Fractions & Decimals.

a. Introduction to Fractions.

* An object divided into NO. of equal parts - FRACTION

Eg: Two fifth of an object.
- 2/5.

Types of fraction:

1. Proper fraction
   num < denom

Eg: 2/5, 1/5, 7/16 etc.

2. Improper fraction
   num ≥ denom

Eg: 7/4, 6/6 etc.

3. Mixed fraction
   Whole Number + fraction.

Eg: $3\frac{4}{5}$, $7\frac{9}{2}$ etc.

Improper fractions & mixed fractions conversion.

Improper fraction → can be expressed as mixed fraction & Vice-Versa.

1. Eg: 13/5 as mixed fraction.

   S1: 13/5 Q→2 R→3.

   S2: $2\frac{3}{5}$

2. Eg: 38/9 Q→4 R→2

   S2: $4\frac{2}{9}$.

3. Express $4\frac{3}{7}$ as improper fraction

   S1: $4 \overset{+}{\underset{\times}{}} \frac{3}{7}$

   S2: $4 \times 7 + 3 = 31/7$.

4. $5\ 5/12$.

S1: $5 \times 12 + 5 = 65$

S2: $65/12$

b. Decimals

→ Decimal number contains a decimal point.
→ contains both whole part & decimal part.

$$\underset{x}{\overleftarrow{abc}}.\underset{y}{\overrightarrow{def}}$$

$x \rightarrow$ powers of 10.

$a \times 10^2 + b \times 10^1 + c \times 10^0$.

$y \rightarrow$ value decreases in reciprocals of power of 10.

d - 1/10 e - 1/100 f - 1/1000. etc.

1. Eg: 25.8

25 → whole part
8 → decimal part.

c. Types of Decimal Numbers.

1. Terminating decimals.

- The decimal numbers which terminate / contain finite digits in the decimal part are called Terminating decimals.

Eg: 15.222345, 9856.784321, 0.147935862.

2. Non-Terminating decimals

- The decimal numbers which do not terminate / contain in-finite digits in the decimal part are called Non-Terminating decimals.

There are 2-types of non-terminating decimals.

a. Recurring decimals

- decimal digits repeats itself / recurs endlessly (infinitely) is called Recurring decimals.

digits → single / multigroup.

Eg: 0.3333... , 28.454545... , 147.25792579... , 7.1333...

The recurring decimals are represented as $0.\dot{3}$, $28.\dot{4}\dot{5}$, $147.\dot{2}5\dot{7}\dot{9}$, $7.1\dot{3}$, $52.3\dot{8}\dot{4}$.

$52.3\dot{8}\dot{4}$, $7.1\dot{3}$: These kind of decimal are also called recurring decimals with terminating part, since one or more digits are terminating and there is recurring part as well.

b. Non-Recurring decimals

- The decimal numbers in which the decimal part contains infinite digits but do not recur are called Non-Recurring decimals.

Eg: $\pi = 3.141592....$, 57.896542310872...

d. Rules to identify the types of decimals

1. Rules to identify terminating decimals.
   * if the denominator has only 2 & 5, sometimes both as some of the factors then it is terminating decimal.

   Eg: $7/2$, $19/5$, $42/10$, $55/16$, $62/25$ etc.

2. Rules to identify non-terminating decimals

   a). Recurring decimals
   * if the denominator has prime numbers as some of the factors but no 2's & 5's, then it is recurring decimal.

   Eg: $22/3$, $34/7$, $63/27$, $85/11$.

   b). Recurring decimals with terminating part
   * if the denominator has prime numbers (i.e. 2,3,5,7) as some of the factors then it is recurring decimal

with terminating part.

Eg: $1/12$, $33/18$, $7/24$ etc.

II Conversions of decimals to fractions

1. For terminating decimals

- Multiply & divide by powers of 10.
- The power of 10 is chosen based on the NO. of digits after the decimal point.
- Suppose there are 3-digits after the decimal point then we multiply & divide by 1000.
(i.e. the NO. of digits after the decimal point & the NO. of ~~digits~~ zeros in the power of 10 must be equal).

a. Eg: 0.24

→ NO. of digits after decimal point is 2

→ SO NO. of zeros in power of 10 must be 2 (i.e. 100)

* Multiply & divide by 100 & simplify to get the fraction equivalent to the decimal.

i.e. $0.24 \times \frac{100}{100} = \frac{24}{100} = 6/25$

b. 34.725

$34 + 0.725$

$34 + 725/1000 = 29/40$

$34\ \frac{29}{40}$

c. 96.72

$96 + 72/100 = 96\ 18/25$

(18/25)

d. 1.73

$1 + 73/100 = 1\ 73/100$

2. For non-terminating decimals (recurring).

* Write the recurring digit as numerator & denominator will contain 9's.
* No. of 9's in denom = No. of digits in the recurring part.

a. Eg: $0.\dot{3}$

* digits recur in decimal part is 3.
* So '3' forms Numerator.
* No. of digits recurring is only one

fraction = $3/9 \Rightarrow 1/3$.

Fraction equivalent of $0.\dot{3} = 1/3$.

b. Eg: $12.\dot{4}\dot{6}$

$\dot{4}\dot{6} = \frac{46}{99}$

$12 + \frac{46}{99} = \frac{(12 \times 99) + 46}{99}$

$= \frac{1234}{99}$

1 × 2
9 9
1098

3. For Recurring decimals with terminating part.

Eg: $21.3\dot{4}\dot{5}$

$= 21.3454545 \rightarrow ①$

Multiply ① by 10

$= 213.4545 \rightarrow ②$

(Multiply until one sequence is onto other part of decimal)

ie. $21345.45 \rightarrow ③$

Eq. ② & ③ have same digits after decimal part.

Subtract these two eqns.

$1000x = 21345.45$
$- 10x \quad\quad 213.45$
$990x \quad\quad 21132.0$

$$x = \frac{21132}{990} = \frac{2348}{110}$$

SR of 21132 is 9, Hence divisible by 9.

Eg: $405.8\dot{7}\dot{1}$

$x = 405.8\dot{7}\dot{1}$

$10x = 4058.\dot{7}\dot{1}$

$1000x = 405871.\dot{7}\dot{1}$

$990x = 405871.\dot{7}\dot{1} - 4058.\dot{7}\dot{1}$

$= 401813$

$$x = \frac{401813}{990}$$

III. Converting fractions into decimals.

1. When denominator is power of 10.

* Count no. of zeros, place the decimal point in the numerator after the digits equal to no. of zeros.

a. $28/10 = 2.8$

b. $7/100 = 0.07$

2. When denominator is 5.

* Multiply both num & den by 2
* Then place decimal in num (as in step 1).

a. $97/5 = \frac{97 \times 2}{5 \times 2} = \frac{194}{10} = 19.4$

b. $13/5 = \frac{13 \times 2}{5 \times 2} = \frac{26}{10} = 2.6$

3. When denominator is 2.

* Multiply by 5 (both Num. & den.).
* follow Step 1.

a. Eg: $\frac{33}{2} = \frac{33 \times 5}{2 \times 5} = \frac{165}{10} = 16.5$

b. Eg: $\frac{81}{2} = \frac{81 \times 5}{2 \times 5} = \frac{405}{10} = 40.5$

4. When the denominator is 4

* Multiply both Num & den. by 5 twice.
* follow step 1.

a. Eg. $\frac{23}{4} = \frac{23 \times 5}{4 \times 5} = \frac{115 \times 5}{20 \times 5} = \frac{575}{100} = 5.75$

b. Eg. $\frac{75}{4} = \frac{75 \times 25}{4 \times 25} = \frac{1875}{100} = 18.75$

IV Highest common factor (HCF).

It's highest common factor present b/n 2 or more Nos.

a. Find the HCF of [22, 26].

S1: Difference b/n 2 Numbers.

$26 - 22 = 4$.

S2: Difference is not prime No., we have to find the factors of 4. Factors of 4: 2 & 4.

S3: check the given numbers are divisible by factors of 4. i.e. both Nos. are divisible by 2, but not by 4.

H.C.F. = 2

b. Find the HCF of [3, 5]

S1: $5 - 3 = 2$

S2: '2' is prime.

S3: Check the given Nos. divisible by '2'
- both Nos. are not divisible by '2'
hence HCF = 1.

c. Find HCF of [138, 161].

S1: 161-138 = 23.

S2: '23' - prime.

S3: 23 divides both Nos.

HCF = 23

d. Find HCF of [20,30].

S1: 30-20 = 10.

S2: Factors of 10 = 2, 5, 10.

S3: All three factors can divide given Nos.

HCF = 10.

Points to Remember:

a. If the difference of 2 Numbers is a prime Number, then check if the numbers are divisible by this number, if yes, then HCF = number
else HCF = 1.

b. difference ≠ prime.
Find the factors.
Check if the numbers are divisible by factors, if it does then the difference itself is HCF otherwise 1.

V. Least Common Multiple (LCM)

A common multiple is a number that is a multiple of 2 or more Numbers.
LCM is a smallest Number (not zero) that is a multiple of both.

Method 1: UT (Urdva Tiryagbhyam).
Meaning: Vertically crosswise.

LCM by UT

a. Find LCM of [12, 18]

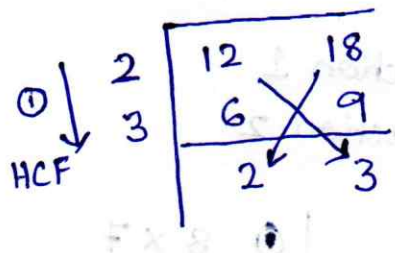
1. General Method. to find LCM

2	12, 18
3	6, 9
	2, 3

2 × 3 × 2 × 3 = 36

LCM = 36

2. To find LCM using UT.



HCF = 2 × 3 = 6

LCM = 12 × 3 = 18 × 2
= 36.

Method 2: Anurupyena method.

Meaning: 'Proportionality'

This is a simple method for finding the lowest common multiple of 2 numbers, when the answer is not obvious.

a. Find LCM of [12, 18].

S1: Write HCF = 6

S2: Divide both nos. by $12/6 = 2$ $18/6 = 3$

S3: $6 \left| \begin{matrix} 12 & 18 \\ 2 & 3 \end{matrix} \right.$

LCM = product of 3 Nos.

= 6 × 2 × 3

= 36

VI Basic operations in fractions:

a. Addition & Subtraction of fractions by UT.

UT → vertically & crosswise

* When the denominators are co-prime

① Eg: Add $5/2 + 7/3$

S1: Cross multiply - 'Num' of fraction 1 × 'Den' of fraction 2

$$5/2 \times\!\!\times 7/3 = \frac{5 \times 3 + 2 \times 7}{?}$$

S2: We get 'den' by multiplying the denominators.

$$\frac{15 + 14}{(2 \times 3)} = \frac{29}{6}$$

2. $\frac{8}{16} - \frac{13}{7}$

S1: Cross multiply - 'Num' of fraction 1 × 'den' of fraction 2.

$\frac{8}{16} \times \frac{13}{7} = \frac{(8\times7)-(13\times16)}{?}$

$= \frac{56-208}{?}$

1×3
1×6
208

S2: Get denominator by multiplying den.

$\frac{56-208}{16\times7} = \frac{-152}{112}$

① 8×7.
$= 1\bar{2} \times 1\bar{3}$
$= 1\bar{2}$
$1\bar{3}$
$1(\bar{5})6$
$= 56$

② 13×16
$1\ 3$
$2\ \bar{4}$
$2\ 2\ \bar{12}$
① 88
$2\ 1\ 88$.
208

* When the denominators are not co-prime

1. Eg: $\frac{3}{8} + \frac{5}{12}$

S1: Write down HCF of denominators

$12 - 8 = 4$

Both divisible by '4'

HCF = 4.

S2: Divide the den. by HCF & write below

i.e. $\frac{3}{8} + \frac{5}{12}$

$\downarrow \quad \downarrow$

$4 \quad 2 \quad 3$

S3: 2 & 3 will be new denominator.

Rewrite as below.

$\frac{3}{2} + \frac{5}{3} = \frac{3\times3 + 2\times5}{4\times2\times3} = \frac{19}{24}$

2. $7\frac{5}{9} - 4\frac{7}{15}$

S1: $7 - 4 = 3$.

S2: $15 - 9 = 6 = 2, 3$

H.C.F. = 3.

S3: $\frac{5}{9}$ ■ $\frac{7}{15}$.

↓ ↓

3 3 5

S4: $\frac{5}{3} - \frac{7}{5} = \frac{5\times5 - 7\times3}{3\times3\times5} = \frac{25-21}{45} = \frac{4}{45}$

③

$7\frac{5}{9} - 4\frac{7}{15} = 3\frac{4}{45}$.

* When the denominator is factor of other.

* When the denominators are same

Eg: $\frac{3}{5} + \frac{19}{5}$

Simply add num & place den as-is.

$$\frac{3+19}{5} = \frac{22}{5}$$

* When denominator is factor of other
- Multiply until denom are same.

① Eg: Add $1/4 + 5/16$

S1: 4 is a factor of 16.

$$\frac{1}{4} \times \frac{4}{4} = \frac{4}{16}$$

S2: $\frac{4}{16} + \frac{5}{16} = \frac{4+5}{16} = \frac{9}{16}$.

2. $\frac{4}{5} - \frac{31}{40}$

$$\frac{4}{5} \times \frac{8}{8} - \frac{31}{40} = \frac{32}{40} - \frac{31}{40} = \frac{1}{40}$$

b. Multiplication of fractions -

- Multiply two 'Nums'
- Multiply two 'Dens'

1. Eg: $\frac{3}{\cancel{4}_1} \times \frac{\cancel{12}^3}{} = 9.$

$\frac{\cancel{3}^1}{4} \times \frac{7}{\cancel{8}_2} = \frac{7}{8}$

2. $1\frac{5}{7} \times 1\frac{1}{3} = \frac{\cancel{12}^4}{7} \times \frac{4}{\cancel{3}_1} = \frac{16}{7} = 2\frac{2}{7}$

c. Division of fractions.

Sutra → 'Transpose & Adjust'

→ keep 1st fraction as-is.

→ Turn 2nd fraction upside down.
& change sign ÷ to '×'.

* paravartya Sutra.

1. Eg $1/2 \div 1/9$

$= 1/\cancel{2}_4 \times \frac{\cancel{9}^3}{1} = 3/4$

2. $2\frac{1}{3} \div 1\frac{3}{4}$

$= 7/3 \div 7/4$

$= \cancel{7}/3 \times \frac{4}{\cancel{7}} = 4/3 = \underline{1\frac{1}{3}}$

EXERCISE

I. convert decimal to Fraction.

a. 0·3 b. 0·45 c. 2·3 d. 0·234 e. 0·5

f. 7·65 g. 0·98 h. 5·8 i. 0·123

II. convert fraction to Decimal.

a. $\frac{4}{10}$ b. $8/5$ c. $\frac{6}{10}$ d. $\frac{61}{2}$ e. $\frac{11}{100}$

f. $\frac{123}{100}$ g. $\frac{17}{4}$ h. $\frac{10}{3}$ i. $\frac{21}{5}$

III. convert Improper fraction to mixed fractions

a. $\frac{9}{4}$ b. $7/2$ c. $\frac{11}{3}$ d. $18/5$ e. $\frac{23}{4}$

f. $\frac{49}{8}$ g. $41/5$ h. $29/3$ i. $\frac{54}{10}$

IV convert mixed fractions to Improper fractions.

a. $3\frac{4}{5}$ b. $2\frac{2}{7}$ c. $4\frac{1}{4}$ d. $2\frac{3}{5}$ e. $3\frac{3}{5}$

f. $7\frac{2}{4}$ g. $8\frac{3}{2}$ h. $4\frac{7}{8}$ i. $6\frac{9}{10}$ j. $5\frac{2}{7}$.

V Find the HCF of the following.

a. 3, 4 b. 32, 36 c. 83, 96 d. 66, 44 e. 48, 32

f. 4, 5 g. 2, 4 h. 6, 12

VI Find the HCF of the following

a. 6, 9 b. 9, 8 c. 83, 84.

d. 77, 66 e. 45, 50 f. 4, 7

VII Find the LCM by UT Method -

a. 18, 24 b. 28, 36 c. 6, 12 d. 15, 20

e. 12, 32 f. 48, 52 g. 72, 84 h. 66, 44.

VIII. Find the LCM by UT Method.

a. 6, 8 b. 6, 9 c. 25, 30.

d. 24, 28 e. 8, 18 f. 9, 15.

IX Find the LCM by Anurupyena method.

a. 12, 8 b. 10, 4 c. 15, 10 d. 40, 60.

e. 8, 14 f. 10, 12 g. 12, 14 h. 6, 20.

X. LCM by UT

a. 5, 12 b. 10, 6 c. 4, 5

d. 4, 16 e. 25, 10 f. 5, 7

XI Add the following

a. $\frac{5}{12} + \frac{1}{5}$ b. $\frac{15}{6} + \frac{2}{3}$ c. $6\frac{1}{18} + 4\frac{1}{16}$ d. $\frac{11}{20} + \frac{1}{4}$

e. $\frac{7}{10} + \frac{1}{12}$ f. $\frac{8}{9} + \frac{3}{8}$ g. $\frac{11}{16} + \frac{5}{32}$ h. $11\frac{11}{16} + 2\frac{3}{8}$

XII Solve the following

a. $\frac{4}{15} + \frac{3}{5}$ b. $\frac{1}{17} + \frac{1}{20}$ c. $\frac{15}{16} + \frac{7}{20}$ d. $2\frac{7}{36} + 94\frac{5}{48}$

e. $3\frac{5}{6} + 2\frac{1}{3}$.

XIII Subtract the following

a. $\frac{7}{9} - \frac{23}{36}$ b. $\frac{5}{8} - \frac{5}{11}$ c. $\frac{5}{6} - \frac{12}{25}$ d. $\frac{7}{8} - \frac{5}{24}$

e. $\frac{11}{12} - \frac{4}{15}$ f. $6\frac{12}{25} - 3\frac{3}{40}$ g. $\frac{22}{36} - \frac{5}{24}$ h. $9\frac{9}{6} - 1\frac{1}{28}$

XIV Solve the following

a. $\frac{7}{8} - \frac{55}{64}$ b. $\frac{31}{530} - \frac{1}{3}$ c. $\frac{21}{250} - \frac{7}{100}$

d. $6\frac{3}{4} - 1\frac{3}{14}$ e. $8\frac{9}{10} - 2\frac{7}{15}$.

XV Multiply the following

a. $\frac{3}{7} \times \frac{2}{5}$ b. $\frac{4}{15} \times \frac{7}{12} \times \frac{25}{21}$ c. $3\frac{8}{9} \times 3\frac{6}{7}$

d. $1\frac{1}{2} \times 1\frac{1}{3} \times 1\frac{1}{5}$ e. $\frac{4}{63} \times \frac{21}{32}$ f. $2\frac{1}{4} \times 5\frac{1}{3}$

g. $2\frac{4}{7} \times 4\frac{2}{3}$ h. $2/3 \times 6/7 \times 3/4$

XVI Solve

a. $1/5 \times 8$ b. $2\frac{1}{4} \times 1/3$ c. $3\frac{1}{16} \times \frac{14}{25} \times 2\frac{1}{7}$

d. $\frac{10}{27} \times \frac{9}{35}$ e. $\frac{4}{7} \times \frac{5}{8}$

XVII Divide the following.

a. $\frac{5}{8} \div \frac{2}{3}$ b. $1 \div 1\frac{1}{2}$ c. $3\frac{1}{7} \div 11$ d. $1\frac{11}{45} \div 10\frac{1}{9}$

e. $\frac{2}{9} \div \frac{4}{3}$ f. $1\frac{5}{7} \div 1\frac{1}{17}$ g. $\frac{5}{8} \div \frac{5}{8}$ h. $\frac{28}{48} \div 4\frac{5}{18}$

XVIII Solve the following.

a. $1\frac{1}{3} \div 2/5$ b. $1/12 \div 1/9$ c. $3\frac{3}{14} \div 1\frac{4}{21}$

d. $5 \div 3\frac{1}{3}$ e. $2\frac{1}{3} \div 1\frac{3}{4}$.