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Individual-Tree Diameter Growth Model for the Northeastern United States

Richard M. Teck
Donald E. Hilt

Abstract

Describes a species-specific, distance-independent individual-tree diameter growth model for the Northeastern United States. Diameter growth is predicted in two steps using a two parameter, sigmoidal growth function modified by a one parameter exponential decay function with species-specific coefficients. Coefficients are presented for 28 species groups. The model accounts for variability in annual diameter growth due to species, tree size, site quality, and the tree's competitive position within the stand. Model performance is evaluated using the mean predicted error and the root mean square error. Results are presented for the calibration data and an independent validation data set. The model has been incorporated into NE-TWIGS, a computerized forest growth model for the Northeastern United States.

The Authors

RICHARD M. TECK, forester, received a B.S. degree in forestry from S.U.N.Y. College of Environmental Science and Forestry in 1983, and an M.S. degree in forestry from West Virginia University in 1987. He joined the Northeastern Forest Experiment Station in 1986 and since then has been involved in the development of individual-tree based forest growth models for mixed-species forest stands of the Northeast at the Northeastern Station's Forestry Sciences Laboratory at Delaware, Ohio.

DONALD E. HILT, research forester, received a B.S. degree in forestry from Iowa State University in 1969, and an M.S. degree in forestry from Oregon State University in 1975. He joined the Northeastern Forest Experiment Station in 1975 and since 1976 has been engaged in research on the growth and yield of managed upland oaks and mixed-species stands at the Northeastern Station's Forestry Sciences Laboratory at Delaware, Ohio.

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Northeastern Forest Experiment Station
5 Radnor Corporate Center
100 Matsonford Road, Suite 200
P.O. Box 6775
Radnor, Pennsylvania 19087

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Introduction

The ability to predict forest stand development accurately over time is essential for forest planning, and a forest growth simulation system that can predict stand development is a desirable forest management tool. Resource managers in the Northeastern United States need forest growth and yield models to: (1) forecast large-scale regional projections of timber resources, (2) evaluate silvicultural prescriptions, and (3) perform economic analyses of multiresource management alternatives.

More than 30 forest cover types are found in the Northeast. The heterogeneous condition of the forests in this region, defined by the large variation in species composition, stand structure, and site quality, can be traced through several centuries of continuous land use change and a variety of harvesting practices. An individual-tree model is well suited for predicting growth under these diverse conditions.

Some of the earlier individual-tree, distance-independent modeling efforts concentrated on even-aged stand structures with single species. Many of these earlier efforts were limited to linear models, since scientific statistical software packages containing nonlinear regression programs were unavailable. The linear model developed by Lemon and Schumacher (1962), for example, predicted diameter growth in the ponderosa pine forest type as a function of stand competition, site quality, tree size, and age.

More recent modeling endeavors have focused on utilizing nonlinear models for predicting the growth of individual trees in mixed-species heterogeneous forest stands. Many biological processes such as population growth and survival can best be described by nonlinear functions. The response function can often be confined within a specified minimum and maximum range, an advantage when one is concerned with the biological feasibility of the prediction.

The mathematical equations developed by Hahn and Leary (1979) predicted the potential diameter growth of individual trees in the Lake States as a function of diameter, site quality, and crown ratio. Holdaway (1984) modified those predicted growth rates to account for inter-tree competition. More recently, Shifley (1987) developed a 9-parameter function for predicting the growth of 22 species groups in the Central States.

Expanding on the concepts formulated by these researchers, we have attempted to extend the geographic range of these models. However, the model we have developed, although similar in concept to those described above, contains fewer parameters. Nonlinear models with few parameters are easier to recalibrate to local conditions.

Species-specific, individual-tree, distance-independent, diameter growth models have been previously developed for the northern New England states (Hilt et al. 1987a, Hilt et al. 1987b, Hilt and Teck 1987). The models performed satisfactorily. Our objective here is to calibrate the model coefficients to other species groups and a much larger geographic area in the Northeast. Model paramaters were calibrated with Northeastern Forest Experiment Station's Forest Inventory and Analysis (FIA) data for 28 species groups in 14 Northeastern states: Connecticut, Delaware, Kentucky, Maine, Maryland, Massachusetts, New Hampshire, New Jersey, New York, Ohio, Pennsylvania, Rhode Island, Vermont, and West Virginia.

Data

Individual-tree measurements collected by the FIA unit were used in developing the model. More than 4,400 1/5-acre permanent plots measured throughout the 14 Northeastern States were used in this study. Data were collected in the 1960's, 1970's, and 1980's. Only one remeasurement period was available for each state except Maine, which was remeasured twice. The remeasurement period averaged 12 years.

The data covered a wide range of age, site, and stocking conditions. Basal area per acre ranged from 30 to 255 ft^2/acre. Site index (base age 50 years), recorded on each 1/5-acre plot for the dominant species, ranged from 30 to 90. Site-index conversion equations were used to assign the appropriate site index to each tree depending on its species. Quadratic mean stand diameter ranged from 5 to 13 inches, indicating a wide range in the age of the stands sampled.

Information recorded for each tree (more than 5 inches d.b.h.) included species, initial diameter, diameter at the end of the remeasurement period, and a status code indicating whether the tree was alive or dead.

Most of the major species had a sufficient number of observations to be modeled independently. Less prevalent species had fewer observations and were grouped with other species exhibiting similar silvical characteristics into one of 28 species groups (Table 1).

Every fourth plot was systematically removed from the data base and was set aside for model validation. The calibration data set contains 51,757 observations (Table 2). The validation data set contains 16,748 observations (Table 3).

The data summarized in Table 2 are a subset of a much larger initial data base. Eliminated from the calibration set data were those plots where catastrophic mortality (more than 70 percent of initial basal area) and/or excessive cutting (residual basal area less than 30 pecent of initial conditions) occurred between remeasurements. These plots were eliminated because we were unable to determine when the events occurred. Without knowing the timing of such events, it is impossible to determine how long the initial stand conditions existed. Since the independent variables in the model are based on initial stand conditions, it was necessary to remove the plots from the data base.

Table 1.—28 species groups and associated species codes used for analysis

Species group	Species	USDA Code[a]
American beech	American beech	531
Balsam fir	Balsam fir	012
Black cherry	Black cherry	762
Black oak	Black oak	837
	Cherrybark oak	813
Chestnut oak	Chestnut oak	832
	Swamp chestnut oak	825
	Swamp white oak	804
Eastern hemlock	Eastern hemlock	261
Hickory	Bitternut hickory	402
	Pignut hickory	403
	Shagbark hickory	407
	Mockernut hickory	409
Loblolly pine	Loblolly pine	131
Noncommercial	Boxelder	313
	Striped maple	315
	Mountain maple	319
	Serviceberry	355
	American hornbeam	391
	Catalpa	450
	Eastern redbud	471
	Flowering dogwood	491
	Hawthorn	500
	Eastern hophornbeam	701
	Plum cherry	760
	Pin cherry	761
N. red oak	N. red oak	833
	Southern red oak	812
N. white-cedar	N. white-cedar	241
	Atlantic white-cedar	043
	Eastern redcedar	068
Other hardwoods	Buckeye	330
	Yellow buckeye	332
	Gray birch	379
	Hackberry	462
	Common persimmon	521
	Honeylocust	552
	American holly	591
	Butternut	601
	Black walnut	602
	Magnolia	650
	Sweetbay	653
	Apple sp	660
	Water tupelo	691
	Blackgum	693
Other hardwoods	Sourwood	711
	Paulownia	712
	Sycamore	731
	Willow oak	831
	Black locust	901
	Black willow	922
	Sassafras	931
	Mountain ash	935
	Basswood	950
	Elm	970
Other pines	Jack pine	105
	Shortleaf pine	110
	Table mountain pine	123
	Pitch pine	126
	Pond pine	128
	Scotch pine	130
Paper birch	Paper birch	375
Quaking aspen	Quaking aspen	746
	Balsam poplar	741
	Eastern cottonwood	742
	Bigtooth aspen	743
Red maple	Red maple	316
Red pine	Red pine	125
Red spruce	Red spruce	097
	Norway spruce	091
	Black spruce	095
Scarlet oak	Scarlet oak	806
	Pin oak	831
Tamarack	Tamarack	071
Virginia pine	Virginia pine	132
White ash	White ash	541
	Black ash	543
	Green ash	544
	Blue ash	546
White oak	White oak	802
	Bur oak	823
	Post oak	835
White pine	White pine	129
White spruce	White spruce	094
Yellow birch	Yellow birch	371
	Sweet birch	372
	River birch	373
Yellow-poplar	Yellow-poplar	621
	Sweetgum	611
	Cucumbertree	651

[a]Standard species codes used by the USDA Forest Service, Forest Inventory and Analysis Unit.

Table 2.—Individual-tree and plot characteristics of the calibration data set

Species group	No. of plots	No. of trees	Site index[a]			Plot TPA[b]			Plot basal area			DBH			BAL[c]			Measurement Interval		
			Min	Avg	Max	Min	Avg	Max	Min	Avg	Max	Min	Avg	Max	Min	Avg	Max	Min	Avg	Max
										Ft^2/acre			Inches			Ft^2/acre			Years	
American beech	587	2,511	32	57	90	40	230	661	32	102	220	5.0	9.2	35.6	4	67	220	4	12	18
Balsam fir	633	4,456	30	50	82	82	327	705	40	116	238	5.0	7.0	18.4	3	76	224	4	11	17
Black cherry	253	812	39	68	90	42	283	621	32	106	210	5.0	8.7	26.1	3	67	195	6	12	17
Black oak	199	664	37	64	90	62	212	571	32	85	163	5.0	10.1	24.8	3	46	151	9	12	17
Chestnut oak	172	821	30	58	90	25	222	506	31	95	192	5.0	9.6	30.4	3	63	180	9	12	18
Eastern hemlock	471	3,431	30	50	83	50	309	806	32	131	255	5.0	9.0	29.2	3	82	255	4	12	18
Hickory	261	1,013	35	60	88	25	231	806	32	92	255	5.0	8.6	22.9	4	66	242	9	12	17
Loblolly pine	41	430	49	70	90	90	221	351	36	87	123	5.0	9.3	24.7	4	55	114	9	10	15
Noncommercial	175	422	30	52	78	58	238	581	32	101	214	5.0	6.6	16.5	5	86	214	7	12	17
N. red oak	478	2,093	30	60	88	52	246	806	30	98	255	5.0	9.9	59.1	3	54	177	7	12	17
N. white-cedar	302	2,804	30	42	70	92	355	716	38	130	234	5.0	8.6	28.0	3	73	224	4	11	17
Other hardwoods	557	2,123	30	64	90	38	246	806	31	96	255	5.0	8.3	32.5	4	68	255	4	11	17
Other pines	62	341	30	47	86	68	201	488	36	79	147	5.0	9.0	17.9	3	47	147	7	13	17
Paper birch	486	1,830	30	55	80	90	290	705	37	103	228	5.0	7.9	23.0	2	61	193	4	11	16
Quaking aspen	298	1,328	36	61	90	75	293	806	32	95	255	5.0	8.1	33.6	3	48	178	7	12	17
Red maple	1,225	6,591	30	58	90	40	278	806	30	108	255	5.0	8.4	33.8	3	71	233	4	12	18
Red pine	31	91	31	68	90	111	259	546	39	100	228	5.0	9.3	18.3	3	51	155	10	12	16
Red spruce	697	4,968	30	46	81	50	332	716	32	118	234	5.0	8.1	25.4	2	68	228	4	11	17
Scarlet oak	133	476	37	59	90	70	206	473	32	76	155	5.0	9.1	23.8	4	46	137	9	13	16
Sugar maple	736	4,237	30	59	90	40	259	661	30	113	237	5.0	9.4	39.3	3	75	219	4	12	18
Tamarack	51	238	30	48	69	105	354	705	36	102	207	5.0	7.5	14.3	2	51	156	6	11	15
Virginia pine	53	664	41	63	87	70	263	385	34	87	144	5.0	8.1	16.4	3	55	144	9	11	15
White ash	454	1.393	32	64	90	38	285	806	30	110	255	5.0	8.7	27.3	3	73	248	4	12	17
White oak	345	1,321	31	59	88	25	229	806	31	90	255	5.0	9.5	37.9	3	58	195	9	12	17
White pine	411	2,914	31	65	90	45	262	690	36	120	238	5.0	10.4	43.3	3	64	201	4	12	18
White spruce	149	481	30	49	80	80	331	645	41	119	238	5.0	8.4	22.3	3	63	197	6	11	15
Yellow birch	746	2,562	30	58	90	45	263	806	32	112	255	5.0	9.3	33.8	3	72	242	4	11	18
Yellow-poplar	182	742	38	69	90	25	180	401	30	94	219	5.0	10.7	32.8	4	62	181	9	11	16
Total		51,757																		

[a]Total height (in feet) at age 50.
[b]Number of trees per acre.
[c]Basal area of trees larger than or equal to subject tree.

Table 3.—Individual-tree and plot characteristics of the validation data set

Species group	No. of plots	No. of trees	Site index[a]			Plot TPA[b]			Plot basal area			DBH			BAL[c]			Measurement Interval		
			Min	Avg	Max	Min	Avg	Max	Min	Avg	Max	Min	Avg	Max	Min	Avg	Max	Min	Avg	Max
									Ft^2/acre			Inches			Ft^2/acre			Years		
American beech	189	771	35	57	86	42	249	645	32	110	200	5.0	9.2	28.8	4	73	200	4	12	17
Balsam fir	211	1,546	30	51	74	70	328	659	40	116	240	5.0	7.0	17.6	3	76	200	4	11	15
Black cherry	70	230	40	70	90	85	273	536	30	106	202	5.0	8.9	23.7	4	68	187	8	12	17
Black oak	77	298	33	61	86	25	211	476	32	92	187	5.0	11.0	41.0	3	51	174	10	12	16
Chestnut oak	59	461	30	56	80	80	265	618	33	105	225	5.0	9.1	26.5	3	70	196	9	11	18
Eastern hemlock	152	1,176	30	50	80	70	319	721	35	128	240	5.0	8.9	27.1	2	79	237	4	12	17
Hickory	102	378	35	62	84	60	234	436	33	87	200	5.0	8.1	28.1	5	66	167	9	12	17
Loblolly pine	19	181	30	53	77	100	200	290	32	92	127	5.0	9.6	20.2	5	63	127	9	10	15
Noncommercial	52	119	30	53	77	80	221	413	36	98	176	5.0	6.9	13.1	6	84	174	8	12	16
N. red oak	162	651	30	60	89	25	262	618	32	105	225	5.0	10.0	31.3	3	58	206	6	12	18
N. white-cedar	91	941	30	43	66	79	375	659	34	143	258	5.0	8.7	23.2	3	82	225	5	11	16
Other hardwoods	205	652	30	61	90	45	224	645	30	94	225	5.0	8.5	25.8	4	70	209	4	12	17
Other pines	10	54	30	57	82	78	275	371	40	98	115	5.0	9.3	17.6	4	59	115	10	13	16
Paper birch	153	518	34	55	86	55	295	658	34	101	200	5.0	8.0	20.7	3	58	178	4	11	16
Quaking aspen	84	367	32	62	84	58	304	659	30	98	238	5.0	7.9	18.7	3	50	196	6	12	17
Red maple	390	2,195	30	58	86	40	293	802	30	110	242	5.0	8.4	31.6	3	71	242	4	12	17
Red pine	9	60	34	65	80	102	400	618	49	135	187	5.1	9.2	17.8	4	54	135	7	12	14
Red spruce	240	1,686	30	47	72	70	352	802	34	123	258	5.0	8.0	21.5	3	71	242	4	11	16
Scarlet oak	50	154	33	58	86	25	235	468	34	84	165	5.0	9.6	22.0	3	49	123	9	12	15
Sugar maple	235	1,318	31	60	86	40	247	721	35	109	232	5.0	9.6	37.2	4	71	232	6	12	17
Tamarack	13	85	35	58	75	120	428	541	49	121	162	5.2	8.0	13.1	4	59	150	10	11	13
Virginia pine	19	128	37	64	90	108	214	363	35	88	124	5.0	9.2	15.9	5	56	116	9	11	15
White ash	138	389	36	65	90	42	252	558	32	102	225	5.0	8.8	26.7	4	69	197	6	12	16
White oak	122	499	30	58	83	45	225	618	32	94	193	5.0	9.6	32.4	4	61	193	9	12	16
White pine	121	730	34	63	90	70	275	802	42	119	242	5.0	10.1	39.1	3	65	211	4	12	17
White spruce	52	162	30	51	69	80	315	659	34	115	238	5.0	8.0	16.7	3	64	200	5	11	15
Yellow birch	262	817	32	57	86	55	236	563	34	104	258	5.0	9.7	28.0	4	66	228	6	12	17
Yellow-poplar	59	182	42	69	94	42	156	351	31	78	150	5.0	10.4	28.1	4	57	158	9	11	16
Total		16,478																		

[a]Total height (in feet) at age 50.
[b]Number of trees per acre.
[c]*Basal area of trees larger than or equal to subject tree.*

Methods

Here we first review the development of the species-specific, individual-tree, distance-independent, diameter growth model previously developed by Hilt and others (1987a). Species-specific coefficients for the model are then calibrated for each of the 28 species groups.

Predicted periodic mean annual diameter growth is modeled using a two-step approach. In the first step, potential periodic mean annual basal area growth is modeled as a function of d.b.h. and site index. The potential basal area growth is then reduced for each tree based on the tree's competitive position within the stand. Basal-area growth is then converted to diameter growth.

Many predictor variables were evaluated for inclusion in our model. Plot variables included site index (SI), basal area per acre, trees per acre, quadratic mean stand diameter (QMD), and stand density. Individual-tree variables included basal area and diameter.

Plots of periodic mean annual diameter-growth over diameter for a given site index and stand density revealed a positive correlation between tree diameter and diameter growth. Additional data analysis also revealed a positive correlation between site index and diameter growth.

Diverse land management practices have resulted in stands with tremendous variation in diameter distributions. The normal bell-shaped diameter distribution associated with even-aged stands, and the reverse J-shaped diameter distribution associated with uneven-aged stands are more the exception than the rule. High frequencies of bimodal. trimodal, and uniform diameter distributions in the plot data owe their existence to a multitude of harvesting practices in the second and third generation forests prevalent in the Northeast.

This diversity of stand conditions negated any correlation associated with diameter growth and mean stand diameter (Hilt et al. 1987a). Furthermore, the elimination of mean stand diameter as a predictor variable reduced the effectiveness of stand basal area for predicting growth. Together, these variables can be used to identify relative stand density. However, basal area alone is an unreliable indicator of relative stand density. One hundred square feet of basal area per acre may represent 100 percent stocking in a stand with a small QMD, but only 60 percent stocking in a stand with a larger QMD.

Since the growth rate of a tree is influenced by its relative position (competitive status) within the stand, we calculated several competition indices including: the ratio of d.b.h. to QMD, ratio of tree basal area to plot basal area, and the number of standard deviations a tree's diameter is from plot QMD. The competition index exhibiting the highest correlation with diameter growth was basal area per acre larger than the subject tree (BAL). BAL has been used as a competition indicator in both the PROGNOSIS growth model (Wykoff et al. 1982) and the Central States TWIGS growth model (Shifley 1987).

The data for each species group were than separated into d.b.h. × BAL × SI cells. The upper and lower boundaries of each cell were selected so that there were approximately equal numbers of trees within each cell. The mean value for each of the three predictor variables and the mean annual periodic individual-tree basal area growth rate within each cell were used in the preliminary analysis to select the model form. Cell means were used to reduce the total number of observations so that various nonlinear model forms could be examined more efficiently.

Numerous model forms and combinations of independent variables were examined (Hilt et al. 1987a). Only the final model selected for application is reported here.

Potential Growth

Individual trees for a given species were sorted in descending order according to their mean annual periodic basal-area growth rates in each d.b.h. × site index class. The top 10 percent of the fastest growers in each class were then used to develop the potential growth function. A modified Chapman-Richards (Richards 1959) sigmoidal growth function was used to predict the potential growth for a given site and tree size:

$$\mathrm{POTBAG} = b_1\mathrm{SI}(1.0 - \exp(-b_2\mathrm{DBH10})) \qquad (1)$$

where POTBAG is the potential basal-area growth for an individual tree, DBH10 is the average d.b.h. of the top 10 percent of the fastest growers, SI is the species specific site index, and b_1 and b_2 are species specific parameters estimated using weighted nonlinear regression. An investigation of the error structure revealed homogeneity among cell variances in relation to d.b.h. and site index. Each observation was weighted by cell frequency (the number of trees in each cell).

The fitted values for b_1 and b_2 are shown in Table 4, and the resulting equation is plotted in Figure 1 for sugar maple. Potential basal-area growth for sugar maple is then compared with several major species groups in Figure 2. Corresponding potential individual-tree diameter growth rates are plotted in Figure 3.

Modifier Function

Graphic analysis of the cell means revealed that individual-tree basal-area growth rates for all trees in a given initial d.b.h. × SI class declined in a negative exponential manner as BAL increased. This trend suggests the following model for a given d.b.h. × SI class:

$$\mathrm{BAG} = \mathrm{POTBAG}(\exp(-b_3(\mathrm{BAL}))) \qquad (2)$$

The intercept term, POTBAG, is the potential basal-area growth estimated from equation (1) for each DBH × site

Table 4.—Individual-tree basal-area growth coefficients[a]

Species group	Potential		Modifier	Variance	
	b_1	b_2	b_3	c_1	c_2
American beech	0.0006911	0.0730441	0.013029	0.0000138	0.0060925
Balsam fir	.0008829	.0602785	.012785	.0000082	.0046624
Black cherry	.0007929	.1568904	.016537	.0000304	.0089725
Black oak	.0008550	.0957964	.020843	.0000131	.0029118
Chestnut oak	.0008238	.0790660	.013762	.0000181	.0147197
Eastern hemlock	.0008737	.0940538	.009149	.0000136	.0098005
Hickory	.0007993	.0779654	.015963	.0000184	.0311543
Loblolly pine	.0009252	.1134195	.017300	.0000209	.0174766
Noncommercial	.0003604	.0328767	.011620	.0000164	.0112692
N. red oak	.0008920	.0979702	.018024	.0000200	.0031519
N. white-cedar	.0009050	.0517297	.012329	.0000056	.0049320
Other hardwoods	.0009567	.1038458	.020653	.0000160	.0027012
Other pines	.0006634	.1083470	.016835	.0000034	.0128994
Paper birch	.0009766	.0832328	.023978	.0000059	.0014599
Quaking aspen	.0011885	.0920050	.016877	.0000140	.0030131
Red maple	.0007906	.0651982	.016191	.0000173	.0031921
Red pine	.0009252	.1134195	.017300	.0000221	.0282408
Red spruce	.0008236	.0549439	.011942	.0000084	.0048774
Scarlet oak	.0008769	.0866621	.018560	.0000159	.0092161
Sugar maple	.0007439	.0706905	.016240	.0000147	.0037588
Tamarack	.0009933	.0816995	.018831	.0000077	.0042517
Virginia pine	.0006634	.1083470	.016835	.0000093	.0042225
White ash	.0008992	.0925395	.015004	.0000264	.0116584
White oak	.0007417	.0867535	.014235	.0000119	.0070636
White pine	.0011303	.0934796	.015496	.0000255	.0028304
White spruce	.0008721	.0578650	.013427	.0000155	.0173732
Yellow birch	.0006668	.0768212	.019046	.0000125	.0080770
Yellow-poplar	.0008815	.1419212	.019904	.0000429	.0090635

[a]$BAG = POTBAG (\exp(-b_3(BAL))$ where: $POTBAG = b_1 SI(1.0 - \exp(-b_2 DBH))$

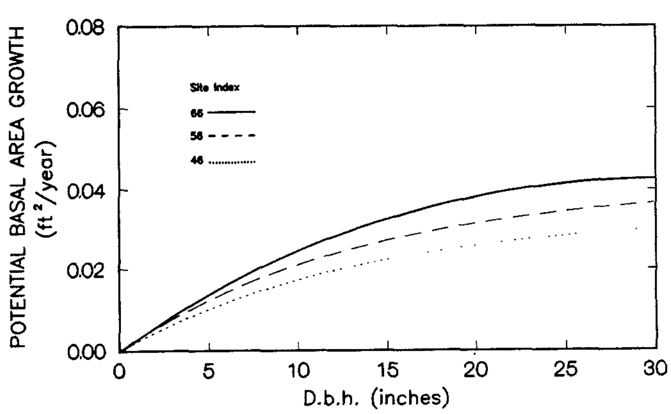

Figure 1.—Effect of site index on the predicted potential basal-area growth rates for sugar maple.

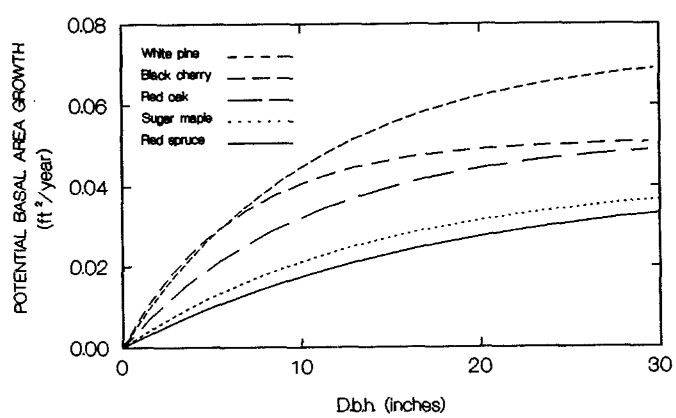

Figure 2.—Predicted potential basal-area growth rates for several important species groups (species specific site index equivalent to SI = 50 for red spruce).

index class. The equation is forced through the potential growth when BAL equals zero.

A two-stage modeling procedure was used to estimate b_3. First, an estimate of b_3 was determined by fitting equation (2) for each d.b.h. × SI class. An investigation of the error structure revealed that within-cell variances were correlated with d.b.h. and BAL. The following model was then fitted for each species to describe the error structure:

$$VARBAG = c_1 DBH(\exp(-c_2(BAL))) \qquad (3)$$

where VARBAG is the variance of the individual-tree basal-area growth rates. Each observation was weighted by the number of observations in the cell divided by VARBAG. Equation (2) was then refit to the weighted observations. The estimated b_3's were plotted over d.b.h. and site index to see if they could be modeled as a function of these two variables. No trends were identified. The mean value of b_3 was used as the final estimate for each species group. Fitted values for b_3, c_1 and c_2 are presented in Table 4. Predicted individual-tree basal-area growth rates for sugar maple are shown in Figure 4 for a range of d.b.h. and BAL values. Corresponding diameter growth rates are presented in Figure 5.

Individual-tree basal-area growth rates are easily calculated using equations (1) and (2). First, compute the potential basal-area growth rate (POTBAG) using equation (1) and the values for b_1 and b_2 from Table 4. Then using the value for b_3 from Table 4, solve equation (2) to determine the individual-tree basal-area growth rate (BAG). Individual-tree basal-area growth rates can then be converted to diameter growth rates (DGROW) using the following conversion formula:

$$DGROW = [\{0.00545415(DBH)^2 + BAG\} / 0.00545415]^{.5} - DBH \qquad (4)$$

Predicted potential diameter growth rates for sugar maple are plotted using a three-dimensional response function in Figure 6. Predicted diameter growth rates are presented in Figure 7 for sugar maple site index 56 for a range of diameters and BAL.

Results

To determine how well the model predicts individual-tree growth, we compared observed and predicted periodic annual basal-area growth and periodic annual diameter growth for each observation in the data base. Observed periodic mean annual diameter growth ranged from a low of 0.064 for the noncommercial species group to a high of 0.159 for white pine. The mean annual diameter growth prediction error (i.e. predicted minus observed growth), based on all 51,757 observations, was -0.013 inches. This is an 11.5 percent underprediction of individual-tree annual diameter growth.

Observed and predicted mean annual periodic basal-area growth rates and diameter growth rates and their associated mean predicted errors are presented by species group in Table 5. The largest discrepency between actual and

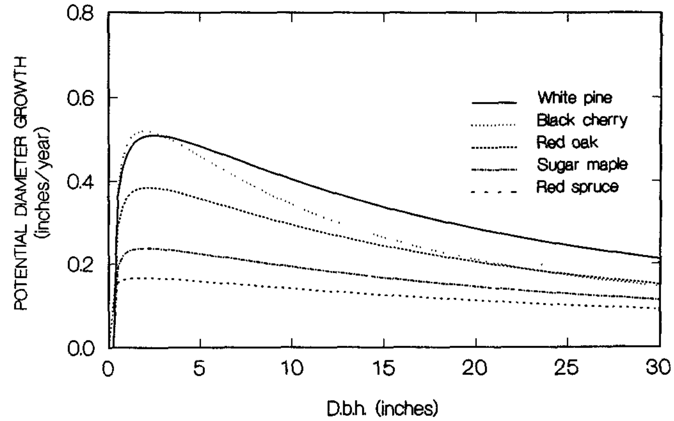

Figure 3.—Predicted potential diameter growth rates for several important species groups (species specific site index equivalent to SI = 50 for red spruce).

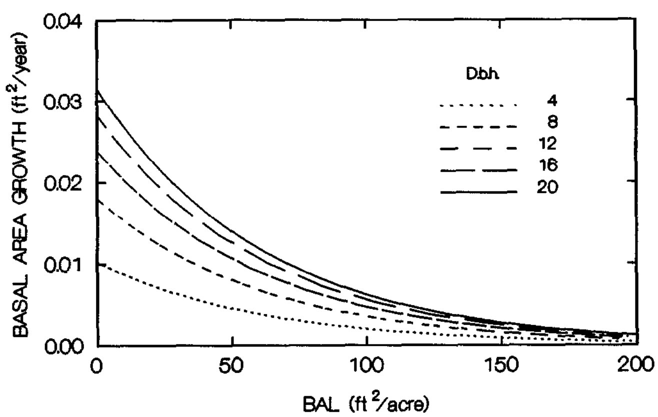

Figure 4.—Predicted basal-area growth rates for sugar maple (SI = 56).

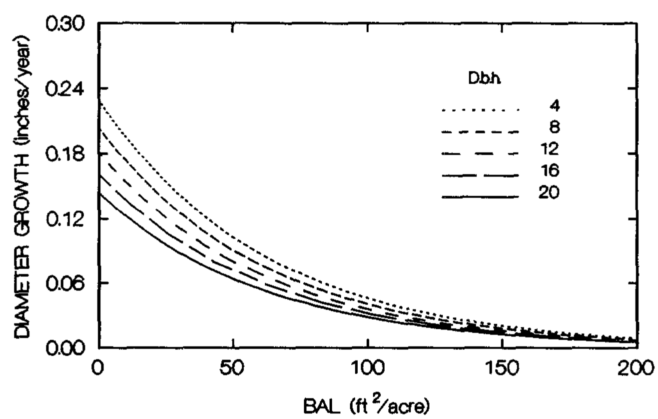

Figure 5.—Predicted diameter growth rates for sugar maple (SI = 56).

predicted individual-tree diameter growth was for quaking aspen. The model overpredicted the mean annual individual-tree diameter growth by 0.062 inches. The model overpredicts annual diameter growth for 16 species groups, and underpredicts annual diameter growth for 12 species groups. The predicted mean annual diameter growth error is within 0.02 inches for 16 of the 28 species groups.

The same evaluation statistics were computed for the validation data (Table 6). The mean annual diameter growth prediction error, based on all 16,748 observations is -0.013 inches. This is an 11.6 percent underprediction of individual-tree annual diameter growth. The model overpredicts annual diameter growth for 12 species groups, and underpredicts annual diameter growth for 16 species groups.

Discussion

Although some of the variability in annual basal-area growth within and among species groups can be explained by the stand and tree variables contained within this model, much of the variability is due to other factors such as the spatial variation of weather, and micro-site conditions.

No simulation model predicting changes to a biological system will ever perfectly represent the system being modeled so long as the environmental conditions within which that system resides continue to change. However, we still need to be concerned with how well the model predicts change relative to alternative models.

At the present time, this model is the only regionally calibrated, species-specific, individual-tree, distance-independent, diameter growth model for mixed-species, multi-aged forest stands in the 14-state Northeastern Region. Other models that have been calibrated for specific forest types within sub-regions of the Northeast are available. Preliminary evaluations comparing this model to some of the locally developed models (U.S. Dept. of Agric. 1990) show promising results regarding model accuracy and precision.

Potential model users should understand that predicted growth rates for a given species are indicative of the average growth rate for that species throughout the region. Growth rates for a given stand may vary considerably from the regional average due to local edaphic conditions. However, for many species, site index accounts for this variation.

The model form has several inherent constraints that should provide biologically reasonable estimates of diameter growth when extrapolated beyond the range of the calibration data base: (1) The growth of an individual tree cannot exceed its potential growth—it equals the potential growth only when BAL = 0; (2) individual-tree basal-area growth rates for a given d.b.h. and site index decrease as BAL increases; and (3) as BAL increases, individual-tree basal-area growth rates for a given d.b.h. and site index

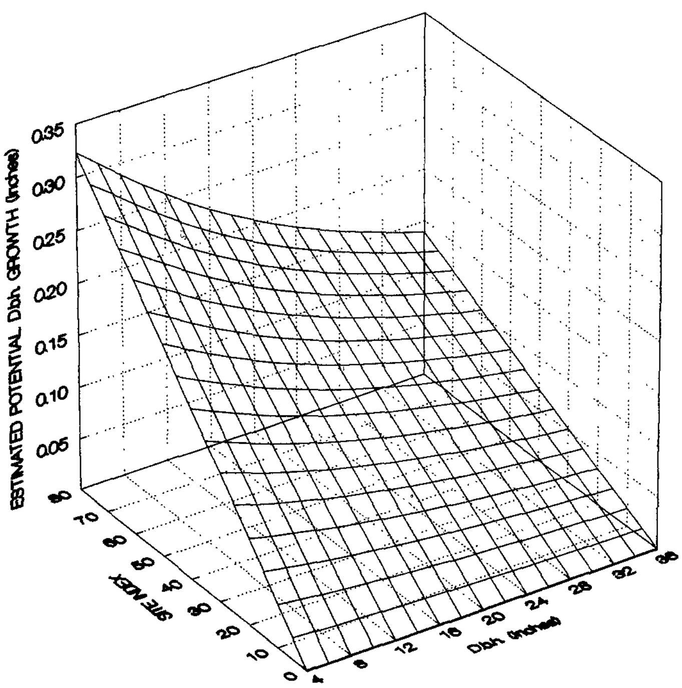

Figure 6.—Predicted potential diameter growth (in inches) for sugar maple.

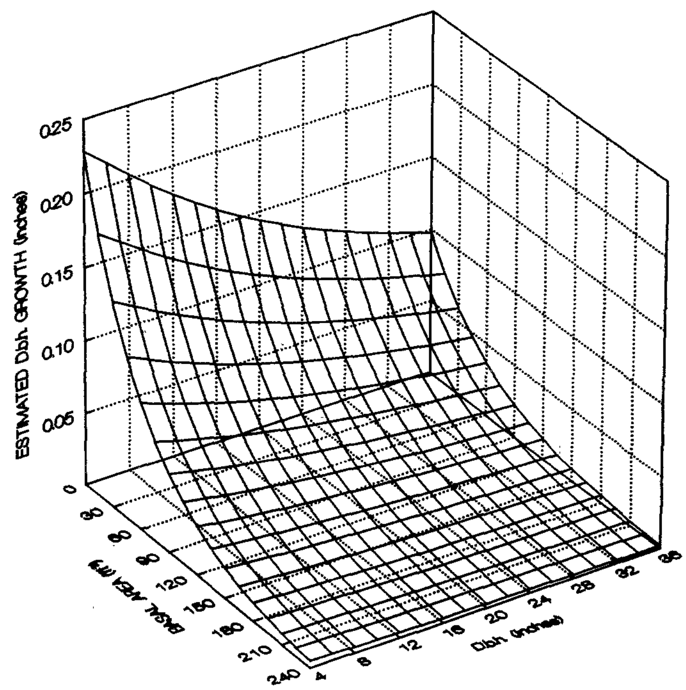

Figure 7.—Predicted diameter growth (in inches) for sugar maple (SI = 56).

Table 5.—Comparison of observed and predicted annual growth rates for calibration data base

Species group	No. of trees	Basal-area growth: Mean observed growth	Basal-area growth: Mean predicted error	Basal-area growth: Root mean square error	Diameter growth: Mean observed growth	Diameter growth: Mean[a] predicted error	Diameter growth: Root[b] mean square error
		Ft^2/tree			Inch/tree		
American beech	2,511	0.0127	−0.0034	0.010	0.113	−0.026	0.080
Balsam fir	4,456	.0082	− .0016	.006	.095	− .013	.066
Black cherry	812	.0164	− .0007	.014	.145	.016	.112
Black oak	664	.0178	− .0026	.013	.140	− .007	.090
Chestnut oak	821	.0128	− .0010	.010	.107	.001	.065
Eastern hemlock	3,431	.0125	.0007	.009	.110	.017	.073
Hickory	1,013	.0103	− .0005	.007	.097	.000	.065
Loblolly pine	430	.0138	.0051	.012	.122	.055	.103
Noncommercial	422	.0053	− .0038	.008	.064	− .043	.077
N. red oak	2,093	.0197	− .0050	.015	.155	− .025	.097
N. white-cedar	2,804	.0075	− .0009	.007	.075	− .010	.068
Other hardwoods	2,123	.0113	.0003	.012	.107	.011	.101
Other pines	341	.0076	.0022	.008	.071	.024	.068
Paper birch	1,830	.0076	.0007	.008	.081	.009	.076
Quaking aspen	1,328	.0149	.0045	.013	.146	.062	.125
Red maple	6,591	.0120	− .0043	.011	.113	.035	.089
Red pine	91	.0154	.0057	.013	.138	.060	.117
Red spruce	4,968	.0093	− .0023	.008	.093	− .019	.069
Scarlet oak	476	.0157	− .0022	.010	.136	− .005	.082
Sugar maple	4,237	.0144	− .0062	.012	.122	− .049	.091
Tamarack	238	.0089	.0017	.009	.095	.022	.080
Virginia pine	664	.0117	− .0005	.008	.116	.002	.076
White ash	1,393	.0156	− .0028	.011	.140	.014	.093
White oak	1,321	.0132	− .0009	.010	.106	.006	.071
White pine	2,914	.0213	− .0008	.016	.159	.013	.112
White spruce	481	.0117	− .0034	.009	.114	− .030	.083
Yellow birch	2,562	.0113	− .0045	.011	.102	− .041	.084
Yellow-poplar	742	.0284	− .0112	.026	.201	− .060	.155
All	51,757	.0125	− .0022	.011	.113	− .013	.086

[a]Predicted minus observed growth. Negative values signify underprediction.

[b]Root mean square error = $[\Sigma(y_i - \hat{y}_i)^2/n]^{0.5}$

asymptotically approach zero.

This diameter growth model has been incorporated into NE-TWIGS, an individual-tree growth projection system for mixed-species forests of the Notheastern United States (Hilt and Teck 1989, Teck 1990). Computerized forest growth projection systems like NE-TWIGS, will allow researchers to quantitatively evaluate stand response to alternative silvicultural treatments. Treatment response comparisons can then be used for developing recommended management guidelines for Northeastern forest stands.

Table 6.—Comparison of observed and predicted annual growth rates for validation data base

Species group	No. of trees	Basal-area growth: Mean observed growth	Basal-area growth: Mean predicted error	Basal-area growth: Root mean square error	Diameter growth: Mean observed growth	Diameter growth: Mean[a] predicted error	Diameter growth: Root[b] mean square error
		Ft²/tree			Inch/tree		
American beech	771	0.0118	−0.0031	0.009	0.106	−0.024	0.076
Balsam fir	1,546	.0084	− .0017	.007	.097	− .014	.068
Black cherry	230	.0178	− .0014	.014	.165	.001	.117
Black oak	298	.0203	− .0062	.017	.142	− .029	.084
Chestnut oak	461	.0105	.0000	.007	.094	.006	.061
Eastern hemlock	1,176	.0128	.0007	.009	.115	.018	.076
Hickory	378	.0107	− .0012	.007	.104	− .004	.062
Loblolly pine	181	.0133	.0039	.014	.118	.037	.106
Noncommercial	119	.0051	− .0034	.006	.060	− .039	.062
N. red oak	651	.0199	− .0050	.014	.154	− .027	.094
N. white-cedar	941	.0074	− .0009	.007	.073	− .010	.061
Other hardwoods	652	.0121	− .0010	.012	.111	− .002	.098
Other pines	54	.0070	.0040	.007	.060	.038	.056
Paper birch	518	.0077	.0014	.009	.082	.013	.081
Quaking aspen	367	.0162	.0032	.012	.160	.055	.127
Red maple	2,195	.0117	− .0039	.010	.112	− .033	.087
Red pine	60	.0155	.0025	.011	.136	.033	.094
Red spruce	1,686	.0089	− .0021	.007	.092	− .018	.072
Scarlet oak	154	.0165	− .0025	.011	.133	− .009	.076
Sugar maple	1,318	.0145	− .0058	.013	.120	− .043	.090
Tamarack	85	.0103	.0013	.008	.103	.016	.074
Virginia pine	128	.0150	− .0031	.010	.136	− .023	.087
White ash	389	.0156	− .0017	.011	.136	.000	.090
White oak	499	.0127	− .0014	.009	.104	.001	.065
White pine	730	.0198	− .0003	.014	.152	.020	.109
White spruce	162	.0138	− .0054	.010	.137	− .048	.095
Yellow birch	817	.0128	− .0051	.012	.112	− .046	.094
Yellow-poplar	182	.0264	− .0074	.023	.188	− .032	.146
All	16,748	.0124	− .0022	.010	.112	− .013	.085

[a]Predicted minus observed growth. Negative values signify underprediction.

[b]Root mean square error = $[\Sigma(y_i - \hat{y}_i)^2/n]^{0.5}$

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Teck, Richard M.; Hilt, Donald E. 1991. **Individual tree-diameter growth model for the Northeastern United States.** Res. Pap. NE-649. Radnor, PA: U.S. Department of Agriculture, Forest Service, Northeastern Forest Experiment Station. 11 p.

Describes a distance-independent individual-tree diameter growth model for the Northeastern United States. Diameter growth is predicted in two steps using a two parameter, sigmoidal growth function modified by a one parameter exponential decay function with species-specific coefficients. Coefficients are presented for 28 species groups. The model accounts for variability in annual diameter growth due to species, tree size, site quality, and the tree's competitive position within the stand. Model performance is evaluated using the mean predicted error and the root mean square error. Results are presented for the calibration data and an independent validation data set. The model has been incorporated into NE-TWIGS, a computerized forest growth model for the Northeastern United States.

Keywords: NE-TWIGS; diameter growth prediction; computer simulation; basal-area growth

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