

Self-Teaching Lesson for Digit Addition

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If you copy all the equations from Table 1 into Table 2, once in the morning and once again in the evening for a whole month, by the end of the month, you will have easily memorised each of the two-digit sums. The most important aspect of this self-teaching lesson is that there are no fears or tears because the result of each sum is known. Adding numbers more than nine is computed by repeatedly using only these 100 sums with the carry rule. This lesson focuses on adding decimal digits, i.e., sums containing zero through nine. Later in school, your teacher will explain the addition carry rule to you.

Addition being *commutative* means: the two digits on the left side of each equation in Table 1 can swap places without changing the result of its sum. For example, $5 + 3 = 8$ is the same as $3 + 5 = 8$; yet, the order in which the 3 and the 5 occur within the sum is different. Addition being commutative creates a visible symmetry in Table 1. To perceive this symmetry notice that the all sums are the same for the equations both above and below the diagonal that runs from the upper left to its lower right. The defining feature of addition's commutativity is that the sum's value is the same, regardless of the order of its addends. There are simple things we do everyday that are not commutative. Is the order of the clothing in which you dress yourself important? Have you ever left the house with your socks worn over the outside of your shoes?

Table 1: Digit Addition Table

$0 + 0 = 0$	$0 + 1 = 1$	$0 + 2 = 2$	$0 + 3 = 3$	$0 + 4 = 4$	$0 + 5 = 5$	$0 + 6 = 6$	$0 + 7 = 7$	$0 + 8 = 8$	$0 + 9 = 9$
$1 + 0 = 1$	$1 + 1 = 2$	$1 + 2 = 3$	$1 + 3 = 4$	$1 + 4 = 5$	$1 + 5 = 6$	$1 + 6 = 7$	$1 + 7 = 8$	$1 + 8 = 9$	$1 + 9 = 10$
$2 + 0 = 2$	$2 + 1 = 3$	$2 + 2 = 4$	$2 + 3 = 5$	$2 + 4 = 6$	$2 + 5 = 7$	$2 + 6 = 8$	$2 + 7 = 9$	$2 + 8 = 10$	$2 + 9 = 11$
$3 + 0 = 3$	$3 + 1 = 4$	$3 + 2 = 5$	$3 + 3 = 6$	$3 + 4 = 7$	$3 + 5 = 8$	$3 + 6 = 9$	$3 + 7 = 10$	$3 + 8 = 11$	$3 + 9 = 12$
$4 + 0 = 4$	$4 + 1 = 5$	$4 + 2 = 6$	$4 + 3 = 7$	$4 + 4 = 8$	$4 + 5 = 9$	$4 + 6 = 10$	$4 + 7 = 11$	$4 + 8 = 12$	$4 + 9 = 13$
$5 + 0 = 5$	$5 + 1 = 6$	$5 + 2 = 7$	$5 + 3 = 8$	$5 + 4 = 9$	$5 + 5 = 10$	$5 + 6 = 11$	$5 + 7 = 12$	$5 + 8 = 13$	$5 + 9 = 14$
$6 + 0 = 6$	$6 + 1 = 7$	$6 + 2 = 8$	$6 + 3 = 9$	$6 + 4 = 10$	$6 + 5 = 11$	$6 + 6 = 12$	$6 + 7 = 13$	$6 + 8 = 14$	$6 + 9 = 15$
$7 + 0 = 7$	$7 + 1 = 8$	$7 + 2 = 9$	$7 + 3 = 10$	$7 + 4 = 11$	$7 + 5 = 12$	$7 + 6 = 13$	$7 + 7 = 14$	$7 + 8 = 15$	$7 + 9 = 16$
$8 + 0 = 8$	$8 + 1 = 9$	$8 + 2 = 10$	$8 + 3 = 11$	$8 + 4 = 12$	$8 + 5 = 13$	$8 + 6 = 14$	$8 + 7 = 15$	$8 + 8 = 16$	$8 + 9 = 17$
$9 + 0 = 9$	$9 + 1 = 10$	$9 + 2 = 11$	$9 + 3 = 12$	$9 + 4 = 13$	$9 + 5 = 14$	$9 + 6 = 15$	$9 + 7 = 16$	$9 + 8 = 17$	$9 + 9 = 18$

Instructions for Copying the Addition Equations from Table 1 into Table 2

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- (1) Choose a square in Table 1 which is shown above, and copy its equation exactly into any empty square you find in Table 2, as given below on your worksheet. Do not change the order of the numbers in the equation, nor change any of their values.
- (2) After your chosen square from Table 1 has been copied into Table 2, then cross out the square in Table 1. Do not also cross out the square in Table 2. When all the squares in Table 1 have been transferred to Table 2, draw a large 'X' entirely across Table 1.
- (3) Then colour the picture on the back of this page. Mathematical reasoning has two components: numerical and geometrical. The tables are the numerical portion and the colouring of the picture is the geometrical component.

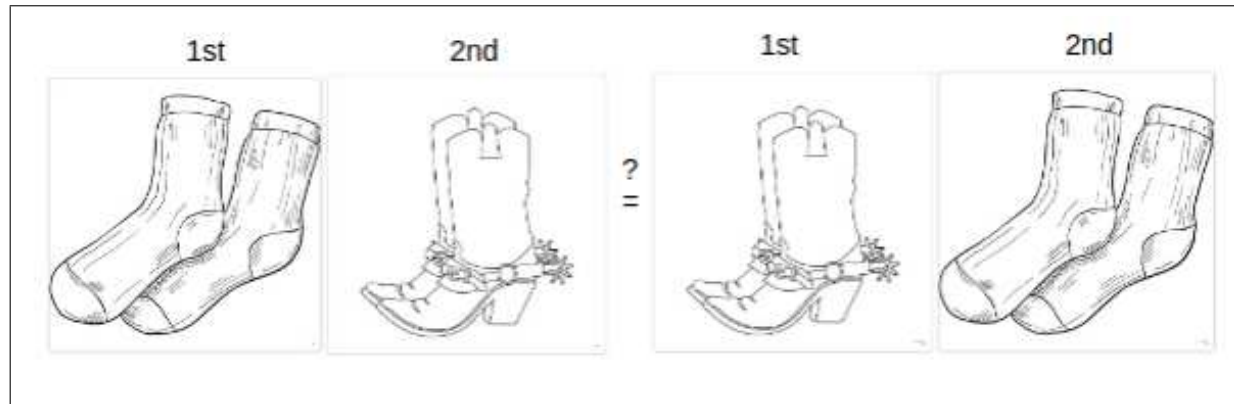


Figure 1: *Is Getting Dressed Commutative?* Pair of Socks and Cowboy Boots, by SuperColoring.com

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Student's Self-Teaching Addition of Digits Worksheet

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Date:

Student's Name:

(Optional) Start Time:

Finish Time:

Table 2: Student's Copy of Digit Addition Table

[illegible]

Student's Reward for Copying the Digit Addition Table

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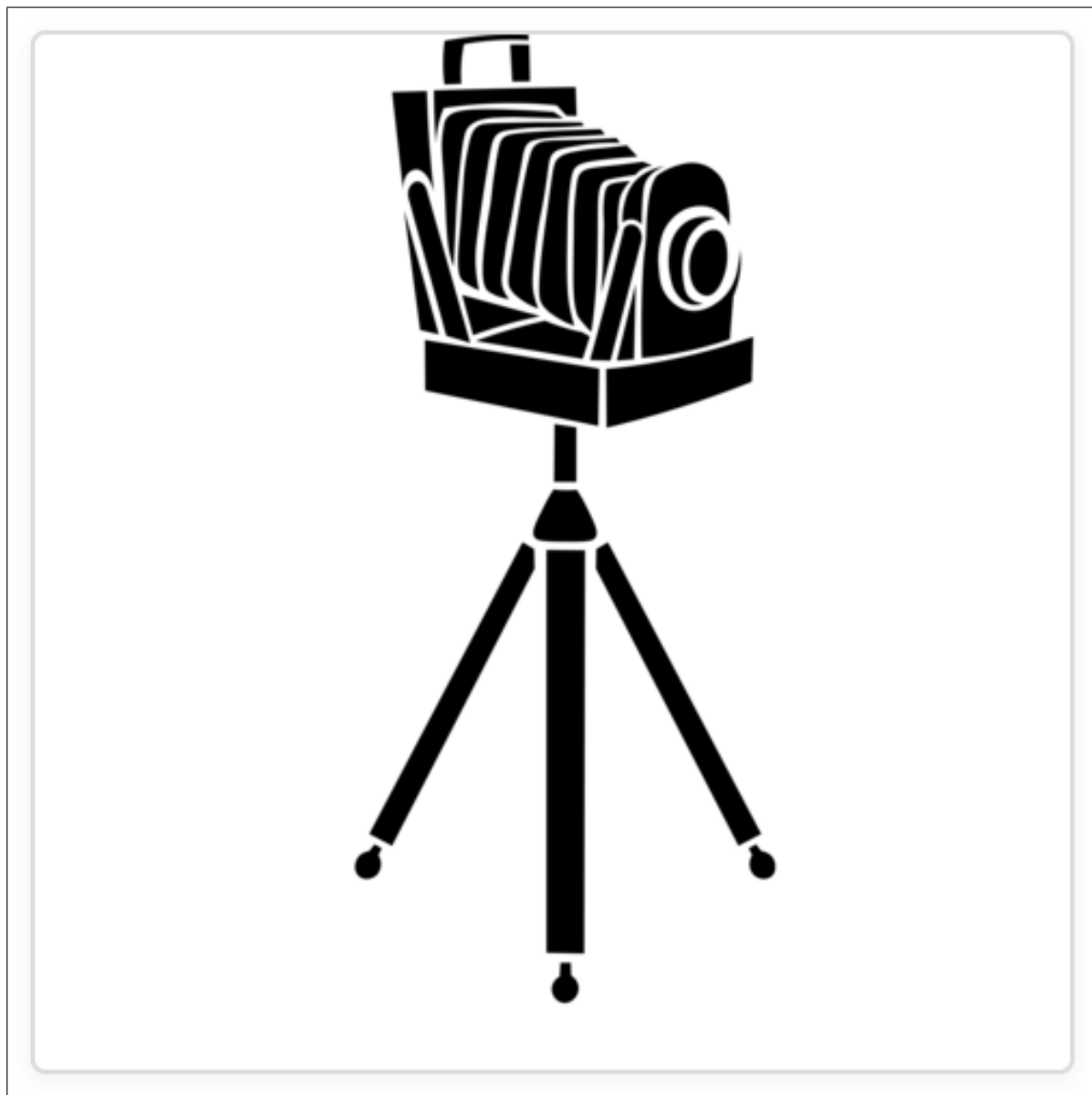


Figure 2: *Stencil of Vintage Camera*, by SuperColoring,
celebrating National Camera Day, June 25, 2025

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Self-Teaching Lesson for Digit Multiplication

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If you copy all the equations from Table 3 into Table 4, once in the morning and once again in the evening for a whole month, by the end of the month, you will have easily memorised each of the two-digit products. The most important aspect of this self-teaching lesson is that there are no fears or tears because the result of each product is known. Multiplying numbers more than nine is computed by repeatedly using only these 100 products with the carry rule. This lesson focuses on multiplying decimal digits, i.e., products containing zero through nine. Later in school, your teacher will explain the multiplication carry rule to you.

Multiplication being *commutative* means: the two digits on the left side of each equations in Table 3 can swap places without changing the result of its sum. For example, $5 \times 3 = 15$ is the same as $3 \times 5 = 15$; yet, the order in which the 3 and the 5 occur in the product is different. Multiplication being commutative creates a symmetry within Table multiplication. To perceive this symmetry notice that all the products are the same for the equations both above and below the diagonal that runs from the upper left to its lower right. The defining feature of multiplication's commutativity is that the product's value is the same, regardless of the order of its factors. There are simple things we do everyday that are not commutative. Is the order of the clothing in which you dress yourself important? Have you ever left the house with your socks worn over the outside of your shoes?

Table 3: Digit Multiplication Table

$0 \times 0 = 0$	$0 \times 1 = 0$	$0 \times 2 = 0$	$0 \times 3 = 0$	$0 \times 4 = 0$	$0 \times 5 = 0$	$0 \times 6 = 0$	$0 \times 7 = 0$	$0 \times 8 = 0$	$0 \times 9 = 0$
$1 \times 0 = 0$	$1 \times 1 = 1$	$1 \times 2 = 2$	$1 \times 3 = 3$	$1 \times 4 = 4$	$1 \times 5 = 5$	$1 \times 6 = 6$	$1 \times 7 = 7$	$1 \times 8 = 8$	$1 \times 9 = 9$
$2 \times 0 = 0$	$2 \times 1 = 2$	$2 \times 2 = 4$	$2 \times 3 = 6$	$2 \times 4 = 8$	$2 \times 5 = 10$	$2 \times 6 = 12$	$2 \times 7 = 14$	$2 \times 8 = 16$	$2 \times 9 = 18$
$3 \times 0 = 0$	$3 \times 1 = 3$	$3 \times 2 = 6$	$3 \times 3 = 9$	$3 \times 4 = 12$	$3 \times 5 = 15$	$3 \times 6 = 18$	$3 \times 7 = 21$	$3 \times 8 = 24$	$3 \times 9 = 27$
$4 \times 0 = 0$	$4 \times 1 = 4$	$4 \times 2 = 8$	$4 \times 3 = 12$	$4 \times 4 = 16$	$4 \times 5 = 20$	$4 \times 6 = 24$	$4 \times 7 = 28$	$4 \times 8 = 32$	$4 \times 9 = 36$
$5 \times 0 = 0$	$5 \times 1 = 5$	$5 \times 2 = 10$	$5 \times 3 = 15$	$5 \times 4 = 20$	$5 \times 5 = 25$	$5 \times 6 = 30$	$5 \times 7 = 35$	$5 \times 8 = 40$	$5 \times 9 = 45$
$6 \times 0 = 0$	$6 \times 1 = 6$	$6 \times 2 = 12$	$6 \times 3 = 18$	$6 \times 4 = 24$	$6 \times 5 = 30$	$6 \times 6 = 36$	$6 \times 7 = 42$	$6 \times 8 = 48$	$6 \times 9 = 54$
$7 \times 0 = 0$	$7 \times 1 = 7$	$7 \times 2 = 14$	$7 \times 3 = 21$	$7 \times 4 = 28$	$7 \times 5 = 35$	$7 \times 6 = 42$	$7 \times 7 = 49$	$7 \times 8 = 56$	$7 \times 9 = 63$
$8 \times 0 = 0$	$8 \times 1 = 8$	$8 \times 2 = 16$	$8 \times 3 = 24$	$8 \times 4 = 32$	$8 \times 5 = 40$	$8 \times 6 = 48$	$8 \times 7 = 56$	$8 \times 8 = 64$	$8 \times 9 = 72$
$9 \times 0 = 0$	$9 \times 1 = 9$	$9 \times 2 = 18$	$9 \times 3 = 27$	$9 \times 4 = 36$	$9 \times 5 = 45$	$9 \times 6 = 54$	$9 \times 7 = 63$	$9 \times 8 = 72$	$9 \times 9 = 81$

Instructions for Copying the Multiplication Equations from Table 3 into Table 4

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- (3) Then colour the picture on the back of this page. Mathematical reasoning has two components: numerical and geometrical. The tables are the numerical portion and the colouring of the picture is the geometrical component.

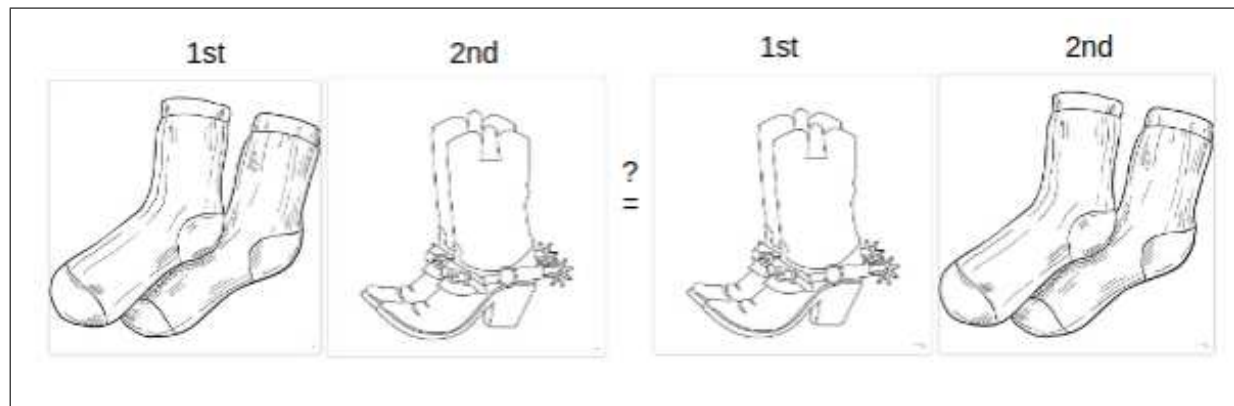


Figure 3: *Is Getting Dressed Commutative?* Pair of Socks and Cowboy Boots, by SuperColoring.com

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Student's Self-Teaching Multiplication of Digits Worksheet

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Date:

Student's Name:

(Optional) Start Time:

Finish Time:

Table 4: Student's Copy of Digit Multiplication Table

[illegible]

Student's Reward for Copying the Digit Multiplication Table

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Figure 4: *Master Baker Pup in Action*, by Vikidigitartkids

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