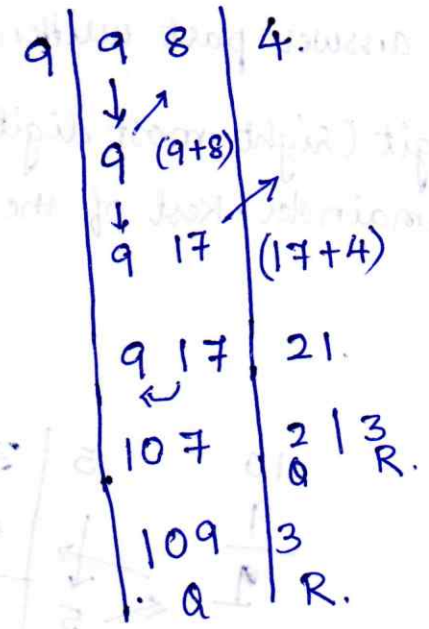
9	1	1	2
	↓	↗	
	11		11+2

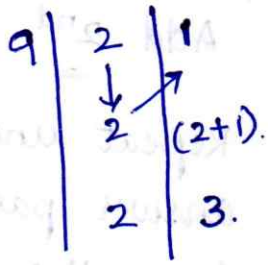
9	1	1	2
	↓	↗	
	1	(1+1)	
	↓	↓	↗
	1	2	2+2
	↓	↓	
	1	2	4

$Q = 12 \quad R = 4$

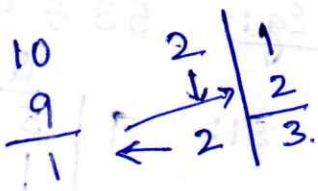
Eg: 3

984 ÷ 9.

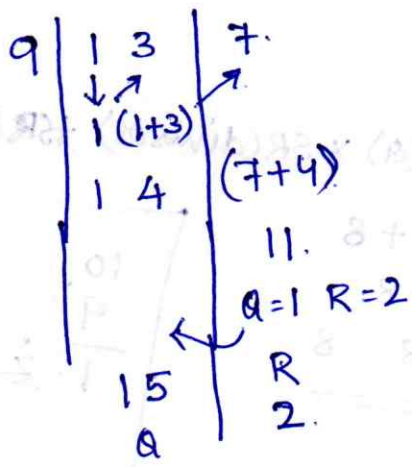


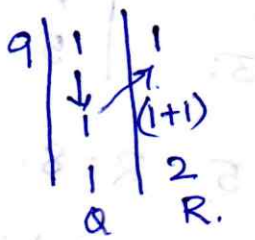


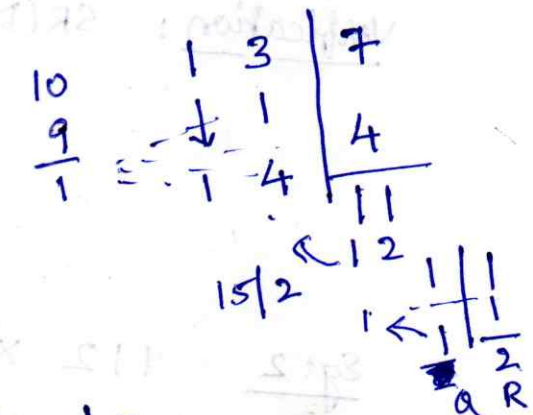
Q = 2 R = 3.



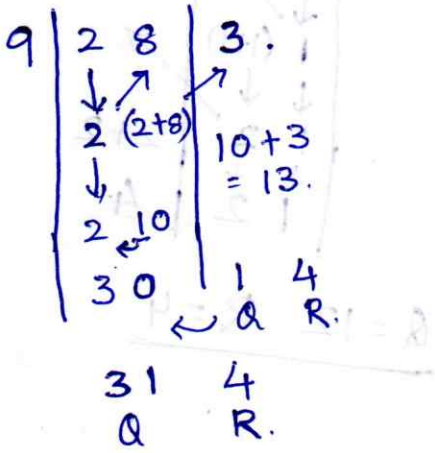
Eg: 4 137 ÷ 9.

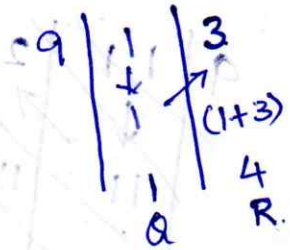






Eg: 5 283 ÷ 9





5. Base Method (Division)

Dividend.

Base

Divisor

Difference

Quotient | Remainder

S1: Divide the Number in 2 parts RHS & LHS.

RHS → Contain No. of zeros in the base.

LHS → Quotient. / RHS → Remainder.

Eg: ① Divide 23 by 9.

No. 9.

RHS → 1 digit (base 10)

(difference 1)

(since one zero).

10

9

1.

2	3.
① →	2
② ×	2
2	5
Q	R.

Eg: ② 31 ÷ 9.

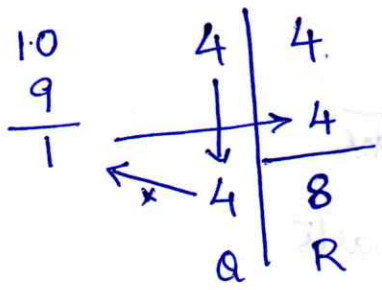
10

9

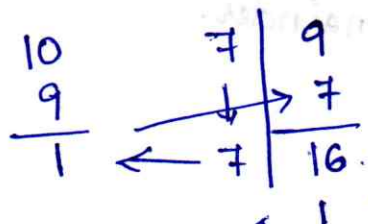
1

3	1
→	3.
← 3	4
Q	R.

Eg: ③ Divide 44 by 9.



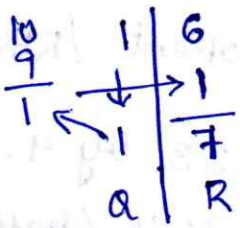
Eg: ④ $79 \div 9$



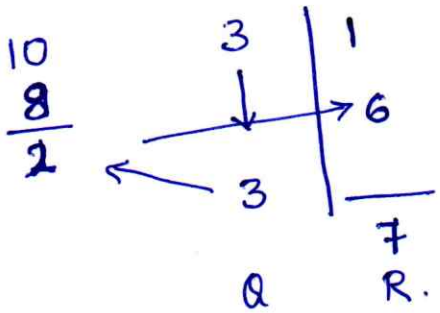
$8 \leftarrow Q \quad 7 \; R.$

7

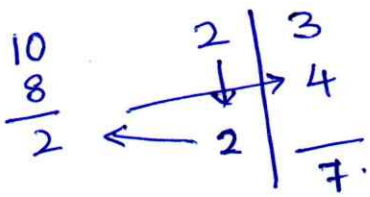
$Q = 8 \quad R = 7.$



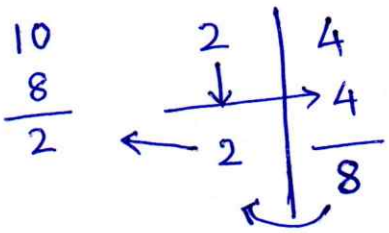
Eg: ⑤ $31 \div 8$



Eg: ⑥ $23 \div 8$

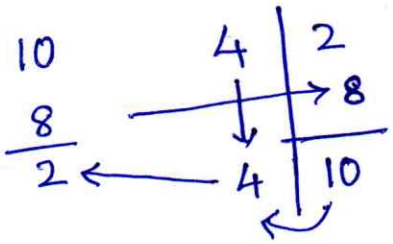


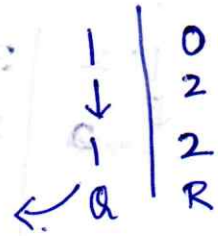
Eg: ⑦ Divide 24 by 8



3 0

Eg: ⑧ Divide 42 × 8.

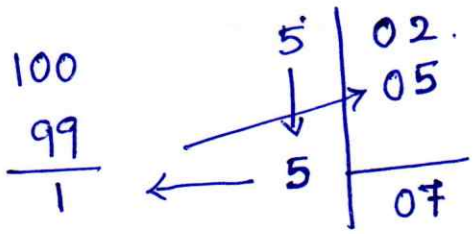




52

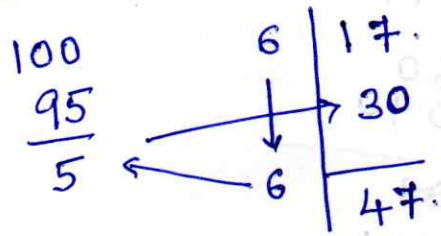
6. Divide by bigger divisors. (Base Method)

Q. Divide 502 by 99.



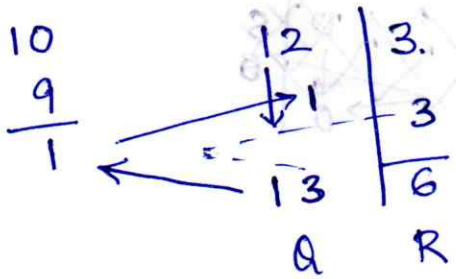
Q=5 R=7

Q. Divide 617 by 95

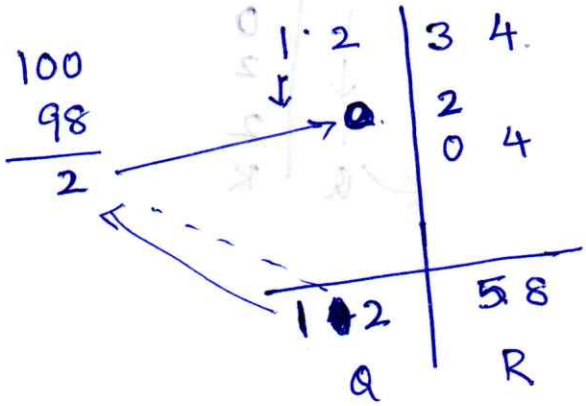


Q R

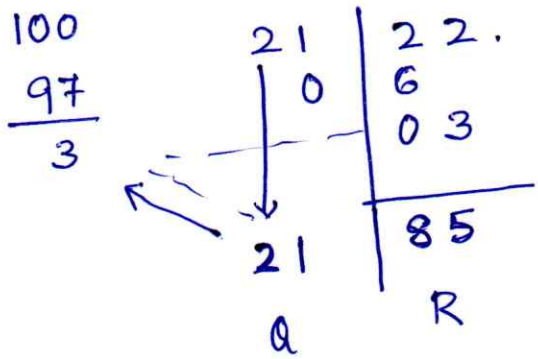
Q. Divide 123 by 9



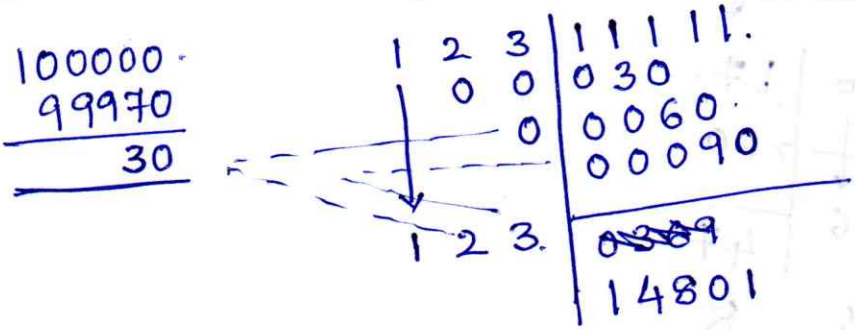
Q. Divide 1234 by 98



Q. Divide 2122 by 97.

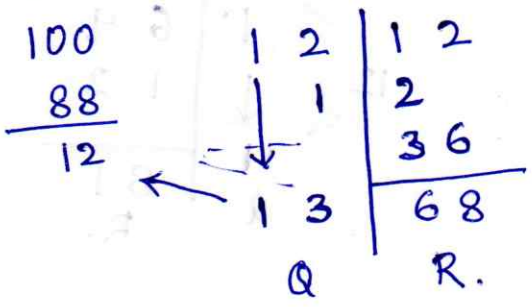


Q. 12311111 by 99970.

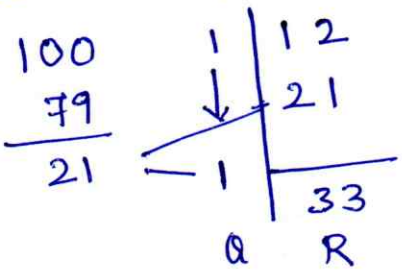


Addional Examples

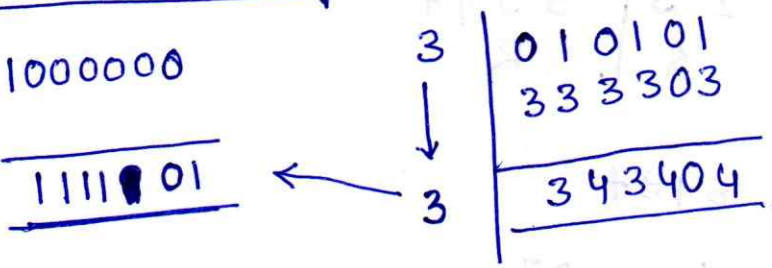
a. 1212 by 88



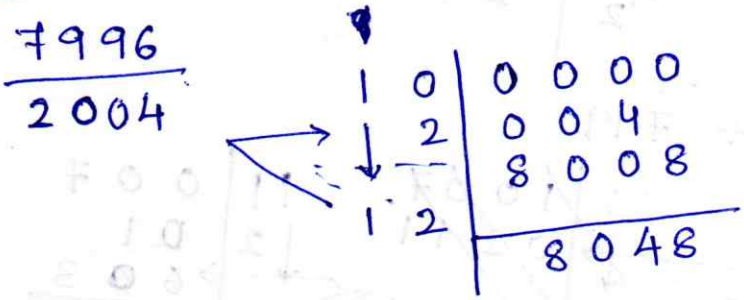
b. 112 by 79.



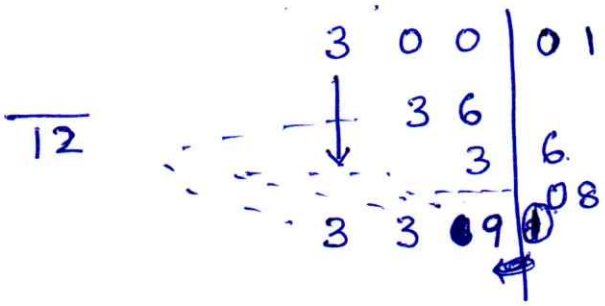
c. 3010101 by 8888899

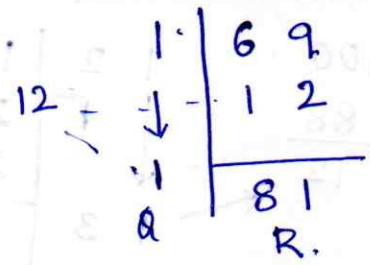


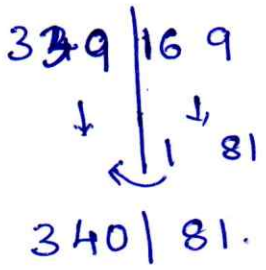
d. Divide 100000 by 7996



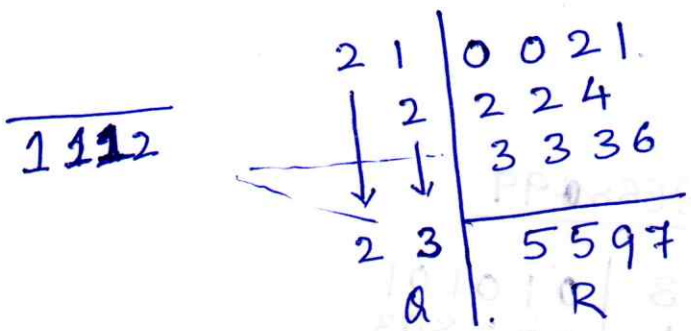
e. 30001 ÷ by 88



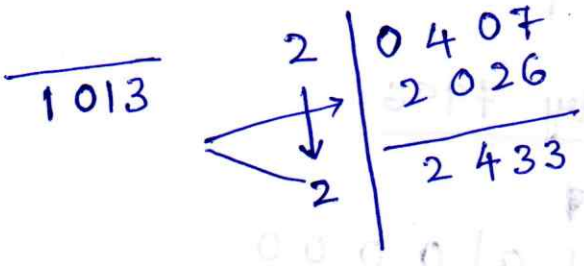




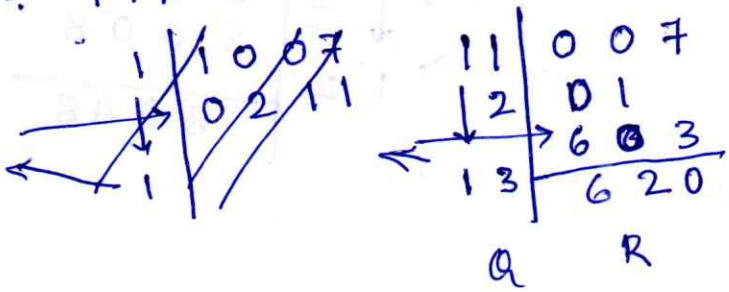
f. 210021 ÷ by 8888



g. 20407 ÷ 8987



h. 11007 ÷ 799.



i. 2211 by 88

12

2 2	1 1
2	4
	4 8
2 4	9 9

24 ① 11

25 | 11.

Q | 99

12

j. 111301 ÷ 897

103

1 1 1	3 0 1
1 0	3
2	0 6
1 2 3	3 0 9
	9 7 0

- 897

① 73

124 | 73

Q | R

K. 30122 ÷ 87

13

301	2 2
39	9.
3	6 9.
33 (13)	1
	2 8 1

343 | Q R

3 20.

346 | 20.

2	8 1
13	2 6
2	1 0 7

3 + 13

1	0 7
1	1 3
1	2 0

EXERCISE

PART A :

a. Divide 102 by 74

b. Divide 10113 by 898

c. Divide 102030 by 7999

d. Divide 1005 by 99.

PART B :

a. Divide 431 by 98.

b. Divide 10301 by 97.

c. Divide 12000 by 889

d. Divide 111099 by 8987

e. Divide 30111 by 87.

06. Paravartya Yojayet – transpose & Apply

a. Divide 3966 by 113

100.
113
-13

3 9	6 6.
-3	-9
	-6 -8
3 6	-6
	-6 -4 -2

b. Divide 1296 by 113.

100
113
-13

1 2	9 6.
-1	-3
	-1 -3
1 1	5 3

c. Divide 2688 by 120

100
-120
-2-0

2 6	8 8.
-4	-0
	-4 -0.
2 2	4 8

d. Divide 113968 by 1023.

1000.
-1023
-023.

1 1 3	9 6 8
-0 -3	-3
-0	-2 -3.
+1 1 1	-0 -2 -3.
111	4 0 5
Q	R.

e. Divide 1999 by 180

100
- 180
-8 -0

1 9	9 9
-8	-0
	-8 -0
1 1	1 9

-ve in Q.

f. Divide 14189 by 102

100
102
-02

2 zeros
2 digits

1 4 1	8 9
-0 -2	
-0	-8
1 4 (-1)	+0 +2
	1 1

(140-1) = 139 | 11

10
1 4 -1
1 3 9

g. Divide 110999 by 1321

1000
1321
-3-2-1

1 1 0	9 9 9
-3 -2	-1
+6	+4 +2
	-12 -8 -4
1 -2 4	0 3 5

(100-20)+4 = 84

10
1 -2 4
0 8 4

h. Divide 1693 by 131

131
-3 -1

1 6	9 3
-3	-1
	-9 -3
1 3	-1 0
1 2	+121

i. $14520 \div 111$

111	145 | 20.	
-1-1	↓ -1-1 | -3.	
	-3 | -1 -1	
	131 | -2 -1	
	111	
	130 | -21	
	90	

j. Divide 16379 by 1222.

-2 -2 -2.	1 6 | 3 7 9.	
	↓ -2 | -2 -2.	
	-8 -8 -8	
	1 4 | -7 -3. 1	
	-700 + -30 + 1	
	-729.	
	13 | 1222 - 729	
	= 493	

07. Substitution Method

In substitution method, we take the difference as below.

Eg: 827 → 173 (Base).
diff

→ (200 - 30 + 3).

k. Eg: Divide 10030 by 827.

a. Normal

827	100 | 30	
173	↓ 17 | 3	
	1 | 73	
	118 | 63	

Substitution method

827	10 | 030.	
2-33.	↓ 2 | -33.	
	4 -6 +6	
	12 | 106	

827	10 | 030.	
173	↓ 1 | 73	
	11 | 173.	
	9.33	
	-827	
12	106	

l. Divide 10000 by 819

Normal

819
1801

1	0	0	0	0.
	↓	1	8	1
1	1	1	8	1
		9	9	1
		−	8	1	9
		1	7	2

12 |.

Subst:

200 − 200 + 1

	10	0	0	0.
2 − 21	↓ 2	−	2	1
	12	4	−	4	1
		2	−	3	1

200 − 30 + 2
= 172.

08. Division by altering divisor

(m). Divide 1459 by 242.

Special case

* Bring 242 closer to 100.
  Eg: By diving $\frac{242}{2} = 121$.
* We divide by 121 first & the Q → ÷ by 2.

121
−2−1

1	4	5	9
↓	−2	−1	
		−4	−2
1	2	0.	7

Q
2) 12
6
Q

07.
7
R.

n. Divide 1112 by 33.

* closer to Base (100) − 33 (×3) = 99.

100
99
01

1 1	1 2
0	1
1 1	0 1
	2.3

×3

33 | 23

o. Divide 12657 by 791.

* 791 to 10 or 100.

÷7 = 113.

100.
−113
−1−3.

1 2 6	5 7.
−1 −3	−3
−1	−2 −6.
1 1 2	
7) 112	0.1
16	01.

p. Divide 1389 by 61.

* closer to 100 − 61 (×2) = 122.

−2 −2

1 3	8 9.
−2	−2
	−2 −2
11	4.7
× 2	
22	47

EXERCISE

PART A

a. Divide 1389 by 113.

b. Divide 145516 by 1321.

c. Divide 136789 by 12131.

d. Divide 246406 by 112

PART B

a. Divide 13592 by 114

b. Divide 25430 by 1230.

c. Divide 15549 by 142

d. Divide 101156 by 808 (Diff. 192.

200 − 10 + 2

2 −1 2

PART C

a. Divide 4949 by 601 (601 × 2 = 1202)

b. Divide 14799 by 492 (492/4 = 123).

09. Quick ways of division

a. Division by 14

S ① ÷ by 2

S ② ÷ by 7.

Split the divisor & divide by each.

Eg ①: Divide 784 by 14.

S1: $\frac{784}{2} = 392$

S2: $\frac{392}{7} = 56$

Eg ②: Divide 1148 by 14.

S1: $\frac{1148}{2} = 574$.

S2: $\frac{574}{14} = 41$.

b. Division by 16 (2 * 8).

Eg ①: 96 ÷ 16.

S1: $\frac{96}{2} = 48$.

S2: $\frac{48}{8} = 6$.

c. Verify the below Answers.

• (i) 1430 ÷ 22. → Q = 65 & R = 0.

SR(Dividend) = SR(Q) × SR(Divisor) + SR(R)

8 = 2 × 4

8 = 8

Hence Ans. is correct.

(ii) $21289 \div 2365 \rightarrow Q = 9$ & $R = 4$.

$$4 = 7 \times 9 + 4$$
$$= 63 + 4$$
$$= 9 + 4$$

✓

01. Squaring Numbers.

A. Criss cross System

1.23^2

2×3

$2 \quad 3$

4 12 9

5 2 9

B. Formula Method.

(i) $(a+b)^2 = a^2 + 2ab + b^2$.

* This applies to close to 10 (multiples).
  Eg: 10, 100, 1000 etc. or similar.

Eg 1: Find the square of 1009.

$1009^2 \rightarrow (1000+9)^2$.

$(1000+9)^2 = (1000)^2 + 2(1000)(9) + 9^2$

$= 1000000 + 18000 + 81$

$= 1018081$.

Eg 2: Find the square of 511.

$511^2 = (500+11)^2$

$(500+11)^2 = 500^2 + 2(500)(11) + 11^2$.

$= 250000 + 11000 + 121$.

$= 261121$

(ii) $(a-b)^2 = a^2 - 2ab + b^2$.

Eg 1: Find the square of 995

$995^2 = (1000-5)^2$

$(1000-5)^2 = 1000^2 - 2(1000)(5) + 5^2$.

$= 1000000 - 10000 + 25$

$= 990000 + 25$

$= 990025$

Eg 2 : Find the Square of 698.

$$698^2 = (700-2)^2$$

$$(700-2)^2 = 700^2 - 2(700)(2) + 2^2$$

$$= 490000 - 2800 + 4$$

$$= 487200 + 4$$

$$= 487204$$

(iii) $a^2 - b^2 = (a+b)(a-b)$.

$$a^2 = (a+b)(a-b) + b^2$$

* Somewhere between numbers.

Eg 1: Find the Square of 72

$\Rightarrow 72^2 = (72+b)(72-b) + b^2$.

Now we have $-b$ which is not known.

Apply / Assume 'b' - such that Further multiplication becomes easy

$b = 2$.

$$72^2 = (72+2)(72-2) + 2^2$$

$$= 74 \times 70 + 2^2$$

$$= (70+4) \times 70 + 2^2$$

$$= 4900 + 280 + 4$$

$$= 5184$$

Eg 2 : Find the Square of 53. $b = 3$

$53^2 = (50+3)(50-3) + 3^2$

$= 53$

$53^2 = (53+3)(53-3) + 3^2$

$= 56 \times 50 + 3^2$

$= 2800 + 9$

$= \underline{2809}$

Eg 3 : Find the Square of 67. $b = 3$

$67^2 = (67+3)(67-3) + 3^2$

$= 70 \times 64 + 9$

$= 4480 + 9$

$= \underline{4489}$ Less than 5.

Eg 4 : Find the Square of 107. $b = 7$

$107^2 = (107+7)(107-7) + 7^2$

$= 114 \times 100 + 49$

$= 11400 + 49$

$= \underline{11449}$

Eg 5 : Find the Square of 94. $b = 4$

$94^2 = (94+4)(94-4) + 4^2$

$= 98 \times 90 + 4^2$

$= (90+8) \times 90 + 4^2$

$= 8100 + 720 + 16$

$= 8820 + 16 = \underline{8836}$

$= \underline{8836}$

EXERCISE

PART A

Q 1. Find the squares of the following numbers using criss-cross system.

(1) 42

(2) 33.

(3) 115

PART B

Q 2. Find the squares using the formula $(a+b)^2$.

(1) 205

(2) 2005

(3) 4050.

Q 3. Find the squares using the formula $(a-b)^2$.

(1) 9991

(2) 9800.

(3) 1090.

Q 4. Find the squares using the formula (a^2-b^2).

(1) 82

(2) 49.

(3) 109

(4) 97.

1. Squaring of Numbers. 11, 22, 33 99

Eg 1: 11^2.

11^2

1 1 — sq.

S1: 01 01.

S2: 01 + 01. = 02.

0101
+ 02
0121

Eg 2: 22^2.

2 2 — sq.

04 04.
08
0484

Eg 3: 33^2.

33

09 09.

0909.
018
1089

Eg 4: 44

4 4

16 16.
032
1936

Eg 5: 55

5 5

2525
050
3025

Eg 6: 66

66

3636.
072
4356

Eg 7: 77

4949.
098
5929

Eg 8: 88

64 64
128
7744

Eg 9: 99

8181.
162
9801

02. Squaring Decimals

Eg: ① $0.54^2 = 0.54 \times 0.54$.

Square 54^2, put decimal at 4th digit.

$54^2 = 50^2 + 2(50)(4) + 16$

$= 2500 + 400 + 16$

$= 2916$

Ans: 0.2916

Eg ②: $0.72^2 = 0.72 \times 0.72$.

$72^2 = (70+2)^2 = 70^2 + 2 \times 70 \times 2 + 2^2$

$= 4900 + 280 + 4$

$= 0.5184$.

72^2.

$a^2 = (a+b)(a-b) + b^2$

$= (72+2)(72-2) + 4$

$= (74 \times 70) + 4$

$= 5180 + 4$.

$= .5184$

Eg ③: 4.3^2.

43^2 ... $a = 40$, $b = 3$.

$43^2 = (43+3)(43-3) + 3^2$

$= 46 \times 40 + 9$.

$= 1849$.

Eg 4: 7.8^2.

78^2.

$78^2 = (78+8)(78-8) + 8^2$.

$= 86 \times 70 + 64$

$= 6020 + 64$

$= 6084$

Cubing Numbers

A. Formula method

* cube of any number can be found out using the formula.

$$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

$$(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3.$$

Eg 1: Find the cube of 102

$$(100+2)^3 = (100)^3 + 3(100)^2 \times 2 + 3(100) \times 2^2 + 2^3.$$

$$= 1000000 + 3(10000) \times 2 + 300 \times 4 + 8$$

$$= 1000000 + 60000 + 1200 + 8.$$

$$= 1061208$$

Eg 2: Find the cube of 97.

$$(100-3)^3 = (100)^3 - 3(100)^2 \times 3 + 3(100) \times 3^2 - 3^3.$$

$$= 1000000 - 9(10000) + 2700 - 27$$

$$= 1000000 - 90000 + 2700 - 27$$

$$= 910000 + 2673$$

$$= 912673$$

Eg 3: Find the cube of 93.

Eg 4: Find the cube of 105

Eg 5: Find the cube of 91.

Eg 6: Find the cube of 108

B. Anurupya Sutra

Eqn: $a^3 + 3a^2b + 3ab^2 + b^3$.

$\Rightarrow a^3 + a^2b + ab^2 + b^3$ — ①

$2a^2b + 2ab^2$ — ②

$= a^3 + 3a^2b + 3ab^2 + b^3$.

From Eqn. ①; we see the pattern below.

1st term — a^3.

2nd term — $a^3 \times \frac{b}{a} = a^2b$.

3rd term — $a^2b \times \frac{b}{a} = ab^2$.

4th term — $ab^2 \times \frac{b}{a} = b^3$.

Hence each term 'x' by $\frac{b}{a}$ — gives next term.

For Eqn. ② 'x' the term twice (from 2, 3 rd terms).

- Add them up. (1) & (2).

Q. Find the cube of 52 : (50+2).

$a = 5 \quad b = 2$

1st term $= a^3 = 5^3 = 125$

1st Row

2nd term $= 125 \times \frac{2}{50} = 50$ ✓

3rd term $= 50 \times \frac{2}{5} = 20$ ✓

4th term $= 2^3 = 8$.

2nd Row

$50 \times 2 + 20 \times 2$

$= 100 + 40$.

125	50	20	8
	100	40	
125	150	60	8

Step 3: add '3' zeros to 125 (append) = 125000
'2' zeros to 150 = 15000
'1' zero to 60 = 600
NO zeros to 8 = 8
= 140608

Q. Find the cube of 12 a=1 b=2

I
$1^3 = 1$.
$1 \times \frac{2}{1} = 2$.
$2 \times \frac{2}{1} = 4$
$2^3 = 8$

II
$2 \times 2 = 4$
$2 \times 4 = 8$

1	2	4	8
	4	8	
1	6	12	8

1000
600
120
8
1728

Q. Find the cube of 31 a=3 b=1

I
$3^3 = 9$.
$9 \times \frac{1}{3} = 3$
$3 \times \frac{1}{3} = 1$
$1^3 = 1$.

II
$2 \times 3 = 6$
$2 \times 1 = 1$.

9	3	1	1
	6	1	
9	9	2	1

9000
900
20
1
9921

I

$3^3 = 27$.

$27 \times \frac{1}{3} = 9$.

$9 \times \frac{1}{3} = 3$.

$1^3 = 1$

II

$2 \times 9 = 18$.

$3 \times 2 = 6$.

27	9	3	1
	18	6	
27	27	9	1

27000
2700
990
1
29791

Q. Find the cube of 13

I

$1^3 = 1$

$1 \times \frac{3}{1} = 3$.

$3 \times \frac{3}{1} = 9$

$3^3 = 27$

II

$3 \times 2 = 6$

$9 \times 2 = 18$.

1	3	9	27
	6	18	
1	9	27	27

1000
900
270
27
2197

Q. Find the cubes of.

a. 14

b. 22

c. 24

d. 33

e. 42

f. 51

g. 62

h. Find the cube of 102 $\overset{a}{(10)}$ $\overset{b}{(2)}$

$10^3 = 1000$.

I $1000 \times \frac{2}{10} = 200$

$200 \times \frac{2}{10} = 40$.

$2^3 = 8$

II $2 \times 200 = 400$.

$2 \times 40 = 80$

1000	200	40	8.
	400	80	
1000	600	120	8

1000000
60000
1200
8
1061208

* Note the above Rule is applicable to only till 999 (3, 2, 1, NO zeros appended).

From 1000 onwards, we put 6, 4, 2 & no zeros.

Eg: Find the cube of 1001. $\overset{a}{10}/\overset{b}{01}$

$10^3 = 1000$.

I $1000 \times \frac{01}{10} = 100$.

$100 \times \frac{1}{10} = 10$.

$1^3 = 1$.

II $2 \times 100 = 200$.

$2 \times 10 = 20$.

1000	100	10	1
	200	20.	
1000	300	30	1

1000 000000
3 000000
3000
1
1003003001

* Very rarely we need to find the nos. more than 1000.

Note: Observe the digits in the question.
& Select the method appropriately.
(the one which can give answer Instantly)
- Number made of 1's, 2's, 3's. or Similar digits - Anurupya Sutra.
- Number is closer to the bases - formula method.

EXERCISE

Q (1). Find the cube of following Nos. using the formula $(a+b)^3$.

(1) 105

(2) 41

(3) 54

(4) 23

Q (2). Find the cube of following Nos. using Anurupya Rule.

(1) 66

(2) 77

(3) 91

(4) 19

Q (3). Find the cube of following Nos. using formula $(a-b)^3$.

(1) 49

(2) 90

(3) 199

(4) 96.

Q.(4) Find the cube of the following Nos. using Anurupya Rule.

(1) 43

(2) 72

(3) 103.

Square Roots

A. Square Roots of perfect Squares.

Squaring → Multiplying No. by itself.

Thus $4 \times 4 \Rightarrow$ Square of '4'. (reps. 4^2)

Eg: $4^2 = 4 \times 4 = 16$.

$5^2 = 5 \times 5 = 25$

Hence - '16' is the 'Square of 4'

& '4' is the 'Square root' of 16'

III^{ly} '5' is the 'Square root of 25'

Method :

S1: Memorize the table below:

Number.	Square
1	1
2	4
3	9
4	16
5	25
6	36
7	49
8	64
9	81
10	100.

Observe: Depending on the ending Number the Sq. Root can be arrived.

Eg: If the Number ends with 1 then the
(In Sq. root) → either 1, or 9.

with 4, → 2, 8.
6, → 4, 6 etc.

Hence we arrive at the table:

Last digit of Square	Last digit of Sq. root
1	1 or 9.
4	2 or 8.
9	3 or 7
6	4 or 6.
5	5
0	0

Square → last digit 9 → last digit of Sq. root = 3 or 7.

* perfect Square will never end with 2, 3, 7 or 8

Table for Number & Square:

Number	Square
10	100
20	400
30	900
40	1600
50	2500
60	3600
70	4900
80	6400
90	8100
100	10000

Q. Find the Square root of 7744

* Number ends with '4' → 2 or 8.

Ans. at this stage } — 2 or — 8

* From Refn. Table.

80 6400]
90 8100] → 7744 lies between.
80 & 90

* Step 1 we know Sq. root ends with 2 or 8.
Hence the No. can be 82 or 88.

* Check the Number, whether it is closer to 6400 or 8100.
Here 7744 is close to 8100.
Hence the No. - 88.

Q: Find the Square root of 9801

① Ending - '1' → 1 or 9

② 90 8100]
100 10000] → 9801 → 91 or 99

③ closer to 10000 → 99.

Q. Find the Square root of 5184

① 4 → 2, 8

② $70^2 = 4900$]
$80^2 = 6400$] 5184 → 72 or 78

③ close to 4900 - 72

Q. Find the Square root of 2304

① __2__ , __8__

② $40^2 = 1600$] 2304 - 42 or 48.
$50^2 = 2500$]

③ 2304 close to 2500 - 48.

Q. Find the Square root of 529

① __3__ or __7__

② $20^2 = 400$.] 529 - 23 or 27.
$30^2 = 900$.]

③ closer to 400 → 23

Q. Find the Square root of 12544

① __2__ or __8__

② 110 - 12100. ⟩ 112 or 118.
120 - 14400. ⟩

③ 112.

Q. Find the Square root of 25281.

① __1__ or __9__

② 150 — 22500. ⟩ 151 or 159.
160 — 25600. ⟩

③ 159.

* Comparision with conventional method.

EXERCISE

PART A

Q (1). Find the Square roots of the following Nos.

(1) 9216

(2) 7569

(3) 5329

(4) 3364

(5) 1681

PART B

Q (1). Find the Square roots of the following Nos.

(1) 9801

(2) 5625

(3) 1936

(4) 3481.

(5) 1369.

PART C

Q (1) Find the Square roots of the following Nos.

(1) 12769

(2) 15625

(3) 23104

(4) 11881.

Cube Roots of perfect cubes.

Cube of $5 = 5 \times 5 \times 5 = 125$.

Cube Root of $125 = 5$.

S1: Memorize the table.

Number	Cube.
1	1
2	8
3	27
4	64
5	125
6	216
7	343
8	512
9	729
10	1000

Cube of $2 \rightarrow 8$ & Cube Root of $8 \rightarrow 2$.

If the last digit of Number ends with 1 $\rightarrow$ last digit of Cube Root - 1.

last digit ends with 2 $\rightarrow$ last digit of Cube Root - 8

etc.

Summarized below :

Last digit of Cube	Last digit of cube root
1	1
2	8
3	7
4	4
5	5
6	6
7	3
8	2
9	9
0	0

Except for the pairs of 3, 7 & 8, 2 all other Numbers end with the same numbers.

* When a no. is given to find the cube root put slash before last 3 digits

Eg: 103|823.

39304 ⇒ 39|304.

Regardless of any no. of digits, put slash before last 3 digits.

Solving the cube Root

NO.	1	2	3	4	5	6	7	8	9	10
cube	1	8	27	64	125	216	343	512	729	1000.

Cube Root - Solved in 2 parts

LHS & RHS.

② ①

Q. Find the cube root of 287496.

S1: Reps. 287 | 496.

S2: NO. ends with. 6, last digit is 6 (RHS).

S3: To find LHS, Take 287.

check for two perfect cubes for 287 (closer)

i.e. 216 & 343.

(6) 7.

= =

Take Smallest NO. & apply. i.e. 6 (LHS).

Hence Answer

(Cube root of 287496) = 66.

Q. Find the cube root of 205379

① 205 | 379.

② Ends with 9, ∴ RHS = 9

③ 205 - between ~~64~~ 125

4 5.

125 216

5 6

Take least NO. 5 (LHS)

Ans: 59

Q. Find the cube root of 681 | 472.

RHS = 8

681 → 512 729

8 9

(LHS).

Ans: 88

Q. Find the cube root of 830 | 584.

RHS = 4.

729 1000. ⇒ 94

9

(LHS)

Q. Find the cube Root of 2197 = 13.

RHS = 3 (for 7).

0 & 1 & 8

LHS.

* Concept of solving cube root remains same & only chance is to get the number line extension.

Number	9	10	11	12	13
Cube	729	1000	1331	1728	2197

Q. Find the cube root of 1157625

1157 | 625

↙ lies between. 1000 & 1331. (10) (11)

↓ LHS.

↓ RHS. = 5

105

Q. Find the cube root of 1404928

1404 | 928

↓ LHS = 2

1331 & 1728. (11) (12).

↓ (RHS).

112

* Comparision with conventional method.

PART A

Q.(1) Find the cube roots of the following numbers.

(1) 970299

(2) 658503

(3) 314432

(4) 110592

(5) 46656

(6) 5832

(7) 421875

(8) 1030301

PART B

Q(2) Find the cube roots of the following numbers.

(1) 132651

(2) 238328

(3) 250047

(4) 941192

(5) 474552

(6) 24389

(7) 32768

(8) 9261