LOCATION ANALYSIS OF NBA DEVELOPMENT LEAGUE TEAMS

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1. INTRODUCTION

The NBA Development League is the minor league of the National Basketball Association. The league (originally known as the NBDL) began in 2001-'02, with eight teams located in Alabama, Georgia, North Carolina, South Carolina, and Virginia [Wikipedia: December, 2008]. Currently, the D-League has 16 teams located in California, Colorado, Iowa, Idaho, Indiana, North Dakota, New Mexico, Nevada, Oklahoma, Pennsylvania, South Dakota, Texas, and Utah. No D-League teams remain in any of the original southeastern states. Figures 1 and 2 display mid-to-high population cities in states where the original teams were located and where current teams are located, respectively. Note that not all mapped cities represent actual team locations.

Figure 1: Mid-to-High Population Cities in States Where ORIGINAL D-League Teams Were Located

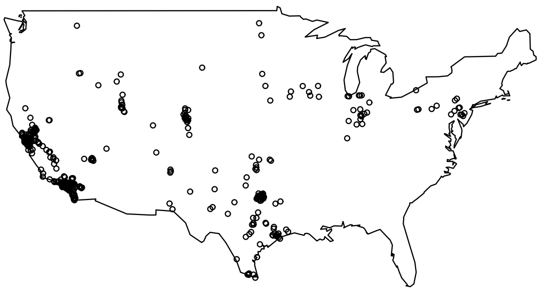

Figure 2: Mid-to-High Population Cities in States Where CURRENT D-League Teams Are Located

2. STUDY OBJECTIVES AND SELECTION CRITERIA

This study has two objectives:

(1) Determine optimal locations to place a complete league of 16 D-League teams.

(2) Determine optimal locations for expansion D-League teams given the placement of the current 16 teams.

Three (often conflicting) criteria were decided upon to assess optimal locations:

(1) D-League teams should be placed in high population cities in order to develop a strong fan base.

(2) D-League teams should be far from NBA cities so that they don't compete for the same fan base as NBA teams.

(3) D-League teams should be located near to each other in order to minimize travel costs.

3. INPUT DATA

All distances in this study were determined from a shortest path distance matrix obtained from a Florida State University website [https://people.scs.fsu.edu/ burkardt/datasets/cities/ cities.html]. The matrix contains the shortest path distances between 257 cities in the lower 48 states. No other cities were considered as potential locations of D-League teams. Population data for the 257 cities were obtained from the U.S. Census Bureau [http://www.census.gov]. In nearly all cases, 2007 population estimates were used; however, in a few cases only 2000 data were available.

4. MODELING THE PLACEMENT OF A COMPLETE 16-TEAM LEAGUE

(i) Constraints and Relaxations

In modeling the placement of a complete 16-team league, cities were only considered as potential locations of D-League teams if they were at least 100 miles from the nearest NBA city, and if they had a population of at least 50,000. The 100-mile constraint was put in place because basketball fans living within driving distance of an NBA team may be unlikely to develop a strong interest in a local minor league team. The 50,000-population constraint was put in place because small cities are not likely to be able to support minor league teams. It is important to note, however, that violations of both of these constraints exist among current D-League teams. Currently, one team is located in an NBA city, and five other teams are located in cities that are less than 100 miles from the nearest NBA city. Additionally, one team is located in a city with fewer than 50,000 people.

One relaxation throughout this study is to exclude the population of neighboring cities from contributing to the fan base of a D-League team. In other words, only the population of the city of a team's location is considered as its fan base. This decision was made due to limitations in the shortest path distance matrix. Since many mid-sized cities were not included in the matrix, large distortions may have resulted from incorporating coverage distances to include neighboring city populations.

(ii) Objective Functions

A multiobjective optimization model was constructed to maximize population in D-League cities and to minimize travel distance between D-League cities. The model was coded in AMPL and solved using the MINLP solver on the NEOS server. The objective function is formulated as a maximization model, with one positive term included for population and one negative term included for travel distance (effectively minimizing travel distance). The model was run with various weights to determine different sets of solutions that represent compromises between the two objectives.

A separate optimization model was constructed to maximize the sum of the distances from each D-League city to its *nearest* NBA city. This objective was not included in any multiobjective model. However, the optimal value from this model provides an additional comparison measure between the compromise solutions of the multiobjective model for population and travel distance. Each solution of the multiobjective model can be evaluated to determine the percentage of optimality obtained for this third objective.

(iii) Multiobjective Model Weighting

Due to the large number of decision variables, an exhaustive analysis of all noninferior solutions for the multiobjective optimization model was deemed impractical. Instead, it was decided that models would be run for a small set of different weights placed on the population component of the multiobjective function. Table 1 displays the weights that were chosen after experimenting with various weights to determine points at which noticeable changes in solutions occurred. The "Maximize Population" weight was chosen to be high enough to ensure that the solution obtained matched the solution obtained for a single objective model for maximizing population. The "Minimize Travel Distance" weight was chosen to be low enough to ensure that the solution obtained matched the solution obtained for a single objective model for minimizing travel distance.

Table 1: Weights for the Population Component of the Multiobjective Function

MODEL	**POPULATION WEIGHT**
Maximize Population	1
Weight 1	$\frac{1}{15}$
Weight 2	$\frac{1}{28}$
Weight 3	$\frac{1}{32}$
Weight 4	$\frac{1}{55}$
Minimize Travel Distance	$\frac{1}{10{,}000}$

5. RESULTS FOR THE PLACEMENT OF A COMPLETE 16-TEAM LEAGUE

(i) Solutions Optimal to a Single Objective

A comparison of the multiobjective model with the "Maximize Population" and "Minimize Travel Distance" weights (which effectively reduce the model to two different single objective models) and the single objective model for maximizing NBA distance are displayed in table 2. Clearly, the relative importance of the criteria for selecting cities greatly impacts model results. The "Maximize Population" model yields an average population over four times as high as the "Minimize Travel Distance" model, while having an average travel distance over five times as high as the "Minimize Travel Distance" model. The "Maximize NBA Distance" model produces an average distance to the nearest NBA city that is over 50% as high as in the other models. Tables 3-5 display the city locations in each of these models, along with each city's population and distance to its nearest NBA city. Figures 3-5 display mid-to-high population cities in the states in which the city locations occur for each model.

Table 2: Comparison of Models Optimal to a Single Objective

MODEL	**AVERAGE POPULATION**	**AVERAGE TRAVEL DISTANCE**	**AVERAGE DISTANCE TO NEAREST NBA CITY**
Maximize Population	560,243	1073	192
Minimize Travel Distance	138,036	199	210
Maximize NBA Distance	221,420	672	326

Table 3: Detailed Solution for "Maximize Population" Model

LOCATION	POPULATION	DISTANCE TO NEAREST NBA CITY
Albuquerque, NM	518,271	329
Bakersfield, CA	315,837	101
Columbus, OH	747,755	126
El Paso, TX	606,913	345
Fresno, CA	470,508	155
Jacksonville, FL	805,605	125
Kansas City, MO	450,375	298
Las Vegas, NV	558,880	228
Nashville, TN	590,807	196
Omaha, NE	424,482	290
Raleigh, NC	375,806	129
Saint Louis, MO	350,759	230
San Diego, CA	1,266,731	111
Seattle, WA	594,210	144
Tucson, AZ	525,529	107
Wichita, KS	361,420	154

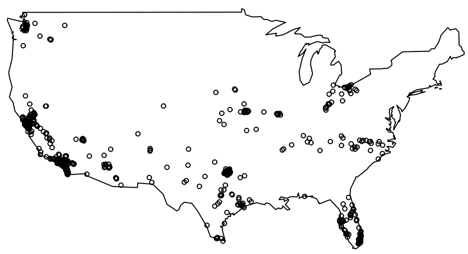

Figure 3: Mid-to-High Population Cities in States to Locate Teams in Order to MAXIMIZE POPULATION IN D-LEAGUE CITIES

Table 4: Detailed Solution for "Minimize Travel Distance" Model

LOCATION	**POPULATION**	**DISTANCE TO NEAREST NBA CITY**
Bloomington, IL	72,416	117
Cedar Rapids, IA	126,396	203
Champaign, IL	75,515	113
Columbia, MO	99,174	291
Decatur, IL	76,674	148
Des Moines, IA	196,998	234
Dubuque, IA	57,313	144
Iowa City, IA	67,062	200
Kansas City, KS	142,320	297
Kansas City, MO	450,375	298
Peoria, IL	113,546	128
Saint Joseph, MO	73,912	331
Saint Louis, MO	350,759	230
Springfield, IL	117,090	175
Topeka, KS	122,642	267
Waterloo, IA	66,387	177

Figure 4: Mid-to-High Population Cities in States to Locate Teams in Order to MINIMIZE TRAVEL DISTANCE BETWEEN D-LEAGUE CITIES

Table 5: Detailed Solution for "Maximize NBA Distance" Model

LOCATION	**POPULATION**	**DISTANCE TO NEAREST NBA CITY**
Albuquerque, NM	518,271	329
Billings, MT	101,876	386
Bismarck, ND	59,503	382
Boise, ID	202,832	295
Columbia, MO	99,174	291
El Paso, TX	606,913	345
Great Falls, MT	58,827	466
Kansas City, KS	142,320	297
Kansas City, MO	450,375	298
Lincoln, NE	248,744	336
Lubbock, TX	217,326	279
Omaha, NE	424,482	290
Rapid City, SD	63,997	313
Saint Joseph, MO	73,912	331
Santa Fe, NM	73,199	284
Spokane, WA	200,975	289

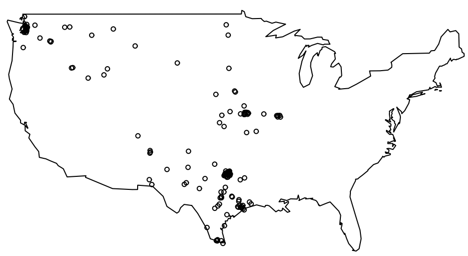

Figure 5: Mid-to-High Population Cities in States to Locate Teams in Order to MAXIMIZE DISTANCE BETWEEN D-LEAGUE CITIES AND NBA CITIES

(ii) Comparison of Non-inferior Solutions for Selected Weights

Table 6 displays the solutions obtained for all multiobjective weighted models. The "Weight 4" and "Minimize Travel Distance" models appear to be too regionalized, locating nearly all teams in the Midwest. When considering only the other four models, the "Weight 2" and "Weight 3" models are the best at minimizing travel distance and the best at maximizing the distance to the nearest NBA city. The "Maximize Population" and "Weight 1" models are the best at maximizing population, but have substantially larger travel distances and locate teams slightly closer to NBA cities.

Table 6: Solutions for All Multiobjective Weighted Models

MAXIMIZE POPULATION	WEIGHT 1	WEIGHT 2	WEIGHT 3	WEIGHT 4	MINIMIZE TRAVEL DISTANCE
Albuquerque, NM	Albuquerque, NM	Albuquerque, NM	Columbia, MO	Cedar Rapids, IA	Bloomington, IL
Bakersfield, CA	Columbus, OH	Columbus, OH	Columbus, OH	Columbia, MO	Cedar Rapids, IA
Columbus, OH	El Paso, TX	Des Moines, IA	Des Moines, IA	Columbus, OH	Champaign, IL
El Paso, TX	Jacksonville, FL	El Paso, TX	Jacksonville, FL	Des Moines, IA	Columbia, MO
Fresno, CA	Kansas City, MO	Jacksonville, FL	Kansas City, KS	Kansas City, KS	Decatur, IL
Jacksonville, FL	Las Vegas, NV	Kansas City, KS	Kansas City, MO	Kansas City, MO	Des Moines, IA
Kansas City, MO	Lexington, KY	Kansas City, MO	Lexington, KY	Lincoln, NE	Dubuque, IA
Las Vegas, NV	Lincoln, NE	Lexington, KY	Lincoln, NE	Nashville, TN	Iowa City, IA
Nashville, TN	Louisville, KY	Lincoln, NE	Louisville, KY	Omaha, NE	Kansas City, KS
Omaha, NE	Nashville, TN	Louisville, KY	Nashville, TN	Peoria, IL	Kansas City, MO
Raleigh, NC	Omaha, NE	Nashville, TN	Omaha, NE	Saint Joseph, MO	Peoria, IL
Saint Louis, MO	Raleigh, NC	Omaha, NE	Saint Louis, MO	Saint Louis, MO	Saint Joseph, MO
San Diego, CA	Saint Louis, MO	Saint Louis, MO	San Diego, CA	Springfield, IL	Saint Louis, MO
Seattle, WA	San Diego, CA	San Diego, CA	Springfield, MO	Springfield, MO	Springfield, IL
Tucson, AZ	Tucson, AZ	Springfield, MO	Topeka, KS	Topeka, KS	Topeka, KS
Wichita, KS	Wichita, KS	Wichita, KS	Wichita, KS	Wichita, KS	Waterloo, IA

Figures 6-8 display differences in measures of performance among all multiobjective weighted models. Note that figures 6 and 7 are monotonic as weight changes because the graphed performance measures in these models reflect the two components of the objective function. Figure 8, however, displays average distance to the nearest NBA city, which is not a measure incorporated in the multiobjective weighted models; and thus does not reflect monotonic changes.

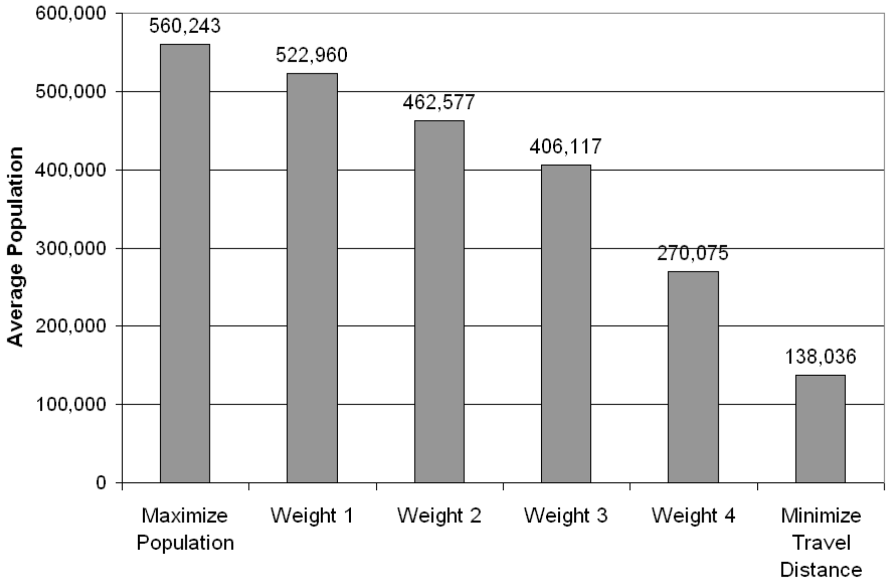

Figure 6: Average Population for All Multiobjective Weighted Models

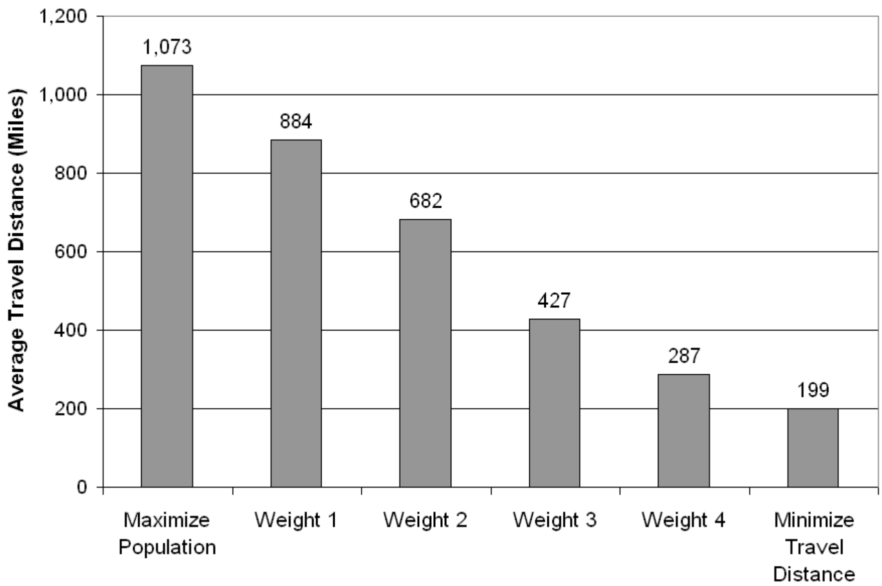

Figure 7: Average Travel Distance for All Multiobjective Weighted Models

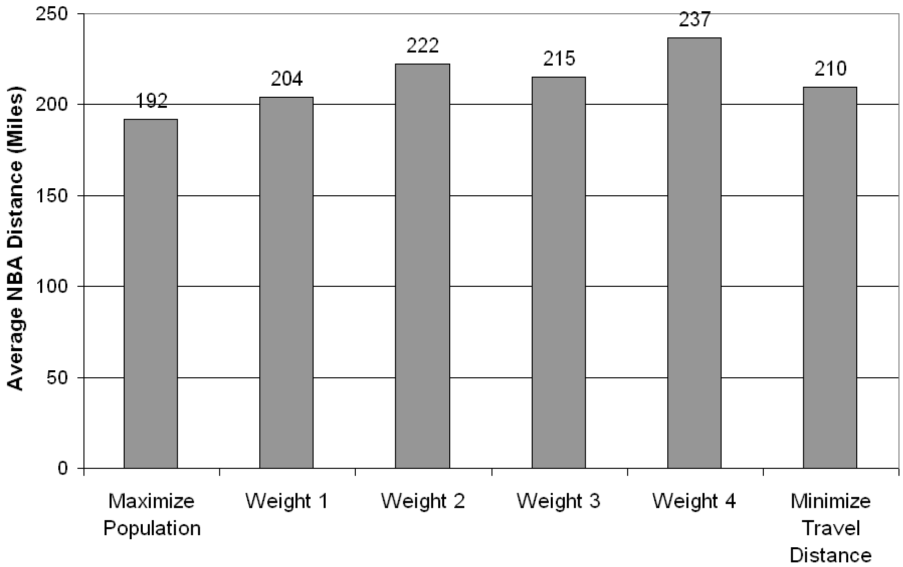

Figure 8: Average Distance to Nearest NBA City for All Multiobjective Weighted Models

Figures 9-12 display the percent of optimality obtained for each of three performance measures across all six multiobjective weighted models. Because travel distance is a quantity desired to be minimized, high percentages of optimality indicate solutions that are poor for this performance measure. The "Weight 2" and "Weight 3" models appear to be the best for balancing all three performance measures. Both models rank among the four best for all three performance measures, including average distance to the nearest NBA city, which is not an objective function component. The "Maximize Population" and "Weight 1" models yield average travel distances that are over 400% of optimality. The "Weight 4" and "Minimize Travel Distance" models achieve less than 50% of optimality for population.

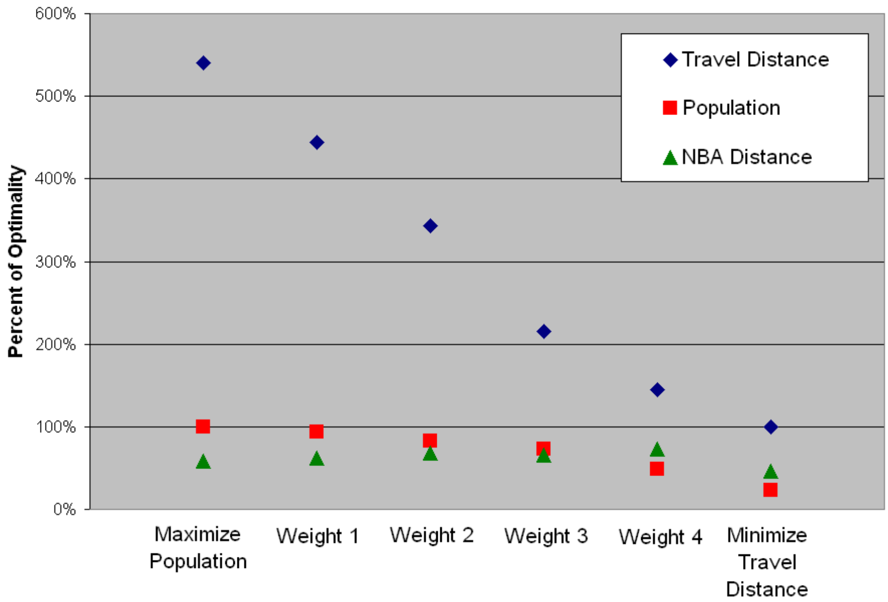

Figure 9: Percent of Optimality Model Comparison for Travel Distance, Population, and NBA Distance

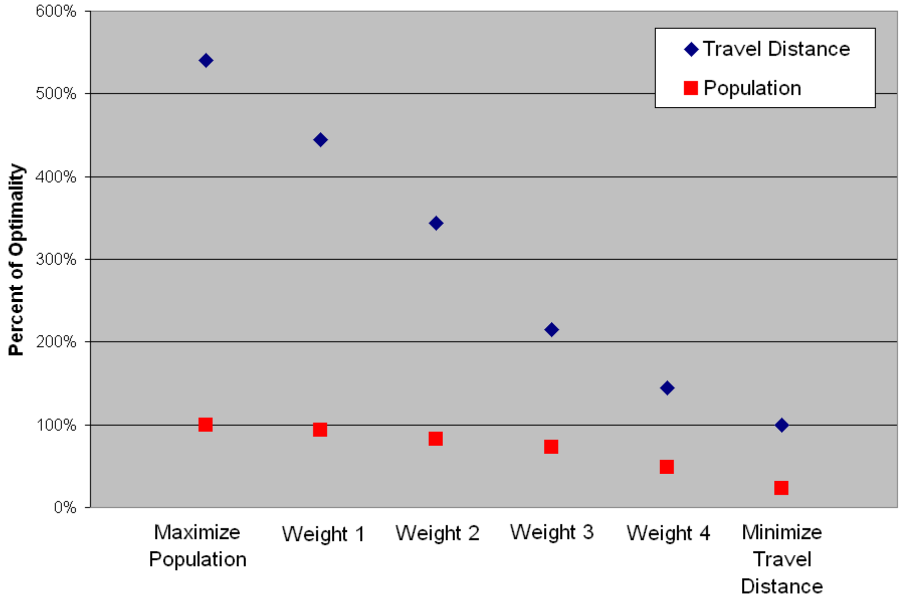

Figure 10: Percent of Optimality Model Comparison for Travel Distance and Population

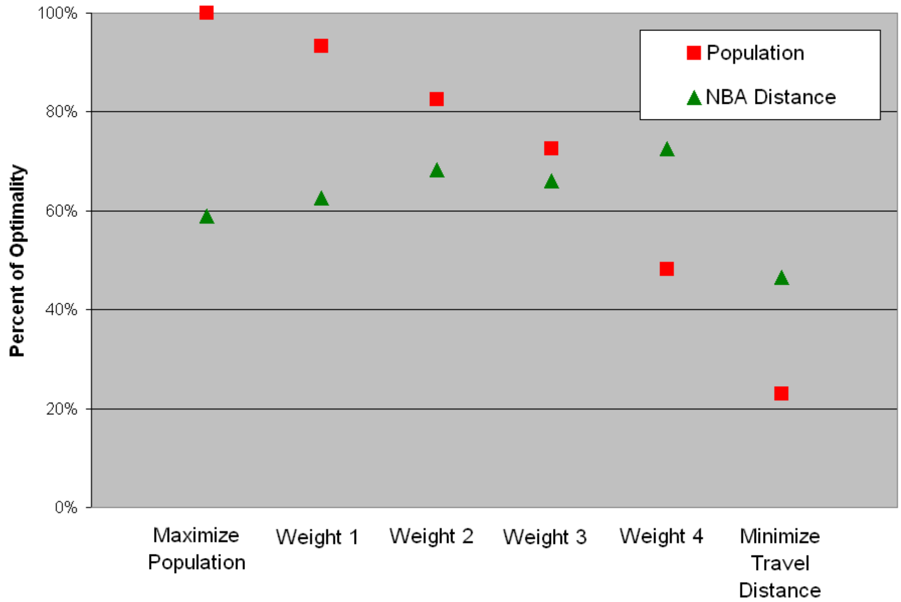

Figure 11: Percent of Optimality Model Comparison for Population and NBA Distance

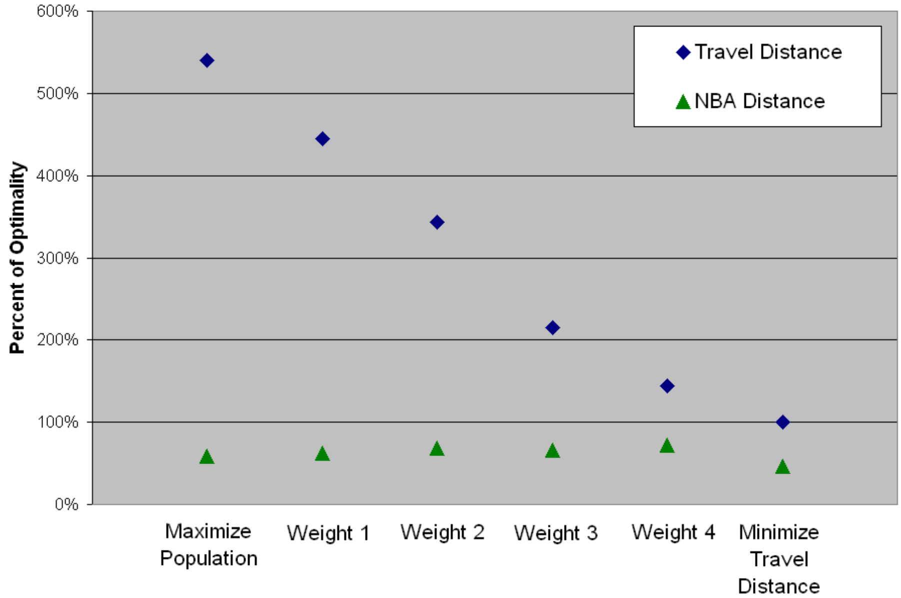

Figure 12: Percent of Optimality Model Comparison for Travel Distance and NBA Distance

6. MODELING THE PLACEMENT OF EXPANSION TEAMS

The multiobjective function implemented in the models for the placement of a complete 16-team league was applied to models for the placement of expansion teams using the same weights. Again, the two objectives are to maximize population in D-League cities and to minimize travel distance. In the expansion teams models, however, the 16 current D-League teams were defaulted to their current cities. In some cases, this involved modifications to the model to force location decisions that otherwise would have violated model constraints. All selected expansion teams, however, were required to meet the same constraints used in the models for the placement of a complete 16-team league. For each weight, separate models were run to determine optimal locations for the addition of one, two, three, and four expansion teams.

7. RESULTS FOR THE PLACEMENT OF EXPANSION TEAMS

San Diego is the obvious first choice if one expansion team is added to the league. It is the first choice for all weights except the "Minimize Travel Distance" weight. If more than one expansion team is added, results vary widely based on weight. Tables 7-10 display all weighted solutions for models for the addition of one, two, three, and four expansion teams. Note that for the "Weight 3" model, Las Vegas is chosen as a location city for the three-team expansion model, but is not chosen as a location city for the corresponding four-team expansion model. This is because minimizing travel distance is an objective for which location decisions made individually rather than jointly can lead to less than optimal decisions. In the four-team expansion "Weight 3" model, Las Vegas is replaced by two Midwest cities (Omaha, NE and Kansas City, MO) which combine to make a sufficient enough impact in minimizing travel distance to replace Las Vegas in the solution, even though each has a smaller population than Las Vegas.

Table 7: Solutions for the Location of One D-League Expansion Team for All Multiobjective Weighted Models

Weight	**Location**
Maximize Population	San Diego, CA
Weight 1	San Diego, CA
Weight 2	San Diego, CA
Weight 3	San Diego, CA
Weight 4	San Diego, CA
Minimize Travel Distance	Pueblo, CO

Table 8: Solutions for the Location of Two D-League Expansion Teams for All Multiobjective Weighted Models

Weight	**Locations**
Maximize Population	San Diego, CA Jacksonville, FL
Weight 1	San Diego, CA Columbus, OH
Weight 2	San Diego, CA El Paso, TX
Weight 3	San Diego, CA El Paso, TX
Weight 4	San Diego, CA Omaha, NE
Minimize Travel Distance	Pueblo, CO Lincoln, NE

Table 9: Solutions for the Location of Three D-League Expansion Teams for All Multiobjective Weighted Models

Weight	**Locations**
Maximize Population	San Diego, CA Jacksonville, FL Columbus, OH
Weight 1	San Diego, CA Columbus, OH El Paso, TX
Weight 2	San Diego, CA El Paso, TX Las Vegas, NV
Weight 3	San Diego, CA El Paso, TX Las Vegas, NV
Weight 4	San Diego, CA Omaha, NE Kansas City, MO
Minimize Travel Distance	Pueblo, CO Lincoln, NE Omaha, NE

Table 10: Solutions for the Location of Four D-League Expansion Teams for All Multiobjective Weighted Models

Weight	**Locations**
Maximize Population	San Diego, CA Jacksonville, FL Columbus, OH El Paso, TX
Weight 1	San Diego, CA Columbus, OH El Paso, TX Las Vegas, NV
Weight 2	San Diego, CA El Paso, TX Las Vegas, NV Tucson, AZ
Weight 3	San Diego, CA El Paso, TX Omaha, NE Kansas City, MO
Weight 4	San Diego, CA Omaha, NE Kansas City, MO Wichita, KS
Minimize Travel Distance	Pueblo, CO Lincoln, NE Omaha, NE Wichita, KS

8. CONCLUSIONS

In locating a complete 16-team league, the weight given to selection criteria greatly impacts the selected cities. Study performance measures reflect large differences between models with different sets of selected cities.

The "Weight 2" and "Weight 3" models appear to be the best at balancing all three selection criteria (large population, small travel distance, and large distance from each city to its nearest NBA city). Both models rank among the four best multiobjective weighted models for all three performance measures, including average distance to the nearest NBA city, which is not an objective function component in these models. Other models may have merit, however, depending on the true relative importance of the selection criteria. Models with weights that favor minimizing travel distance over maximizing population tend to locate teams in the Midwest. As more weight is placed on maximizing population, locations become more spread out. San Diego (CA), Jacksonville (FL), and Columbus (OH) have the three highest populations among cities that are at least 100 miles from the nearest NBA city. All three of these cities are chosen in the “Weight 2” and “Weight 3” models in addition to the higher population weight models. None of these cities, however, currently has a D-League team.

No more than two current D-League cities were selected in any of the multiobjective weighted models. Three current D-League cities were selected in the single objective “Maximize NBA Distance” model. In all of the complete 16-team league models, however, seven current D-League cities were excluded from selection based on model constraints that require every selected location to be at least 100 miles from the nearest NBA city and to have a population of at least 50,000.

San Diego is the recommended location for the addition of a single D-League expansion team. It is the first choice for an expansion team in five of six multiobjective weighted models that take into account the locations of the current 16 D-League teams. San Diego has a population of approximately 1.3 million and is 111 miles from the nearest NBA city. The NBA’s Houston Rockets franchise was originally located in San Diego from 1967-1971 [Wikipedia: December, 2008].

Two weaknesses of the formulated models in this study are (1) that no measure of regional basketball interest is included, and (2) that neighboring city populations are not taken into account. These relaxations likely lead to many cities in which city population is a misleading indicator for potential fan base. This is particularly important to consider in models in which population is given a high weight.

APPENDIX A

MULTIOBJECTIVE AMPL MODEL FOR THE PLACEMENT OF A COMPLETE 16-TEAM LEAGUE

```
set demnodes;
set sitenodes;
set notlower48;
set nba;

param P;
param W {demnodes};
param spdmat {demnodes, sitenodes} >= 0;
param population {demnodes};
param pop50plus {demnodes};
param nbacity {sitenodes};
param dleaguecity {sitenodes};


var X {sitenodes} binary >= 0;
var Y {demnodes} binary >= 0;
var Binum {demnodes} binary >= 0;


maximize TotalDemand: sum {i in demnodes} population[i]*X[i] - sum {i in demnodes, j in sitenodes} spdmat[i,j]*Y[i]*X[j];

subject to locatePteams:
 sum {j in sitenodes} X[j] = P;

subject to locatePpop50teams:
 sum {j in sitenodes} X[j] * pop50plus[j] = P;

subject to lower48:
 sum {k in notlower48} X[k] = 0;

subject to notnba:
 sum {n in nba} X[n] = 0;

subject to hundredmiles {i in demnodes}:
  W[i]*X[i] >= 100*Binum[i];

subject to bicnst {i in demnodes}:
  Binum[i] >= X[i];

subject to yandx {i in demnodes}:
  Y[i] >= X[i];
```

APPENDIX B

SINGLE OBJECTIVE AMPL MODEL FOR THE PLACEMENT OF A COMPLETE 16-TEAM LEAGUE

```
set demnodes;
set sitenodes;
set notlower48;
set nba;

param P;
param W {demnodes};
param spdmat {demnodes, sitenodes} >= 0;
param population {sitenodes};
param pop50plus {sitenodes};
param nbacity {sitenodes};
param dleaguecity {sitenodes};


var X {sitenodes} binary >= 0;
var Binum {demnodes} binary >= 0;

maximize NBADist: sum {i in demnodes} W[i]*X[i];

subject to locateteams:
 sum {j in sitenodes} X[j] = P;

subject to locatePpop50teams:
 sum {j in sitenodes} X[j] * pop50plus[j] = P;

subject to lower48:
 sum {k in notlower48} X[k] = 0;

subject to notnba:
 sum {n in nba} X[n] = 0;

subject to hundredmiles {i in demnodes}:
  W[i]*X[i] >= 100*Binum[i];

subject to bicnst {i in demnodes}:
  Binum[i] >= X[i];
```

APPENDIX C

MULTIOBJECTIVE AMPL MODEL FOR THE PLACEMENT OF EXPANSION TEAMS

```
set demnodes;
set sitenodes;
set notlower48;
set nba;
set nbdl;

param P;
param W {demnodes};
param forcein {demnodes};
param spdmat {demnodes, sitenodes} >= 0;
param population {demnodes};
param pop50plus {demnodes};
param nbacity {sitenodes};
param dleaguecity {sitenodes};


var X {sitenodes} binary >= 0;
var Y {demnodes} binary >= 0;
var Binum {demnodes} binary >= 0;


maximize TotalDemand: sum {i in demnodes} population[i]*X[i] - sum {i in demnodes, j in sitenodes} spdmat[i,j]*Y[i]*X[j];

subject to locatePteams:
 sum {j in sitenodes} X[j] = P;

subject to locatePpop50teams:
 sum {j in sitenodes} X[j] * pop50plus[j] = P;

subject to lower48:
 sum {k in notlower48} X[k] = 0;

subject to notnba:
 sum {n in nba} X[n] <= 2;

subject to currentteams:
 sum {m in nbdl} X[m] = 14;

subject to hundredmiles {i in demnodes}:
  W[i]*X[i] >= 100*Binum[i];

subject to bicnst {i in demnodes}:
  Binum[i] >= X[i];

subject to yandx {i in demnodes}:
  Y[i] >= X[i];
```