Self–Teaching Lesson for Digit Addition

The repetition of copying all of the sums shown below in Table 1, to Table 2, an empty tableaux, results in their easy memorization. For this reason, we provide both an AM and a PM arithmetic lesson. By completing each of our worksheets over 30 consecutive days, once in the morning and once in the evening, you will memorize all of these sums without effort. The most important aspect of this self–teaching method is that there are no fears or tears because the sum of each is readily visible. These 100 sums form all the building blocks needed for adding together any two numbers larger than nine. However, bricks require mortar, and so too do both addends, which are connected together with the carry rule for addition. This lesson focuses on adding the decimal digits, i.e., sums containing zero through nine, inclusive. Later, either your parent or your school teacher will explain the carry rule for addition to you.

Addition is commutative, which means both digits on the left side of each equation in Table 1 can swap places with each other, and that such a position change does not affect its sum. For example, the equation $5 + 3 = 8$ has the same sum as the equation $3 + 5 = 8$; yet, the order of the addends 3 and 5 within the equation is different. Addition's commutativity creates a visible symmetry in Table 1. Notice all the sums are the same for the addition equations that are positioned above and below the main diagonal, which runs from the tableau's upper left to its lower right. There are simple things we do every day that are not commutative. Is the order of the clothing in which you dress yourself essential? Do you ever leave the house with your socks worn over the outside of your shoes?

Table 1: Digit Addition Table

$0 + 0 = 0$	$0 + 1 = 1$	$0 + 2 = 2$	$0 + 3 = 3$	$0 + 4 = 4$	$0 + 5 = 5$	$0 + 6 = 6$	$0 + 7 = 7$	$0 + 8 = 8$	$0 + 9 = 9$
$1 + 0 = 1$	$1 + 1 = 2$	$1 + 2 = 3$	$1 + 3 = 4$	$1 + 4 = 5$	$1 + 5 = 6$	$1 + 6 = 7$	$1 + 7 = 8$	$1 + 8 = 9$	$1 + 9 = 10$
$2 + 0 = 2$	$2 + 1 = 3$	$2 + 2 = 4$	$2 + 3 = 5$	$2 + 4 = 6$	$2 + 5 = 7$	$2 + 6 = 8$	$2 + 7 = 9$	$2 + 8 = 10$	$2 + 9 = 11$
$3 + 0 = 3$	$3 + 1 = 4$	$3 + 2 = 5$	$3 + 3 = 6$	$3 + 4 = 7$	$3 + 5 = 8$	$3 + 6 = 9$	$3 + 7 = 10$	$3 + 8 = 11$	$3 + 9 = 12$
$4 + 0 = 4$	$4 + 1 = 5$	$4 + 2 = 6$	$4 + 3 = 7$	$4 + 4 = 8$	$4 + 5 = 9$	$4 + 6 = 10$	$4 + 7 = 11$	$4 + 8 = 12$	$4 + 9 = 13$
$5 + 0 = 5$	$5 + 1 = 6$	$5 + 2 = 7$	$5 + 3 = 8$	$5 + 4 = 9$	$5 + 5 = 10$	$5 + 6 = 11$	$5 + 7 = 12$	$5 + 8 = 13$	$5 + 9 = 14$
$6 + 0 = 6$	$6 + 1 = 7$	$6 + 2 = 8$	$6 + 3 = 9$	$6 + 4 = 10$	$6 + 5 = 11$	$6 + 6 = 12$	$6 + 7 = 13$	$6 + 8 = 14$	$6 + 9 = 15$
$7 + 0 = 7$	$7 + 1 = 8$	$7 + 2 = 9$	$7 + 3 = 10$	$7 + 4 = 11$	$7 + 5 = 12$	$7 + 6 = 13$	$7 + 7 = 14$	$7 + 8 = 15$	$7 + 9 = 16$
$8 + 0 = 8$	$8 + 1 = 9$	$8 + 2 = 10$	$8 + 3 = 11$	$8 + 4 = 12$	$8 + 5 = 13$	$8 + 6 = 14$	$8 + 7 = 15$	$8 + 8 = 16$	$8 + 9 = 17$
$9 + 0 = 9$	$9 + 1 = 10$	$9 + 2 = 11$	$9 + 3 = 12$	$9 + 4 = 13$	$9 + 5 = 14$	$9 + 6 = 15$	$9 + 7 = 16$	$9 + 8 = 17$	$9 + 9 = 18$

Instructions for Copying the Addition Equations from Table 1 into Table 2

(1) Choose a square in Table 1 which is shown above, and copy its equation exactly into any empty square you find in Table 2, as given below on your worksheet. Do not change the order of the numbers in the equation, nor change any of their values.

(2) After your chosen square from Table 1 has been copied into Table 2, then cross out the square in Table 1. Do not also cross out the square in Table 2. When all the squares in Table 1 have been transferred to Table 2, draw a large 'X' entirely across Table 1.

(3) Then color the picture on the back of this page. Mathematical reasoning has two components: numerical and geometrical. The tables are the numerical portion and the coloring of the picture is the geometrical component.

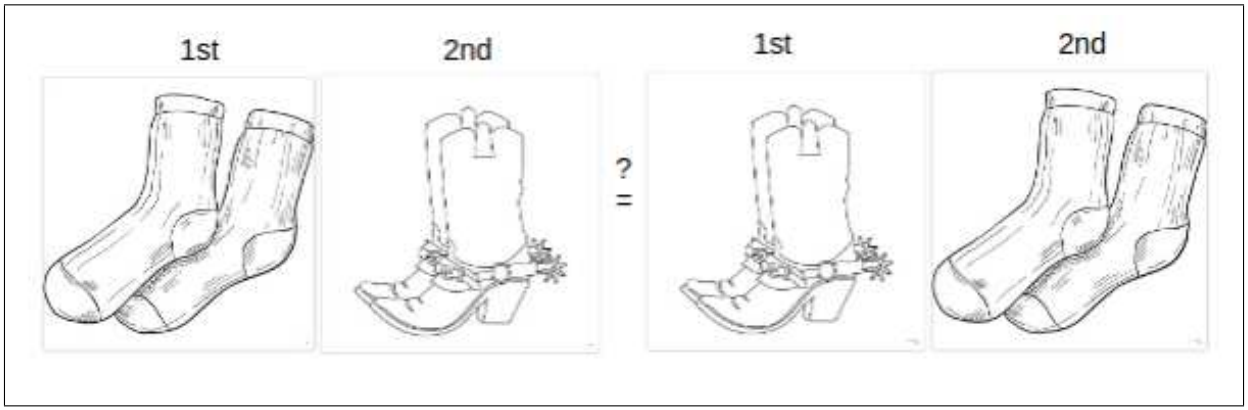

Figure 1: *Is Getting Dressed Commutative?* Pair of Socks and Cowboy Boots, by SuperColoring.com

Student's Self–Teaching Addition of Digits Worksheet

Date:

Student's Name:

(Optional) Start Time: Finish Time:

Table 2: Student's Copy of Digit Addition Table

Student's Reward for Copying the Digit Addition Table

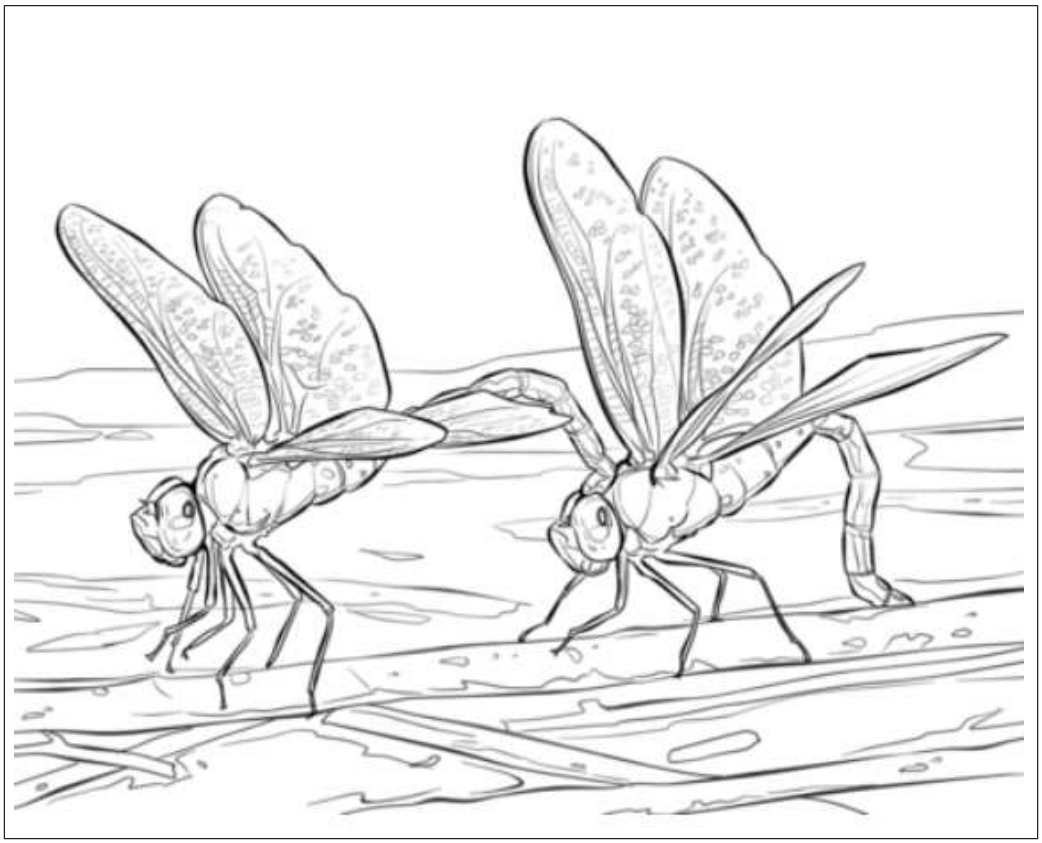

Figure 2: *Two Green Darner Dragonflies*, by Yulia Znayduk

Self–Teaching Lesson for Digit Multiplication

The repetition of copying all of the products shown below in Table 3, to Table 4, an empty tableaux, results in their easy memorization. For this reason, we provide both an AM and a PM arithmetic lesson. By completing each of our worksheets over 30 consecutive days, once in the morning and once in the evening, you will memorize all of these products without effort. The most important aspect of this self–teaching method is that there are no fears or tears because the product of is readily visible. These 100 products form all the building blocks needed for multiplying together any two numbers larger than nine. However, bricks require mortar, and so too do both factors, which are connected together with a product carry rule and the alignment of partial products necessary for long multiplication. This lesson focuses on multiplying the decimal digits, i.e., sums containing zero through nine, inclusive. Later, either your parent or your school teacher will explain the details of long multiplication to you.

Multiplication is commutative, which means both digits on the left side of each equation in Table 3 can swap places with each other, and that such a position change does not affect its sum. For example, the equation $5 \times 3 = 15$ has the same product as the equation $3 \times 5 = 15$; yet, the order of the factors 3 and 5 within the equation is different. Multiplication's commutativity creates a visible symmetry in Table 3. Notice all the products are the same for the multiplication equations that are positioned above and below the main diagonal, which runs from the tableau's upper left to its lower right. There are simple things we do every day that are not commutative. Is the order of the clothing in which you dress yourself essential? Do you ever leave the house with your socks worn over the outside of your shoes?

Table 3: Digit Multiplication Table

$0 \times 0 = 0$	$0 \times 1 = 0$	$0 \times 2 = 0$	$0 \times 3 = 0$	$0 \times 4 = 0$	$0 \times 5 = 0$	$0 \times 6 = 0$	$0 \times 7 = 0$	$0 \times 8 = 0$	$0 \times 9 = 0$
$1 \times 0 = 0$	$1 \times 1 = 1$	$1 \times 2 = 2$	$1 \times 3 = 3$	$1 \times 4 = 4$	$1 \times 5 = 5$	$1 \times 6 = 6$	$1 \times 7 = 7$	$1 \times 8 = 8$	$1 \times 9 = 9$
$2 \times 0 = 0$	$2 \times 1 = 2$	$2 \times 2 = 4$	$2 \times 3 = 6$	$2 \times 4 = 8$	$2 \times 5 = 10$	$2 \times 6 = 12$	$2 \times 7 = 14$	$2 \times 8 = 16$	$2 \times 9 = 18$
$3 \times 0 = 0$	$3 \times 1 = 3$	$3 \times 2 = 6$	$3 \times 3 = 9$	$3 \times 4 = 12$	$3 \times 5 = 15$	$3 \times 6 = 18$	$3 \times 7 = 21$	$3 \times 8 = 24$	$3 \times 9 = 27$
$4 \times 0 = 0$	$4 \times 1 = 4$	$4 \times 2 = 8$	$4 \times 3 = 12$	$4 \times 4 = 16$	$4 \times 5 = 20$	$4 \times 6 = 24$	$4 \times 7 = 28$	$4 \times 8 = 32$	$4 \times 9 = 36$
$5 \times 0 = 0$	$5 \times 1 = 5$	$5 \times 2 = 10$	$5 \times 3 = 15$	$5 \times 4 = 20$	$5 \times 5 = 25$	$5 \times 6 = 30$	$5 \times 7 = 35$	$5 \times 8 = 40$	$5 \times 9 = 45$
$6 \times 0 = 0$	$6 \times 1 = 6$	$6 \times 2 = 12$	$6 \times 3 = 18$	$6 \times 4 = 24$	$6 \times 5 = 30$	$6 \times 6 = 36$	$6 \times 7 = 42$	$6 \times 8 = 48$	$6 \times 9 = 54$
$7 \times 0 = 0$	$7 \times 1 = 7$	$7 \times 2 = 14$	$7 \times 3 = 21$	$7 \times 4 = 28$	$7 \times 5 = 35$	$7 \times 6 = 42$	$7 \times 7 = 49$	$7 \times 8 = 56$	$7 \times 9 = 63$
$8 \times 0 = 0$	$8 \times 1 = 8$	$8 \times 2 = 16$	$8 \times 3 = 24$	$8 \times 4 = 32$	$8 \times 5 = 40$	$8 \times 6 = 48$	$8 \times 7 = 56$	$8 \times 8 = 64$	$8 \times 9 = 72$
$9 \times 0 = 0$	$9 \times 1 = 9$	$9 \times 2 = 18$	$9 \times 3 = 27$	$9 \times 4 = 36$	$9 \times 5 = 45$	$9 \times 6 = 54$	$9 \times 7 = 63$	$9 \times 8 = 72$	$9 \times 9 = 81$

Instructions for Copying the Multiplication Equations from Table 3 into Table 4

(1) Choose a square in Table 3 which is shown above, and copy its equation exactly into any empty square you find in Table 4, as given below on your worksheet. Do not change the order of the numbers in the equation, nor change any of their values.

(2) After your chosen square from Table 3 has been copied into Table 4, then cross out the square in Table 3. Do not also cross out the square in Table 4. When all the squares in Table 3 have been transferred to Table 4, draw a large 'X' entirely across Table 3.

(3) Then color the picture on the back of this page. Mathematical reasoning has two components: numerical and geometrical. The tables are the numerical portion and the coloring of the picture is the geometrical component.

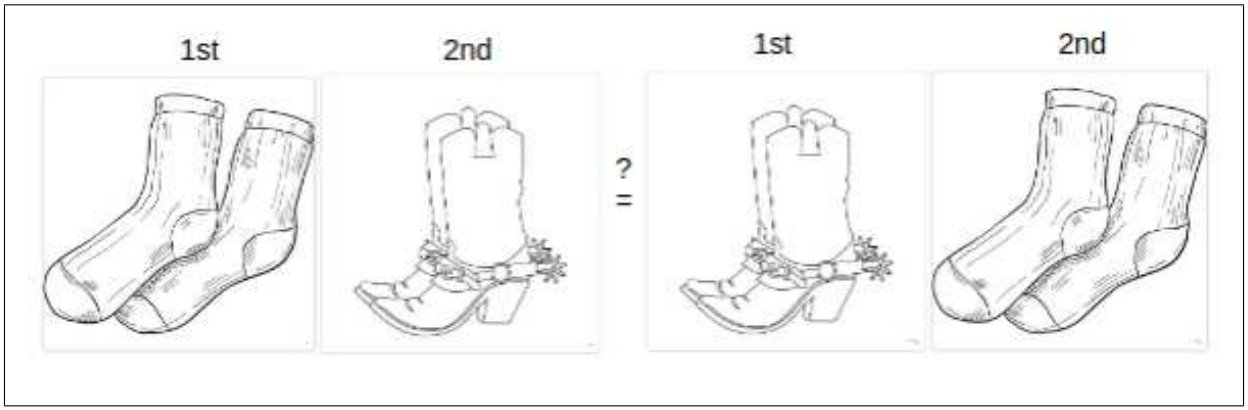

Figure 3: *Is Getting Dressed Commutative?* Pair of Socks and Cowboy Boots, by SuperColoring.com

Student's Self–Teaching Multiplication of Digits Worksheet

Date:

Student's Name:

(Optional) Start Time: Finish Time:

Table 4: Student's Copy of Digit Multiplication Table

Student's Reward for Copying the Digit Multiplication Table

Figure 4: California Poppy, *Celebrating Wildflowers Coloring Book*, by Karl Urban