Self-Teaching Digit Addition Table

If you copy all the equations from Table 1 into Table 2, once in the morning and once again in the evening for a whole month, by the end of the month, you will have easily memorized each of the two–digit sums. The most important aspect of this self–teaching lesson is that there are no fears or tears because the result of each sum is known. Adding numbers more than nine is computed by repeatedly using only these 100 sums with the carry rule. This lesson focuses on adding decimal digits, i.e., sums containing zero through nine. Later in school, your teacher will explain the addition carry rule to you.

Addition, being commutative means that the two digits on the left side of each equation in Table 1 can swap places without changing the answer. For example, 5 + 3 = 8 is the same as 3 + 5 = 8; yet, the order in which the three and the five occur within the sum is different. The commutativity of addition creates a visible symmetry in Table 1. To perceive this symmetry, notice that all the answers are the same for the equations both above and below the diagonal that runs from the upper left to the lower right. The defining feature of addition's commutativity is that the sum is the same, regardless of the order of its addends. There are simple things we do every day that are not commutative. Is the order of the clothing in which you dress yourself essential? Have you ever left the house with your socks worn over the outside of your shoes?

Table 1: Addition Table for Decimal Digits, Zero to Nine Inclusive

0 + 0 = 0	0 + 1 = 1	0 + 2 = 2	0 + 3 = 3	0 + 4 = 4	0 + 5 = 5	0 + 6 = 6	0 + 7 = 7	0 + 8 = 8	0 + 9 = 9
1 + 0 = 1	1 + 1 = 2	1 + 2 = 3	1 + 3 = 4	1 + 4 = 5	1 + 5 = 6	1 + 6 = 7	1 + 7 = 8	1 + 8 = 9	1 + 9 = 10
2 + 0 = 2	2 + 1 = 3	2 + 2 = 4	2 + 3 = 5	2 + 4 = 6	2 + 5 = 7	2 + 6 = 8	2 + 7 = 9	2 + 8 = 10	2 + 9 = 11
3 + 0 = 3	3 + 1 = 4	3 + 2 = 5	3 + 3 = 6	3 + 4 = 7	3 + 5 = 8	3 + 6 = 9	3 + 7 = 10	3 + 8 = 11	3 + 9 = 12
4 + 0 = 4	4 + 1 = 5	4 + 2 = 6	4 + 3 = 7	4 + 4 = 8	4 + 5 = 9	4 + 6 = 10	4 + 7 = 11	4 + 8 = 12	4 + 9 = 13
5 + 0 = 5	5 + 1 = 6	5 + 2 = 7	5 + 3 = 8	5 + 4 = 9	5 + 5 = 10	5 + 6 = 11	5 + 7 = 12	5 + 8 = 13	5 + 9 = 14
6 + 0 = 6	6 + 1 = 7	6 + 2 = 8	6 + 3 = 9	6 + 4 = 10	6 + 5 = 11	6 + 6 = 12	6 + 7 = 13	6 + 8 = 14	6 + 9 = 15
7 + 0 = 7	7 + 1 = 8	7 + 2 = 9	7 + 3 = 10	7 + 4 = 11	7 + 5 = 12	7 + 6 = 13	7 + 7 = 14	7 + 8 = 15	7 + 9 = 16
8 + 0 = 8	8 + 1 = 9	8 + 2 = 10	8 + 3 = 11	8 + 4 = 12	8 + 5 = 13	8 + 6 = 14	8 + 7 = 15	8 + 8 = 16	8 + 9 = 17
9 + 0 = 9	9 + 1 = 10	9 + 2 = 11	9 + 3 = 12	9 + 4 = 13	9 + 5 = 14	9 + 6 = 15	9 + 7 = 16	9 + 8 = 17	9 + 9 = 18

Instructions for Copying the Addition Equations from Table 1 into Table 2

(1) Choose a square in Table 1 which is shown above, and copy its equation exactly into any empty square you find in Table 2, as given below on your worksheet. Do not change the order of the numbers in the equation, nor change any of their values.

(2) After your chosen square from Table 1 has been copied into Table 2, then cross out the square in Table 1. Do not also cross out the square in Table 2. When all the squares in Table 1 have been transferred to Table 2, draw a large 'X' entirely across Table 1.

(3) Then color the picture on the back of this page. Mathematical reasoning has two components: numerical and geometrical. The tables are the numerical portion and the coloring of the picture is the geometrical component.

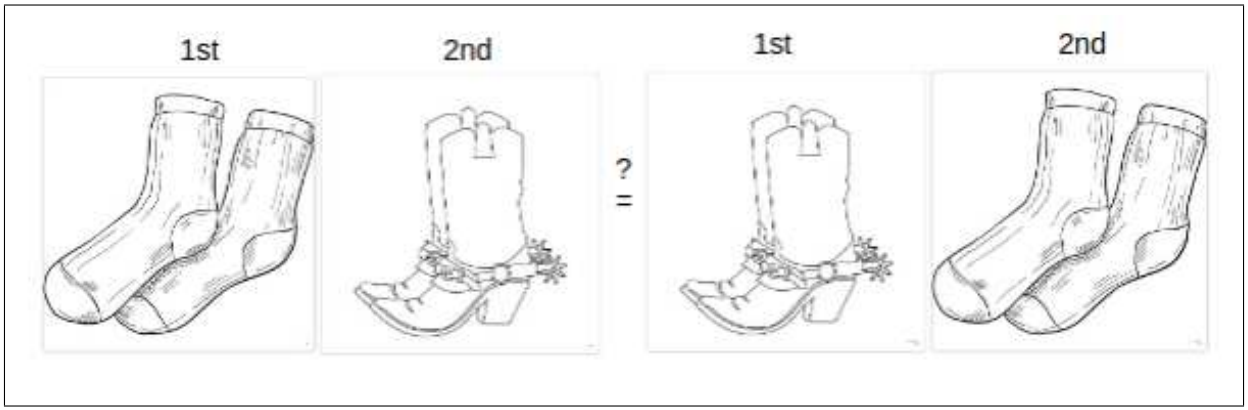

Figure 1: *Is Getting Dressed Commutative?* Pair of Socks and Cowboy Boots, by SuperColoring.com

Student's Self–Teaching Addition of Digits Worksheet

Date:
Student's Name:
(Optional) Start Time: Finish Time:

Table 2: Student's Copy of Addition Table for Decimal Digits, Zero to Nine Inclusive

Student's Reward for Copying the Addition Digits Table

Figure 2: *Circus Elephant Playing with a Ball*, by Painter

Self-Teaching Digit Multiplication Table

If you copy all the equations from Table 3 into Table 4, once in the morning and once again in the evening for a whole month, by the end of the month, you will have easily memorized each of the two–digit products. The most important aspect of this self–teaching lesson is that there are no fears or tears because the result of each product is known. Multiplying numbers more than nine is computed by repeatedly using only these 100 products with the carry rule. This lesson focuses on multiplying decimal digits, i.e., products containing zero through nine. Later in school, your teacher will explain the multiplication carry rule to you.

Multiplication, being commutative, means that the two digits on the left side of each equation in Table 3 can swap places without changing the answer. For example, $5 \times 3 = 15$ is the same as $3 \times 5 = 15$; yet, the order in which the three and the five occur in each product is different. The commutativity of multiplication creates a visible symmetry in Table 3. To perceive this symmetry, notice that all the answers are the same above and below the main diagonal that runs from the upper left to the lower right. The defining feature of multiplication's commutativity is that the product is the same, regardless of the order of its factors. There are simple things we do every day that are not commutative. Is the order of the clothing in which you dress yourself essential? Have you ever left the house with your socks worn over the outside of your shoes?

Table 3: Multiplication Table for Decimal Digits, Zero to Nine Inclusive

$0 \times 0 = 0$	$0 \times 1 = 0$	$0 \times 2 = 0$	$0 \times 3 = 0$	$0 \times 4 = 0$	$0 \times 5 = 0$	$0 \times 6 = 0$	$0 \times 7 = 0$	$0 \times 8 = 0$	$0 \times 9 = 0$
$1 \times 0 = 0$	$1 \times 1 = 1$	$1 \times 2 = 2$	$1 \times 3 = 3$	$1 \times 4 = 4$	$1 \times 5 = 5$	$1 \times 6 = 6$	$1 \times 7 = 7$	$1 \times 8 = 8$	$1 \times 9 = 9$
$2 \times 0 = 0$	$2 \times 1 = 2$	$2 \times 2 = 4$	$2 \times 3 = 6$	$2 \times 4 = 8$	$2 \times 5 = 10$	$2 \times 6 = 12$	$2 \times 7 = 14$	$2 \times 8 = 16$	$2 \times 9 = 18$
$3 \times 0 = 0$	$3 \times 1 = 3$	$3 \times 2 = 6$	$3 \times 3 = 9$	$3 \times 4 = 12$	$3 \times 5 = 15$	$3 \times 6 = 18$	$3 \times 7 = 21$	$3 \times 8 = 24$	$3 \times 9 = 27$
$4 \times 0 = 0$	$4 \times 1 = 4$	$4 \times 2 = 8$	$4 \times 3 = 12$	$4 \times 4 = 16$	$4 \times 5 = 20$	$4 \times 6 = 24$	$4 \times 7 = 28$	$4 \times 8 = 32$	$4 \times 9 = 36$
$5 \times 0 = 0$	$5 \times 1 = 5$	$5 \times 2 = 10$	$5 \times 3 = 15$	$5 \times 4 = 20$	$5 \times 5 = 25$	$5 \times 6 = 30$	$5 \times 7 = 35$	$5 \times 8 = 40$	$5 \times 9 = 45$
$6 \times 0 = 0$	$6 \times 1 = 6$	$6 \times 2 = 12$	$6 \times 3 = 18$	$6 \times 4 = 24$	$6 \times 5 = 30$	$6 \times 6 = 36$	$6 \times 7 = 42$	$6 \times 8 = 48$	$6 \times 9 = 54$
$7 \times 0 = 0$	$7 \times 1 = 7$	$7 \times 2 = 14$	$7 \times 3 = 21$	$7 \times 4 = 28$	$7 \times 5 = 35$	$7 \times 6 = 42$	$7 \times 7 = 49$	$7 \times 8 = 56$	$7 \times 9 = 63$
$8 \times 0 = 0$	$8 \times 1 = 8$	$8 \times 2 = 16$	$8 \times 3 = 24$	$8 \times 4 = 32$	$8 \times 5 = 40$	$8 \times 6 = 48$	$8 \times 7 = 56$	$8 \times 8 = 64$	$8 \times 9 = 72$
$9 \times 0 = 0$	$9 \times 1 = 9$	$9 \times 2 = 18$	$9 \times 3 = 27$	$9 \times 4 = 36$	$9 \times 5 = 45$	$9 \times 6 = 54$	$9 \times 7 = 63$	$9 \times 8 = 72$	$9 \times 9 = 81$

Instructions for Copying the Multiplication Equations from Table 3 into Table 4

(1) Choose a square in Table 3 which is shown above, and copy its equation exactly into any empty square you find in Table 4, as given below on your worksheet. Do not change the order of the numbers in the equation, nor change any of their values.

(2) After your chosen square from Table 3 has been copied into Table 4, then cross out the square in Table 3. Do not also cross out the square in Table 4. When all the squares in Table 3 have been transferred to Table 4, draw a large 'X' entirely across Table 3.

(3) Then color the picture on the back of this page. Mathematical reasoning has two components: numerical and geometrical. The tables are the numerical portion and the coloring of the picture is the geometrical component.

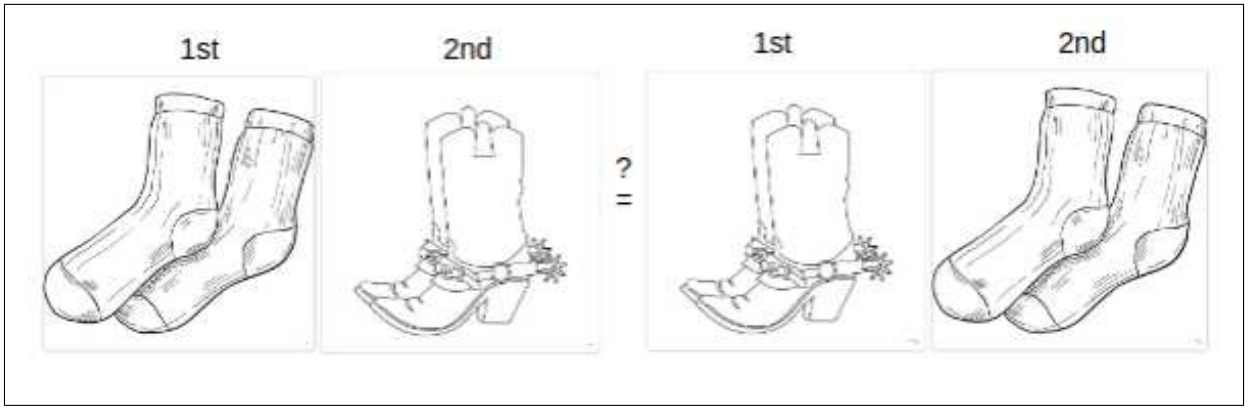

Figure 3: *Is Getting Dressed Commutative?* Pair of Socks and Cowboy Boots, by SuperColoring.com

Student's Self–Teaching Multiplication of Digits Worksheet

Date:
Student's Name:
(Optional) Start Time: Finish Time:

Table 4: Student's Copy of Multiplication Table for Decimal Digits, Zero to Nine Inclusive

Student's Reward for Copying the Multiplication Digits Table

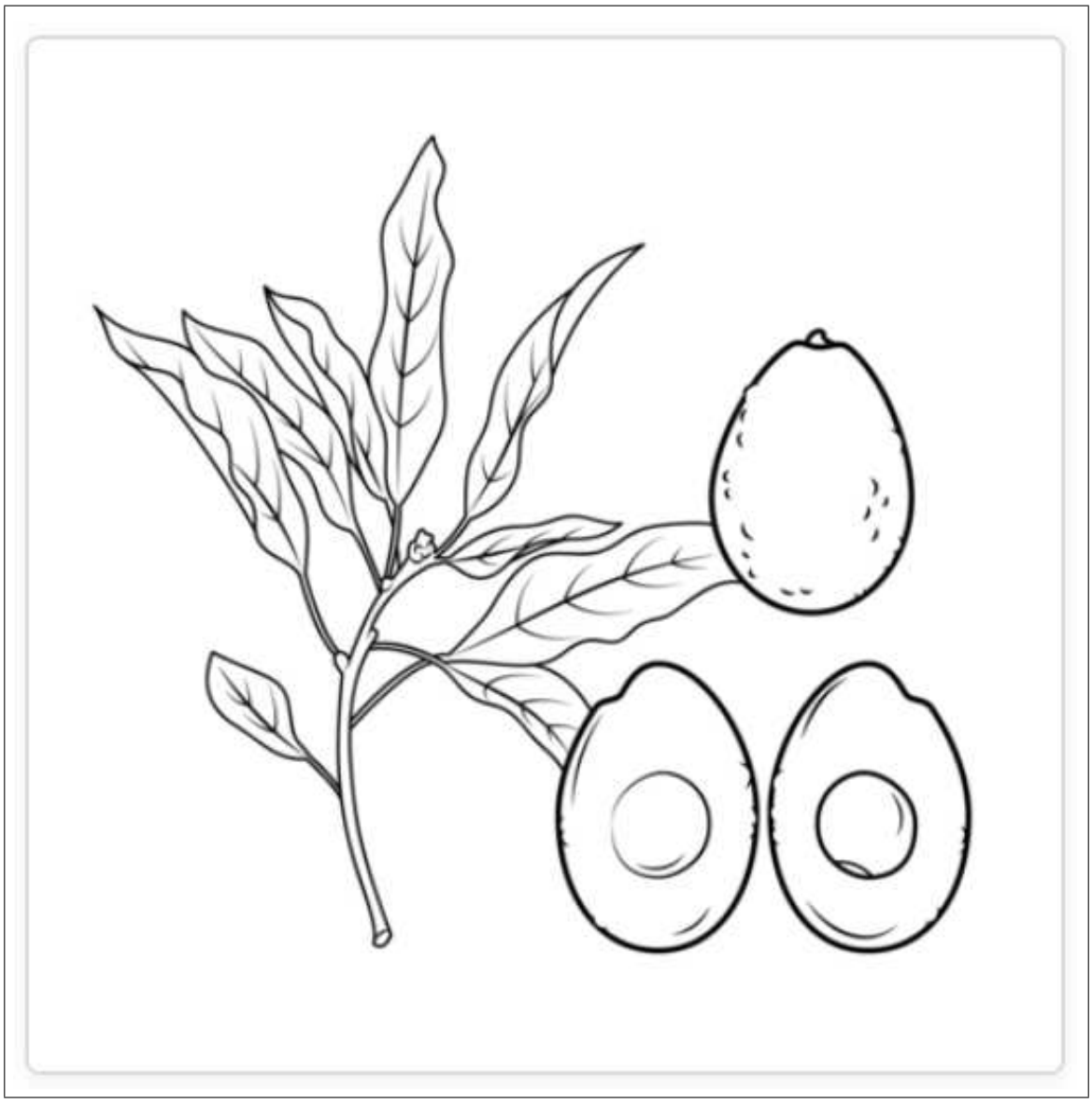

Figure 4: *Avocado*, by Artsashina, original image credit:
Avocado photo by Edrean