Geometry

a. Introduction

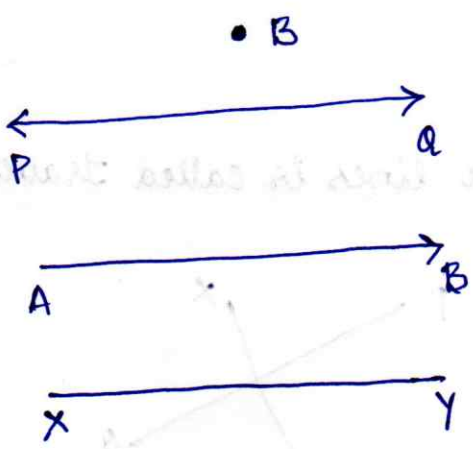
point : A point is a location on the plane or space. It is represented by a dot (•)

Line : A collection of points from a line. It extends on both the directions (i.e. it has no end points)

Ray: A ray is also a collection of points but it extends only in one direction (i.e. it has one end point)

Line Segment : A part of line is called line segment. It has two end points.
A line segment can also be defined as line joining two points.

Representations :



B → point

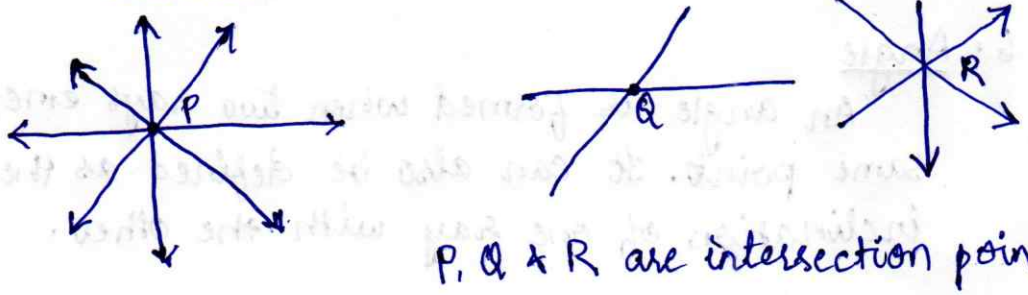
Line → PQ (Here P & Q are not end points.

Ray → AB (A - End point).

Line Segment → XY (X & Y both have end points).

b. classification of lines.

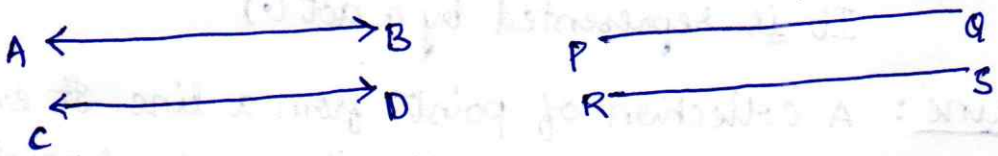
1. **Intersecting Lines** : Two or more lines which meet at a point is called Intersecting lines.
The point where they meet is called the point of Intersection.



P, Q & R are intersection points.

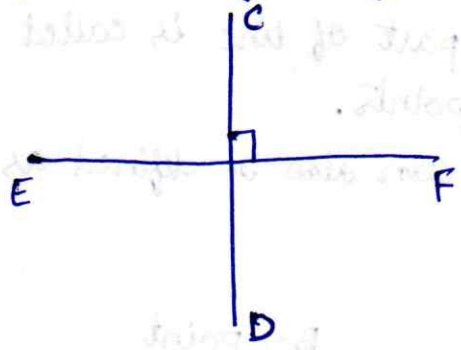
2. Parallel lines

Two lines are called parallel lines, if they never intersect even when extended indefinitely.



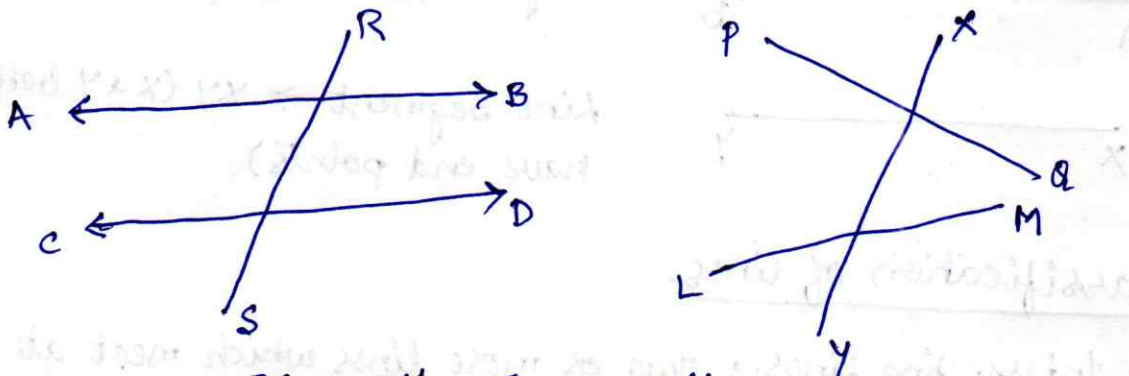
3. Perpendicular lines

Two lines are said to be perpendicular if they intersect at right angle. (i.e. right angle formed) between 2 lines.



4. Transversal lines

A line intersecting two lines is called Traversal line.



RS & XY - Traversal line

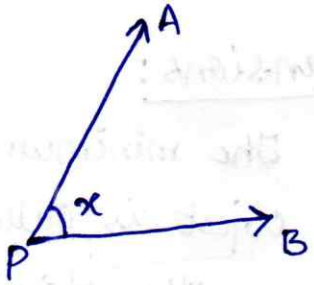
5. Vertex

A point at which two lines meet is called vertex.

6. Angle

An angle is formed when two rays emerge from the same point. It can also be defined as the measure of inclination of one ray with the other.

Here PA & PB are two rays & P is a vertex. 'x' is the angle formed by the rays PA & PB.



7. Surface

A surface is a boundary in a space. It divides the space into 2 regions or it bounds a region in space.

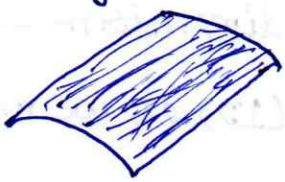


fig 1

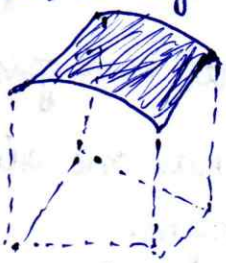


fig 2

Here fig 1 is the surface & fig 2 is the region bounded by the surface

8. Plane :

A plane is a flat surface which extends infinitely. It is a two dimensional surface.

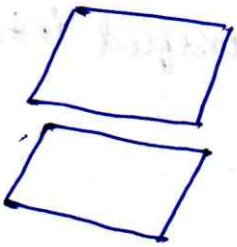


fig 3.

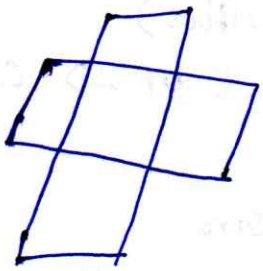


fig 4

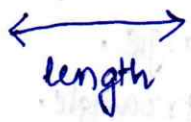
fig 3 : Represents two parallel planes to each other

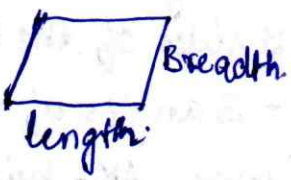
fig 4 : Represents two intersecting planes.

c. Geometric shapes :

A set of points or vertices connected by a set of lines or curves form geomitric shapes.

Eg :





Dimensions:

The minimum No. of co-ordinates required to define the object is called dimensions.

The objects may be points, lines, rays or geometric shapes.

Dimensions of geometric shapes:

1. Point: A point has zero or no-dimension.
2. Line: A line has one dimension (1D) as we measure only length.
3. Plane: A plane has 2 dimensions (2D) as we measure length & breadth.
4. Solid: A solid has 3 dimensions (3D) as we measure length, breadth & width.

2D Geometric shapes:

(Examples with properties).

Geometric shapes in 2D can be classified into 2-main categories.

a. Polygons
b. Curved shapes.

Polygons is derived from the greek word 'Poles' meaning many and 'gon' meaning angle or corner.

Types of Polygons:

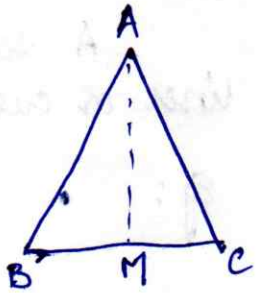
a. Triangle:

The simplest polygon with three sides & three angles is called triangle.

AB, BC, AC - 3 sides of the triangle.

A, B & C are - 3 angles of the triangle.

BC - base of triangle AM - height of triangle.



* Perimeter of triangle:

let a, b & c be the lengths of the triangle

Perimeter = a+b+c.

* Area of triangle.

Area = $\frac{1}{2}$ x base x height.

= $\frac{1}{2}$ x b x h.

b - base length.

h - height length.

b. Quadrilateral

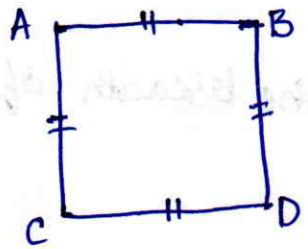
The polygon with four sides & four angles is called. Quadrilateral.

Quadrilateral are further classified based on the length of their sides & angles.

1. Square:

A quadrilateral in which all the sides are equal & interior angles are all right angles is called Square.

- The opposite sides are parallel to each other.



- AB, BD, AC & CD are equal sides.
- AB, CD & AC, BD are pairs of parallel sides.

A, B, C & D are right angles.

Perimeter of Square:

Let the length of the side of the square is 'a'

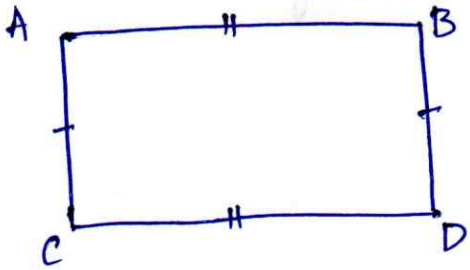
Perimeter = 4 x a = 4a.

Area of the Square = a x a = a^2

2. Rectangle:

A quadrilateral in which only the opposite sides are equal & interior angles are right angles is called Rectangle.

* The opposite sides are parallel to each other.



- AB & CD, AC & BD are parallel sides where AB = CD & AC = BD.
- A, B, C & D are right angles.

Perimeter of Rectangle:

Let the length be 'l' & breadth 'b' of a rectangle.

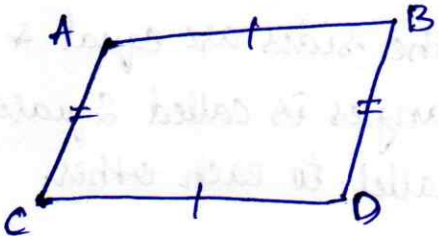
Perimeter = $2 \times (l + b)$.

Area of the rectangle = $l \times b$.

3. Parallelogram:

A quadrilateral in which opposite sides are equal & the interior opposite angles are equal is called parallelogram.

* The opposite sides are parallel to each other.



- CD & AB, AC & BD are pairs of equal & parallel sides.
- A & D, C & B are opposite angles & are equal.

Perimeter of Parallelogram:

let 'l' be the length & 'b' be the breadth of the Parallelogram.

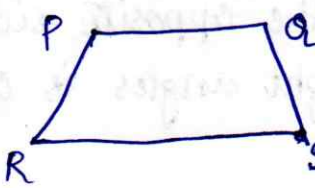
Perimeter = $2 \times (l + b)$.

Area = $l \times b$.

4. Trapezium:

A quadrilateral in which one pair of opposite sides are parallel is called trapezium.

All four sides & four angles are not equal.



- PQ & SR are parallel sides.
- All sides are of different length.
- All the interior angles are not equal.

Perimeter of Trapezium:

let 'a' & 'b' be the length of parallel sides of the trapezium & 'c' & 'd' be the length of the other two sides of the trapezium.

Perimeter = $a+b+c+d$.

Area = $\frac{h}{2}(a+b)$.

d. General formula for area of 2D shapes:

Let us consider the height of each of the shapes as 'h' & 'b' as the base.

Formula in 2-Steps:

S1: Find the average of the parallel sides of 2D shape

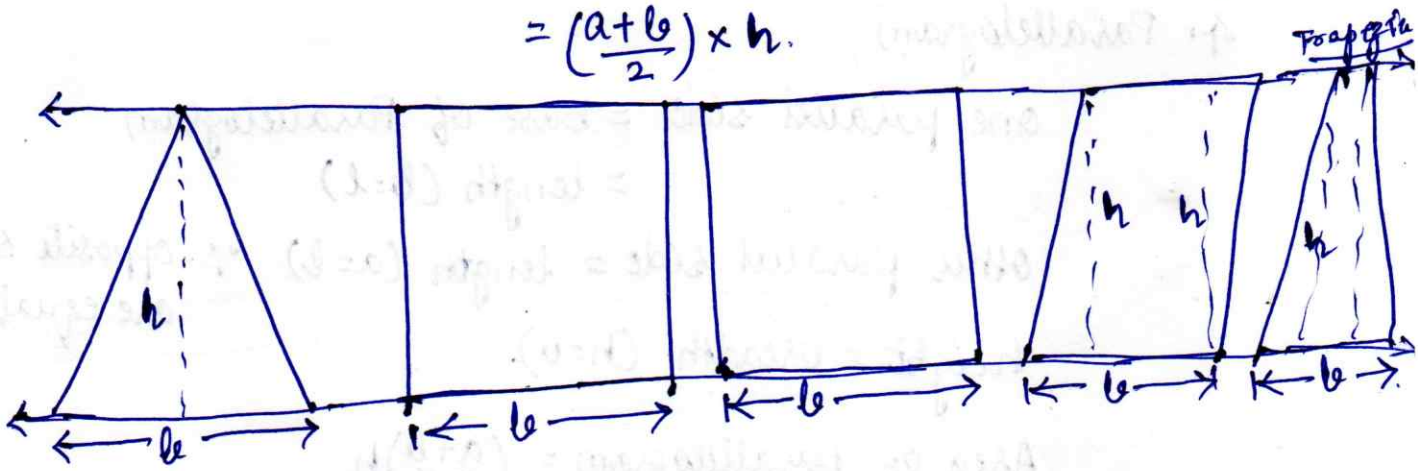
S2: Find the height of the 2D shape.

* Multiply the answers of steps 1 & 2 to get the area.

Let 'a' & 'b' be the parallel sides (where 'b' is the base, & 'h' be the height of the 2D shapes).

Area of 2D shape = Average of parallel sides x height

$= \left(\frac{a+b}{2}\right) \times h.$



Find the area of each geometrical shapes:

1. Triangle

one parallel side = base of triangle (b).
other parallel side = Zero (i.e. a=0).

height of triangle = h.

Area of triangle = $\left(\frac{a+b}{2}\right) \times h = \left(\frac{b+0}{2}\right) \times h$.

$= \frac{1}{2} \times b \times h$.

2. Square

one parallel side = b (base of square)

other parallel side = a ($a = b$).

Height of the Square = h ($h = b$)

Area of Square = $\left(\frac{a+b}{2}\right) \times h = \left(\frac{b+b}{2}\right) \times b$.

$= b \times b = b^2$.

3. Rectangle

one parallel side = base of length (b) = l.

other parallel side = base of ~~length~~ breadth (b) = b.

height of Rectangle = $h = b$.

Area of Rectangle = $\left(\frac{a+b}{2}\right) \times h$

$= \left(\frac{l+b}{2}\right) \times b = \underline{l \times b}$.

4. Parallelogram

one parallel side = Base of Parallelogram

= length ($b = l$)

other parallel side = length ($a = l$) ∵ opposite sides are equal.

height = breadth. ($h = b$).

Area of parallelogram = $\left(\frac{a+b}{2}\right) h$

$= \left(\frac{l+l}{2}\right) \times b$

$= \underline{l \times b}$.

5. Trapezium

One parallel side = base ($b = a$)

Other parallel side = other side ($a = b$).

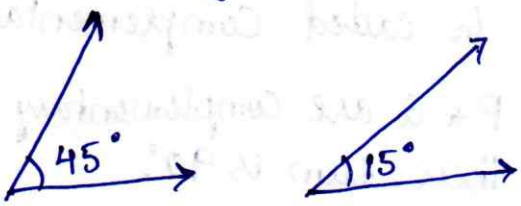
height of trapezium = h.

Area of trapezium = $\left(\frac{a+b}{2}\right) \times h$.

e. Classification of Angles.

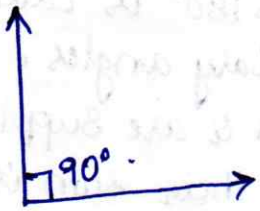
I. Based on measurement of Angles:

1. Acute Angle:



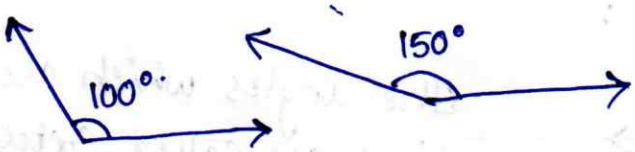
* if the angle measures b/n 0° to 90°.

2. Right Angle.



* If an angle measures exactly 90°.

3. Obtuse Angle

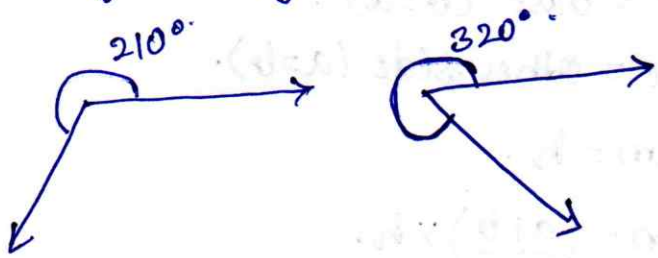


* if an angle measures b/n 90° to 180°.

4. Straight angle.

* if an angle measures exactly - 180°.

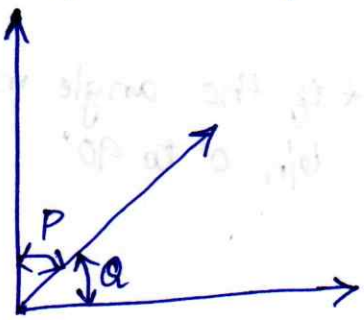
5. Reflex Angle.



* if an angle measures b/n 180° to 360°.

II Based on Sum of Angles

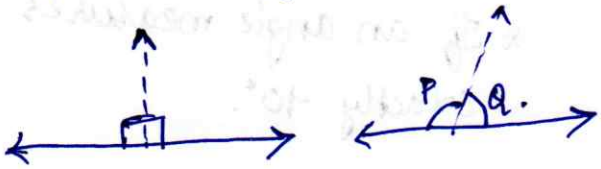
1. Complementary Angles



* pair of angles whose sum is 90° is called Complementary angles.

P & Q are complementary angles as their sum is 90°.

2. Supplementary Angles

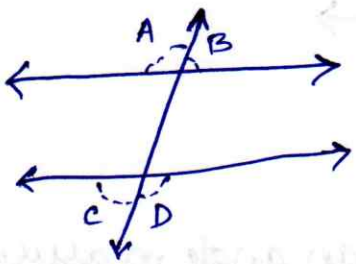


* pair of angles whose sum is 180° is called Supplementary angles.

P & Q are Supplementary angles as their sum is 180°.

III Based on Traversal line :.

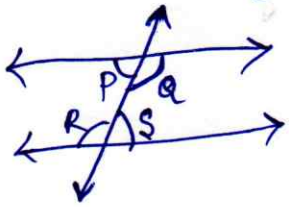
1. Exterior Angles :



The angles which are outside the lines are called Exterior Angles.

A, B, C & D are exterior Angles.

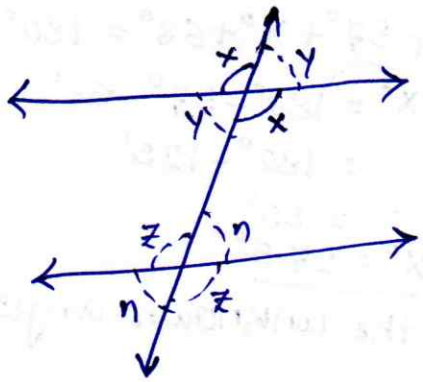
2. Interior Angles.



The angles which are inside the lines are called Interior Angles.

P, Q, R & S are interior Angles.

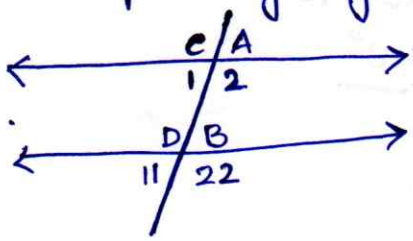
3. Vertically opposite Angles.



A pair of angles which are opposite when two lines intersect are called Vertically opposite angles.

x, y, n, z are opposite angles.

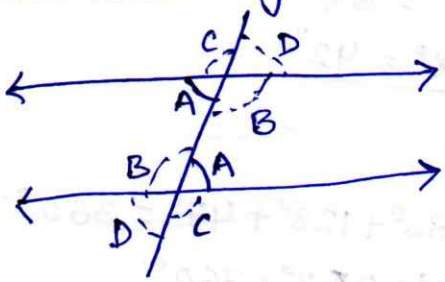
4. corresponding Angles.



Two angles on the same side of the traversal line, among which one is interior & other one is exterior are called corresponding angles.

A&B, C&D, 1&11, 2&22 are corresponding angles.

5. Alternate Angles.



Two angles on the opposite sides of the traversal line, either exterior or interior are called Alternate Angles.

A, B, C & D are Alternate Angles.

6. Finding of Angles.

Sankalana Vyavakalanabhyam Sutra.

1. Missing angle in Straight line

Vedic maths has a sutra for solving these types of questions & i.e. Sankalana Vyavakalanabhyam Sutra. It means "By addn. or Sub.".

Eg 1: Find the answer in ??.

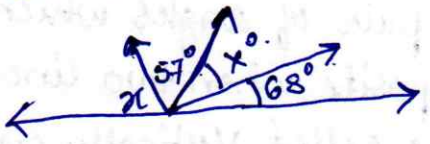
Angle of St. line = 180°.

$97° + x = 180°$

$x = 180° - 97°$

$= 83°$.

Eg: 2 Find what should be the answer in place of x°.



$x^\circ + 57^\circ + x^\circ + 68^\circ = 180^\circ$.

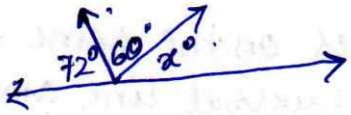
$2x^\circ = 180^\circ - 57^\circ - 68^\circ$.

$= 180^\circ - 125^\circ$.

$= 55^\circ$.

$x^\circ = 27.5^\circ$.

Eg: 3 Find the measurement of the unknown angle which is denoted by x°.



$x^\circ + 72^\circ + 60^\circ = 180^\circ$.

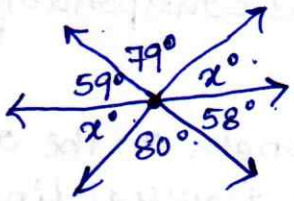
$x^\circ = 180^\circ - 132^\circ$.

$= 48^\circ$.

2. Missing angle at a point

A point has angle of 360°.

Eg 1:



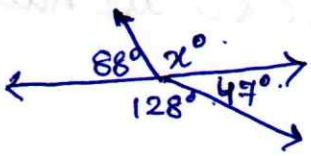
$2x^\circ + 79^\circ + 59^\circ + 58^\circ + 80^\circ = 360^\circ$.

$2x^\circ + 276^\circ = 360^\circ$.

$2x^\circ = 84^\circ$.

$x^\circ = 42^\circ$.

Eg 2: Find the value of x°.

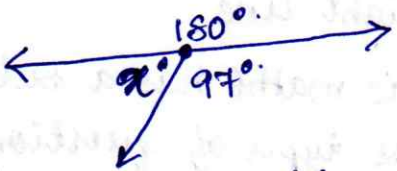


$x^\circ + 88^\circ + 128^\circ + 47^\circ = 360^\circ$.

$x^\circ + 263^\circ = 360^\circ$.

$x^\circ = 97^\circ$.

Eg 3: Find the value of x°.



$x^\circ + 97^\circ + 180^\circ = 360^\circ$.

$x + 277^\circ = 360^\circ$.

$x = 83^\circ$.

3. Missing Angles in Triangle

Triangle has.

a. Three line segments

b. Three angles.

c. Three vertices or Edges.

Triangles are of many types based on two parameters.

a. Based on angles.

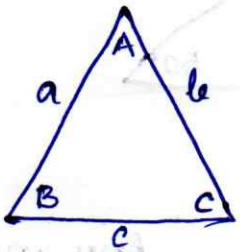
b. Based on sides.

1. Based on sides.

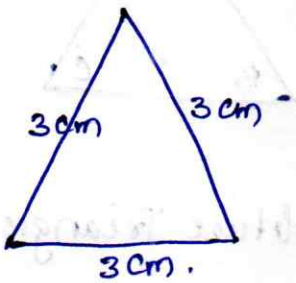
(i) Equilateral Triangle

In this triangle all the three sides are same (measurement).

Eg:



$a = b = c$.

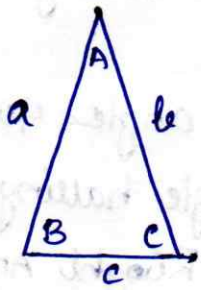


Note: In equilateral triangle, All the 3 sides & angles (= 60°).

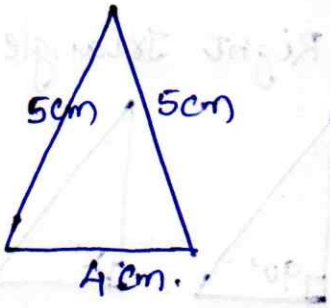
$A = B = C = 60°$.

(ii) Isoceles Triangle

In this triangle any two sides are equal. (measurements).

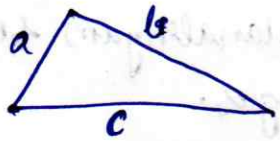


$a = b$

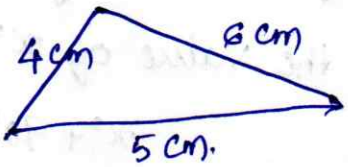


Also, $a = c$ or $b = c$ apply.

(iii) Scalene triangle



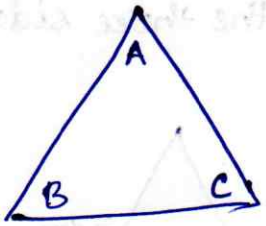
a, b & c are of different sides with different measurements



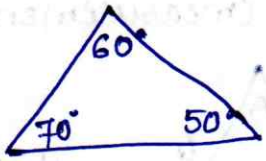
2. Based on Angles.

(i) Acute Triangle

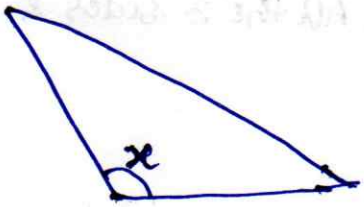
Acute triangle (angle) measure angle b/n 0° & 90°.
So when the angles in a triangle are acute angles then these triangles are known as Acute triangles.

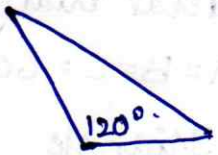


$A < 90°$, $B < 90°$ & $C < 90°$.



(ii) Obtuse Triangle



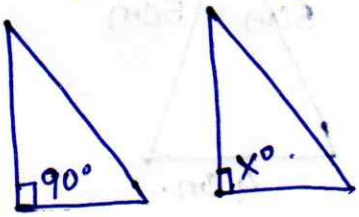


Any angle $> 90°$ is called Obtuse Angle ($< 180°$)

Obtuse Angle $> 90° - x - < 180°$

A triangle having an Obtuse angle is called Obtuse Angle Triangle.

(iii) Right Triangle



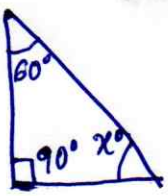
$x = 90°$.

A right angle = 90°.
A triangle having 90° angle is called Right Angle Triangle.

Note: Sum of all angles (3) in a triangle = 180°.

→ Using Sankalana Vyavakalanabhyam sutra to finding the missing angles.

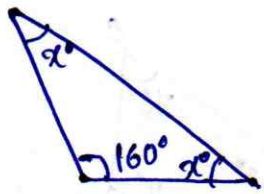
Eg 1: Find the value of $x°$.



$x° + 90° + 60° = 180°$

$x° = 30°$

Eg 2: Find the value of x°.



$x^\circ + 160^\circ + x^\circ = 180^\circ$.

$2x^\circ = 20^\circ$.

$x = 10^\circ$.

9. Finding exterior angle of a triangle

* concept is a combination of both straight line and triangles. We know that straight angle is 180°. and a triangle is also made by straight lines.

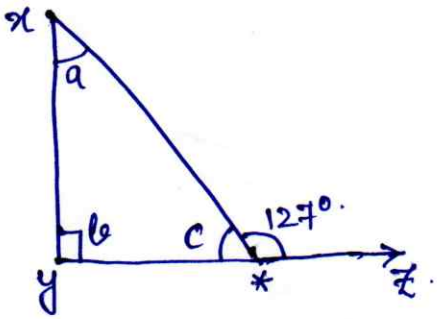


fig a.

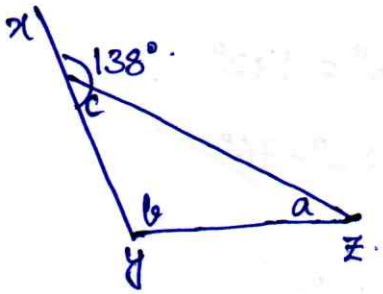


fig b

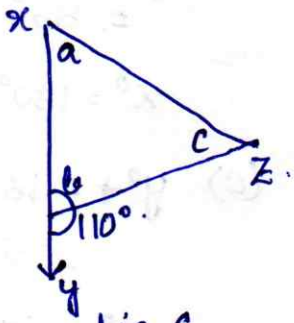


fig c.

Here a, b & c angles are known as interior angles.

127°, 138° & 110° → Exterior angles.

In fig a, a line segment "yz" is extended outside to make exterior angle of 127° but if you see at point (star mark) that point forms a straight line angle is 180°.

$\angle c + 127^\circ = 180^\circ$

$\angle c = 180^\circ - 127^\circ$

$= \underline{53^\circ}$

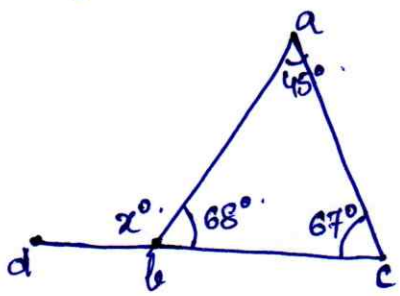
fig b. $\angle c = 180^\circ - 138^\circ$

$\angle c = 42^\circ$.

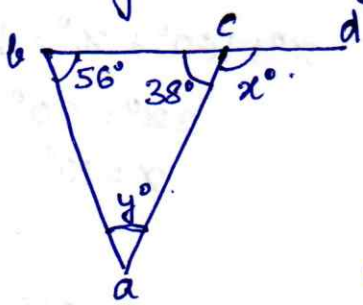
fig c. $\angle c = 180^\circ - 110^\circ$

$\angle c = 70^\circ$.

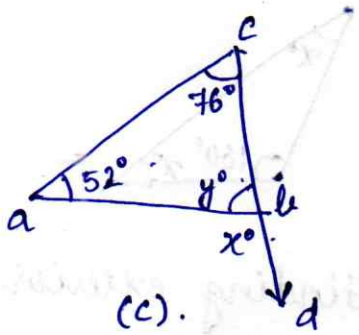
Eg 1: Find the exterior angle in the given triangles.



(a)



(b)



(c).

(a) $x^\circ = 180^\circ - 68^\circ = 112^\circ$.

(b) $y^\circ = 180^\circ - 56^\circ - 38^\circ$.

$= 86^\circ$.

$x^\circ = 180^\circ - 38^\circ = 142^\circ$.

(c) $y^\circ = 180^\circ - 52^\circ - 76^\circ$

$= 52^\circ$

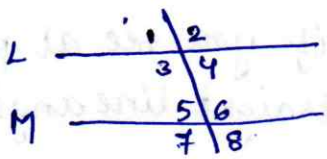
$x^\circ = 180^\circ - 52^\circ$.

$= 128^\circ$.

h. Finding angles using Vilokanam Sutra.

Vilokonam – Observation.

If a traversal line cuts two parallel lines then.

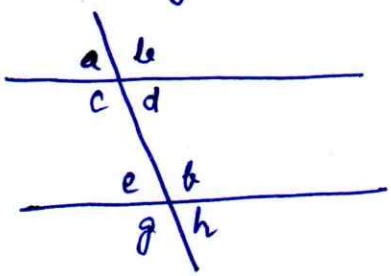


Then $\angle 1 = \angle 8$, $\angle 3 = \angle 6$,

$\angle 4 = \angle 5$, $\angle 2 = \angle 7$.

Interior & Exterior Angles:

Eg 1: using Vilokanam sutra show the vertically opposite angles



$\angle a = \angle d$

$\angle e = \angle h$

$\angle b = \angle g$

$\angle b = \angle c$

2. Eg: Show the Supplementary angles using Vilokanam Sutra

$\angle a + \angle b = 180°$

$\angle c + \angle d = 180°$ etc.

3. Eg: Name Interior & Exterior angles

Interior angles = c, d, e, f.

Exterior angles = a, b, g, h.

Alternate interior angles = c & f, e & d.

Alternate exterior angles = a & h, b & g.

4. Find the corresponding angles

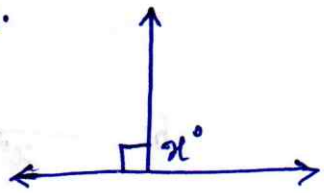
corresponding angles - b & f.

- d & h
- a & e.
- c & g.

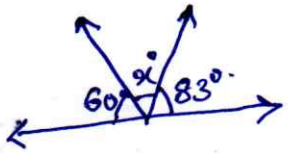
EXERCISE.

I. Find the missing angle in a straight line.

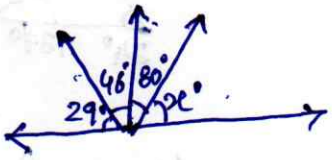
a.



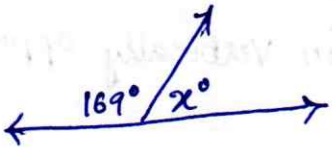
b.



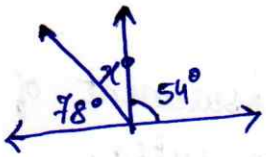
c.



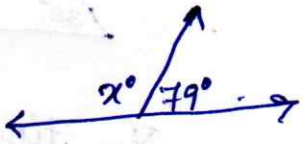
d.



e.

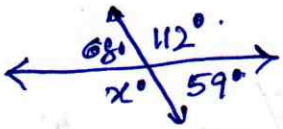


f.



II. Find the missing angle of a point

a.



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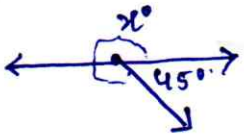
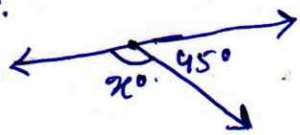
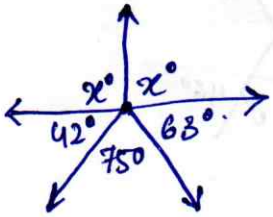


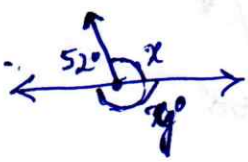
c.



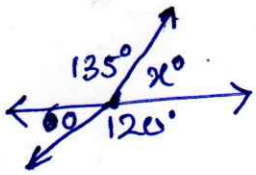
d.



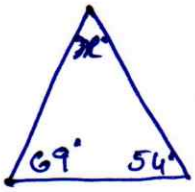
e.

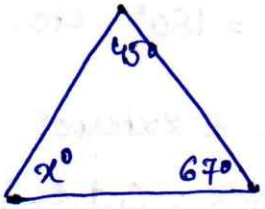


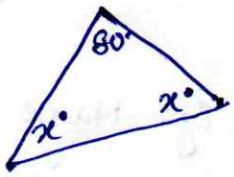
f.

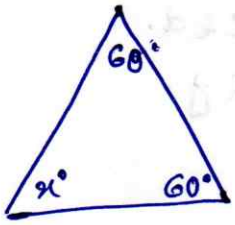


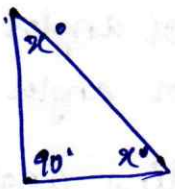
III. Find the Measurement of missing angle in a triangle.

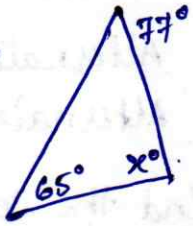
a. 

b. 

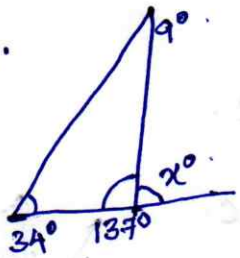
c. 

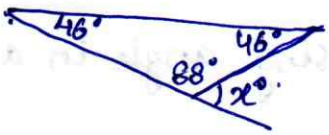
d. 

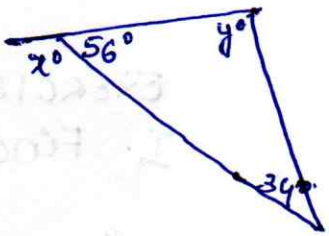
e 

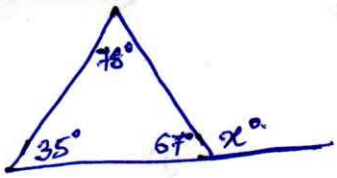
f. 

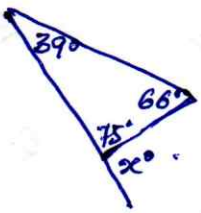
IV Find the measurement of Exterior angles

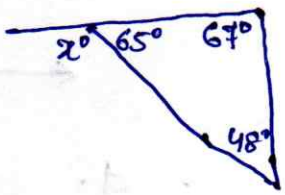
a. 

b. 

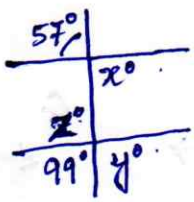
c. 

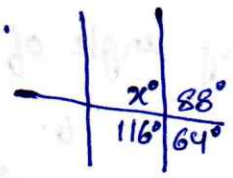
d. 

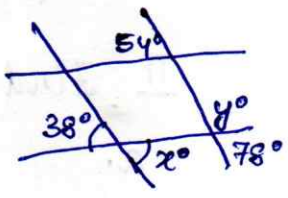
e. 

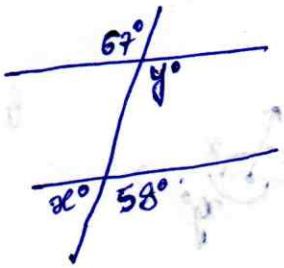
b. 

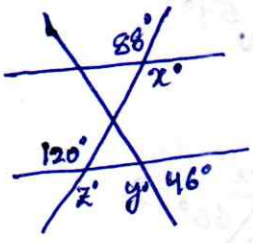
V Find the measurement of unknown Vertically opposite angles by Vikolanam sutra.

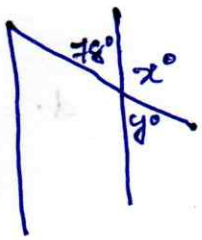
a. 

b. 

c. 

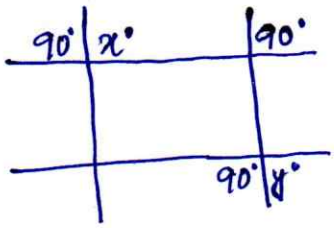
d. 

e. 

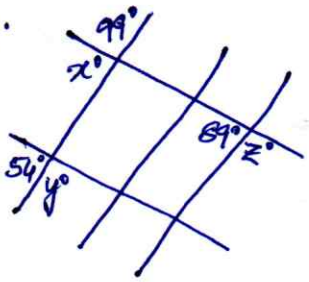
f. 

VI. Find the measurement of unknown supplementary angles by Vilokanam sutra: 10

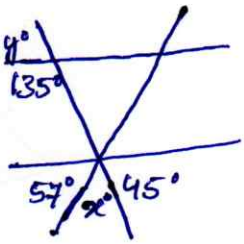
a.



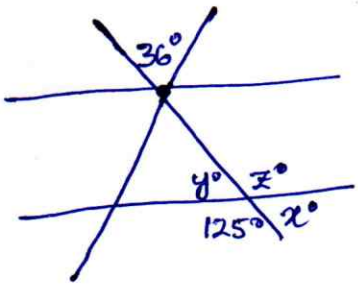
b.



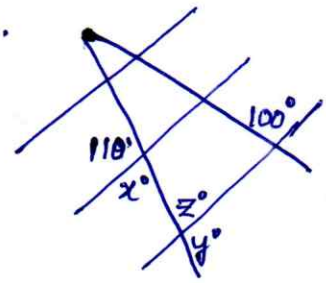
c.



d.



e.



f.