

## Self-Teaching Digit Addition Table

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If you can copy Table 1 onto Table 2, once in the morning and once again in the evening for a month, you will have easily memorized all the two digit sums within the month. The most important attribute of this self-teaching procedure is a learning process that creates no fears or tears because the answers are not known. The addition of larger numbers is always computed by repeatedly using only these 100 sums. Therefore, when these simple 100 digit sums are memorized then the addition of any pair of larger numbers becomes a very easily implemented technique using the carry rule.

Lesson's Objective: Addition of Digits and an Introduction to the Addition's Commutative Property

What does addition being commutative mean? Notice that in Table 1, given any two digits, their sum is always the same regardless on which side of the plus sign the number appears. For example,  $2 + 3 = 5$  is the same as  $3 + 2 = 5$ , where you may also have noticed the symmetry of the answers above and below the main diagonal in Table 1. By way of a contrast, highways in general, do not commute. Is this because legally driving down one side of a two lane road in the USA will not always get you to the same place as legally driving down the other side of the same road? No. While it is true that humans can commute on highways; no highway beneath our vehicles is doing any commuting, because highways do not travel by changing their position.

Table 1: Addition Table for Digits Zero to Nine, Inclusive

|             |              |              |              |              |              |              |              |              |              |
|-------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|
| $0 + 0 = 0$ | $0 + 1 = 1$  | $0 + 2 = 2$  | $0 + 3 = 3$  | $0 + 4 = 4$  | $0 + 5 = 5$  | $0 + 6 = 6$  | $0 + 7 = 7$  | $0 + 8 = 8$  | $0 + 9 = 9$  |
| $1 + 0 = 1$ | $1 + 1 = 2$  | $1 + 2 = 3$  | $1 + 3 = 4$  | $1 + 4 = 5$  | $1 + 5 = 6$  | $1 + 6 = 7$  | $1 + 7 = 8$  | $1 + 8 = 9$  | $1 + 9 = 10$ |
| $2 + 0 = 2$ | $2 + 1 = 3$  | $2 + 2 = 4$  | $2 + 3 = 5$  | $2 + 4 = 6$  | $2 + 5 = 7$  | $2 + 6 = 8$  | $2 + 7 = 9$  | $2 + 8 = 10$ | $2 + 9 = 11$ |
| $3 + 0 = 3$ | $3 + 1 = 4$  | $3 + 2 = 5$  | $3 + 3 = 6$  | $3 + 4 = 7$  | $3 + 5 = 8$  | $3 + 6 = 9$  | $3 + 7 = 10$ | $3 + 8 = 11$ | $3 + 9 = 12$ |
| $4 + 0 = 4$ | $4 + 1 = 5$  | $4 + 2 = 6$  | $4 + 3 = 7$  | $4 + 4 = 8$  | $4 + 5 = 9$  | $4 + 6 = 10$ | $4 + 7 = 11$ | $4 + 8 = 12$ | $4 + 9 = 13$ |
| $5 + 0 = 5$ | $5 + 1 = 6$  | $5 + 2 = 7$  | $5 + 3 = 8$  | $5 + 4 = 9$  | $5 + 5 = 10$ | $5 + 6 = 11$ | $5 + 7 = 12$ | $5 + 8 = 13$ | $5 + 9 = 14$ |
| $6 + 0 = 6$ | $6 + 1 = 7$  | $6 + 2 = 8$  | $6 + 3 = 9$  | $6 + 4 = 10$ | $6 + 5 = 11$ | $6 + 6 = 12$ | $6 + 7 = 13$ | $6 + 8 = 14$ | $6 + 9 = 15$ |
| $7 + 0 = 7$ | $7 + 1 = 8$  | $7 + 2 = 9$  | $7 + 3 = 10$ | $7 + 4 = 11$ | $7 + 5 = 12$ | $7 + 6 = 13$ | $7 + 7 = 14$ | $7 + 8 = 15$ | $7 + 9 = 16$ |
| $8 + 0 = 8$ | $8 + 1 = 9$  | $8 + 2 = 10$ | $8 + 3 = 11$ | $8 + 4 = 12$ | $8 + 5 = 13$ | $8 + 6 = 14$ | $8 + 7 = 15$ | $8 + 8 = 16$ | $8 + 9 = 17$ |
| $9 + 0 = 9$ | $9 + 1 = 10$ | $9 + 2 = 11$ | $9 + 3 = 12$ | $9 + 4 = 13$ | $9 + 5 = 14$ | $9 + 6 = 15$ | $9 + 7 = 16$ | $9 + 8 = 17$ | $9 + 9 = 18$ |

### Instructions for Copying the Addition Equations from Table 1 into Table 2

- (1) Choose a square in Table 1, shown above, and exactly copy its equation into any empty square in Table 2. Do not change the order of the numbers in the equation, nor change any of their values.
- (2) After your chosen square from Table 1 has been copied into Table 2, then cross out the square in Table 1. Do not also cross out the square in Table 2. When all the squares in Table 1 have been transferred to Table 2 then draw a large 'X' entirely across Table 1.
- (3) Then color the picture on the back of this page. Reasoning in mathematics has two components: numerical and geometrical. The tables are the numerical portion and the coloring of the picture is the geometrical component.



## Student's Reward for Copying the Addition Digits Table



Figure 1: *Magnolia x Soulangeana*, by Nata Silina

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## Self-Teaching Digit Multiplication Table

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If you can copy Table 3 onto Table 4, once in the morning and once again in the evening for a month, you will have easily memorized all the two digit products within the month. The most important attribute of this self-teaching procedure, is a learning process that creates no fears or tears because the answers are not known. The multiplication of larger numbers is always computed by repeatedly using only these 100 sums. Therefore, when these simple 100 digit products are memorized then the multiplication of any pair of larger numbers becomes a very easily implemented technique using the carry rule.

Lesson's Objective: Multiplication of Digits and an Introduction to the Multiplication's Commutative Property

What does multiplication being commutative mean? Notice that in Table 3, given any two digits, their product is always the same regardless on which side of the multiplication sign the number appears. For example,  $5 \times 3 = 15$  is the same as  $3 \times 5 = 15$ , where you may also have noticed the symmetry of the answers above and below the main diagonal in Table 3. By way of a contrast, highways in general, do not commute. Is this because legally driving down one side of a two lane road in the USA will not always get you to the same place as legally driving down the other side of the same road? No. While it is true that humans can commute on highways; no highway beneath our vehicles is doing any commuting, because highways do not travel by changing their position.

Table 3: Multiplication Table for Digits Zero to Nine, Inclusive

|                  |                  |                   |                   |                   |                   |                   |                   |                   |                   |
|------------------|------------------|-------------------|-------------------|-------------------|-------------------|-------------------|-------------------|-------------------|-------------------|
| $0 \times 0 = 0$ | $0 \times 1 = 0$ | $0 \times 2 = 0$  | $0 \times 3 = 0$  | $0 \times 4 = 0$  | $0 \times 5 = 0$  | $0 \times 6 = 0$  | $0 \times 7 = 0$  | $0 \times 8 = 0$  | $0 \times 9 = 0$  |
| $1 \times 0 = 0$ | $1 \times 1 = 1$ | $1 \times 2 = 2$  | $1 \times 3 = 3$  | $1 \times 4 = 4$  | $1 \times 5 = 5$  | $1 \times 6 = 6$  | $1 \times 7 = 7$  | $1 \times 8 = 8$  | $1 \times 9 = 9$  |
| $2 \times 0 = 0$ | $2 \times 1 = 2$ | $2 \times 2 = 4$  | $2 \times 3 = 6$  | $2 \times 4 = 8$  | $2 \times 5 = 10$ | $2 \times 6 = 12$ | $2 \times 7 = 14$ | $2 \times 8 = 16$ | $2 \times 9 = 18$ |
| $3 \times 0 = 0$ | $3 \times 1 = 3$ | $3 \times 2 = 6$  | $3 \times 3 = 9$  | $3 \times 4 = 12$ | $3 \times 5 = 15$ | $3 \times 6 = 18$ | $3 \times 7 = 21$ | $3 \times 8 = 24$ | $3 \times 9 = 27$ |
| $4 \times 0 = 0$ | $4 \times 1 = 4$ | $4 \times 2 = 8$  | $4 \times 3 = 12$ | $4 \times 4 = 16$ | $4 \times 5 = 20$ | $4 \times 6 = 24$ | $4 \times 7 = 28$ | $4 \times 8 = 32$ | $4 \times 9 = 36$ |
| $5 \times 0 = 0$ | $5 \times 1 = 5$ | $5 \times 2 = 10$ | $5 \times 3 = 15$ | $5 \times 4 = 20$ | $5 \times 5 = 25$ | $5 \times 6 = 30$ | $5 \times 7 = 35$ | $5 \times 8 = 40$ | $5 \times 9 = 45$ |
| $6 \times 0 = 0$ | $6 \times 1 = 6$ | $6 \times 2 = 12$ | $6 \times 3 = 18$ | $6 \times 4 = 24$ | $6 \times 5 = 30$ | $6 \times 6 = 36$ | $6 \times 7 = 42$ | $6 \times 8 = 48$ | $6 \times 9 = 54$ |
| $7 \times 0 = 0$ | $7 \times 1 = 7$ | $7 \times 2 = 14$ | $7 \times 3 = 21$ | $7 \times 4 = 28$ | $7 \times 5 = 35$ | $7 \times 6 = 42$ | $7 \times 7 = 49$ | $7 \times 8 = 56$ | $7 \times 9 = 63$ |
| $8 \times 0 = 0$ | $8 \times 1 = 8$ | $8 \times 2 = 16$ | $8 \times 3 = 24$ | $8 \times 4 = 32$ | $8 \times 5 = 40$ | $8 \times 6 = 48$ | $8 \times 7 = 56$ | $8 \times 8 = 64$ | $8 \times 9 = 72$ |
| $9 \times 0 = 0$ | $9 \times 1 = 9$ | $9 \times 2 = 18$ | $9 \times 3 = 27$ | $9 \times 4 = 36$ | $9 \times 5 = 45$ | $9 \times 6 = 54$ | $9 \times 7 = 63$ | $9 \times 8 = 72$ | $9 \times 9 = 81$ |

Instructions for Copying the Multiplication Equations from Table 3 into Table 4

- (1) Choose a square in Table 3, shown above, and exactly copy its equation into any empty square in Table 4. Do not change the order of the numbers in the equation, nor change any of their values.
- (2) After your chosen square from Table 3 has been copied into Table 4, then cross out the square in Table 3. Do not also cross out the square in Table 4. When all the squares in Table 3 have been transferred to Table 4 then draw a large 'X' entirely across Table 3.
- (3) Then color the picture on the back of this page. Reasoning in mathematics has two components: numerical and geometrical. The tables are the numerical portion and the coloring of the picture is the geometrical component.



## Student's Reward for Copying the Multiplication Digits Table



Figure 2: *Joey Votto*, by Nata Silina

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