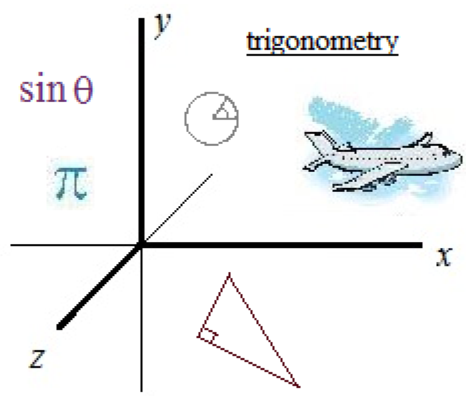
Trigonometry Identities I

Introduction

Includes notes, formulas, examples, and practice test (with solutions)



Intro to Trigonometry Identities: Notes and Examples

Definition of Identity: An equation which is true for every value of the variable.

Example: $3(x + 4) = 3x + 12$

(Every value of x is true)

Definitions

"Reciprocal Identities"

$$\frac{1}{\text{Sin } x} = \text{Csc } x \qquad \frac{1}{\text{Csc } x} = \text{Sin } x$$

$$\frac{1}{\text{Cos } x} = \text{Sec } x \qquad \frac{1}{\text{Sec } x} = \text{Cos } x$$

$$\frac{1}{\text{Tan } x} = \text{Cot } x \qquad \frac{1}{\text{Cot } x} = \text{Tan } x$$

"Quotient Identities" or "Ratio Identities" (or, "Tangent Identities")

$$\text{Tan } x = \frac{\text{Sin } x}{\text{Cos } x}$$

$$\text{Cot } x = \frac{\text{Cos } x}{\text{Sin } x}$$

"Pythagorean Identities"

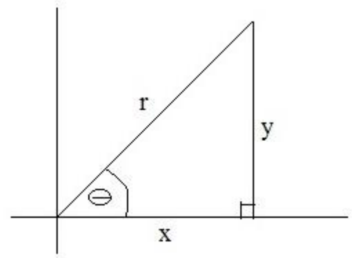
$$\text{Sin}^2\theta + \text{Cos}^2\theta = 1$$

$$1 + \text{Tan}^2\theta = \text{Sec}^2\theta$$

$$1 + \text{Cot}^2\theta = \text{Csc}^2\theta$$

Note: $(\text{Cos X})^2 = \text{Cos}^2\text{X}$

Notes



$$\text{Sin }\theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{y}{r}$$

$$\text{Csc }\theta = \frac{\text{hypotenuse}}{\text{opposite}} = \frac{r}{y}$$

$$\frac{1}{\frac{y}{r}} = \frac{r}{y}$$

$$\frac{\text{Sin }\theta}{\text{Cos }\theta} = \frac{\frac{y}{r}}{\frac{x}{r}} = \frac{y}{\cancel{r}} \cdot \frac{\cancel{r}}{x} = \frac{y}{x} = \text{Tan }\theta$$

(Pythagorean Theorem) In the right triangle above, we know

$$x^2 + y^2 = r^2$$

$$\text{Sin}^2 + \text{Cos}^2 = \left(\frac{y}{r}\right)^2 + \left(\frac{x}{r}\right)^2 = \frac{x^2 + y^2}{r^2} = \frac{r^2}{r^2} = 1$$

$$\frac{\text{Sin}^2 + \text{Cos}^2 = 1}{\text{Sin}^2} \longrightarrow \frac{\text{Sin}^2}{\text{Sin}^2} + \frac{\text{Cos}^2}{\text{Sin}^2} = \frac{1}{\text{Sin}^2}$$

$$1 + \text{Cot}^2 = \text{Csc}^2$$

Definitions

"Sign Identities" or "Odd/Even Identities"
(Negative Angles)

'Odd Functions'

Sin (-x) = − Sin x

Csc (-x) = − Csc x

Tan (-x) = −Tan x

Cot (-x) = − Cot x

'Even Functions'

Cos (-x) = Cos x

Sec (-x) = Sec x

"Double Angles"

Sin 2X = 2SinXCosX

$\text{Cos } 2X = \text{Cos}^2 X - \text{Sin}^2 X$

$= 2\text{Cos}^2 X - 1$

$= 1 - 2\text{Sin}^2 X$

$$\text{Tan } 2X = \frac{2\text{Tan}X}{1 - \text{Tan}^2 X}$$

"Sum and Difference Identities"

sin (A + B) = sinAcosB + cosAsinB

cos (A + B) = cosAcosB − sinAsinB

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

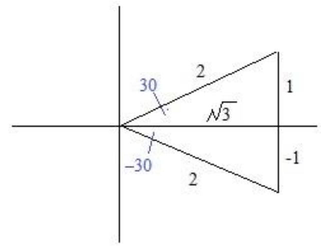
sin (A - B) = sinAcosB − cosAsinB

cos (A - B) = cosAcosB + sinAsinB

$$\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

Notes

Note: X^2 is an even function
X^3 is an odd function



Sin (-30) = -1/2 = −(Sin 30)

Cos (-30) = $\sqrt{3}/2$ = Cos 30

Tan (-30) = $-1/\sqrt{3}$ = − (Tan 30)

Examples:

1) Sin 2(90) ≠ 2 Sin (90) = 2 ✗

Sin 2(90) = Sin (180) = 0 ✓

= 2 Sin (90) Cos (90) = 2 (1) (0) = 0 ✓

2) Sin 2(30) ≠ 2 Sin 30 = 2 · 1/2 = 1 ✗

Sin 2(30) = Sin 60 = $\sqrt{3}/2$ ✓

or

2 Cos(30)Sin(30) = 2 · $\sqrt{3}/2$ · 1/2 = $\sqrt{3}/2$ ✓

3) sin 90 ≠ sin 30 + sin 60

1 ≠ 1/2 + $\sqrt{3}/2$ ✗

sin 90 = sin (30 + 60) = sin(30)cos(60) + sin(60)cos(30)

= 1/2 · 1/2 + $\sqrt{3}/2 \cdot \sqrt{3}/2$ ✓

= 1/4 + 3/4 = 1

Trigonometry Identities: Examples and Strategies

1) Simplify: $\cos(-x) \cdot \tan(-x)$

Strategy:
1) get rid of the negatives
2) try to change terms to sin's and cos's
3) simplify

$\cos(-x) \cdot \tan(-x)$ — cosine is an "even" identity; tan is an "odd" identity

$\cos x \cdot -\tan x$ — quotient identity (for tangent)

$\cos x \cdot \dfrac{-\sin x}{\cos x}$ — algebra/simplify

$-\sin x$

2) Prove: $\dfrac{\tan x}{\sin x} = \sec x$

Strategy:
1) choose 'more complex' side to simplify
2) try to change terms to sines and cosines
3) simplify

$\dfrac{\tan x}{\sin x} = \sec x$ — quotient identity (for tangent)

$\dfrac{\left(\dfrac{\sin x}{\cos x}\right)}{\sin x} = \sec x$ — simplify

$\dfrac{1}{\cos x} = \sec x$ — reciprocal identity

$\sec x = \sec x$

3) Prove: $\tan y + \cot y = \csc y \cdot \sec y$

Strategy:
1) recognize the reciprocals
2) try to change terms to sines and cosines
3) simplify (moving toward the objective)

$\tan y + \cot y = \csc y \cdot \sec y$ — ratio/quotient identities

$\dfrac{\sin y}{\cos y} + \dfrac{\cos y}{\sin y} = \csc y \cdot \sec y$ — common denominator/add fractions

$\dfrac{\sin^2 y + \cos^2 y}{\cos y \sin y} = \csc y \cdot \sec y$ — Pythagorean identities

$\dfrac{1}{\cos y \sin y} = \csc y \cdot \sec y$ — split the fraction

$\dfrac{1}{\cos y} \cdot \dfrac{1}{\sin y} = \csc y \cdot \sec y$ — reciprocal identites

$\csc y \cdot \sec y = \csc y \cdot \sec y$

4) Solve: $\text{Tanx} + 1 = \text{Secx}$

Try squaring both sides.

$$(\text{Tanx} + 1)^2 = \text{Sec}^2 x$$

$$\text{Tan}^2 x + 2\text{Tanx} + 1 = \text{Sec}^2 x \qquad 1 + \text{Tan}^2 x = \text{Sec}^2 x$$

$$2\text{Tanx} + \text{Sec}^2 x = \text{Sec}^2 x$$

$$2\text{Tanx} = 0$$

$$\text{Tanx} = 0$$

$$x = 0 \text{ and } \pi$$

Check answers!
(extraneous solutions?)

$$\text{Tanx} + 1 = \text{Secx} \qquad \text{Tan}(0) + 1 = \text{Sec}(0)$$

$$0 + 1 = 1 \checkmark$$

$$\text{Tan}(\pi) + 1 = \text{Sec}(\pi)$$

$$0 + 1 = -1 \times$$

$x = 0 + 2\pi k$

where k is an integer...

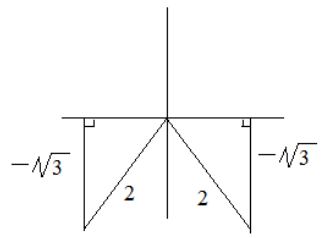
5) Solve: $2\sin\frac{\theta}{3} + \sqrt{3} = 0$

$$\sin\frac{\theta}{3} = \frac{\sqrt{3}}{-2}$$

Use substitution

$$U = \frac{\theta}{3}$$

$$\sin U = \frac{\sqrt{3}}{-2} \qquad \frac{\text{(opposite)}}{\text{(hypotenuse)}}$$



$$U = \frac{4\pi}{3}, \frac{5\pi}{3}$$

$$\text{Since } U = \frac{\theta}{3} = \frac{4\pi}{3} + 2\pi k$$

$$U = \frac{\theta}{3} = \frac{5\pi}{3} + 2\pi k$$

$\theta = 4\pi + 6\pi k$

$5\pi + 6\pi k$

6) Verify: $\dfrac{\tan x - \cot x}{\tan^2 x - \cot^2 x} = \sin x \cos x$

$$\frac{\frac{\sin x}{\cos x} - \frac{\cos x}{\sin x}}{\frac{\sin^2 x}{\cos^2 x} - \frac{\cos^2 x}{\sin^2 x}}$$

Change into sines and cosines
(quotient identity)

$$\frac{\frac{\sin x}{\sin x}\frac{\sin x}{\cos x} - \frac{\cos x}{\sin x}\frac{\cos x}{\cos x}}{\frac{\sin^2 x}{\cos^2 x}\frac{\sin^2 x}{\sin^2 x} - \frac{\cos^2 x}{\sin^2 x}\frac{\cos^2 x}{\cos^2 x}}$$

Common denominators

$$\frac{\frac{\sin^2 x - \cos^2 x}{\sin x\cos x}}{\frac{\sin^4 x - \cos^4 x}{\sin^2 x \cos^2 x}}$$

Combine numerator terms
and denominator terms

Invert and multiply

$$\frac{\sin^2 x - \cos^2 x}{\sin x\cos x} \cdot \frac{\sin^2 x \cos^2 x}{\sin^4 x - \cos^4 x}$$

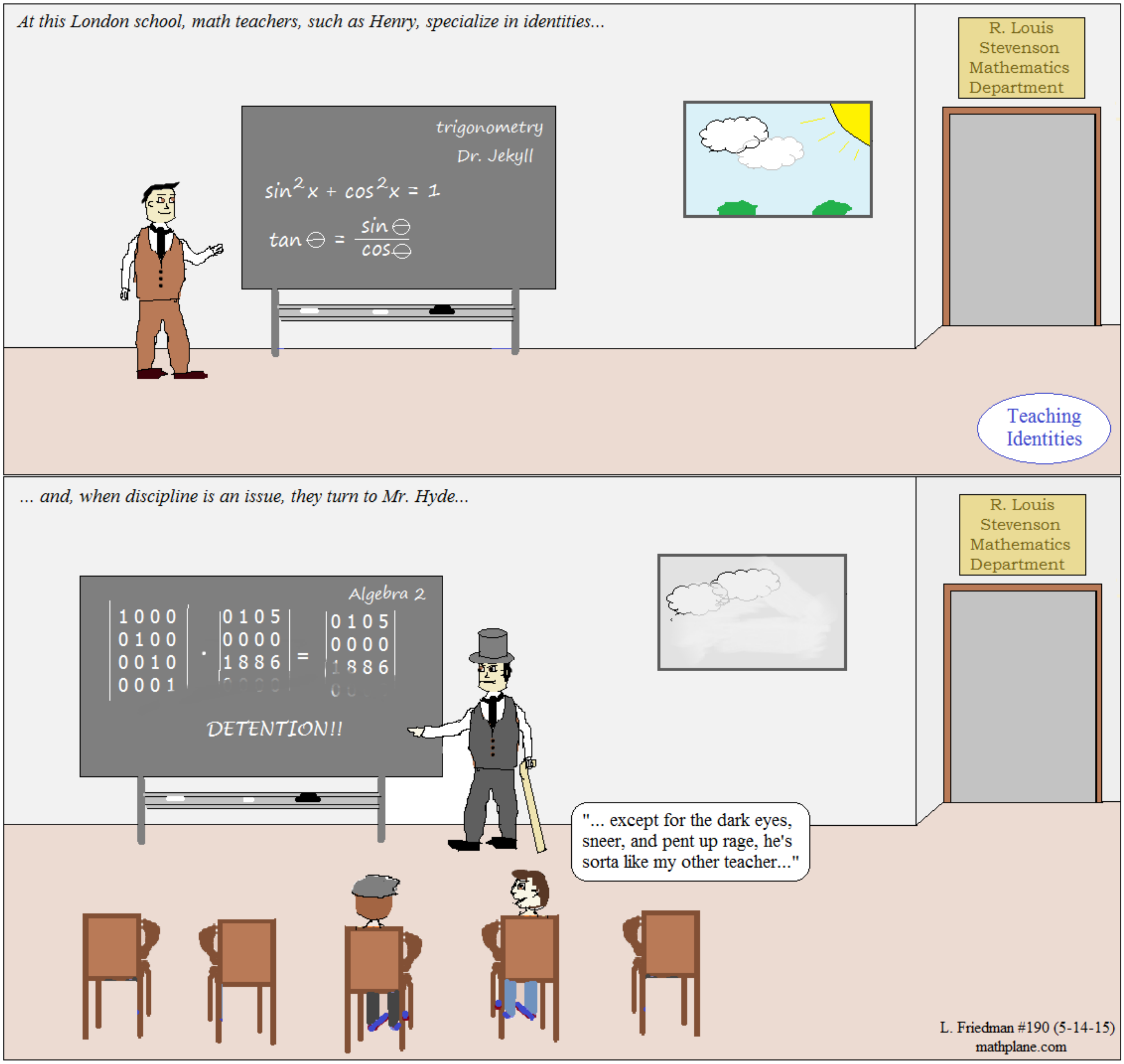
Simplify and
Factor (difference of squares)

$$\frac{\sin^2 x - \cos^2 x}{1} \cdot \frac{\sin x \cos x}{(\sin^2 x - \cos^2 x)(\sin^2 x + \cos^2 x)}$$

$$\frac{1}{1} \cdot \frac{\sin x\cos x}{(\sin^2 x + \cos^2 x)} = \sin x\cos x$$

$\text{Sin}^2 x + \text{Cos}^2 x = 1$
(trig identity)

$$\sin x\cos x = \sin x\cos x$$


At this London school, math teachers, such as Henry, specialize in identities...
R. Louis Stevenson Mathematics Department
trigonometry
Dr. Jekyll
sin²x + cos²x = 1
tan ⊖ = sin ⊖ / cos ⊖
Teaching Identities
... and, when discipline is an issue, they turn to Mr. Hyde...
R. Louis Stevenson Mathematics Department
Algebra 2
DETENTION!!
"... except for the dark eyes, sneer, and pent up rage, he's sorta like my other teacher..."
L. Friedman #190 (5-14-15)
mathplane.com

Using the Conjugate: $\dfrac{\sin A}{1-\cos A} = \dfrac{1+\cos A}{\sin A}$

$$\frac{\sin A}{1-\cos A} \cdot \frac{1+\cos A}{1+\cos A}$$

$$\frac{\sin A \cdot (1+\cos A)}{1-\cos^2 A}$$

$$\frac{\cancel{\sin A} \cdot (1+\cos A)}{\sin^{\cancel{2}} A} = \frac{1+\cos A}{\sin A}$$

Combine Fractions: $\dfrac{1}{1-\sin B} - \dfrac{1}{1+\sin B} = 2\sec B\tan B$

$$\frac{1+\sin B}{1+\sin B} \cdot \frac{1}{1-\sin B} - \frac{1}{1+\sin B} \cdot \frac{1-\sin B}{1-\sin B}$$

$$\frac{1+\sin B}{1-\sin^2 B} - \frac{1-\sin B}{1-\sin^2 B}$$

$$\frac{2\sin B}{1-\sin^2 B}$$

$$\frac{2\sin B}{\cos^2 B}$$

$$\frac{1 \cdot 2\sin B}{\cos B \cdot \cos B} = 2\sec B\tan B$$

Factoring Terms: $1 - \dfrac{\sin^2 x}{1-\cos x} = -\cos x$

$$\frac{1-\cos x}{1-\cos x} - \frac{\sin^2 x}{1-\cos x}$$

$$\frac{1-\sin^2 x - \cos x}{1-\cos x}$$

$$\frac{\cos^2 x - \cos x}{1-\cos x}$$

$$\frac{\cos x(\cos x - 1)}{1-\cos x}$$

$$\frac{\cos x(\cos x - 1)}{(-1)(\cos x - 1)} = -\cos x$$

Recognize Identity and Factors: $\dfrac{1+\sin A}{\sin A} = \dfrac{\cot^2 A}{\csc A - 1}$

$$\frac{\csc^2 A - 1}{\csc A - 1}$$

$$\frac{(\csc A - 1)(\csc A + 1)}{\csc A - 1}$$

$$\csc A + 1$$

$$\frac{1}{\sin A} + \frac{\sin A}{\sin A} = \frac{1+\sin A}{\sin A}$$

Using conjugate to shrink expression: $\dfrac{\csc x - 1}{1 - \sin x} = \csc x$

$$\frac{\csc x - 1}{1-\sin x} \cdot \frac{1+\sin x}{1+\sin x}$$

$$\frac{\csc x + \csc x \sin x - 1 - \sin x}{1 - \sin^2 x}$$

$$\frac{\csc x - \sin x}{\cos^2 x}$$

$$\frac{\dfrac{1}{\sin x} - \dfrac{\sin^2 x}{\sin x}}{\cos^2 x}$$

$$\frac{\dfrac{1 - \sin^2 x}{\sin x}}{\cos^2 x}$$

$$\frac{\dfrac{\cos^2 x}{\sin x}}{\cos^2 x} = \csc x$$

Multiple approaches: $\sin^2 x + \cot^2 x \sin^2 x = 1$

1) factoring

$$\sin^2 x\,(1 + \cot^2 x) = 1$$

$$\sin^2 x\,(\csc^2 x) = 1$$

$$1 = 1$$

2) change to sines and cosines

$$\sin^2 x + \frac{\cos^2 x}{\sin^2 x}\sin^2 x = 1$$

$$\sin^2 x + \cos^2 x = 1$$

Combine fractions to condense expression: $\frac{\tan x}{1+\sec x} + \frac{1+\sec x}{\tan x} = 2\csc x$

Trigonometry Identity Techniques

$$\frac{\tan^2 x + (1+\sec x)^2}{\tan x(1+\sec x)}$$

$$\frac{\tan^2 x + 1 + 2\sec x + \sec^2 x}{\tan x(1+\sec x)}$$

$$\frac{\sec^2 x + 2\sec x + \sec^2 x}{\tan x(1+\sec x)}$$

$$\frac{2\sec x(1+\sec x)}{\tan x(1+\sec x)}$$

$$\frac{2\frac{1}{\cos x}}{\frac{\sin x}{\cos x}}$$

$$\frac{2}{\sin x} = 2\csc x$$

Recognizing unusual identity: $\frac{\sin^2 A}{1+(\sin\frac{\pi}{2}-A)} + \cos A = 1$

$$\frac{\sin^2 A}{1+\cos A} + \cos A$$

$$\frac{(1+\cos A)(1-\cos A)}{1+\cos A} + \cos A$$

$$1-\cos A + \cos A = 1$$

Change to sines and cosines: $2\sec^2 x - 2\sec^2 x\sin^2 x - \sin^2 x - \cos^2 x = 1$

$$\frac{2}{\cos^2 x} - \frac{2\sin^2 x}{\cos^2 x} - \sin^2 x - \cos^2 x$$

$$\frac{2-2\sin^2 x}{\cos^2 x} - \sin^2 x - \cos^2 x$$

$$\frac{2(1-\sin^2 x)}{\cos^2 x} - \sin^2 x - \cos^2 x$$

$$2 - \sin^2 x - \cos^2 x$$

$$1 - \sin^2 x + 1 - \cos^2 x$$

$$\cos^2 x + \sin^2 x = 1$$

Using Trig Identities, factoring, and Unit Circle Angles

A) $\tan X + \cot X = -2$

Step 1: I want to get rid of the inverse.. (CotX) Multiply everything by TanX

$\tan^2 X + 1 = -2\tan X$

Step 2: Move everything to one side and factor

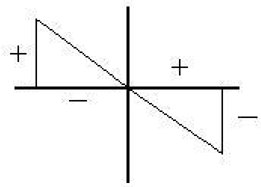
$\tan^2 X + 2\tan X + 1 = 0$

$(\tan X + 1)(\tan X + 1) = 0$

Step 3: Find solutions and check

$\tan X = -1$

$X = \frac{-\pi}{4} \quad \frac{3\pi}{4}$



Tan (-45) = -1
Tan (135) = -1

Check: $\tan \frac{-\pi}{4} + \cot \frac{-\pi}{4}$

$= -1 + -1 = -2$

B) $2\cos^2 X - \cos X = 2 - \sec X$

Step 1: Factor the left side to see if it leads to anything.

$\cos X\,(2\cos X - 1) = \frac{2\cos X}{\cos X} - \frac{1}{\cos X}$

Step 2: Combine the elements on the right side.

$(2\cos X - 1)\cos X = \frac{2\cos X - 1}{\cos X}$

Step 3: Move everything to one side and factor.

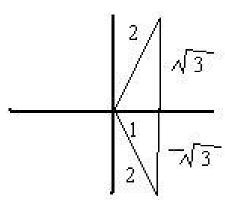
$(2\cos X - 1)\cos X - (2\cos X - 1)\left(\frac{1}{\cos X}\right) = 0$

(distributive property to rearrange and regroup)

$(2\cos X - 1)\left(\cos X - \frac{1}{\cos X}\right) = 0$

Step 4: Solve and check.

$2\cos X - 1 = 0$
$\cos X = 1/2$
X = 60, 300



Check X = 60

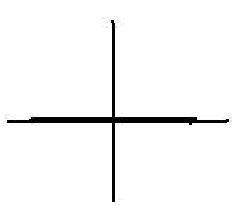
$2(1/2)^2 - 1/2 = 2 - 2$
$2(1/4) - 1/2 = 0$
$1/2 - 1/2 = 0$
$0 = 0$

$\cos X = 1/\cos X$ (multiply both sides by Cosine)

$\cos^2 X = 1$ (square root both sides)

$\cos X = 1, -1$

X = 0, 180



C) Cot X = CscX -2

Step 1: Try to simplify and get a common trig sign.

Multiply by TanX (to get rid of the CotX)
--- since $1 + Tan^2 = Sec^2$

$$1 = (TanX)\, CscX - 2TanX$$

$$1 + 2TanX = \frac{Sin\ X}{Cos\ X} \cdot \frac{1}{SinX}$$

$$1 + 2TanX = SecX$$

simplify
square both sides
and substitute

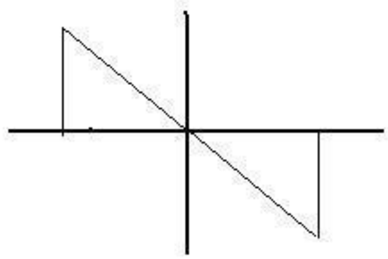
$$1 + 4TanX + 4Tan^2 X = Sec^2 X$$

$$1 + 4TanX + 4Tan^2 X = 1 + Tan^2 X$$

Step 2: Factor and solve.

$$3Tan^2 X + 4TanX = 0$$

$$TanX\,(3TanX + 4) = 0$$



TanX = 0
X = 0, 180

~~0~~, ~~π~~ radians

3TanX + 4 = 0
TanX = -4/3
X = -53, 127

~~.927~~, 2.214
Radians

**Must check your answers!

Cot X = CscX - 2

Cot 0 is undefined.. Cot of 180 is undefined...

Reminder: $0 \le$ Sine X ≤ 1
therefore $1 \le$ CscX
.927 cannot work..

Finally, 2.214 or 127 degrees..

Cot (127) = Csc (127) - 2
-.75 = 1.25 -2

-.75 = -.75

Examples: Solve the following:

1) $\text{Cos}\,\theta\,\text{Cot}\,\theta = 2\text{Cos}\,\theta$ For $0^\circ \leq \theta \leq 360^\circ$

$\text{Cos}\,\theta\,\text{Cot}\,\theta = 2\text{Cos}\,\theta$

Subtract $2\text{Cos}\,\theta$ from both sides
(This produces an equation = 0)

$\text{Cos}\,\theta\,\text{Cot}\,\theta - 2\text{Cos}\,\theta = 0$

Factor out $\text{Cos}\,\theta$

$\text{Cos}\,\theta\,(\text{Cot}\,\theta - 2) = 0$

Separate and solve

$\text{Cos}\,\theta = 0$

$\theta = 90^\circ$ or 270°

$(\text{Cot}\,\theta - 2) = 0$

$\text{Cot}\,\theta = 2$

$\theta = 26.6^\circ$ or 206.6°

*Note: At the beginning, we didn't divide both sides by Cosine. (this could cancel possible solutions). Instead, we moved everything to one side and factored out the Cosine.

2) $2\text{Cos}^2\theta + 3\text{Sin}\,\theta = 3$ For $0^\circ \leq \theta \leq 360^\circ$

$2\text{Cos}^2\theta + 3\text{Sin}\,\theta = 3$

We see a Cosine term and a Sine term. To make them the same, we can substitute the identity: $(1 - \sin^2) = \cos^2$

$2(1 - \text{Sin}^2\theta) + 3\text{Sin}\,\theta - 3 = 0$

Then, simplify the equations, placing all terms on the left equal to zero.

$2 - 2\,\text{Sin}^2\theta + 3\text{Sin}\,\theta - 3 = 0$

$2\text{Sin}^2\theta - 3\text{Sin}\,\theta + 1 = 0$

Separate and Solve

$(2\text{Sin}\,\theta - 1)(\text{Sin}\,\theta - 1) = 0$

$2\text{Sin}\,\theta - 1 = 0$

$\text{Sin}\,\theta = 1/2$

$\theta = 30^\circ$ or 150°

$\text{Sin}\,\theta - 1 = 0$

$\text{Sin}\,\theta = 1$

$\theta = 90^\circ$

Solve the following Trig Problems Algebraically.
Then, verify your solutions Graphically.

1) $y = \cos 2\theta$ and $y = \sin\theta$ $\quad 0^\circ \le \theta < 360^\circ$

2) $y = \cos 2x$ and $y = \cos x + 2$ $\quad 0 \le x < 2\pi$

SOLUTIONS

1) $y = \cos 2\theta$ and $y = \sin\theta$

(To find solutions, set equations equal to each other)

$\cos 2\theta = \sin\theta$ — Substitution (Double Angle Identity)

$1 - 2\sin^2\theta = \sin\theta$ — Set equation equal to zero

$1 - 2\sin^2\theta - \sin\theta = 0$ — Re-arrange the polynomial

$2\sin^2\theta + \sin\theta - 1 = 0$ — Factor

$(2\sin\theta - 1)(\sin\theta + 1) = 0$ — Solve

$2\sin\theta - 1 = 0$

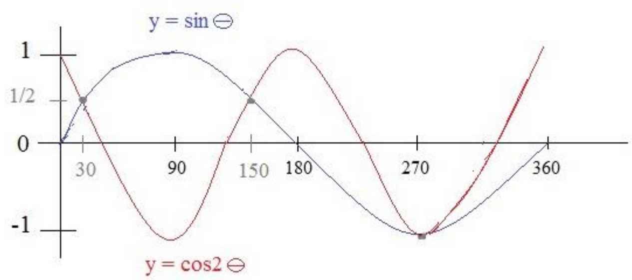
$\sin\theta = \frac{1}{2}$

$\theta = 30^\circ$ or 150°

$\sin\theta + 1 = 0$

$\sin\theta = -1$

$\theta = 270^\circ$



2) $y = \cos 2x$ and $y = \cos x + 2$

(Set equations equal to each other)

$\cos 2x = \cos x + 2$ — Substitution (Double Angle Identity)

$2\cos^2 x - 1 = \cos x + 2$ — Set equation equal to zero

$2\cos^2 x - 1 - \cos x - 2 = 0$ — Re-arrange the polynomial

$2\cos^2 x - \cos x - 3 = 0$ — Factor

$(2\cos x - 3)(\cos x + 1) = 0$ — Solve

$2\cos x - 3 = 0$

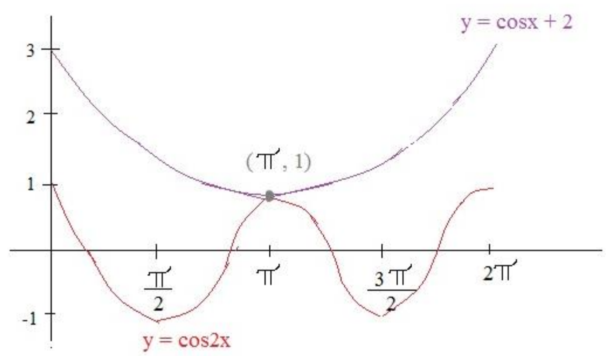
~~$\cos x = \frac{3}{2}$~~

No Solution!
($\cos \not> 1$)

$\cos x + 1 = 0$

$\cos x = -1$

$x = \pi$



$x = \pi \quad \begin{cases} \cos 2\pi = 1 \ ✓ \\ \cos\pi + 2 = 1 \ ✓ \end{cases}$

Study Break:
Math Snacks

LanceAF #35 6-3-12
www.mathplane.com

Preferable to ordinary computer cookies...

Essential part of a well-rounded, academic diet.

Try with (t), or any beverage...

Also, look for Honey Graham Squares
in the geometry section of your local store...

Practice Quiz -→

Quiz: Factoring and Trig Identities

I. Solve for $0 \leq \theta < 360$ and $0 \leq X < 2\pi$

1) $\tan^2\theta + \tan\theta - 12 = 0$

2) $2\cos^2 X = 3\sin X$

3) $5 - 7\sin X = 2\cos^2 X$

4) $2\sec^2 X - 2\tan X = 6$

5) $4\cos^2\theta = -2\cos\theta$

6) $\cos X + \sin X \tan X = 2$

Quiz: Factoring and Trig Identities

II. Simplify the following

1) $\dfrac{\tan^2\theta}{\sec\theta + 1} + 1$

2) $(\sec^2\theta - 1)(\csc^2\theta - 1)$

3) $\dfrac{\cot A + \tan A}{\csc^2 A}$

4) $\dfrac{1 + \cos 2x}{\cot x}$

Quiz: Factoring and Trig Identities

III. Solving more trig equations

1) Solve for x where $0 \le x < 2\pi$

$$5\sin x - \sqrt{3} = 3\sin x$$

2) Solve for y where $0^\circ \le y < 360^\circ$

$$\sqrt{2}\sin y\cos y - \sin y = 0$$

3) Solve for B where $0 \le B < 2\pi$

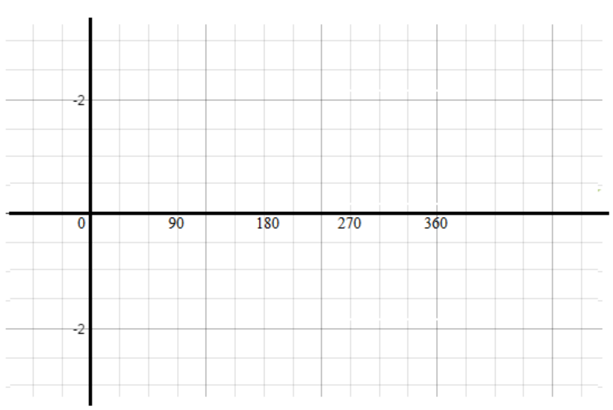
$$\sin B \cdot \tan^2 B = \sin B$$

4) Find a general solution (in degrees)

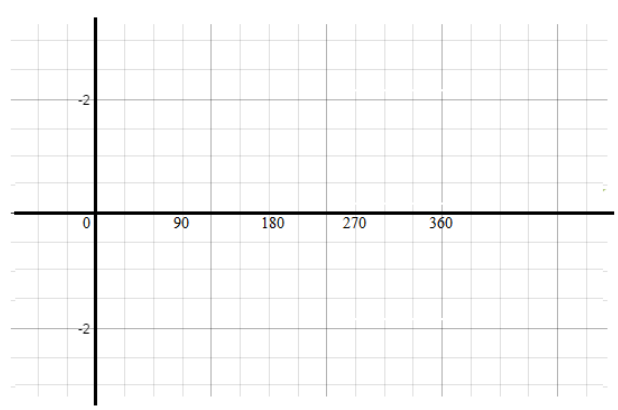
$$12\tan^2\theta - 4 = 0$$

5) Find all solutions and graph:

$$2\sin x - \csc x = 0$$



6) Solve and graph: $1 - \sin x = \sqrt{3}\cos x$



Quiz: Factoring and Trig Identities

SOLUTIONS

I. Solve for $0 \le \theta < 360$ and $0 \le X < 2\pi$

1) $\tan^2\theta + \tan\theta - 12 = 0$

$(\tan\theta + 4)(\tan\theta - 3) = 0$

$\tan\theta + 4 = 0$ $\quad$ $\tan\theta - 3 = 0$

$\tan\theta = -4$ $\quad$ $\tan\theta = 3$

$\theta = 104^\circ, 284^\circ$ $\quad$ $\theta = 71.5^\circ, 251.5^\circ$

2) $2\cos^2 X = 3\sin X$

$2(1 - \sin^2 X) = 3\sin X$

$2 - 2\sin^2 X = 3\sin X$

$2\sin^2 X + 3\sin X - 2 = 0$

$(2\sin X - 1)(\sin X + 2) = 0$

$(\sin X + 2) = 0$

~~$\sin X = -2$~~ Extraneous

$2\sin X - 1 = 0$

$\sin X = \frac{1}{2}$

$X = \frac{\pi}{6} \quad \frac{5\pi}{6}$

3) $5 - 7\sin X = 2\cos^2 X$

$5 - 7\sin X = 2(1 - \sin^2 X)$

$5 - 7\sin X = 2 - 2\sin^2 X$

$2\sin^2 X - 7\sin X + 3 = 0$

$(2\sin X - 1)(\sin X - 3) = 0$

$(\sin X - 3) = 0$

~~$\sin X = 3$~~ Extraneous

$(2\sin X - 1) = 0$

$\sin X = \frac{1}{2}$

$X = \frac{\pi}{6} \quad \frac{5\pi}{6}$

4) $2\sec^2 X - 2\tan X = 6$

$2(1 + \tan^2 X) - 2\tan X = 6$

$2 + 2\tan^2 X - 2\tan X - 6 = 0$

$2\tan^2 X - 2\tan X - 4 = 0$

$\tan^2 X - \tan X - 2 = 0$

$(\tan X - 2)(\tan X + 1) = 0$

$\tan X - 2 = 0$

$\tan X = 2$

$X = 63.4^\circ \quad 243.4^\circ$

1.11 Radians $\quad$ 4.25 Radians

$\tan X + 1 = 0$

$\tan X = -1$

$X = 135^\circ \quad 315^\circ$

$\frac{3\pi}{4}$ Rad $\quad$ $\frac{7\pi}{4}$ Rad

5) $4\cos^2\theta = -2\cos\theta$

$4\cos^2\theta + 2\cos\theta = 0$

$2\cos^2\theta + \cos\theta = 0$

$\cos\theta(2\cos\theta + 1) = 0$

$\cos\theta = 0$

$\theta = 90^\circ \quad 270^\circ$

$2\cos\theta + 1 = 0$

$\cos\theta = -\frac{1}{2}$

$\theta = 120^\circ \quad 240^\circ$

6) $\cos X + \sin X \tan X = 2$

$\cos X + \sin X \frac{\sin X}{\cos X} = 2$

$\cos X + \frac{\sin^2 X}{\cos X} = 2$

$\cos^2 X + \sin^2 X = 2\cos X$

$1 = 2\cos X$

$\cos X = \frac{1}{2}$

$X = \frac{\pi}{3} \quad \frac{5\pi}{3}$

**Note: To check these answers, simply plug solutions into original equation.

II. Simplify the following

Solutions

Quiz: Factoring and Trig Identities

1) $\frac{\tan^2\theta}{\sec\theta + 1} + 1$

$\frac{(\sec^2\theta - 1)}{\sec\theta + 1} + 1$

Identity:

$1 + \tan^2 x = \sec^2 x$

$\frac{(\sec\theta + 1)(\sec\theta - 1)}{\sec\theta + 1} + 1$

$\sec\theta - 1 + 1 = \boxed{\sec\theta}$

2) $(\sec^2\theta - 1)(\csc^2\theta - 1)$

Identities:

$1 + \tan^2 x = \sec^2 x$

$1 + \cot^2 x = \csc^2 x$

$(\tan^2\theta)(\cot^2\theta) = \boxed{1}$

tan and cotangent are reciprocals

3) $\frac{\cot A + \tan A}{\csc^2 A}$

Identities:

$\tan = \frac{\sin}{\cos}$

$\cot = \frac{\cos}{\sin}$

$\frac{\frac{\cos A}{\sin A} + \frac{\sin A}{\cos A}}{\csc^2 A}$

$\frac{\frac{\cos^2 A}{\sin A\cos A} + \frac{\sin^2 A}{\cos A\sin A}}{\csc^2 A}$

$\frac{\frac{\cos^2 A + \sin^2 A}{\sin A\cos A}}{\csc^2 A}$

$\frac{\frac{1}{\sin A\cos A}}{\frac{1}{\sin^2 A}}$

$\frac{\sin^2 A}{\sin A\cos A} = \frac{\sin A}{\cos A} = \boxed{\tan A}$

4) $\frac{1 + \cos 2x}{\cot x}$

Identities:

$\cos 2x = \cos^2 x - \sin x$

$\cos 2x = 2\cos^2 x - 1$

$\sin 2x = 2\sin x\cos x$

$\frac{1 + 2\cos^2 x - 1}{\cot x}$

$\frac{2\cos^2 x}{\frac{\cos x}{\sin x}} = 2\cos^2 x \cdot \frac{\sin x}{\cos x} = 2\cos x\sin x = \boxed{\sin 2x}$

III. Solving more trig equations

SOLUTIONS

Quiz: Factoring and Trig Identities

1) Solve for x where $0 \le x < 2\pi$

$$5\sin x - \sqrt{3} = 3\sin x$$

$$2\sin x = \sqrt{3}$$

$$\sin x = \frac{\sqrt{3}}{2}$$

$$x = \frac{\pi}{3}, \frac{2\pi}{3}$$

2) Solve for y where $0^\circ \le y < 360^\circ$

$$\sqrt{2}\sin y\cos y - \sin y = 0$$

$$\cos y(\sqrt{2}\sin y - 1) = 0$$

$\cos y = 0$ 90 and 270 degrees

$$\sqrt{2}\sin y - 1 = 0$$

$\sin y = \frac{1}{\sqrt{2}}$ 45 and 135 degrees

3) Solve for B where $0 \le B < 2\pi$

$$\sin B \cdot \tan^2 B = \sin B$$

NOTE: Do not divide both sides by sinB (because you may cancel a solution)... Instead factor the expression.

$$\sin B \cdot \tan^2 B - \sin B = 0$$

$$\sin B(\tan^2 B - 1) = 0$$

$\sin B = 0$ $B = 0, \pi$

$$\tan^2 B - 1 = 0$$

$$\tan^2 B = 1$$

$$\sqrt{\tan^2 B} = \sqrt{1}$$

$\tan B = 1$ or $\tan B = -1$

$$B = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$$

4) Find a general solution (in degrees)

$$12\tan^2\theta - 4 = 0$$

$$12\tan^2\theta = 4$$

"the square root of a square is plus and minus"

$$\tan^2\theta = \frac{1}{3}$$

$$\tan\theta = \pm\frac{1}{\sqrt{3}}$$

$$\tan^{-1}\theta = -\frac{1}{\sqrt{3}}$$

$$\tan^{-1}\theta = +\frac{1}{\sqrt{3}}$$

$-30^\circ, 150^\circ, 330^\circ, \ldots$

$30^\circ, 210^\circ, 390^\circ, \ldots$

$30^\circ + 180^\circ k$

$-30^\circ + 180^\circ k$

where k is any integer

5) Find all solutions and graph:

$$2\sin x - \csc x = 0$$

(multiply by sinx)

$$2\sin^2 x - 1 = 0$$

$$\sin^2 x = \frac{1}{2}$$

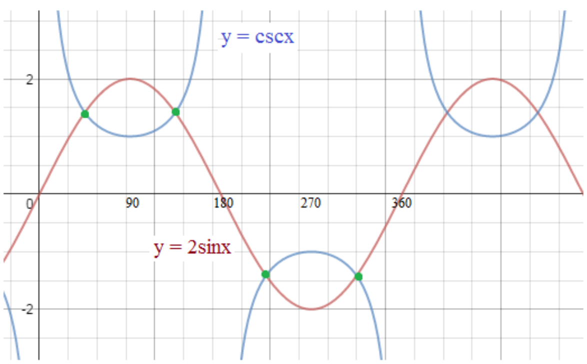
$$\sin x = +\frac{1}{\sqrt{2}} \quad -\frac{1}{\sqrt{2}}$$

$x = 45, 135, 225, 315$

(then, check for extraneous)

Graph 2sinx and cscx

The intersections of these 2 equations are the solutions!



6) Solve and graph: $1 - \sin x = \sqrt{3}\cos x$

$$1 - 2\sin x + \sin^2 x = 3\cos^2 x$$

$$1 - 2\sin x + \sin^2 x = 3(1 - \sin^2 x)$$

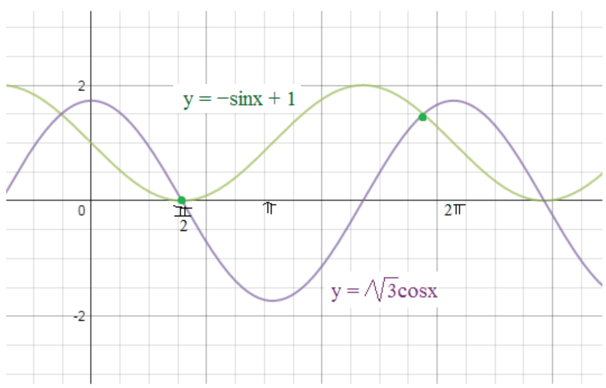
$$4\sin^2 x - 2\sin x - 2 = 0$$

$$2(2\sin x + 1)(\sin x - 1) = 0$$

$$x = \cancel{\frac{7\pi}{6}}, \frac{11\pi}{6}, \frac{\pi}{2}$$

Then, check for extraneous solutions....

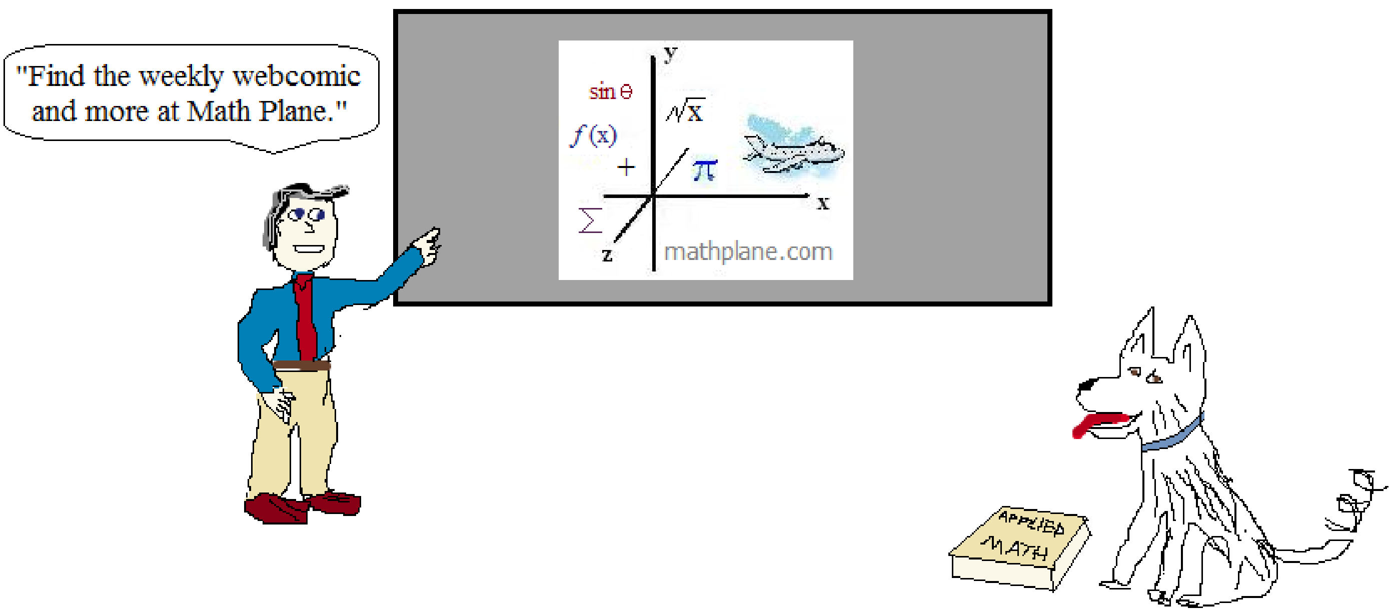
$\frac{\pi}{2}$ and $\frac{11\pi}{6}$



Thanks for visiting. (Hope it helped!)

If you have questions, suggestions, or requests, let us know.

Good luck!



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