Relativistic Beaming of Gravity and the Missing Mass Problem

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ABSTRACT

In this paper I develop a theory based on the principal of relativistic beaming of the gravitational force. By using Gauss's Law, I show that compact objects such as neutron stars, stellar mass black holes, and the supermassive black hole at SgrA*, if rotating relativistically, and in alignment with the rotation vector of the galaxy, could possibly explain the missing mass problem for the galaxy. Further, if the theory is considered in the context of the universe as a whole, then it is consistent with studies of the Bullet Cluster, because neutron stars, black holes and supermassive black holes would not be stripped off with the gas in the collision of clusters, but would remain with the stars.

Keywords: Milky Way dynamics (1051), Gravitation (661), Dark matter distribution (356), Relativity (1393), Mond (1069)

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1. INTRODUCTION

The problem of the missing mass is to understand why galaxies such as the Milky Way rotate with flat rotation curves and also why galactic clusters do not fly apart, despite the appearance of insufficient mass to maintain those structures. By flat rotation curves, I mean that high surface brightness spiral galaxies tend to rotate in such a manner that the stars and gas move at similar velocities out to the edge of a given galaxy, rather than moving more slowly near the edge as we would expect from Kepler's laws. The easy solution to these problems is to assume that there exists some new kind of matter, termed dark matter, that exists in halos around the galaxies, and which provides the needed mass. The problem with the easy solution is that no one has been able to find or identify this dark matter, despite considerable theoretical and experimental effort over the last 50 years or so. And I really think that Occam would put away his razor and let his beard grow out when told that dark matter halo theory requires about five times more dark matter than normal baryonic matter to explain the flat rotation curves of spirals and the stability of galactic clusters. (Planck Collaboration et al. 2014)

In this paper I will propose a solution to the missing mass problem that differs starkly from dark matter halos. I call it *Directed Gravity*, by which I mean relativistic beaming of the gravitational force. The key idea is that when compact objects such as neutron stars, stellar mass black holes and supermassive black holes rotate at nearly the speed of light, the gravitation they emit is relativistically beamed in new angles as would be light from such a compact object rotating at the same speed. As I will show in this paper, if a compact object rotates relativistically, we should expect the gravitational flux emitted by the compact object to be concentrated in the equatorial plane of the object, rather than being emitted isotropically. Further, as I will show in the discussion below, this will result in a gravitational field that will decrease in the equatorial directions by approximately $1/r$, rather than the usual $1/r^2$.

As I will show below, a 1/r gravitational field can result in a flat rotation curve.

2. RELATIVISTIC BEAMING OF GRAVITY

When a light source is moving relativistically, the emitted light experiences relativistic beaming, which is called *the headlight effect*. The widely used text "An Introduction to Modern Astrophysics" by Carroll and Ostlie explains the headlight effect using only the facts that light travels at the speed *c*, and that light is subject to the formula for the relativistic addition of velocities. No other properties of light are involved. (Carroll & Ostlie 2007, p. 101) But gravity travels at the speed of light, *c*, according to Einstein's general relativity, and according to experimental observation as well (Abbott et al. 2017). Further, one should note that the relativistic addition of velocities formula is not a property of light, per se, but is a property of spacetime. Anything that travels through spacetime must be subject to it, including gravity. Thus, we must conclude that gravity must be subject to relativistic beaming, just as light!

It might be argued that gravity does not experience relativistic beaming, because such beaming is a form of aberration, and gravity does not experience aberration. However, gravity does experience aberration! With a little thought, one realizes that the time light effect and aberration both apply to gravity, but we do not readily perceive them because we cannot see the path that gravity takes in the same way that we can see the path of light. Since we know that the gravity originated at the previous (that is, the retarded) position of the source object, we know that the path the gravity took would have been the same path that light would have taken, thus, the time light effect should apply to gravity. And if we were standing on a moving body, such as the earth, looking in the direction of a source of gravity, such as the sun, and if we could see the apparent path of the gravitational force in the same way we can see the apparent path of light, we would undoubtedly perceive gravitational aberration!

The fact that the gravitational force points to the instantaneous position of the object rather than the retarded position does not deny the fact that the gravity travelled from the retarded position. Instead, gravity points to the instantaneous position because gravity has a velocity dependent component. (Carlip 2000)

2.1. Relativistically Rotating Compact Objects

Consider a relativistically rotating compact object, that is, a relativistically rotating stellar mass black hole or supermassive black hole, or a neutron star with a core that is relativistically rotating. If the compact object were not rotating, we would expect gravitational rays to emit equally in all directions from each point on the compact object. Let us consider a point "A" on the equator of a compact object such that "A" is moving at almost the speed of light, *c*, directly toward our observer as in figure 1 below.

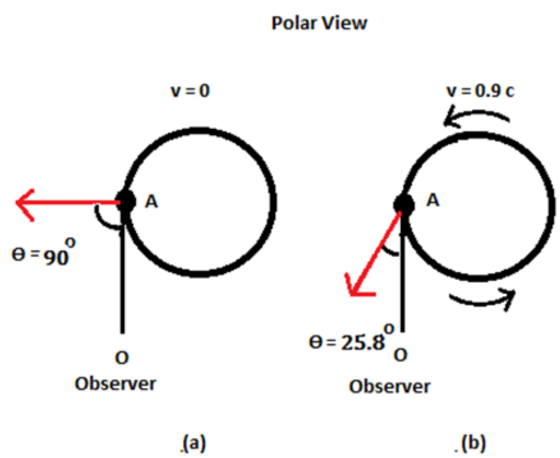

Figure 1. Consider point "A" on a spherical compact object. In 1(a), the object is not rotating, and emits a ray of gravity at a 90° angle. In 1(b), the object is now rotating at 0.9*c.* If it emits the same ray of gravity, it will be relativistically beamed to an angle of 25.8°.

Consider a ray of gravity that would be emitted from point A at the angle $\theta' = 90°$ if the compact object were not rotating. At what angle θ would that ray of gravity be emitted if the object were rotating relativistically at 0.9 *c*? To solve this problem, we use Einstein's relativistic beaming formula. (Einstein 1905) Here, we use the inverse version where v is replaced by –v and θ by θ'.

$$\cos(\theta) = \frac{\cos(\theta') + \frac{v}{c}}{1 + \left(\frac{v}{c}\right)(\cos(\theta'))} \qquad (1)$$

If we insert θ′ = 90° and v = 0.9 *c* into the above equation, we obtain the result of θ = 25.8°. In other words, a ray of gravity that would have been emitted at 90° will instead be emitted at 25.8°! Further, rays of gravity emitted from point A in every direction will be relativistically beamed toward the instantaneous direction of v. Figure 2 illustrates the gravitational beaming from point A for different velocities v.

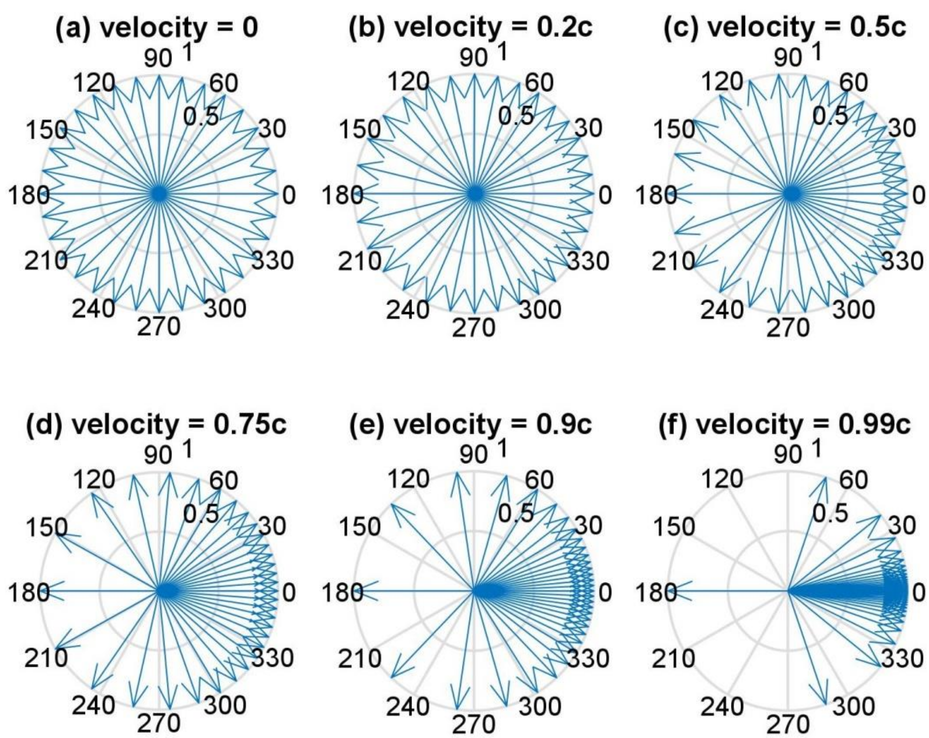

Figure 2. This figure illustrates the amount of relativistic beaming of gravity for different values of the velocity *v.* Figure 2(a) shows that there is no beaming for *v* = 0. Figures 2(b) through 2(f) show increasing amounts of beaming for successively higher values of *v* as it approaches *c.* I created this figure using the compass program from MATLAB® by adjusting the inputted angles using Einstein's relativistic beaming formula, Eq. 1. The

compass program from MATLAB® has copyright 1984-2005 The Mathworks, Inc.

Now, one might argue that the strong gravity in a compact object might change the above result. However, you should note that in general relativity, on the smallest scale, the metric is locally flat and the local manifold will always be Minkowski space. (Hobson et al. 2006, pp. 149, 151) Thus, at the point A where the beaming takes place, the metric will be locally flat, and the beaming will take place according to Einstein's formula.

3. DERIVATION OF THE FORMULA FOR DIRECTED GRAVITY

In this paper we are interested in whether relativistic beaming of gravity could solve the missing mass problem. To that end, we are most interested in compact objects that are rotating at velocities very near *c,* and that are additionally rotating such that their angular momentum vectors line up with that of the galaxy. That some such compact objects should exist seems likely, because in recent years researchers have demonstrated stellar alignment in open clusters. (Corsaro et al. 2017, Kovacs 2018) Further, at least one of the two clusters studied by Corsaro showed close alignment with galactic north. (Kamann et al. 2019) Compact objects that are rotating close to *c* and are in alignment with galactic north are important in that their directed gravity should mostly stay in the galactic disk, with only very small leakage at the top and bottom of the disk. We will now develop a theory for these particular objects. We will use Gauss's law to develop this theory. By the way, we will need to assume that the mass density of the compact objects is constant for any particular value of the radius *r*, *r* being the distance from the central axis of the stellar disk. This will result in the directed gravitational force being a function of r, such that $\boldsymbol{g} = \boldsymbol{g}(r)$.

First, we will model the stellar disk of the galaxy as a cylinder. Because the stellar disk is shaped as a cylinder, its outer surface is a closed surface for the purpose of applying Gauss's law of gravity. Thus, by Gauss's law:

$$\oint_S \boldsymbol{g} \cdot \boldsymbol{dA} = -4\pi GM \qquad (2)$$

The formula is from Simpson (2006), where the left hand side of the equation represents the surface integral of $\boldsymbol{g} \cdot \boldsymbol{dA}$ over the entire surface of the stellar disk's cylinder; $\boldsymbol{g}$ being the gravitational acceleration vector while $\boldsymbol{dA}$ = $\widehat{\boldsymbol{n}}$ dA is a vector that represents the differential unit of surface area along the outer surface *S* of the cylinder, directed outwards. M is the total mass of the relativistically rotating compact objects described above within radius r of the central axis of the cylinder and G is the usual gravitational constant. Note also that $\widehat{\boldsymbol{n}}$ is the unit normal vector, *h* is the height of the stellar disk, and $\bar{G}$ = 4πG.

Now, since virtually all of the gravitational flux will exit the sides of the disk, and none at the top or bottom, we have that $\widehat{\boldsymbol{n}}$ is perpendicular to $\boldsymbol{g}$ for the top and bottom of the cylinder. Thus, for the top and bottom, we have:

$$\boldsymbol{g} \cdot \widehat{\boldsymbol{n}} = 0 \tag{3}$$

So that the top and bottom of the cylinder will not contribute to the integrated surface area, only the outer edge will. Note also that the $\boldsymbol{g}$ points in the opposite direction of $\widehat{\boldsymbol{n}}$ for the outer edge of the cylinder, so that $\boldsymbol{g} \cdot \widehat{\boldsymbol{n}}$ will equal –*g(r)* around the cylinder's outer edge.
Thus we have,

$$\begin{aligned} \oint_S \boldsymbol{g} \cdot \boldsymbol{dA} &= \oint_S \boldsymbol{g} \cdot \widehat{\boldsymbol{n}} \, \mathrm{dA} = \oint_S \big(-g(r)\big)\mathrm{dA} \\ &= -\, g(r) \oint_S \mathrm{dA} = -g(\mathrm{r})2\pi\mathrm{rh} \end{aligned} \tag{4}$$

And since *g(r)* is constant at any particular value of r, and does not depend on A, it comes out of the integral. Now, by Eq. 2 and Eq. 4 above, we have:

$$-g(\mathrm{r})\pi 2\mathrm{rh} = -4\pi \mathrm{GM} \tag{5}$$

Which after simplification gives us our formula for the acceleration *g*(r) caused by directed gravity:

$$g(\mathrm{r}) = \frac{4\pi GM}{2\pi rh} = \frac{\bar{\mathrm{G}}M}{2\pi rh} \tag{6}$$

By multiplying each side of the equation by the test mass *m*, we obtain the force equation for directed gravity:

$$\mathrm{F} = \mathrm{m}g(\mathrm{r}) = \frac{\bar{\mathrm{G}}Mm}{2\pi rh} \tag{7}$$

F is the force of directed gravity caused by the highly relativistically rotating compact objects that are in alignment with galactic north. The force points toward the center of the stellar disk which is modelled as a cylinder of height h and radius r. M is the mass of the highly relativistically rotating compact objects that are aligned with galactic north and within radius r, and m is a test mass. Note that since h is a constant, the denominator of the force equation increases by r, rather than the usual r^2. Note also that the denominator of the force equation is equal to the area of the outer edge of the cylinder through which the directed gravity passes.

4. COULD DIRECTED GRAVITY FROM THE GALAXY'S SUPERMASSIVE BLACK HOLE EXPLAIN THE MISSING MASS?

We can use our acceleration formula, Eq. 6, derived above to test whether directed gravity from the supermassive black hole at SgrA* in the center of the galaxy could be the source of the missing mass. That is, could it be responsible for the approximately 170 km s^{-1} of rotation speed at the edge of the galaxy that is attributed to missing mass or dark matter. (Kafle et al. 2014, p.10 at figure 8, reading off the circular velocity attributed to the dark halo at approximately 15 kpc.) (de Salas et al. 2019, pp.10 & 11 at figures 3 and 4, reading off the rotation curves at 15 kpc for two different dark halo models.) The mass of the supermassive black hole at SgrA* has been measured to be $4.02 \pm 0.16 \times 10^6$ $M_\odot$ (Boehle et al. 2016). I am modelling the stellar disk of the galaxy as a cylinder of radius 50,000 ly and height 1,000 ly. (Coffey 2010; Rix & Bovy 2013). That translates to a radius of 4.7304×10^{20} m and a height of 9.461×10^{18} m.

First, we will determine the amount of acceleration necessary to create 170 km s^{-1} of rotation speed (circular velocity) using the formula for centripetal acceleration, a_c.

$$a_c = \frac{v^2}{r} = \frac{(170 \times 10^3 m\, s^{-1})^2}{4.73 \times 10^{20} m} = 6.11 \times 10^{-11} \text{ m s}^{-2} \quad (8)$$

Now, if all of the measured mass of SgrA* were rotating at nearly the speed of light, and if it were in alignment with galactic north, then the formula for g(r) above would apply to SgrA*. Thus, we will insert the figures for the mass of SgrA* and the dimensions of the galactic disk into the formula for $g(r)$, Eq.6, to determine whether SgrA* could be the source of the missing mass.

$$g(\mathrm{r}) = \frac{4\pi \mathrm{GM}}{2\pi r h} = \frac{4\pi(6.67 \times 10^{-11})(7.996 \times 10^{36} kg)}{2\pi\,(4.73 \times 10^{20} m)(9.46 \times 10^{18} m)} = 2.38 \times 10^{-13} \text{ m s}^{-2} \quad (9)$$

The result of our calculations is that SgrA* appears to be too small to provide the necessary directed gravity to explain the missing mass. If we divide the calculated acceleration g(r) by the expected acceleration from missing mass, a_c, we obtain:

$$\frac{g(r)}{a_c} = \frac{2.38 \times 10^{-13}}{6.11 \times 10^{-11}} = 0.0039 \quad (10)$$

which is clearly inadequate.

However, the problem with this analysis is that the figure for the mass of SgrA* represents only the mass which is gravitating isotropically. Any mass that is moving near the speed of light, and thus has its gravitational force beamed into the equatorial plane, will not be included in the figure for the mass. Consider for example the orbit of the S-2, a star whose orbit has been recently used to measure the mass of SgrA*. S-2's orbit will be unaffected by relativistically moving mass throughout the entirety of its orbit, except for those times when it crosses the equatorial plane. The result of crossing the plane will be small changes in the orbital parameters, which will be the subject of a future paper.

Therefore, we cannot rule out the possibility that SgrA* is the source of the missing mass. However, for this to be the case, SgrA* must be approximately 1000 times more massive than has been measured, with the additional mass being relativistically moving matter.

If a person concludes that the Supermassive black hole is too small to be the source of missing mass, then there are other possible sources of directed gravity in the galaxy: neutron stars and stellar mass black holes that are compact remnants of supernovae. Could the neutron stars or stellar mass black holes be the source of the missing mass?

5. COULD DIRECTED GRAVITY FROM NEUTRON STARS OR STELLAR MASS BLACK HOLES EXPLAIN THE MISSING MASS

Our galaxy has probably produced approximately 10^9 compact remnants of supernovae consisting of neutron stars and stellar mass black holes. (Branch & Wheeler 2017, p. 597) With neutron stars and stellar mass black holes each more massive than 1 $M_\odot$, this would represent about three orders of magnitude more mass than is present in SgrA*. Could directed gravity from these compact remnants be the source of gravitational acceleration attributed to dark matter?

5.1. Brief Discussion of Neutron Stars and Stellar Mass Black Holes

While neutron stars and stellar mass black holes are the compact remnants of supernovae, neutron stars are believed to result from supernovae of moderate sized stars, while stellar mass black holes would result from supernovae from very large stars. Neutron stars range in mass from about 1 $M_\odot$ to 2 $M_\odot$, with the canonical size considered to be 1.4 $M_\odot$. (Branch & Wheeler 2017, p. 598) Stellar mass black holes range from about 5 $M_\odot$ to perhaps 100 $M_\odot$, with the canonical size being 10 $M_\odot$. (Branch & Wheeler 2017, p. 602) The outer shells of neutron stars rotate at less than the speed of light, with the fastest discovered so far rotating at 715 Hz (Hessels et al. 2006) which translates to a linear speed at the equator of about 0.24 *c*. (See

Appendix 1) However, the cores of many neutron stars are believed to be composed of neutrons in a superfluid state that rotate at greater speeds than the stellar surfaces. (Haskell et al. 2018; Haskell & Melatos 2015; Alpar et al. 1984) As for stellar mass black holes, many rotate with spin parameters between 0.7 and 1, which are believed to be relativistic. (Branch & Wheeler 2017, p. 599) For the relative numbers, neutron stars should outnumber stellar mass black holes by about an order of magnitude. (Branch & Wheeler 2017, p. 597)

As before, we are most interested in compact remnants that are rotating at nearly the speed of light, *c*, and that are in close alignment with galactic north. Of the approximately 10^9 compact remnants, we would like to know how many satisfy these conditions, and what their total mass would be. For these questions we can only speculate as to the answers. So that is what we will do!

5.2. Missing Mass Calculation with Neutron Stars and Stellar Mass Black Holes

We shall speculate that there is a class of neutron stars and stellar mass black holes that rotate at nearly *c* and are in close alignment with galactic north, and that their total mass is approximately $10^9\ M_\odot$. The reason for this speculation will become clear as we perform calculations below. Additionally, we will assume that the mass density of this class of neutron stars and stellar black holes is a function of the distance *r* to the central axis of the galactic disk, with the consequence that ***g*** = ***g(r)***. Thus the conditions for our formula for *g*(r) are satisfied. Therefore, we can use Eq.6 to figure the acceleration at the edge of the disk due to directed gravity:

$$g(r) = \frac{4\pi GM}{2\pi rh} = \frac{4\pi(6.674\times10^{-11})10^{9}(1.9891\times10^{30}kg)}{2\pi(4.73\times10^{20}m)(9.46\times10^{18}m)} = 5.93\times10^{-11}\ \mathrm{ms}^{-2} \qquad (11)$$

We can translate that acceleration to circular velocity at the edge of the stellar disk by once again using the formula for centripetal acceleration, Eq. 8, noting that the centripetal acceleration equals the gravitational acceleration.

$$g(r) \; = \; a_c = \frac{v^2}{r} \tag{12}$$

That implies:

$$v_c = \sqrt{g(r)r} = \sqrt{(5.93 \times 10^{-11} m s^{-2})(4.73 \times 10^{20} m)} = 167 \text{ kms}^{-1} \tag{13}$$

Of course, 167 km s^{-1} is very close to 170 km s^{-1} which is the expected value for the circular velocity caused by dark matter at the edge of the stellar disk. So if our speculation is correct, directed gravity from neutron stars and stellar mass black holes that are rotating at nearly *c* and are in close alignment with galactic north could explain the missing mass problem for the galaxy.

5.3. Plotting a Circular Velocity Curve for Directed Gravity

We would like to plot the circular velocity for each value of *r* within the stellar disk and also beyond the edge of the disk. However, we first need to know whether directed gravity from compact objects outside of the radius *r* will affect the gravitational field at *r*. It turns out that for a cylindrically shaped stellar disk, with ***g=g(r)***, the directed gravity from the compact objects outside of the radius *r* cancels out to zero. (See Appendix 2) Thus, to compute the acceleration at each value of *r* within the stellar disk, we simply need to apply the formula for *g(r)*, Eq. 6, keeping in mind that the value for M = M(r) will be a function of *r*, in that M will be the mass of the compact objects that are within the distance *r* of the central axis, and of course are rotating at near *c* and in alignment with galactic north. A further complication is that finding M(r) will be difficult unless we know how the mass density of our class of compact objects varies with r. To make the calculation possible, we will assume for this particular diagram that the mass density of our class of neutron stars and stellar mass black holes is uniform throughout the stellar disk.

After obtaining *g* for each value of *r*, we will compute each value for the circular velocity, $v_c(r)$. Note that beyond the edge of the stellar disk, that is, beyond about 15 kpc, M will be a constant. Thus, beyond the disk,

$$v_c = \sqrt{g(r)r} = \sqrt{\frac{4\pi GM}{2\pi h}} \tag{14}$$

$$= \sqrt{\frac{4\pi(6.674\times10^{-11})10^{9}(1.9891\times10^{30}kg)}{2\pi(9.46\times10^{18}m)}} = 167\text{kms}^{-1}$$

Thus, we have the following graph for the contribution to circular velocity by directed gravity, where we have made the additional assumption of uniformity of the mass density of our special class of compact objects throughout the stellar disk.

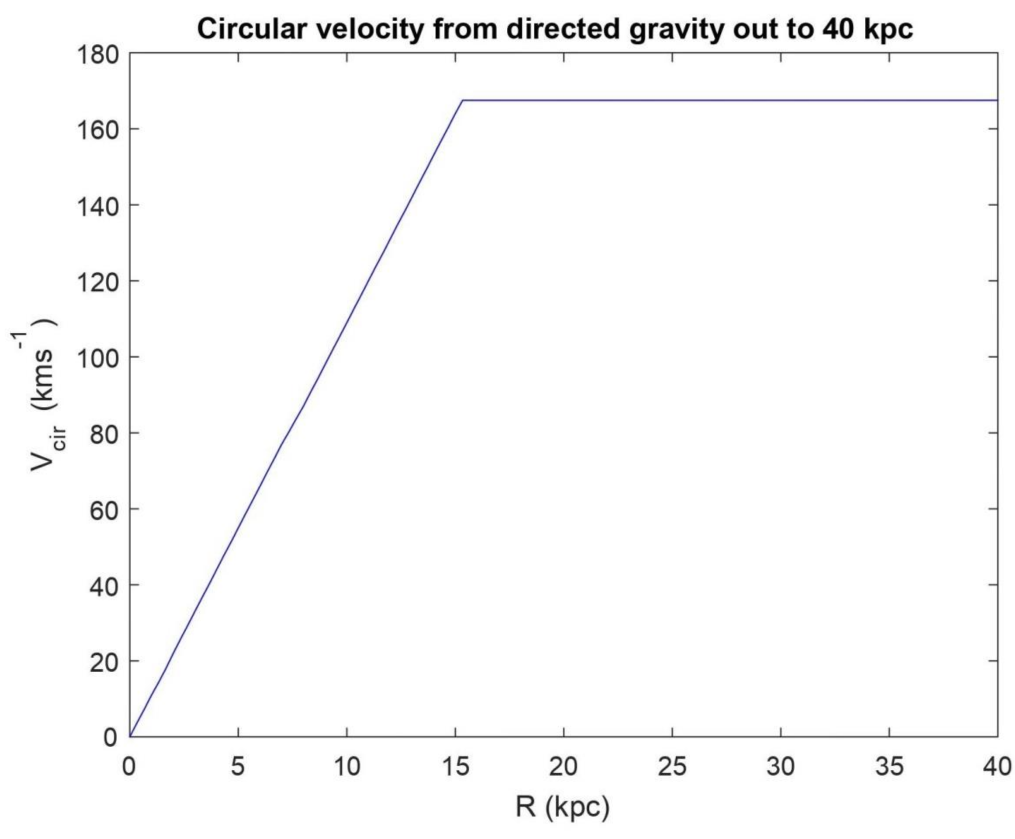

Figure 3. This figure shows the circular velocity curve of directed gravity in the Milky Way Galaxy from the special class of compact objects, assuming that their mass density is constant throughout the stellar disk, and that

their total mass is 10^9 $M_\odot$. Once the edge of the stellar disk is reached at approximately 15 kpc, the curve flattens out to a constant value of 167 kms^{-1}.

You may note that this graph looks very similar to the dark halo contribution to circular velocity from figure 3 of Sofue et al (2009).

6. CONCLUDING REMARKS

6.1. The Bullet Cluster

In a celebrated paper, Clowe et al. (2006) determined that observations of the Bullet Cluster were inconsistent with modified Newtonian dynamics, (MOND), and claimed that this presented direct empirical evidence of dark matter. During the collision of two clusters, the stellar components and the X-ray emitting plasma were essentially segregated. However, gravitational lensing maps did not trace the plasma, which was the dominant baryonic component, but instead approximately traced the distribution of galaxies. (Clowe et al. 2006) Do these results contradict the theory of directed gravity? The answer is "No." Directed gravity comes from compact stellar objects such as neutron stars, stellar mass black holes and supermassive black holes. All of these compact objects would have stayed with the distribution of galaxies, along with the stars, and would not have been stripped off by ram pressure to join the X-ray emitting plasma. Thus, the observations of the Bullet Cluster, although arguably disproving MOND, in no way prove the existence of dark matter in halos. Instead, those observations are beautifully consistent in principle with the theory of directed gravity.

6.2. General Conclusion

In this paper we have endeavored to show that gravitation should be subject to relativistic beaming in the same manner as light. Because both light and gravity are believed to travel at the speed *c*, and because both must be subject to the relativistic addition of velocities which is a property of the spacetime through which they pass, we conclude that gravity must be subject to relativistic beaming. Further, we can calculate the amount of beaming by applying Einstein's formula for relativistic beaming of light.

As a practical matter, relativistic beaming of gravity, or *directed gravity*, will most easily be observed with regards to rotating compact objects such as stellar mass black holes, supermassive black holes, and possibly neutron stars. Directed gravity from such compact objects should result in an increased concentration of gravitational flux in the equatorial planes of such objects, as we demonstrated in the discussions accompanying Fig. 1 and Fig. 2.

Concentration of gravity in a plane, in particular, in the stellar plane of the galaxy, suggests the possibility of directed gravity providing an efficient solution to the missing mass problem. Thus, we became interested in a particular class of compact objects: those which are rotating at very near the speed of light, *c*, and which are also in close alignment with galactic north. The directed gravity from compact objects in this class would stay almost entirely in the stellar disk. Thus, we developed a theory for this particular class of compact objects.

We used Gauss's law of gravity to derive formulas for the gravitational force and acceleration due to directed gravity in the galaxy from this class of objects. The acceleration formula, Eq.6, is:

$$g(\mathrm{r}) = \frac{4\pi GM}{2\pi rh} = \frac{\overline{\mathrm{G}}M}{2\pi rh} \tag{15}$$

And the force formula, Eq. 7, is:

$$F = mg(r) = \frac{\overline{G}Mm}{2\pi rh} \tag{16}$$

Note that both of these formulas depend inversely on the radius *r*, rather than the usual $1/r^2$ dependence. Since gravity decreasing as 1/r could result in a flat rotation curve, we proceeded to investigate the possible connection of directed gravity with the missing mass problem.

The first question to address was whether directed gravity from the central supermassive black hole in the galaxy, at SgrA*, could be responsible for the additional gravitation attributed to missing mass. To investigate this, we hypothesized that the supermassive black hole might satisfy the conditions for the application of the formula for directed gravity acceleration, *g(r)*, which we developed above at Eq. 6. That is, we assumed for sake of argument that the supermassive black hole spins at nearly the speed of light, *c*, and that its angular momentum vector is aligned with that of the galaxy. The other condition, that its mass density is a function of *r*, the distance from the central axis, would seem to be trivially true.

Thus, we could apply the formula for *g(r)*, Eq. 6, derived above. The result was that the acceleration from the directed gravity of the central supermassive black hole would be at best only 0.004 of that needed to explain the contribution of missing mass to the rotation curve of the galaxy. (Eq. 10) Thus, we concluded that the supermassive black hole at SgrA* was probably too small to explain the missing mass problem. We noted, however, that if SgrA* were about 1000 times more massive than measured, with that additional mass moving at nearly the speed of light, then SgrA* could explain the missing mass!

We next considered the possibility that compact objects resulting from supernovae: neutron stars and stellar mass black holes, could provide the necessary directed gravity. We first assumed that that the mass density of the special class of stellar mass black holes and neutron stars was a function of the distance *r* from the central axis of the stellar disk. Then we made a speculation: that the combined mass of neutron stars and stellar mass black holes that were rotating at nearly *c*, and were in alignment with galactic north, equaled approximately 10^9 $M_\odot$. With that speculation, we were ready to apply the formula for *g(r)* of directed gravity at Eq. 6.

The result of that calculation showed that the gravitational acceleration of missing mass could be explained by neutron stars and stellar mass black holes that satisfied the above conditions, if their total mass was about 10^9 $M_\odot$. (Eq.11) We calculated that this would result in a contribution to the circular velocity at the edge of the galaxy of 167 km s^{-1}, which is very close to the expected 170 km s^{-1} from missing mass. (Eq.12)

By making a further assumption, that the mass density of the compact objects in our special class was uniform throughout the stellar disk, we were able to calculate the circular velocity at each value of *r.* Thus we proceeded to plot a circular velocity curve for directed gravity for each value of *r* within the stellar disk, and extended the curve to 40 kpc. (Fig. 3) We noted the similarity between the curve we derived for circular velocity from directed gravity and the expected dark halo circular velocity curve from Sofue et al. (2009).

Personally, I think that the answer to the missing mass will turn out to be some combination of directed gravity from the Supermassive black hole and from other compact objects in the galaxy. However, it would be somewhat rash to claim that we have solved the missing mass problem. After all, the additional mass needed in SgrA* subject to directed gravity would be quite large. And as for neutron stars and stellar mass black holes, we had to speculate as to the total mass of neutron stars and stellar mass black holes that satisfied the conditions for application of the formulas for directed gravity, especially regarding the speed and direction of rotation. However, we have suggested a possible theory that could efficiently explain the missing mass problem without increasing the mass of the universe by a factor of five as required for dark matter halos. Further, our theory surpasses MOND as it is compatible with the observations of the Bullet Cluster.

Appendix 1

Calculation showing that a neutron star rotating at 716 Hz spins with a linear velocity at its equator of at most 0.24 c.

Here we provide a calculation of the approximate linear velocity at the equator of Pulsar PSR J1748-2446ad, which rotates at 716 Hz. (Hessels et al. 2006) This translates to a linear speed at the equator of about 0.24 *c,* as the following computation shows. The reference states that, if the pulsar mass is less than $2M_\odot$, then the radius must be less than 16 km. (Hessels et al. 2006) A 16 km radius would imply a circumference of $C = 2\pi R = 2\pi(16{,}000m) = 1.00531 \times 10^5\ m$. Thus, the distance travelled in one second is $D = (716)(1.00531 \times 10^5\ m) = 7.198 \times 10^7\ m$. Thus, the maximum linear velocity of the neutron star's surface at the equator is $v = 7.198 \times 10^7\ m\ s^{-1} \approx 0.24\ c$.

Appendix 2

Proof that the directed gravitational forces from the special class of compact objects that are outside of radius r sum to zero.

In Figure 4(a), we are looking at the galactic disk from above. We are interested in the directed gravitational forces on the point P at radius *r* from the center of the disk. We will show that the sum of the directed gravitational forces from the special class of compact objects located outside the radius *r*, that is, from the compact objects located in the shaded region of Figure 4, sum to zero.

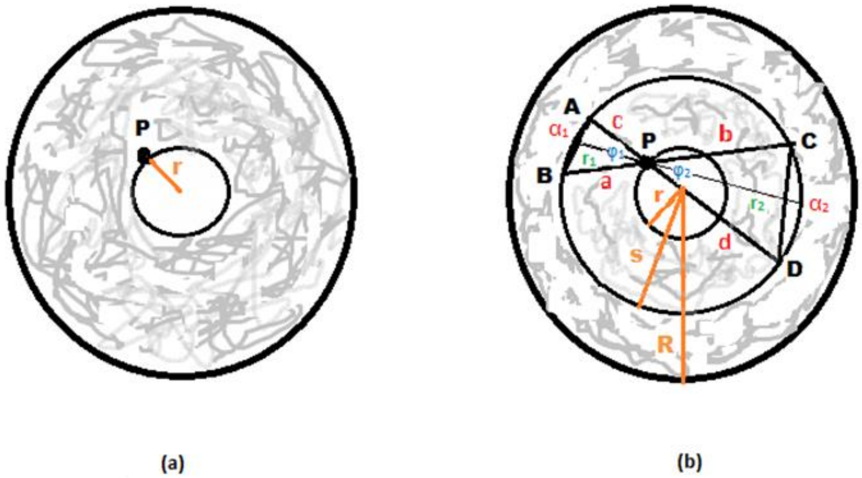

Figure 4. In 4(a), we see a polar view of the stellar disk. We are considering the directed gravitational forces from the special class of compact objects that are outside of radius *r.* In 4(b), we see depicted a circular ring of compact objects of the special class at a distance *s* from the central axis of the stellar disk. Note that points A, B, C and D lie on the ring. The argument below will show that the directed gravitational forces from the ring upon the point P balance to zero, and further, that the directed gravitational forces from all of the special class of compact objects in the shaded area balance to zero.

First we note that this is a two dimensional problem. Because the special class of compact objects includes only those that are rotating at nearly *c*, and have their rotation axes strongly aligned with the galaxy, the directed gravity travels horizontally and stays within a narrow plane within the galactic disk. Thus, the directed gravity from above and below that particular plane will have little effect on the forces at the point P. Thus we have the following proof.

Figure 4(b), redrawn from Figure 4(a) but with additional information, shows

a ring of stellar mass black holes at distance s from the center of the stellar disk. Points A, B, C and D lie upon the ring. The linear mass density of stellar mass black holes along the ring will be $\rho(r)$, but since r is constant along the ring, ρ will be a constant. $\widehat{AB}$ is an arc along the ring of arc length α_1, so that

$M_1 = \rho\alpha_1$ will be the mass of the special class of black holes along the arc $\widehat{AB}$. I have constructed the line r_1, which bisects the arc $\widehat{AB}$ and intersects P, such that r_1 is the radial distance between the arc $\widehat{AB}$ and P. Extending the lines from $\widehat{AB}$ through P, we find another arc $\widehat{CD}$ at radius r_2 from P on the other side of the ring. Note that by construction, the angles at P labeled ϕ_1 and ϕ_2 are congruent.

Note that we have two triangles, $\overline{ABP}$ and $\overline{CDP}$ which appear to be similar. By similar, we mean that the triangles' corresponding sides are in proportion, and their corresponding angles are congruent. (Polyanin 2007, p. 44, No.3°) To prove they are similar, we use the Intersecting Chord Theorem for a circle, by which we conclude:

$$ab = cd \tag{17}$$

(Polyanin 2007, p. 58, No. 12). Now, since b and d are nonzero, this of course implies:

$$\frac{a}{d} = \frac{c}{b} \tag{18}$$

Thus the corresponding sides of the triangles are in proportion, and since the included angles of the sides, ϕ_1 and ϕ_2 are congruent, we can conclude by the Side Angle Side Theorem that the triangles $\overline{ABP}$ and $\overline{CDP}$ are similar! (Polyanin 2007, p. 44, No.3°3) Since the triangles are similar, we know that $\overline{AB}$ and $\overline{CD}$ are also in proportion!

Next, we compare the directed gravitational forces from the arcs $\widehat{AB}$ and $\widehat{CD}$ on the point P. We use Eq. 7 to determine the respective forces.

$$F_1 = \frac{\bar{G}M_1 m}{2\pi r_1 h};\ F_2 = \frac{\bar{G}M_2 m}{2\pi r_2 h} \tag{19}$$

Where F_1 is the force of the arc $\widehat{AB}$ on the mass m at P, M_1 is the mass of the arc $\widehat{AB}$, h is the height of the stellar disk, $\bar{G} = 4\pi G$, G being the gravitational constant. We of course have similar definitions for F_2.

Now recall that $M_1 = \rho\alpha_1$ and $M_2 = \rho\alpha_2$. Thus we have:

$$F_1 = \frac{\bar{G}\rho\alpha_1 m}{2\pi r_1 h};\ F_2 = \frac{\bar{G}\rho\alpha_2 m}{2\pi r_2 h} \tag{20}$$

Now, by the similarity of triangles $\overline{ABP}$ and $\overline{CDP}$, we can conclude that:

$$\frac{|\overline{AB}|}{|\overline{AP}|} = \frac{|\overline{CD}|}{|\overline{PC}|} \tag{21}$$

If we consider the limit as the arc length α_1 approaches zero, we see that $\widehat{AB}$ approaches the line segment $\overline{AB}$, $\widehat{CD}$ approaches the line segment $\overline{CD}$, $\overline{AP}$ approaches r_1, and $\overline{PC}$ approaches r_2.

Thus, in the limit as α_1 approaches zero,

$$\frac{|\widehat{AB}|}{r_1} = \frac{|\widehat{CD}|}{r_2} \tag{22}$$

Which is equivalent to:

$$\frac{\alpha_1}{r_1} = \frac{\alpha_2}{r_2} \tag{23}$$

Thus, in the limit as α_1 approaches zero,

$$\vec{F}_1 = \frac{\bar{G}\rho\alpha_1 m}{2\pi r_1 h}\hat{r}_1 = -\frac{\bar{G}\rho\alpha_2 m}{2\pi r_2 h}\hat{r}_2 = -\vec{F}_2 \tag{24}$$

Where the minus sign arises because the two Forces are oppositely directed, and $\hat{r}_1$ and $\hat{r}_2$ are unit vectors in the direction of $\vec{F}_1$ and $\vec{F}_2$ respectively.

Now to find the net force of the ring upon P, we just integrate the sum of the two force differentials for all values of the angle ϕ.

$$\overrightarrow{F_P} = \frac{1}{2}\int_0^{2\pi}\left(d\overrightarrow{F_1} + d\overrightarrow{F_2}\right) d\varphi = 0 \qquad (25)$$

The integral equals zero because the integrand is identically zero everywhere.

Now, we can add up the force $\overrightarrow{F_P}$ for each ring from *r* to R. As all rings will contribute zero force upon P, the directed gravity from all of the special class of compact objects outside the radius *r* will sum to zero. This is what we set out to prove!

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