The Foundations of Relativistic Beaming of Gravity

Bradford C Blake
Independent Researcher
PO Box 773
Central City, CO 80427
bblake7882@gmail.com

ABSTRACT

Dr Carlip showed that in General Relativity, gravity has a velocity-based component. This means that in General Relativity, gravity must depend on the source velocity. (Carlip, 2000) Thus, we can deduce that for any valid theory of gravity, this must also be true. Therefore, it must be true as well for Newton's Law of Universal Gravitation, that gravity depends on the source velocity. I show herein by a geometrical argument that while Newton's gravity appears to be an instantaneous force, in actuality the velocity-based component of the force means that it is not instantaneous, but that the direction of the force is towards the instantaneous position rather than the retarded position of the test object. I also show that Newton's gravity is the simplest particle-exchange gravity or *quantum* gravity. Of course, I assume the existence of the graviton, as the exchange particle or force carrier for gravity. Next, I examine the traditional setup for relativistic beaming of light, but instead

apply it to gravity. I show that gravity emanating from a source moving near the speed of light toward us, must be beamed into a cone, because of the velocity-based component of the Newtonian gravity, even when we use Newtonian addition of velocities. Although that cone is not as severe as a relativistic cone, when we consider that the gravity is actually streams of gravitons, which are quantum particles much like photons, we realize the necessity for using Einstein's relativistic addition of velocities. Doing so results in relativistic beaming of gravity which is mathematically identical to relativistic beaming of light! Next, I briefly describe some of the problems in physics for which relativistic beaming of gravity may provide the solution.

INTRODUCTION

Wave-particle duality is an important characteristic of quantum particles. Although the graviton has not yet been observed, many scientists believe that it is the force carrier for gravity. The belief is that it is most likely a spin 2 boson with no mass. It should be a quantum particle, and as such, it should exhibit wave-particle duality.

General Relativity is generally considered the best theory of gravity. It often gives the most accurate computational results for certain problems such as the precise orbit of Mercury, the bending of light around the Sun, the existence of gravitational waves, and timing of clocks in a gravitational field. However, Newton's Universal Law of Gravitation, for centuries, has provided the means to simply compute most gravitational problems. Where General Relativity has an advantage appears to be situations where the wave-nature of gravity is important.

If gravity experiences wave-particle duality, as I contend it must, then we should consider the consequences of treating gravity as a flow of particles, that is, gravitons. It is easy to devise a simple formula that we would expect to describe a particle exchange, or quantum, theory of gravity. For two massive objects, M_1 and M_2, of mass $m_1$and m_2 respectively, we note that doubling the mass $m_1$should double the gravitons traveling to M_2, while doubling the mass m_2 should have a similar effect on M_1. With r being the radius between the centers of mass of M_1 and M_2, we would expect the number of gravitons going from $M_1$to M_2 to be $\frac{1}{r^2}$ Because, assuming gravitons are emitted equally in all directions, the surface area of the sphere of radius r goes as r^2. Thus, the gravitons reaching M_2 fall off as $\frac{1}{r^2}$ Similarly for gravitons reaching M_1 from M_2. All we need now is a constant, K, and we have the following formula for the gravitational force between $M_1 and\ M_2$

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$$F = K\frac{m_1 m_2}{r^2}$$

But this formula is just Newton's Law of Universal Gravitation, where K is replaced by the experimentally derived gravitational constant G. For this reason, I assert that Newton's law of gravity is actually the simplest particle exchange, or quantum, form of gravity. People often think of Newton's law of gravity as just an approximation of General Relativity, but it is much more than that! It represents the particle nature of gravity in the wave-particle dichotomy.

1. Why the Force in Newton's Law of Universal Gravitation Appears to be Instantaneous

One of the reasons people doubt the fundamental significance of Newton's law of gravity is the way the force appears to be instantaneous. We know that it is impossible for the gravitational force to propagate instantaneously, because special relativity provides that matter particles, including quantum force particles, cannot travel faster than c, the speed of light. And yet when we solve problems with Newton's law, we assume that the force is instantaneous. Not only that, but if the force is not instantaneous, then we have obvious problems, such as the Solar System would not persist for any reasonable time. The solution is that the force of gravity does not propagate instantaneously. Instead, the direction of the force, when it arrives at the destination object, does not pull that object toward the retarded position of the source, but toward the instantaneous position of the source. Here follows a geometrical demonstration.

For our demonstration we will consider the Sun's gravitational attraction to the Earth. It takes about 8.33 minutes for light from the Sun to reach Earth, so we will consider an 8.33 minute period of Earth's rotation about the Sun. Note that it is well established that gravity propagates at c, the speed of light. (Abbott et al. 2017) During 8.33 minutes, the Earth will travel only a very small portion of its orbit, so we can approximate that portion of the orbit as a line segment. We want to examine gravity that originates at the Earth, and arrives at the Sun.

I have found that the easiest way to work this problem is to consider what I call the "gravitational flashlight." By using this technique, we will account for all of the gravity, including gravity from every possible angle, that leaves the Earth and travels to the Sun. Figure 1 shows the Earth and the Sun, except in this figure, the

Earth and the Sun are not moving with respect to each other, i.e. the Earth is not in orbit about the Sun. We can easily see in Figure 1, that all gravity leaving the Earth that goes to the Sun is accounted for.

Figure 1. The Gravitational Flashlight

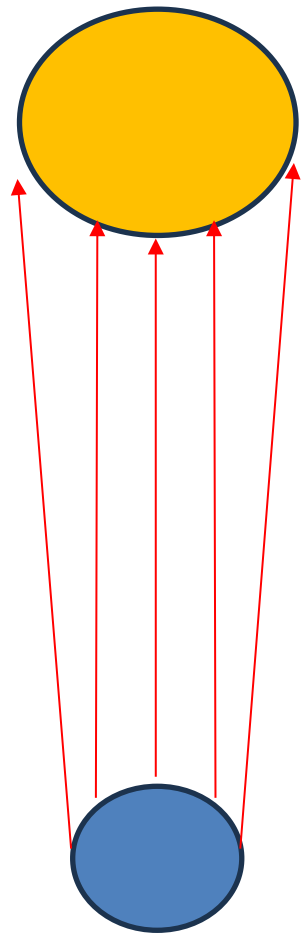


Figure 1: Here we see the Earth and the Sun. For this figure, the Earth and the Sun are not moving with respect to each other, i.e. the Earth is not orbiting the Sun. We can see in the picture that all gravity going from the Earth to the Sun is accounted for. Note that the figure is not to scale.

In Figure 2, we show the Earth in orbit about the Sun. However, we only depict 8.33 minutes of the orbit. With such a small time compared to the total orbital period, we can approximate the Earth's motion as a line segment. We see the Earth in its original (retarded) position, and also in its new position. Once again, we shine the gravitational flashlight directly toward the Sun. However, since gravity has a velocity-based component, i.e. gravity moves with its source, we see that the gravitational flashlight veers to the right, either missing the Sun completely, or at least not square to the Sun. (remember, these diagrams are not to scale. The Sun is much larger than the Earth, and much further away.)

Figure 2

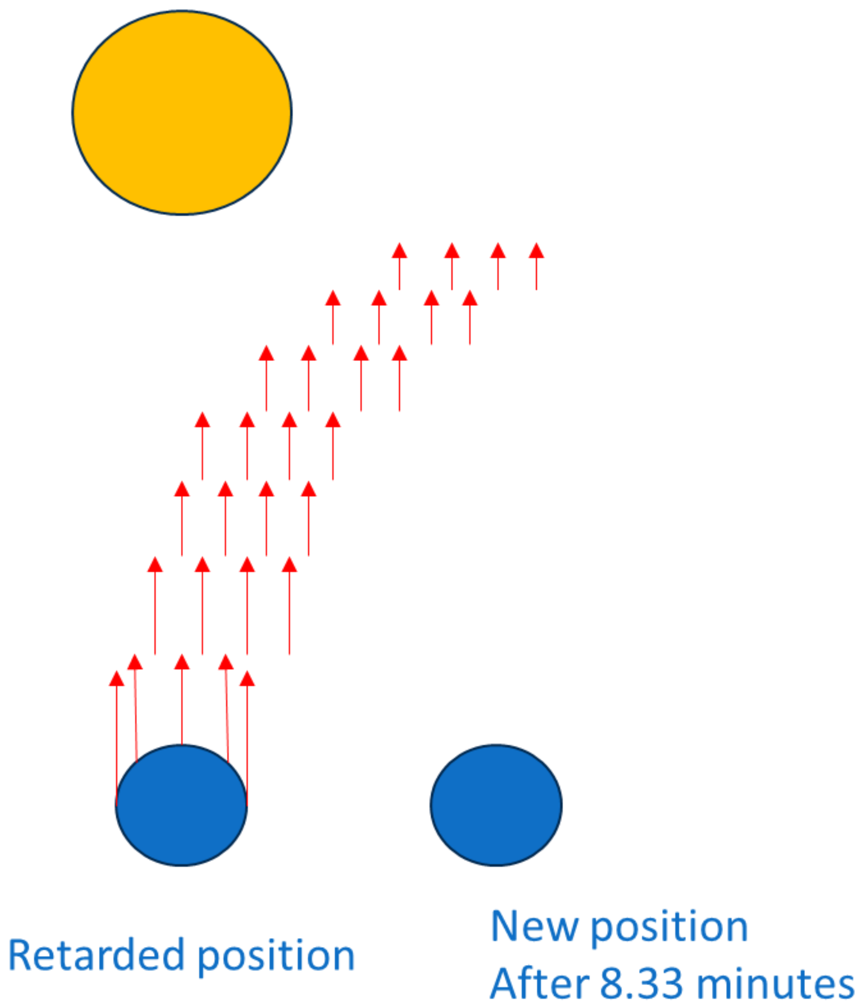


Figure 2: In this figure, the gravitational flashlight is shined directly towards the Sun. However, the Earth is orbiting around the Sun. Because the Earth's gravity moves with the source, the gravity veers to the right, and will either miss the Sun, or not hit it squarely. (Figure is not to scale.)

In Figure 3, we measure the angle α, from the new position of the Earth, to the Sun.

Figure 3

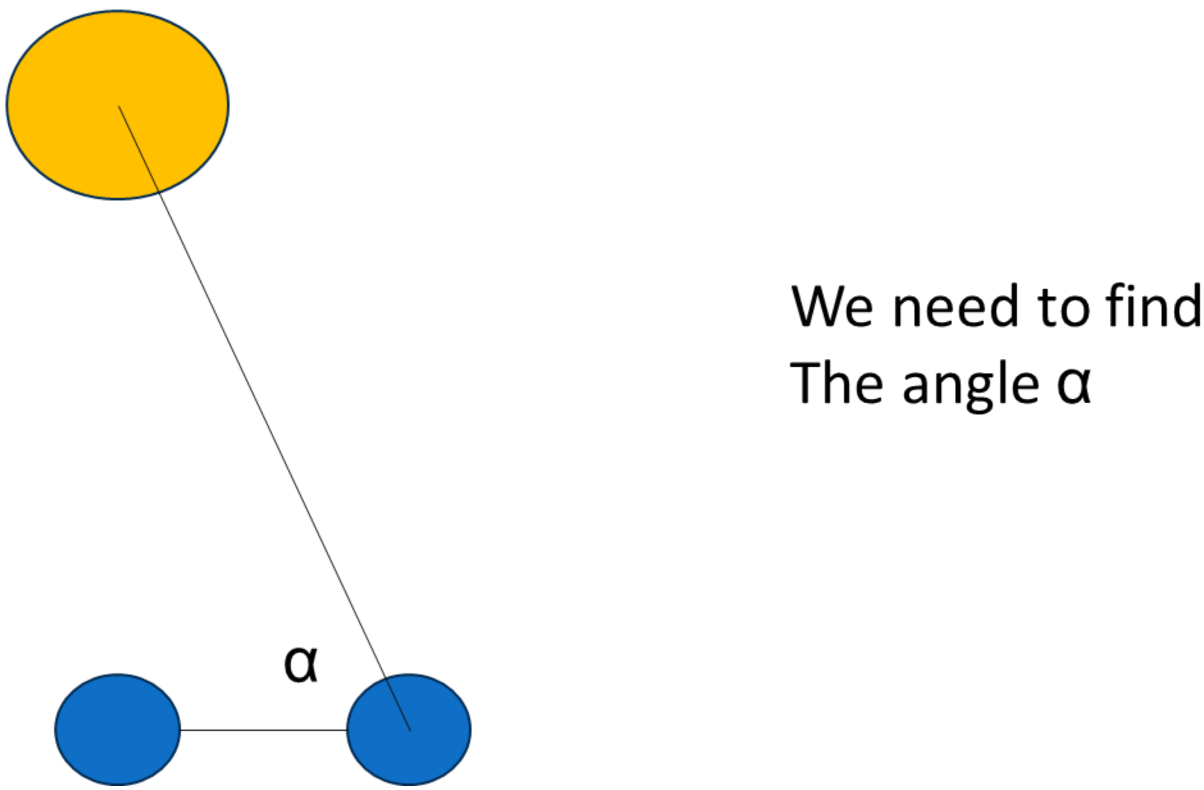
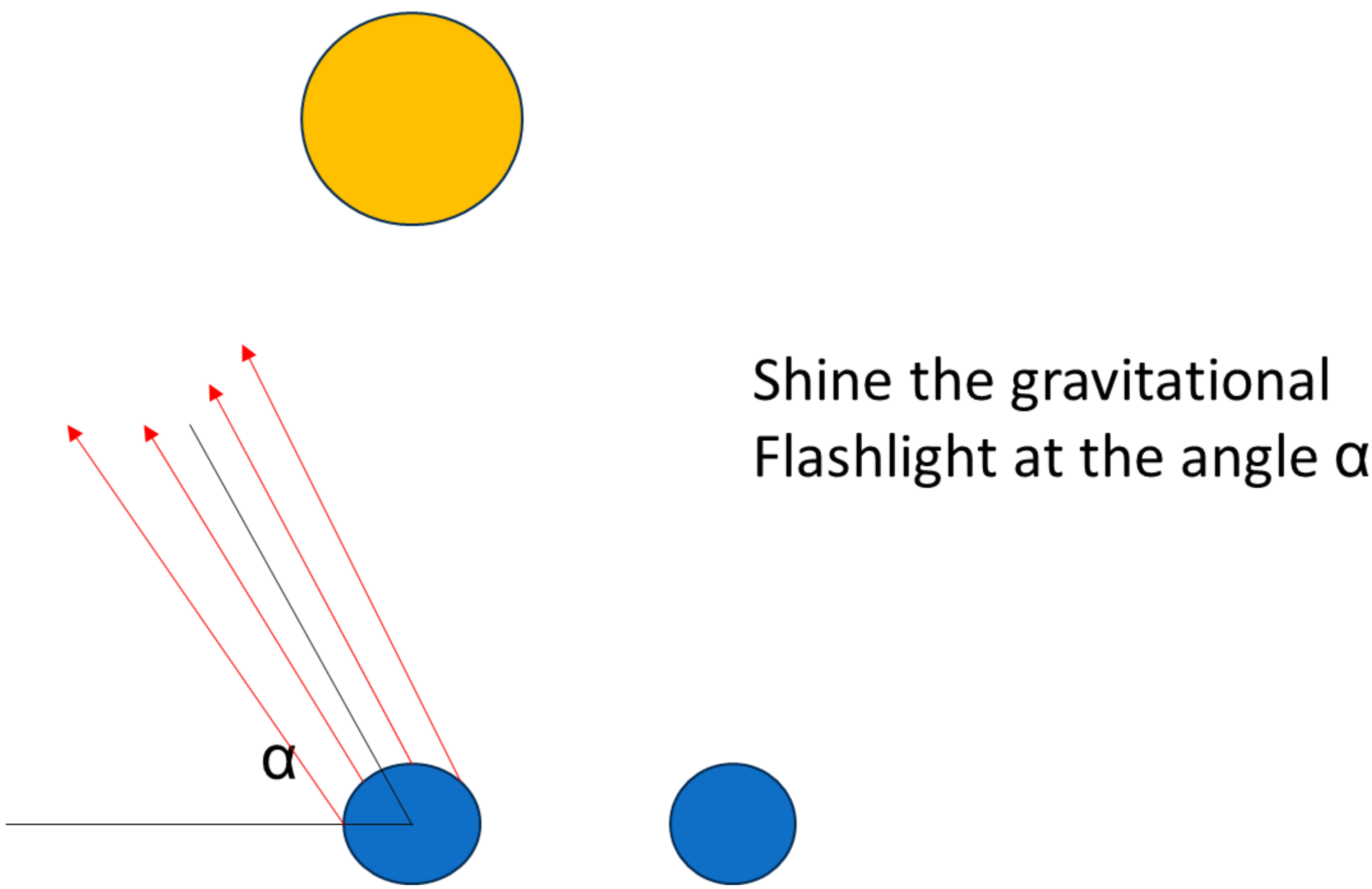


Figure 3: In the figure, we measure the angle α, from the new position of the Earth to the Sun.

In Figure 4, we now shine the gravitational flashlight from the retarded position of the Earth at the angle α, rather than directly at the Sun.

Figure 4



In Figure 5, we see that the force arrows drift to the right because gravity moves with the source. The result is that the gravity arrives square to the Sun, and by examining the force arrows, you can see that the Earth's gravity that left Earth from the retarded position at angle α, pulls the Sun towards the Earth's new position.

Figure 5

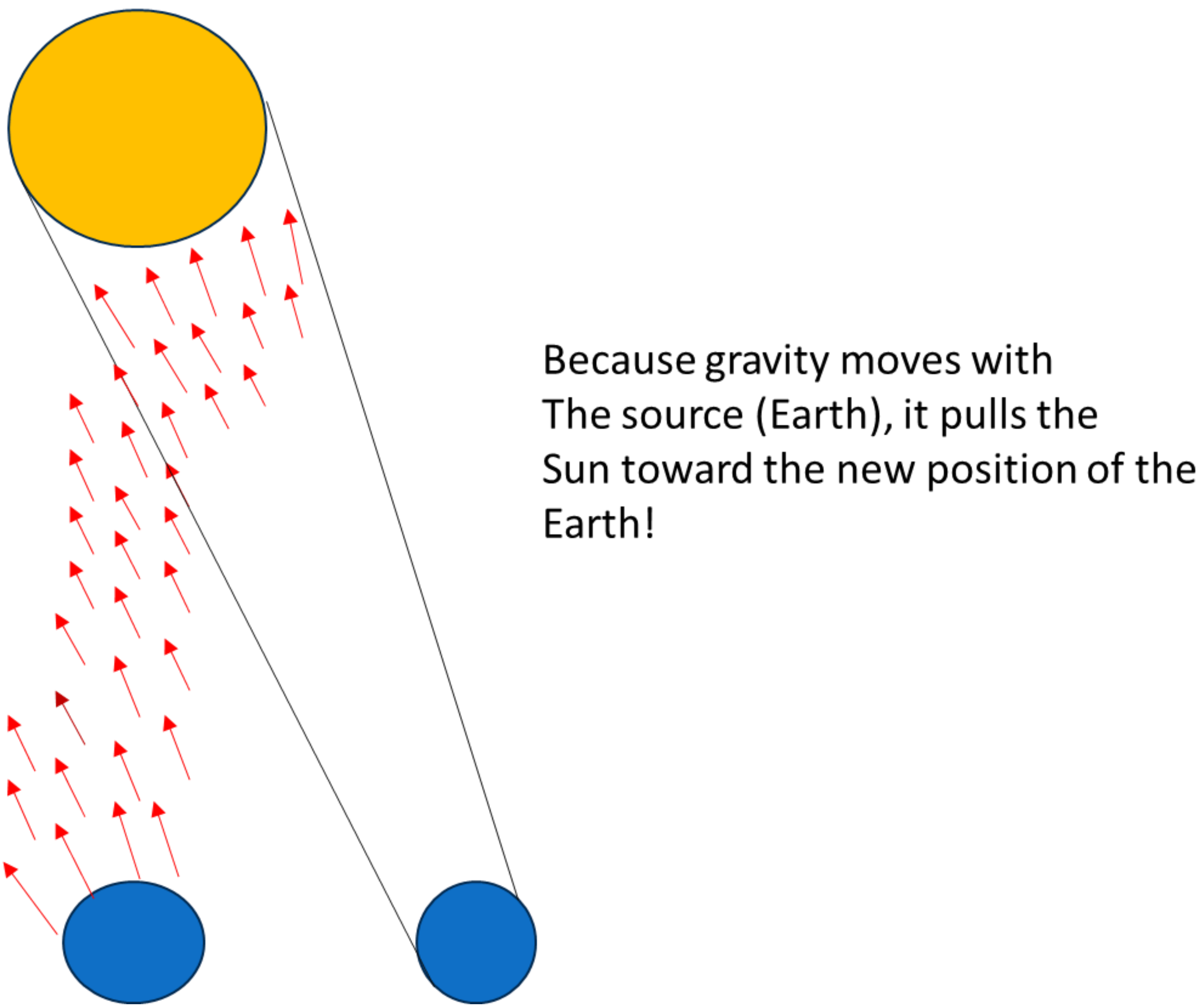


Figure 5, in this figure, we see that because gravity moves with the source, here, the Earth, and because the force arrows point back to the new position of the Earth, we see that the Sun is pulled toward the new position of the Earth, not the retarded position.

As we see from Figures 1-5, the pull of gravity in Newton's Law of Universal Gravitation is directed toward the instantaneous position of the source object, not the retarded position. The reason this is true is because gravity has a velocity-based component. (Carlip, 2000). Thus, gravitation in Newton's Law of Universal Gravitation is not instantaneous, and does not need to be.

Reversing our problem above, and showing that the Sun's gravitational pull on the Earth is towards the Sun's center of mass, is trivial. That is because we have tacitly assumed that the reference frame for our problem above is the Sun's, and that the Earth's gravity is not enough to move the Sun an appreciable amount in 8.33 minutes. Thus, when the Earth reaches its new position after 8.33 minutes, the gravitational pull of the Sun upon the Earth will automatically pull the Earth squarely toward the Sun.

Finally, we should consider the case of two objects that have similar but different masses. Let's say it takes 10 minutes for light (or gravity!) to pass from one object to the other. Once again, we will assume that the distance travelled by each object in ten minutes is small compared to the size of the orbits, so we can approximate the change in the position by two different line segments, one for each orbit. Using the same technique as in the first case, we will compute the two angles from each object's new (instantaneous) position to the new position of the other object. Armed with those angles, we will shine the gravitational flashlight from each object's retarded position at the appropriate angle, such that it will arrive squarely at the opposing object at its new position of the orbit. We discover that the gravitational attraction for each object is towards the new, instantaneous, position of its counterpart, rather than the retarded position. This is because gravity has a velocity-based component, and is affected by the motion of its source. Thus, we see that in all three cases, the pull of Newton's Law of Universal Gravitation is toward the instantaneous position, not the retarded position, of the test object.

2. Gravity must be Relativistically Beamed

We shall now examine the phenomena of relativistic beaming. We shall look at the classic situation, where a source is moving toward us, at close to the speed of light, and emits light equally in all directions from the point of view of an observer on the source. The result for light is of course the headlight effect. (Carroll and Ostlie 2007, p101) However, since we are interested in gravity, we will instead consider that the source is emitting gravity, equally in all directions according to an observer on the source. Because half of the gravity is emitted in our direction, and half in the opposite direction, according to an observer on the source, we want to examine a ray of gravity emitted on the plane between the forward and the backward directions. We will choose the ray along the z axis. From our perspective, that ray is emitted upward at the speed of light, however, because gravity has a velocity-based component, it is also moving at nearly the speed of light in our direction

Figure 6

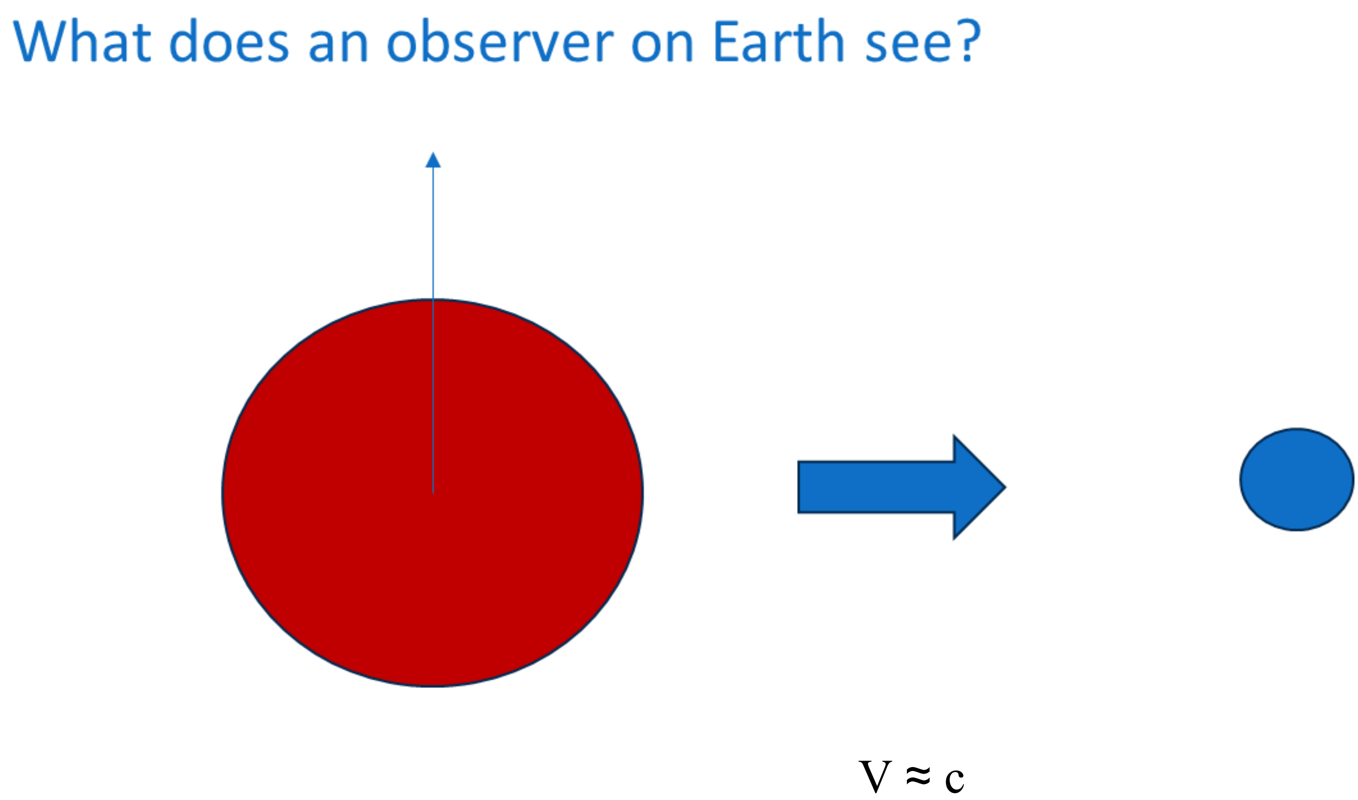


Figure 6: In this figure, an observer on Earth sees the force arrow in the z direction going upwards at velocity c, but also sees the brown object approaching Earth at almost velocity c. Because gravity is affected by the velocity of its source, we need to be able to add the two velocities to get the resultant velocity of the gravitational vector.

Thus, to figure the actual trajectory of the ray, we must add the velocities. We will presume for a moment that we can use Newton's usual summation of velocities. We then get, in the limit, a 45 degree angle to the x-axis. Thus, we find that all of the light in the forward direction is beamed into a cone of half-angle 45 degrees. Although this is not relativistic beaming, it is still beaming, and it occurs because gravity has a velocity-based component. See Figure 7.

Figure 7

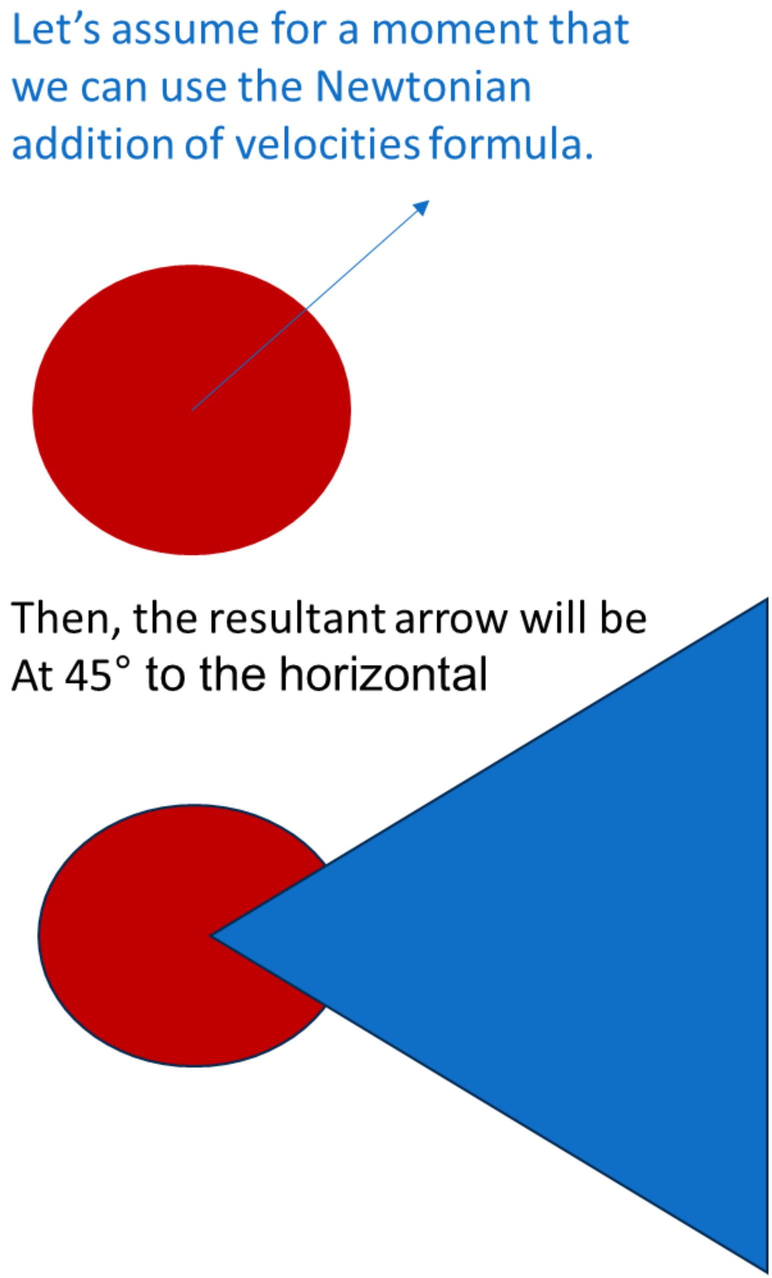


Figure 7: in this figure, assuming we can use the Newtonian addition of velocities formula, we see that the resultant gravitational vector will be at 45° to the

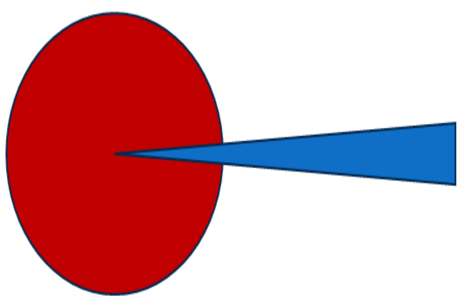
horizontal. So, all of the gravity in the Earth's direction will be beamed into a cone of half angle 45°. This is classical, without relativity!

Now consider that the ray of gravity discussed above is actually a stream of gravitons. Gravitons are quantum particles travelling at the speed of light, like photons, and as such, we cannot perform the addition of velocities of gravitons with Newtonian summation. Instead, we must use Einstein's addition of velocities formula. (Einstein, 1905) When we do so, we get exactly the same formula as for relativistic beaming of light! Thus, the gravity cone we observe from Earth is no longer restricted to 45 degrees, but as the source velocity becomes closer to the speed of light, the cone becomes much tighter. For source velocities very close to c, the speed of light, the sine of the angle equals γ^{-1} , where γ is the Lorentz factor. This is the same result we get for light! (Carroll and Ostlie 2007, p 101) Thus, we indeed have relativistic beaming of gravity! See Figure 8.

Figure 8

However, gravity must be relativistically beamed!

1. We know (or assume) that gravity is a flow of gravitons.
2. Gravitons are quantum particles that travel at c.
3. Thus, we must use Einstein's formula for relativistic addition of velocities.



As v approaches c, the half angle equals $1/\gamma$, where γ is the Lorentz factor.

Figure 8: In this figure, we see that because gravity is (assumed to be) a flow of gravitons which travel at c, we must use Einstein's relativistic addition of velocities formula. This means that gravity must be relativistically beamed!

2.1 Does Relativistic Beaming of Gravity Somehow Contradict General Relativity?

I believe the answer to this is "no." I make a distinction between the General Relativity of Einstein, and the Kerr Solution. We might ask, "What was Einstein's opinion of the Kerr Solution." We will never know the answer, because Einstein passed away about eight years before Roy Kerr proposed the Kerr Solution in 1963. (Kerr 1963) I am not an expert on the Kerr Solution, but I suspect that that solution is only valid for slow rotations, that is, rotations that are not near the speed of light. Firstly, it does not appear to me that Dr Kerr was considering limitations of the speed of light when he developed the Kerr solution and the corresponding metric. Secondly, many solutions to General Relativity exist that are valid only for limited domains. (Rendall, 2000) Thus, it would not be surprising if that were the case for the Kerr Solution. Also, the Kerr metric's behavior for fast rotations predicts naked singularities. (Hobson 2006, pp 301, 324) While science has reluctantly accepted the Cosmic Censorship Hypothesis to explain the absence of naked singularities, I think the better approach is to realize that the prediction by the Kerr metric of naked singularities shows that the Kerr Solution is invalid for fast rotations, i.e. rotations near the speed of light.

3. Successes of Relativistic Beaming of Gravity

Although in this paper I have introduced relativistic beaming of gravity by considering the usual beaming arrangement, that is: a distant source approaching us at nearly the speed of light, much more useful for problem solving is a different arrangement. We usually consider a compact object such as a neutron star, a stellar mass black hole, an intermediate mass black hole, or a supermassive black hole. If the core of the compact object is composed primarily of superfluid neutrons, and if the core is rotating at close to the speed of light, then we can surmise that the gravity from the core will be beamed into the rotational plane of the compact object, and will approximate $\frac{1}{r}$ gravity rather than $\frac{1}{r^2}$ gravity. (Blake, 2022b)

As is the case with much of my research, we will have to make reasonable extrapolations from known facts. For example, it is not known exactly how fast the cores of neutron stars are rotating. We do know that the cores are likely rotating faster than the outer shells of the given neutron star. This is because during glitches, which are times when neutron stars suddenly change rotation speed, usually by speeding up, it appears that angular momentum is transferred from the core of the neutron star to its outer shell or crust. (Haskell et al, 2018) (Basu, Char, Nandi, et al, 2018) (Andersson et al, 2012) Also, young neutron stars' cores would seem more likely to be rotating at near their original, natal speed than the cores of older neutron stars. So, I would surmise that at least some neutron stars' cores are rotating near the speed of light.

Similarly, some of my work requires that there be considerable alignment between the rotation of compact objects and the rotation vector of the galaxy. Here too, we will have to make extrapolations from known facts. For example, researchers have discovered alignment among the stars in some open clusters. (Corsaro, 2017) (Kovacs, 2018) (Kamann, 2019) Also, considerable alignment has been shown for the jets of protostars in the Serpens Nebula. (Green et al, 2024) Based on this kind of evidence, I extrapolate to expect considerable alignment in closed clusters, although direct evidence of this is not yet available.

So, with a nod toward the necessity of some speculation in my work, we can examine some of the successes of relativistic beaming of gravity! I will briefly describe some of those successes, which are described in more detail in my other papers.

3.1 The Missing Mass Problem

This problem concerns why galaxies, including The Milky Way, appear to have considerably more mass than we observe by tallying up the mass of stars and gas clouds, etc., within the given galaxy. Also, galaxy clusters appear to have more mass than can be attributed to the visible mass inside the clusters. Science has basically settled on the idea of vast clouds of so-called dark matter as an explanation for this problem. These clouds being called dark matter halos. However, despite great effort during the last 100 years, none of this dark matter has been identified.

3.1.1 The Bullet Cluster

Dark matter halos have been the preferred solution ever since the discovery and analysis of the Bullet Cluster. (Clowe, Bradač, Gonzalez, et al 2006) The Bullet Cluster is actually two clusters of galaxies that passed through each other, with much of the baryonic matter consisting of gas that was stripped off of both clusters by ram pressure. The result is a high concentration of baryonic matter, gas, which was left in the middle, between the two clusters, while the stellar matter, too dense to be stripped off by the ram pressure, stayed with each of the clusters. The surprising thing about the Bullet Cluster is that observations show that the gravitational lensing stayed with the stars, instead of with the gas, which is believed to represent the majority of the baryonic matter.

Presumably, dark matter, which by assumption can only interact with other matter gravitationally, would have stayed with the stars, and not have been stripped off by the ram pressure. Thus, most scientists believe that dark matter halos explain the results of the Bullet Cluster. The problem with this solution is that it is basically a solution by fiat. In other words, we are solving the surprising result of the gravitational lensing staying with the stars, by postulating a special kind of matter that only interacts gravitationally, even though we have never identified this special dark matter in any way, shape or form.

Contrast this with how relativistic beaming of gravity would address the results of the Bullet Cluster. Instead of solving the problem by fiat, we just have to note that relativistic beaming of gravity comes from compact objects, which are stellar objects, generally the remnants of collapsed stars, and therefore would not be stripped off by the ram pressure. Instead, they would stay with the stars, and thus the extra gravitational lensing would stay with the stars as well. The solution with relativistic beaming of gravity is simple and elegant.

3.1.2 The Missing Mass in the Milky Way

So, how does relativistic beaming of gravity solve the missing mass problem? For this I refer you to my first published paper, (Blake, 2022b). The basic idea is that there is enough mass locked up in compact objects within the Milky Way Galaxy to provide a 1/r gravitational field at the outer edges of the Galaxy, if enough of those compact objects have cores rotating near the speed of light, and if they are rotating in alignment with the rotation of the Galaxy. This 1/r gravitational field can explain the motion of stars and gas in the outer reaches of the Galaxy.

3.2 Jets from Compact Objects

Relativistic beaming of gravity can explain why compact objects such as neutron stars, stellar mass black holes, and supermassive black holes, and possibly even protostars, have jets at the poles. For this, I refer you to (Blake, 2023). The basic idea is that the cores of these compact objects consist primarily of superfluid neutrons. As a superfluid, they must rotate by rotating around vortices in the core. If the rotation is close to the speed of light, then the gravitation from the core will be beamed into the rotational plane. Thus, there will be less gravity at the poles of the compact object. We can calculate the differential pressure at the poles, and we find that, if the core is large enough, then the differential pressure at the poles will overcome the reduced gravity at the poles, which results in streams of superfluid neutrons escaping the compact object at the poles. These streams of superfluid neutrons result in jets, which become visible as some of the neutrons decay into protons and electrons, resulting in synchrotron radiation.

3.3 The Mass Gap between Neutron Stars and Black Holes

This is admittedly some of my most speculative work. Never-the-less, I consider myself a conservative physicist, and yet I believe that this work is probably true. There is a mass gap between neutron stars and black holes, based on observations over the last one hundred years or so, which is hard to explain. Neutron stars range in mass between about 1.1 $M_{\odot}$ and 2.2 $M_{\odot}$, while all verified black holes have masses at least about 6 $M_{\odot}$. Why are there no neutron stars or black holes of mass between about 2.2 $M_{\odot}$ and about 6 $M_{\odot}$, the mass gap? Certainly, there should be stars that when they collapse, should leave remnants within the mass gap. What happens to those remnants? Very recently, there have been scientists questioning whether there is a mass gap, based upon gravitational wave observations, but I will discuss that a little later.

I will refer you to my paper, (Blake, 2022a). To understand this mass gap, we need to ignore some of the more esoteric developments in black hole theory. First, it is a very easy proof that if a neutron star's physical radius is smaller than it's Schwarzschild radius, then there will be an event horizon, and the neutron star will be inside a black hole. This proof came with assistance from the Google AI.

$v_{esc} = \sqrt{\frac{GM}{r}}$, Newtonian escape velocity

$v_{esc(GR)} = c = \sqrt{\frac{GM}{R_S}}$, by General Relativity

So, if r < R_S , then $\sqrt{\frac{GM}{r}} > \sqrt{\frac{GM}{R_S}}$, which implies $v_{esc} = \sqrt{\frac{GM}{r}} > c$

Where r is the physical radius of the neutron star and R_S is the neutron star's Schwarzschild radius.

Thus, we see that the physical radius r, being smaller than the Schwarzschild radius, implies that the escape velocity is greater than the speed of light. This means there is an event horizon, and a black hole.

I will assert that to have a black hole resulting from collapse of a star, this must be the case: that the physical radius of the resulting neutron star is smaller than it's

Schwarzschild radius. If we ignore the idea that gravitational collapse continues beyond the neutron star stage, then this can explain the mass gap. Make the reasonable assumption that all neutron stars have approximately the same density. (Ignore the TOV assumption that neutron stars' radii decrease as neutron stars become more massive, which is a remnant of Chandrasekhar's analysis of white dwarfs.) Then set the neutron star physical radius equal to its Schwarzschild radius, and you find that happens at about 5.3 $M_{\odot}$. This means that for a remnant of mass greater than 5.3 $M_{\odot}$, you have a black hole. If the remnant is in the mass gap, it must be a neutron star. However, it will be an overweight neutron star, and thus will lose mass due to instabilities. The extra mass will most likely go into the wind nebula. The overweight neutron star will quickly (within a thousand years or so) reach the usual mass range for neutron stars. See the Crab Pulsar and the Crab Nebula!

Some researchers doubt that there is a mass gap, based upon several findings from gravitational wave research. The problem with these findings is that if the remnant appears to be in the mass gap, it may only be a temporary object, as explained above. It likely would be an overweight neutron star that would soon lose mass and finish in the usual mass range of neutron stars. If instead the object is one of the merging objects, the actual mass of the object may be in doubt, because relativistic beaming of gravity will make merging objects appear to be more massive than they actually are.

The existence of the mass gap provides evidence that black holes are actually neutron stars whose physical radii are smaller than their Schwarzschild radii. In my paper, (Blake, 2022a), I consider each of the differences between black holes and neutron stars, and show how the existence of event horizons explains those differences.

4. Conclusion

It is well established by Dr Carlip's work that gravity has a velocity-based component, i.e., that gravity is affected by the motion of its source. This means that for a source moving towards an observer at close to the speed of light, the observer will observe the gravity in the forward direction to be within a cone. If one believes that gravity is composed of gravitons that are quantum particles, then it follows that the cone of gravity is mathematically identical to that for light. Thus, the fact that gravity has a velocity-based component implies that it must be beamed, and if the nature of gravity is a stream of gravitons, then that beaming must be relativistic. Thus, it seems to be almost a given fact that gravity is relativistically beamed.

Relativistic beaming of gravity makes possible the solution of many outstanding problems in physics. I have summarized my attempts to solve three of those problems in this paper. The first is the missing mass problem. Science so far has preferred attempts to solve that problem with Dark Matter Halos, but those attempts have not yet borne fruit. Although some speculation is involved in the relativistic beaming solution, the difficulties seem to be the current limitations of observation, not of theory. The second problem I have tackled is the existence of jets, particularly relativistic jets, emitted from the poles of many compact objects. Current theory has great difficulty explaining how mass appears in the jets, especially from black holes. Relativistic beaming of gravity explains away this problem, and describes the composition of the jets. The third problem I have dealt with concerns the existence of the mass gap between neutron stars and black holes. Relativistic beaming of gravity is the only explanation I am aware of for the mass gap. Further, it predicts the extent of the gap. It does require some adjustment in our understanding of black hole theory. In particular, it requires us to reject the commonly assumed continual gravitational collapse to a singularity, and accept that gravitational collapse stops at the neutron star stage, at least for stellar collapse. Although this may prove difficult for many scientists to accept, it appears to me to explain the similarities and the differences between neutron stars and black holes.

These problems are certainly not the only ones that can be solved by relativistic beaming of gravity. However, I would prefer not to speculate regarding other problems, until I have further researched them, and put pen to paper.

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