Trigonometry Identities V: Honors

Examples and Practice Test (with Solutions)

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WHOLESALE
"What did you get in the bag?"
"Math stuff: calculator, pencils, paper. I'm taking an honors class."
Back to School Supplies
"What's in the big basket?"
"Erasers... Lots of erasers..."
Sometimes it's best to buy an item in bulk!
LanceAF #287 (7-20-17)
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Examples-→

Verifying Trigonometry Identities

Example: $$\frac{\tan x - \cot x}{\tan^2 x - \cot^2 x} = \sin x \cos x$$

Using Pythagorean Identities
and Quotient Identities

$$\frac{\dfrac{\sin x}{\cos x} - \dfrac{\cos x}{\sin x}}{\dfrac{\sin^2 x}{\cos^2 x} - \dfrac{\cos^2 x}{\sin^2 x}}$$

Convert to sines and cosines using quotient identities

$$\frac{\dfrac{\sin^2 x - \cos^2 x}{\cos x \sin x}}{\dfrac{\sin^4 x - \cos^4 x}{\cos^2 x \sin^2 x}}$$

Combine numerator

and

Combine denominator

Simplify entire rational expression

$$\frac{\sin^2 x - \cos^2 x}{\cancel{\cos x \sin x}} \bullet \frac{\cancel{\cos^2 x \sin}^2 x}{\sin^4 x - \cos^4 x}$$

$$\frac{\cancel{\sin^2 x - \cos^2 x}}{1} \bullet \frac{\cos x \sin x}{\cancel{(\sin^2 x - \cos^2 x)}(\sin^2 x + \cos^2 x)}$$

$$\frac{\cos x \sin x}{\cancel{(\sin^2 x + \cos^2 x)}} = \sin x \cos x$$ ✓

Example: $$\frac{\cos(x)}{1 - \tan(x)} + \frac{\sin(x)}{1 - \cot(x)} = \cos(x) + \sin(x)$$

change to sines and cosines (quotient identities)

$$\frac{\cos(x)}{1 - \dfrac{\sin(x)}{\cos(x)}} + \frac{\sin(x)}{1 - \dfrac{\cos(x)}{\sin(x)}}$$

condense the denominators

$$\frac{\cos(x)}{\dfrac{\cos(x) - \sin(x)}{\cos(x)}} + \frac{\sin(x)}{\dfrac{\sin(x) - \cos(x)}{\sin(x)}}$$

divide each rational expression

$$\frac{\cos(x)}{1} \cdot \frac{\cos(x)}{\cos(x) - \sin(x)} + \frac{\sin(x)}{1} \cdot \frac{\sin(x)}{\sin(x) - \cos(x)}$$

$$\frac{\cos^2(x)}{\cos(x) - \sin(x)} + \frac{\sin^2(x)}{\sin(x) - \cos(x)}$$

multiply 2nd expression by -1/-1

$$\frac{\cos^2(x)}{\cos(x) - \sin(x)} - \frac{\sin^2(x)}{\cos(x) - \sin(x)}$$

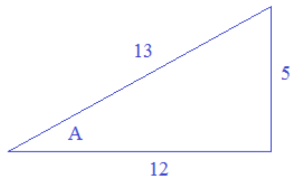
$$\frac{\cos^2(x) - \sin^2(x)}{\cos(x) - \sin(x)}$$

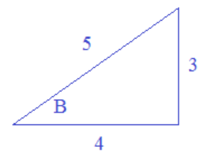
factor the numerator, then simplify...

$$\frac{(\cos(x) + \sin(x))(\cos(x) - \sin(x))}{\cos(x) - \sin(x)} = \cos(x) + \sin(x)$$ ✓

Example: $\sin\left(\sin^{-1}\frac{5}{13} - \cos^{-1}\frac{4}{5} \right)$

$\sin(A - B)$ Label angles and draw triangles





$\sin A\cos B - \cos A\sin B$

$$\frac{5}{13}\cdot\frac{4}{5} - \frac{12}{13}\cdot\frac{3}{5}$$

$$\boxed{\frac{-16}{65}}$$

calculator check:

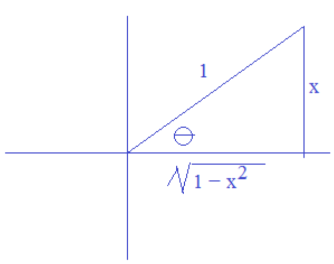
$\sin(22.62^\circ - 36.87^\circ)$

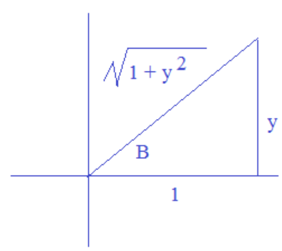
$\sin(-14.25^\circ) = -.246$

approx. = -16/65

Example: $\cos(\sin^{-1}x - \tan^{-1}y)$

$\cos(\theta - B)$





$\cos\theta\cos B + \sin\theta\sin B$

$$\frac{\sqrt{1-x^2}}{1}\cdot\frac{1}{\sqrt{1+y^2}} + \frac{x}{1}\cdot\frac{y}{\sqrt{1+y^2}} \Longrightarrow \boxed{\frac{\sqrt{1-x^2} + xy}{\sqrt{1+y^2}}}$$

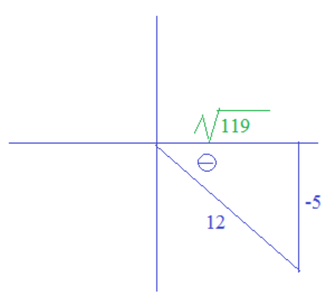
Example: Evaluate $\sin(\theta + B)$ if $\theta = \csc^{-1}\left(\frac{-12}{5}\right)$ inverse function for csc is in quadrant IV

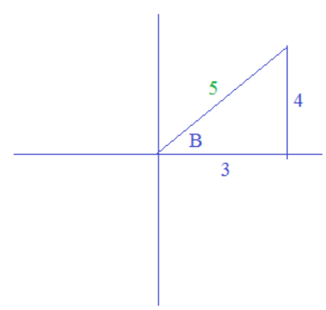
$B = \cot^{-1}\left(\frac{3}{4}\right)$ inverse function for cot is in quadrant I

$\sin\theta\cos B - \cos\theta\sin B$

$$\frac{-5}{12}\ \frac{3}{5} - \frac{\sqrt{119}}{12}\ \frac{4}{5}$$

$$\boxed{\frac{-1}{4} - \frac{\sqrt{119}}{15}}$$





Example: $\tan^2 x = \frac{3}{2}\sec x$

Since we want a "common trig function", we'll use the identity:

$1 + \tan^2 x = \sec^2 x$

$\sec^2 x - 1 = \frac{3}{2}\sec x$ (multiply by 2 for convenience)

$2\sec^2 x - 2 = 3\sec x$ (rearrange)

$2\sec^2 x - 3\sec x - 2 = 0$ (factor)

$(2\sec x + 1)(\sec x - 2) = 0$ (solve)

$2\sec x + 1 = 0$

$\sec x = \frac{-1}{2}$

Not possible

$\sec x - 2 = 0$

$\sec x = 2$

$\boxed{x = 60, 300 \text{ degrees} \quad \text{or} \quad \frac{\pi}{3} \quad \frac{5\pi}{3}}$

Example: $-11 = -3 - 4\csc(\theta + 225^\circ)$ find $0^\circ < \theta < 360^\circ$

$2 = \csc(\theta + 225^\circ)$

$\csc^{-1}(2) = (\theta + 225^\circ)$

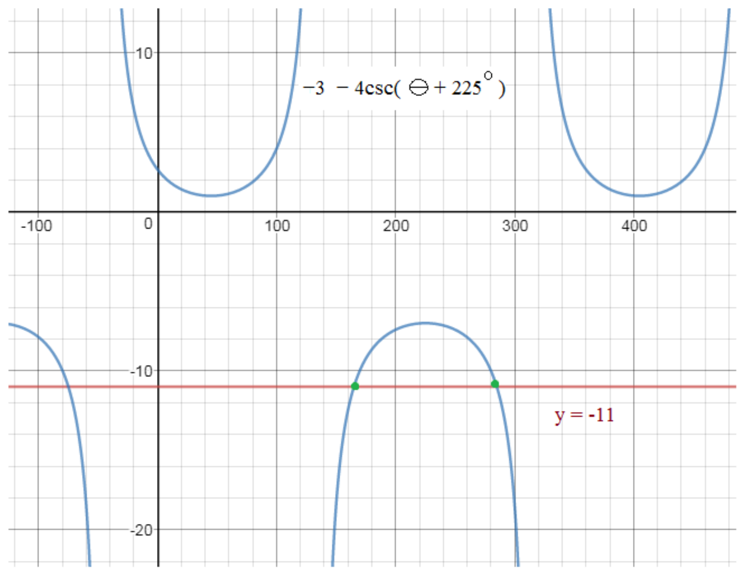
$30^\circ = (\theta + 225^\circ)$ $\theta = -195$

$150^\circ = (\theta + 225^\circ)$ $\theta = -75$

$390^\circ = (\theta + 225^\circ)$ $\boxed{\theta = 165^\circ}$

$510^\circ = (\theta + 225^\circ)$ $\boxed{\theta = 285^\circ}$

etc...



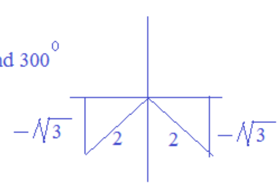
Example: $\sin(2\theta + 30^\circ) = \frac{-\sqrt{3}}{2}$

Find θ, where $0^\circ < \theta < 360^\circ$

Let $A = (2\theta + 30^\circ)$

$\sin A = \frac{-\sqrt{3}}{2}$

$A = 240^\circ$ and 300°



Also, $A = 600^\circ$ and 660°

$\boxed{105, 135, 285, 315}$

$(2\theta + 30^\circ) = 240^\circ$

$\theta = 105^\circ$

$(2\theta + 30^\circ) = 300^\circ$

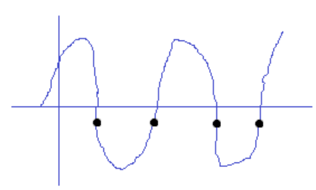
$\theta = 135^\circ$

$(2\theta + 30^\circ) = 600^\circ$

$\theta = 285^\circ$

$(2\theta + 30^\circ) = 660^\circ$

$\theta = 315^\circ$



Solving Trig Equations: Techniques

Example: $\cos x + \sin x = 0$

method 1

square both sides

$$\underbrace{\cos^2 x + \sin^2 x}_{1} + 2\sin x\cos x = 0$$

$2\sin x\cos x = -1$

$\sin(2x) = -1$

$2x = 270, 630, \text{etc..}$

$x = 135, 315, \ldots$

OR

method 2

use trig quotient identity...

$\cos x = -\sin x$

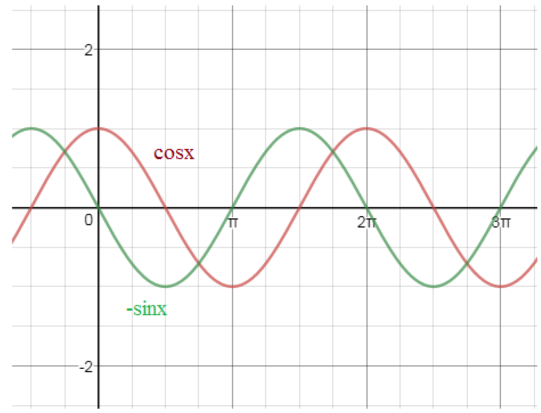
$-1 = \dfrac{\sin x}{\cos x}$

$-1 = \tan x$

$x = \dfrac{3\pi}{4}$ and $\dfrac{7\pi}{4}$

To solve by graphing, find the intersections of cosx and −sinx

$\cos x + \sin x = 0 \Longrightarrow \cos x = -\sin x$



Example: $\tan\theta = 2\sin\theta$

$\dfrac{\sin\theta}{\cos\theta} = 2\sin\theta$

Important: Don't divide by sine... (It may erase a solution!)

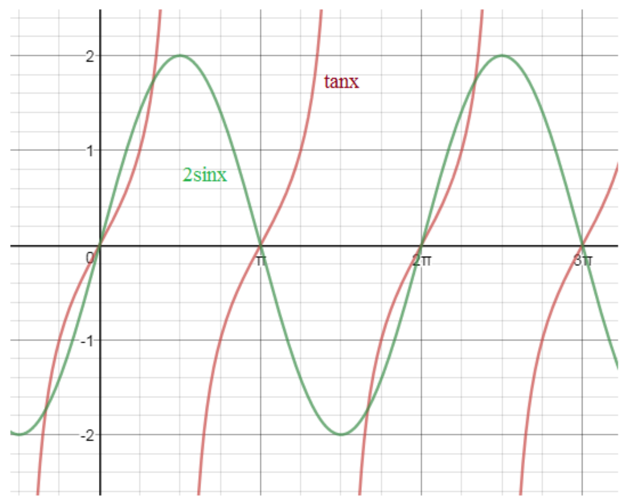
Instead, factor out the sine...

$\dfrac{\sin\theta}{\cos\theta} - 2\sin\theta = 0$

$\sin\theta\left(\dfrac{1}{\cos\theta} - 2\right) = 0$

$\sin\theta = 0 \qquad \sec\theta = 2$

$\theta = 0, 180 \qquad \theta = 60, 300$



Example: $\cos 2A = \cos A$

In this form, cosine is NOT the common factor...

Use trig identity to change double angle...

$\cos^2 A - \sin^2 A = \cos A$

$\cos^2 A - (1 - \cos^2 A) = \cos A$

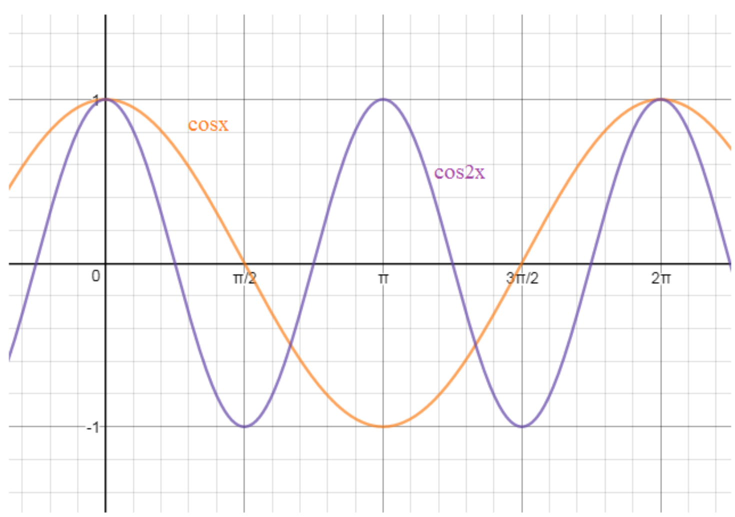
Now, the equation includes common factors, cosA

$2\cos^2 A - \cos A - 1 = 0$

$(2\cos A + 1)(\cos A - 1) = 0$

$\cos A = -1/2 \qquad \cos A = 1$

$A = 120, 240 \qquad A = 0, 360$

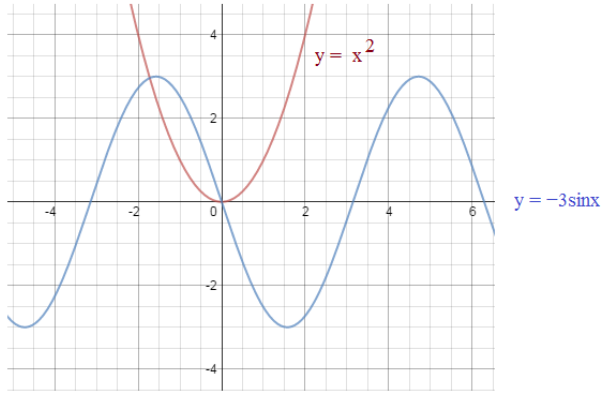


Random Trig Extras

Example: Graph and solve (with calculator)

$x^2 + 3\sin x = 0$

$x = -1.722$ or $x = 0$



Example: $\tan x = \cot x$ (solve and graph, using degrees or radians)

method 1: use quotient identities

$$\frac{\sin x}{\cos x} = \frac{\cos x}{\sin x}$$

$$\sin^2 x = \cos^2 x$$

$$\cos^2 x - \sin^2 x = 0$$

double angle identity

$$\cos 2x = 0$$

$2x = 90, 270, 450, 630,$ etc..

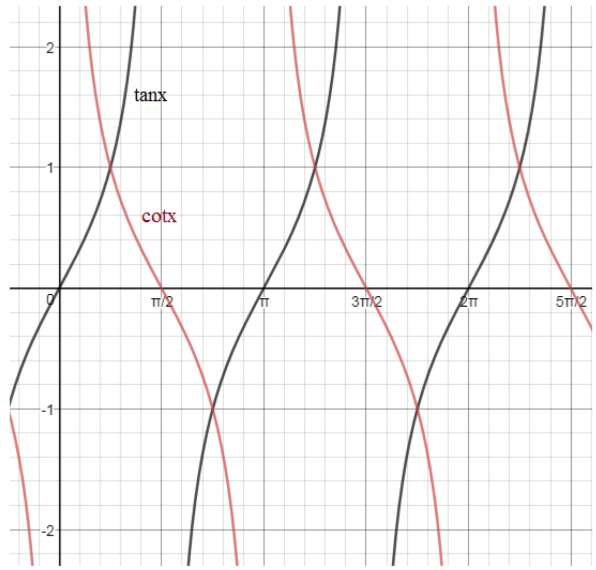
$x = 45, 135, 225, 315,$ etc..

method 2: use reciprocal

$$\tan^2 x = 1$$

$\tan x = 1$ $\tan x = -1$

$\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$, etc..



Notes:

$\sin\frac{\pi}{4} + \sin\frac{\pi}{6} = ?$

$\sin\frac{5\pi}{12} = \sin\left(\frac{\pi}{4} + \frac{\pi}{6}\right) = ?$

Are these the same?!?!?

$$\frac{\sqrt{2}}{2} + \frac{1}{2} = \frac{\sqrt{2}+1}{2}$$

$$\frac{\sqrt{2}}{2}\,\frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2}\,\frac{1}{2} = \frac{\sqrt{6}+\sqrt{2}}{4}$$

$\sin(45)\cos(30) + \cos(45)\sin(30)$

$\sin(45) + \sin(30) = 1.207$

$\sin(75) = .966$

These are NOT the same answers!

"Lowering of Powers"

Example: $\sin^2 x\cos^2 x \Rightarrow (\sin x\cos x)^2$

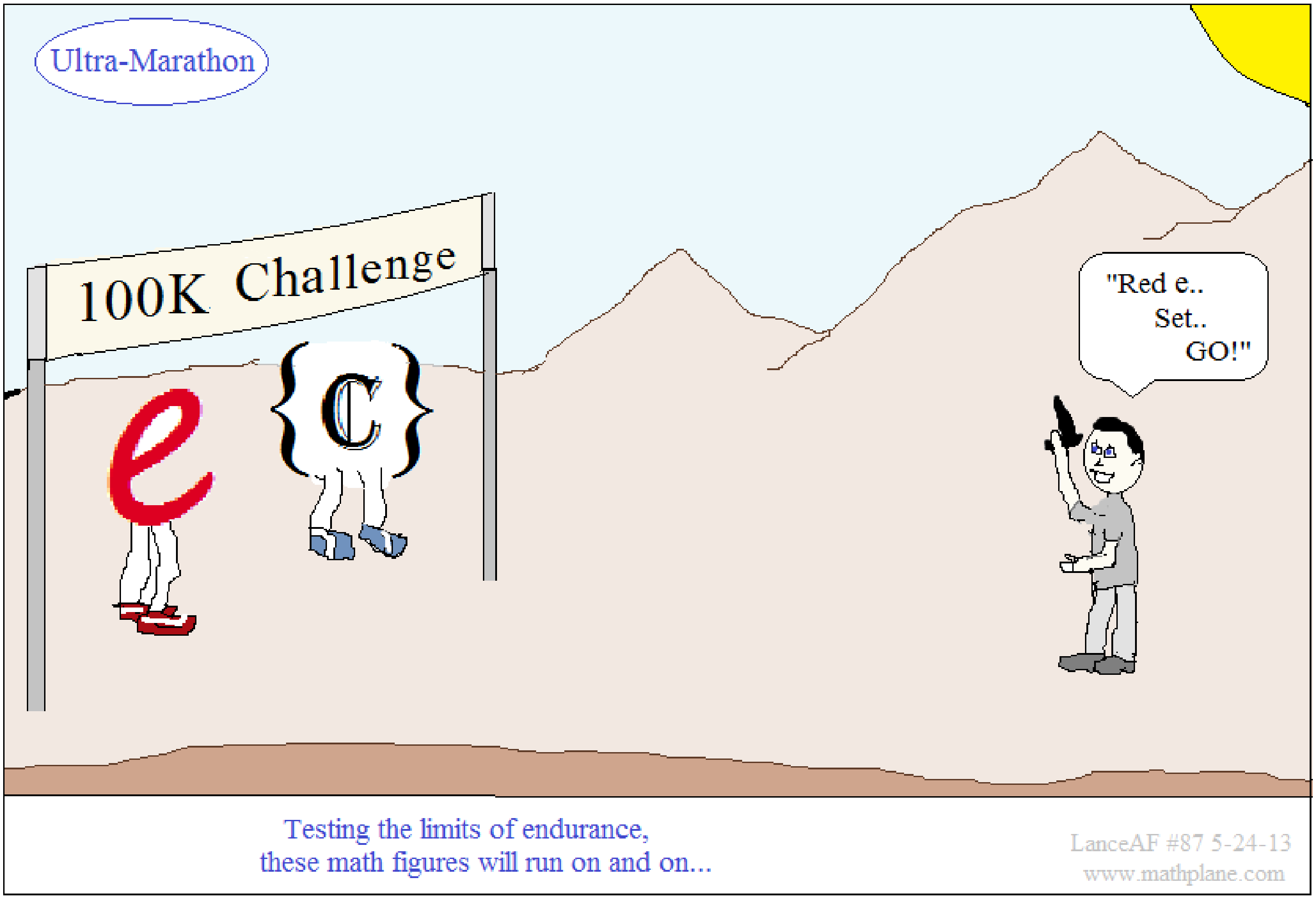
$$\frac{4(\sin x\cos x)^2}{4}$$

$$\frac{(2\sin x\cos x)^2}{4}$$

$$\frac{(\sin 2x)^2}{4}$$

apply the identity

$$\frac{(\sin 2x)^2}{4} \Rightarrow \frac{1 - \cos 2(2x)}{2(4)} = \frac{1 - \cos 4x}{8}$$



Practice Test-→

Verify the Trig Identities...

Trigonometry Review Test (Honors)

1) $$\frac{1 + \cos x + \sin x}{1 + \cos x - \sin x} = \sec x + \tan x$$

2) $$\frac{\cos(x)}{1 - \tan(x)} + \frac{\sin(x)}{1 - \cot(x)} = \cos(x) + \sin(x)$$

Evaluate the expressions....

3) $\tan\left(\sin^{-1}\frac{3}{5} - \cos^{-1}\frac{-4}{5}\right)$

4) $\tan^{-1}\left(\cos\frac{17\pi}{14}\right)$

5) $\cos\left(\sin^{-1}\frac{x}{5} + \cos^{-1}\frac{3}{x}\right)$

6) $\sin\left(\tan^{-1}\left(\frac{5}{8}\right) - A\right)$ where $\sin A = -3/5$ and $\tan A > 0$

7) $\sin 2\theta \sin\theta = \cos\theta$ for $0^\circ < \theta < 360^\circ$

8) $\sin(30)\cos(-10) - \cos(30)\sin(10)$

Rewrite as a single trig value....

9) $2(\sin 53\cos 23 - \cos 53\sin 23)$

Simplify....

10) If θ is an acute angle, and $\sin\theta = 3x$,

what is $\tan\theta$?

11) $4\sin^2 x + 2\cos^2 x = 3$ for all real numbers...

12) $\sec(2x) = \dfrac{\sec^2 x}{2 - \sec^2 x}$ Verify the identity...

13) $\sin\frac{1}{2}A + \cos A = 1$

where $0 \leq A < 2\pi$

14) $\cos\left(\sin^{-1}\left(\frac{5}{13}\right) + \cot^{-1}\left(\frac{3}{4}\right)\right)$ Evaluate...

15) $\tan(\frac{\theta}{2}) = \sqrt{3}$

where $0 < \theta < 360$

16) $\sin(y) + \cos(y) = 1$ Find y (in degrees or radians)

17) $\dfrac{\sin(x+y) - \sin(x-y)}{\cos(x+y) + \cos(x-y)} = \tan y$ Verify the identity...

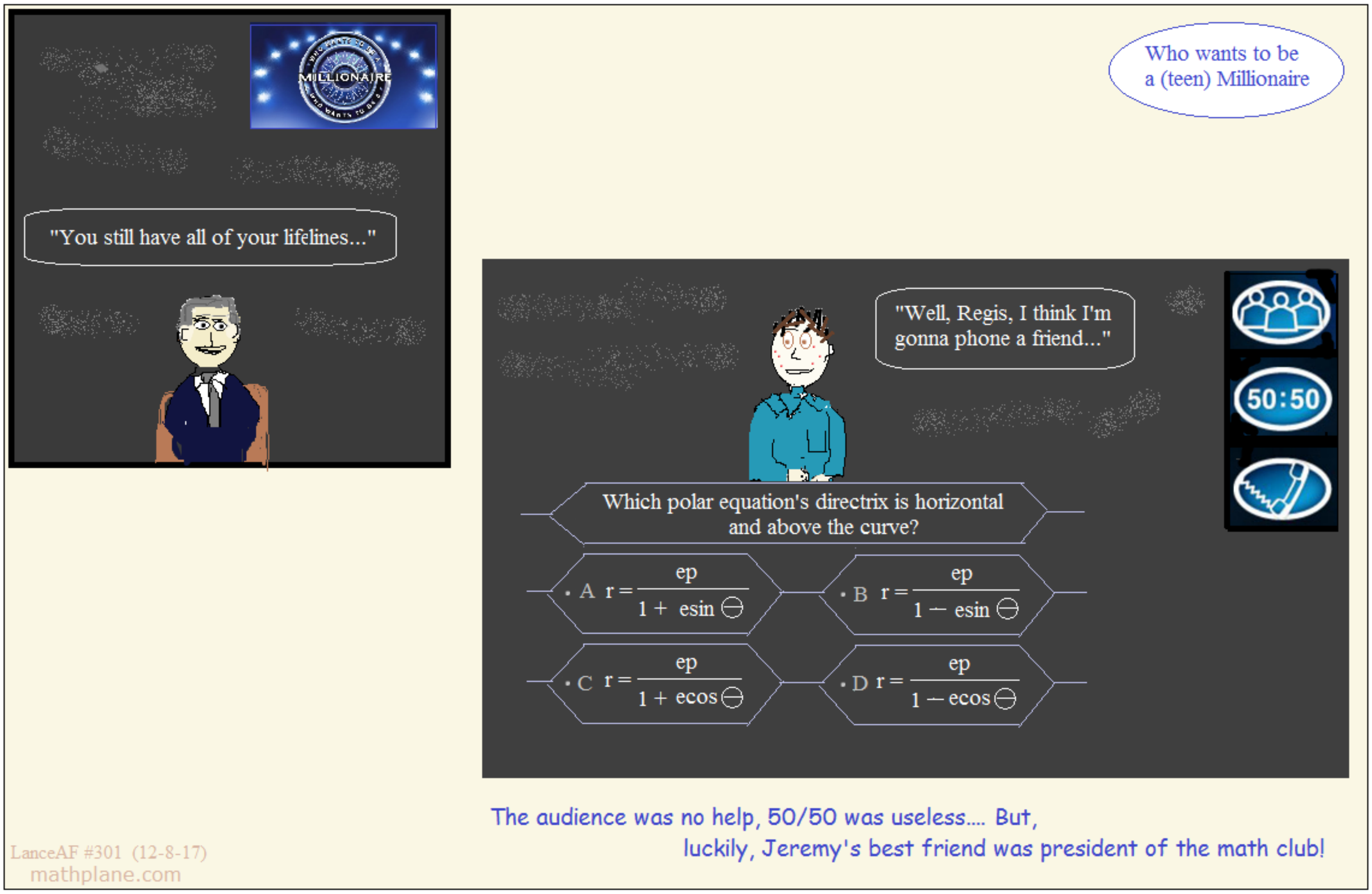
18) Using trig addition/subtraction properties, find $\cos(165)$

19) $2\sin\left(\frac{y}{3}\right) + \sqrt{3} = 0$ Find all solutions for y.

20) -1/4(cosx) = -1/2

21) $\sin\left(2 \cdot \sin^{-1}\left(\frac{-4}{5}\right)\right)$ Evaluate..

22) $\cos\left(\frac{\cos^{-1}\left(\frac{12}{13}\right)}{2}\right) =$


Who wants to be
a (teen) Millionaire
MILLIONAIRE
"You still have all of your lifelines..."
"Well, Regis, I think I'm
gonna phone a friend..."
50:50
Which polar equation's directrix is horizontal
and above the curve?
A $r = \frac{ep}{1 + e\sin\theta}$
B $r = \frac{ep}{1 - e\sin\theta}$
C $r = \frac{ep}{1 + e\cos\theta}$
D $r = \frac{ep}{1 - e\cos\theta}$
The audience was no help, 50/50 was useless.... But,
luckily, Jeremy's best friend was president of the math club!
LanceAF #301 (12-8-17)
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ANSWERS-→

Verify the Trig Identities...

SOLUTIONS

Trigonometry Review Test (Honors)

1) $$\frac{1 + \cos x + \sin x}{1 + \cos x - \sin x} = \sec x + \tan x$$

$$\frac{1 + \cos x + \sin x}{1 + \cos x - \sin x} \cdot \frac{(1+\cos x) + \sin x}{(1+\cos x) + \sin x} = \frac{1 + \cos x + \sin x + \cos x + \cos^2 x + \cos x \sin x + \sin x + \sin x \cos x + \sin^2 x}{(1+\cos x)^2 - \sin^2 x}$$

multiply using conjugate

$$= \frac{1 + 2\cos x + 2\sin x + 2\sin x\cos x + \cos^2 x + \sin^2 x}{1 + 2\cos x + \cos^2 x - \sin^2 x}$$

$$= \frac{2 + 2\cos x + 2\sin x + 2\sin x\cos x}{1 + 2\cos x + \cos^2 x - 1 + \cos^2 x}$$

$$= \frac{2(1 + \cos x + \sin x + \sin x\cos x)}{2(\cos x + \cos^2 x)}$$

$$= \frac{(1+\cos x)(1+\sin x)}{\cos x(1+\cos x)} = \frac{1 + \sin x}{\cos x} = \sec x + \tan x$$

2) $$\frac{\cos(x)}{1-\tan(x)} + \frac{\sin(x)}{1-\cot(x)} = \cos(x) + \sin(x)$$

change to sines and cosines (quotient identities)

$$\frac{\cos(x)}{1 - \frac{\sin(x)}{\cos(x)}} + \frac{\sin(x)}{1 - \frac{\cos(x)}{\sin(x)}}$$

condense the denominators

$$\frac{\cos(x)}{\frac{\cos(x)-\sin(x)}{\cos(x)}} + \frac{\sin(x)}{\frac{\sin(x)-\cos(x)}{\sin(x)}}$$

divide each rational expression

$$\frac{\cos(x)}{1} \cdot \frac{\cos(x)}{\cos(x)-\sin(x)} + \frac{\sin(x)}{1} \cdot \frac{\sin(x)}{\sin(x)-\cos(x)}$$

$$\frac{\cos^2(x)}{\cos(x)-\sin(x)} + \frac{\sin^2(x)}{\sin(x)-\cos(x)}$$

multiply 2nd expression by -1/-1

$$\frac{\cos^2(x)}{\cos(x)-\sin(x)} - \frac{\sin^2(x)}{\cos(x)-\sin(x)}$$

$$\frac{\cos^2(x) - \sin^2(x)}{\cos(x)-\sin(x)}$$

factor the numerator, then simplify...

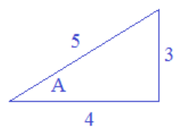
$$\frac{(\cos(x)+\sin(x))(\cos(x)-\sin(x))}{\cos(x)-\sin(x)} = \cos(x) + \sin(x)$$

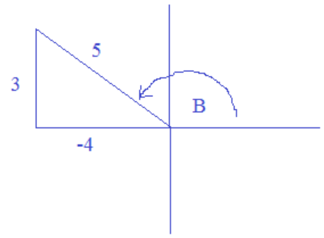
Evaluate the expressions....

3) $\tan\left(\sin^{-1}\frac{3}{5} - \cos^{-1}\frac{-4}{5}\right)$

SOLUTIONS

Let $A = \sin^{-1}\frac{3}{5}$ Let $B = \cos^{-1}\frac{-4}{5}$





$$\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B} = \frac{3/4 - -3/4}{1 + (3/4)(-3/4)} = \frac{3/2}{7/16} = \boxed{24/7}$$

4) $\tan^{-1}\left(\cos\frac{17\pi}{14}\right)$

$\frac{17\pi}{14}$ is in quadrant III

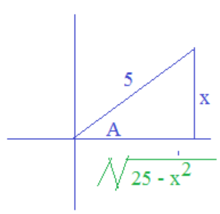
with a negative value and reference angle $\frac{3\pi}{14}$

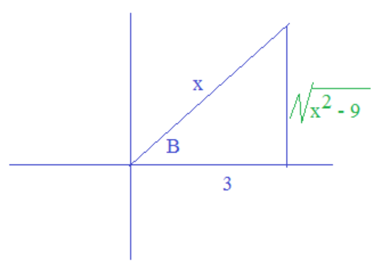
therefore, the inverse tangent function, of a negative value, must be in quadrant IV... with a reference value of

$$\frac{3\pi}{14} \Rightarrow \boxed{-\frac{3\pi}{14}}$$

5) $\cos\left(\sin^{-1}\frac{x}{5} + \cos^{-1}\frac{3}{x}\right)$

A B





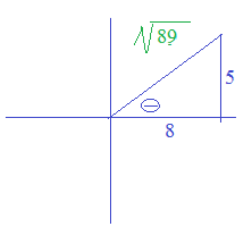
$$\cos(A + B) = \cos A \cos B - \sin A \sin B$$

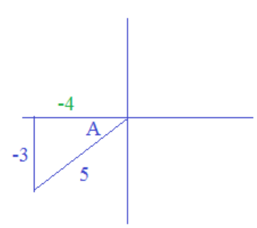
$$\frac{\sqrt{25 - x^2}}{5} \cdot \frac{3}{x} - \frac{x}{5} \cdot \frac{\sqrt{x^2 - 9}}{x}$$

$$\boxed{\frac{3\sqrt{25 - x^2} - x\sqrt{x^2 - 9}}{5x}}$$

6) $\sin\left(\tan^{-1}\left(\frac{5}{8}\right) - A\right)$ where $\sin A = -3/5$

θ A and $\tan A > 0$





$$\sin(\theta - A) = \sin\theta \cos A - \sin A \cos\theta$$

$$\frac{5}{\sqrt{89}} \cdot \frac{-4}{5} - \frac{-3}{5} \cdot \frac{8}{\sqrt{89}}$$

$$\boxed{\frac{4}{5\sqrt{89}}}$$

7) $\sin 2\theta \sin\theta = \cos\theta$ for $0^\circ < \theta < 360^\circ$

$2\sin^2\theta\cos\theta - \cos\theta = 0$

$\cos\theta\,(2\sin^2\theta - 1) = 0$

$\cos\theta = 0$ $\qquad$ $2\sin^2\theta - 1 = 0$

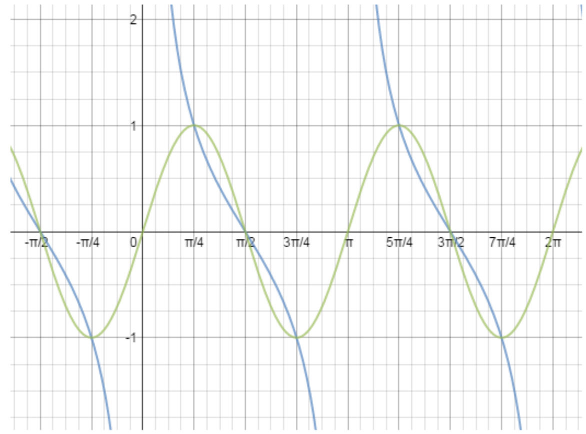
$\boxed{\theta = 90 \text{ and } 270}$

$\sin^2\theta = \frac{1}{2}$

$\sin\theta = \frac{\pm 1}{\sqrt{2}}$

$\boxed{\theta = 45, 135, 225, \text{ and } 315}$

The graph shows the system of $\sin 2\theta$ and $-\cot\theta$



8) sin(30)cos(-10) − cos(30)sin(10)

Rewrite as a single trig value....

Since cos(-10) = cos(10), we can rewrite

cofunction identities

sin(30)cos(10) − cos(30)sin(10)

SOLUTIONS

$\sin(30 - 10) = \boxed{\sin(20)}$

addition/ subtraction properties

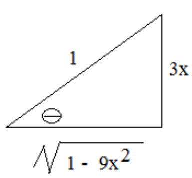
9) 2(sin53cos23 − cos53sin23)

Simplify....

$2(\sin 30) = \boxed{1}$

10) If θ is an acute angle, and $\sin\theta = 3x$,

what is $\tan\theta$?



$\boxed{\tan\theta = \frac{3x}{\sqrt{1 - 9x^2}}}$ $\qquad \frac{\text{opposite}}{\text{adjacent}}$

11) $4\sin^2 x + 2\cos^2 x = 3$ for all real numbers...

"split the sines"

$2\sin^2 x + 2\sin^2 x + 2\cos^2 x = 3$

$2\sin^2 x + 2(\sin^2 x + \cos^2 x) = 3$

$2\sin^2 x + 2(1) = 3$

$\sin^2 x = \frac{1}{2}$

$\sin x = \frac{1}{\sqrt{2.}}$ or $\frac{-1}{\sqrt{2.}}$

x = 45 , 135, 225, 315, ...

$\boxed{\frac{\pi}{4} + \frac{\pi}{2}k \quad \text{where k is any integer}}$

SOLUTIONS

12) $\sec(2x) = \dfrac{\sec^2 x}{2 - \sec^2 x}$ Verify the identity...

$$\sec(2x) = \frac{1}{\cos(2x)} = \frac{1}{\cos^2 x - \sin^2 x} \cdot \frac{\sec^2 x}{\sec^2 x}$$

double angle identity

$$\frac{\sec^2 x}{1 - \tan^2 x}$$

$$\frac{\sec^2 x}{1 - (\sec^2 x - 1)} \qquad \frac{\sec^2 x}{2 - \sec^2 x}$$

13) $\sin\frac{1}{2}A + \cos A = 1$

where $0 \leq A < 2\pi$

using 1/2 angle identity.... $\sqrt{\dfrac{1 - \cos A}{2}} = 1 - \cos A$

$$\frac{1 - \cos A}{2} = 1 - 2\cos A + \cos^2 A$$

1/2 angle identity

$$1 - \cos A = 2 - 4\cos A + 2\cos^2 A$$

$$2\cos^2 A - 3\cos A + 1 = 0$$

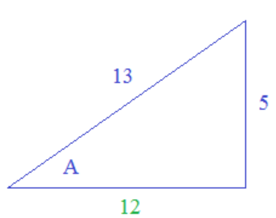
$$(2\cos A - 1)(\cos A - 1) = 0$$

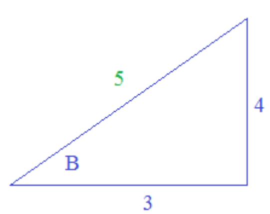
$\cos A = 1$ $\boxed{A = 0}$

$\cos A = 1/2$ $\boxed{A = \dfrac{\pi}{3} \quad \dfrac{5\pi}{3}}$

14) $\cos\left(\sin^{-1}\left(\frac{5}{13}\right) + \cot^{-1}\left(\frac{3}{4}\right)\right)$ Evaluate...

A B





$\cos(A + B) = \cos A\cos B - \sin A\sin B$

$$\frac{12}{13} \cdot \frac{3}{5} - \frac{5}{13} \cdot \frac{4}{5} = \boxed{\frac{16}{65}}$$

15) $\tan\left(\frac{\theta}{2}\right) = \sqrt{3}$

where $0 < \theta < 360$

Let $\dfrac{\theta}{2} = A$

$\tan A = \sqrt{3}$ $A = -60, 60, 240,$

$\theta = -120, \boxed{120,} 480..$ only 120 degrees...

16) $\sin(y) + \cos(y) = 1$ Find y (in degrees or radians)

square both sides

$\sin^2(y) + 2\sin(y)\cos(y) + \cos^2(y) = 1$

1

$2\sin(y)\cos(y) = 0$

$\sin(2y) = 0$

$2y = 0, 180, 360, 540,$ etc...

$y = 0, 90, 180, 270, 360, \ldots$

Important! Since we multiplied variables, we may have added extraneous solutions... Check the answers..

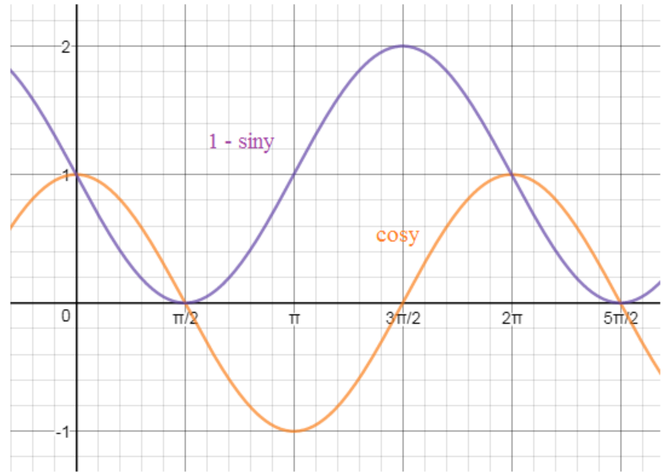
If $y = 0$, $\sin(0) + \cos(0) = 1$ ✓

$y = 90$, $\sin(90) + \cos(90) = 1$ ✓

$y = 180$, $\sin(180) + \cos(0) = 1$ ✗

$y = 270$, $\sin(270) + \cos(270) = 1$ ✗

To solve by graphing, find the intersections of $\cos(y)$ and $1 - \sin(y)$...



$$\boxed{\begin{aligned} y &= 0^\circ + 360k \\ y &= 90^\circ + 360k \end{aligned}}$$

SOLUTIONS

17) $$\frac{\sin(x+y) - \sin(x-y)}{\cos(x+y) + \cos(x-y)} = \tan y$$ Verify the identity...

Using Addition and Subtraction Identities

$$\frac{\sin x\cos y + \sin y\cos x - (\sin x\cos y - \sin y\cos x)}{\cos x\cos y - \sin x\sin y + (\cos x\cos y + \sin x\sin y)} = \frac{2\sin y\cos x}{2\cos x\cos y} = \frac{\sin y}{\cos y} = \tan y$$

18) Using trig addition/subtraction properties, find $\cos(165)$

method 1: $135 + 30$

$\cos(165) = \cos(135 + 30)$

$= \cos(135)\cos(30) - \sin(135)\sin(30)$

$$= \frac{-\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \cdot \frac{1}{2}$$

$$\boxed{\frac{-\sqrt{6} - \sqrt{2}}{4}}$$

method 2: $\cos(15) = -(\cos 165)$

$\cos(15) = \cos(45 - 30)$

$= \cos(45)\cos(30) + \sin(45)\sin(30)$

$$= \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \cdot \frac{1}{2}$$

$$\cos(15) = \frac{\sqrt{6} + \sqrt{2}}{4}$$

then, flip the signs to switch to 165...

$$\boxed{\frac{-\sqrt{6} - \sqrt{2}}{4}}$$

19) $2\sin\left(\frac{y}{3}\right) + \sqrt{3} = 0$ Find all solutions for y.

SOLUTIONS

$$2\sin\left(\frac{y}{3}\right) = -\sqrt{3}$$

$$\sin\left(\frac{y}{3}\right) = \frac{-\sqrt{3}}{2}$$

$$\frac{y}{3} = \sin^{-1}\left(\frac{-\sqrt{3}}{2}\right)$$

$$\frac{y}{3} = \frac{4\pi}{3},\ \frac{5\pi}{3},\ \frac{10\pi}{3},\ \frac{11\pi}{3},\ \ldots$$

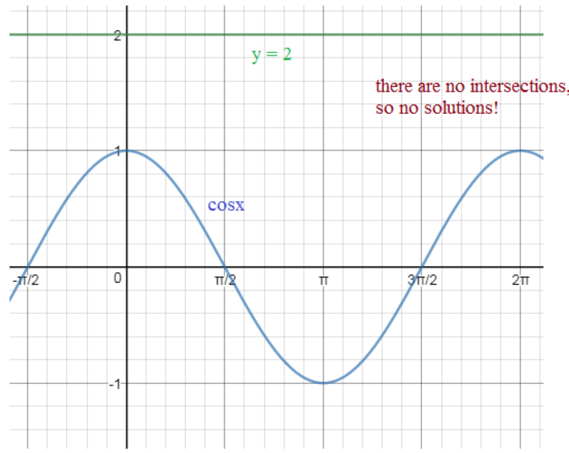
$$y = 4\pi,\ 5\pi,\ 10\pi,\ 11\pi, \ldots$$

$$\boxed{\begin{array}{ll} 4\pi + 6\pi K & \text{where K are integers} \\ 5\pi + 6\pi K & \end{array}}$$

20) $-1/4(\cos x) = -1/2$

$$\cos x = 2$$

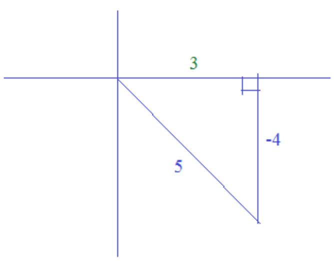
$$\boxed{\text{NO SOLUTION}}$$



21) $\sin\left(2 \cdot \sin^{-1}\left(\frac{-4}{5}\right)\right)$ Evaluate..

working with double angles and inverse trig values

let $A = \sin^{-1}\left(\frac{-4}{5}\right)$



$$\sin(2A) = 2\sin A\cos A$$

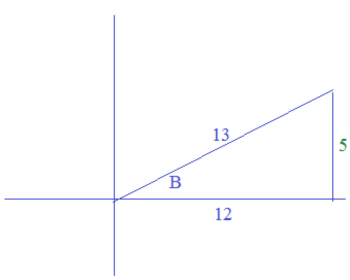
$$2 \cdot \frac{-4}{5} \cdot \frac{3}{5} = \boxed{\frac{-24}{25}}$$

22) $\cos\left(\frac{\cos^{-1}\left(\frac{12}{13}\right)}{2}\right) =$

working with 1/2 angles and inverse trig values

let $B = \cos^{-1}\left(\frac{12}{13}\right)$

create the triangle...



$$\cos\left(\frac{B}{2}\right) = \sqrt{\frac{1 + \cos B}{2}}$$

(since the angle is in quad I, the result is positive)

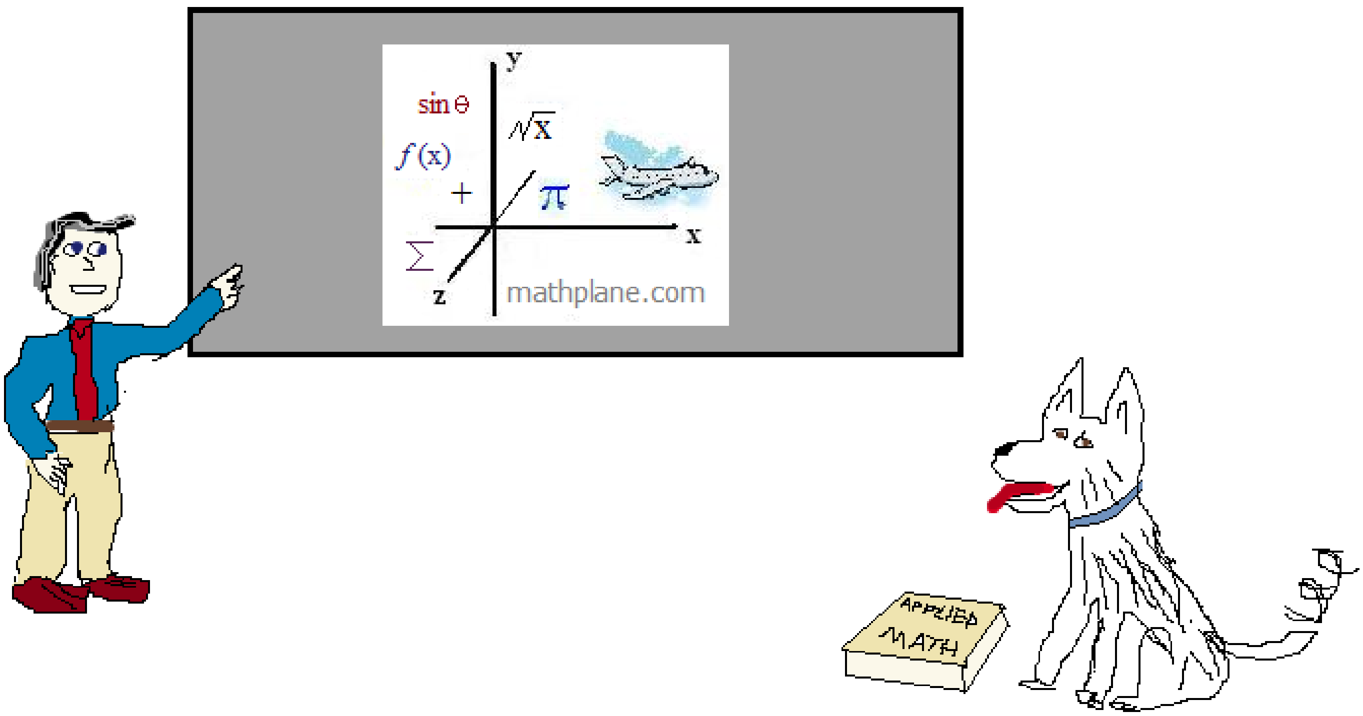
$$= \sqrt{\frac{1 + 12/13}{2}}$$

$$\boxed{\sqrt{\frac{25}{26}}}$$

Thanks for visiting. (Hope it helped!)

If you have questions, suggestions, or requests, let us know.

Cheers



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