Algebra 1: Linear Equations Test

(And, Solutions)

Topics include slope, parallel lines, intercepts, coordinates, distance, midpoints, graphing, and more.

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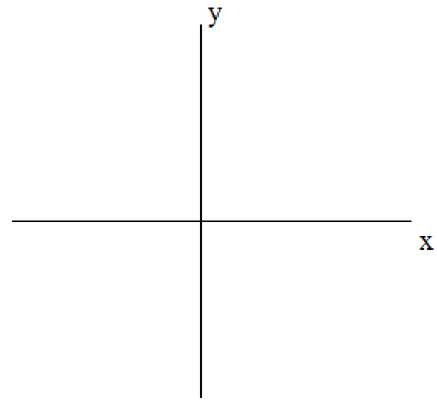
Linear Equations Test

Part 1:

5 points 1) Plot the coordinates (2, 6) (5, -3) on the plane.



10 points 2) What is the slope of a line passing through these two points?

15 points 3) Write the equation of this line in

a) Point slope form:

b) Slope intercept form:

c) Standard form:

10 points 4) What are the intercepts?
(i.e. x-intercept & y-intercept)

10 points 5) Is the point (10, -17) on this line?

Linear Equations Test (continued)

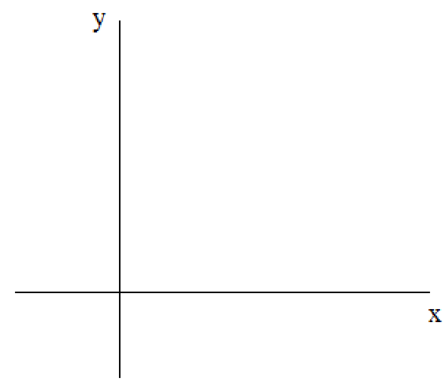
Assume line l is $y = -3x + 17$

15 points 6) Write an equation for the line <u>parallel</u> to l
that passes through (-3, 6)

10 points 7) Write the equation for a line <u>perpendicular</u> to l
that passes through (2, 6)

Part 2:

5 points 1) Plot and connect the following points:
(4, 2) (10, 2) (10, 10)



10 points 2) What is the midpoint of (4, 2) and (10, 10)?

10 points 3) What is the distance between (4, 2) and (10, 2)?

Extra Credit

(10 bonus points) Find the *area* of the triangle formed
by the three points (in Part 2).

(10 bonus points) Find the *perimeter* of the triangle
formed by the three points.

Part 3: Concepts

Linear Equations Test

1) What is the sum of the y-intercept and the slope of $4x - 8y = 6$?

a) -2
b) -1/2
c) -1/4
d) 2
e) 6

2) What is the equation of a line perpendicular to $x = 2$ and goes through (-1, 4)?

a) $y = 2$
b) $x = -1$
c) $y = 4$
d) $x = 4$
e) $y = -1$

3) Which equation creates an infinite number of solutions when solved for a system with $y = 8x - 9$?

a) $y = 9x - 8$
b) $3y - 24x = -36$
c) $4y + 24x = -27$
d) $4y - 32x = -36$
e) $2y + 16x = -18$

4) If you shifted $y = 3x + 6$ five units to the right, what would the new linear equation be?

a) $y = 3x + 11$
b) $y = 8x + 6$
c) $y = 3x + 1$
d) $y = 3x - 9$
e) $y = 8x + 11$

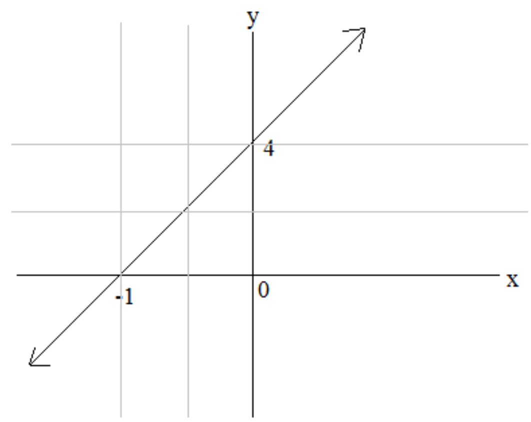
5) Which is the equation of the line?

a) $y = -x + 4$

b) $y = 8x + 4$

c) $y = x + 4$

d) $y = 4x + 4$



6) Write the equation of a line that bisects quadrants II and IV.

7) Find the missing term:

x	y
-12	17
-2	-3
-1	-5
0	$\boxed{}$
6	-19

8) What is the equation of a line that is parallel to the x-axis and passes through (2, -3)?

9) What is the equation of a line that is perpendicular to the y-axis and passes through the (-4, 5)?

Identify the parts of each linear equation. Then, graph.

Linear Equations Exercise

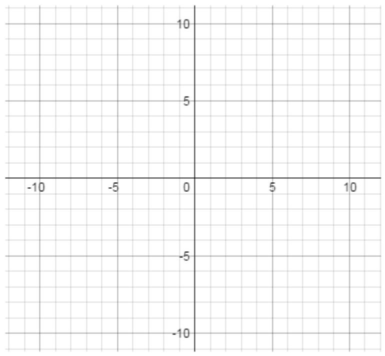
A) $2x + 7y = 14$

Linear Form:

Slope:

x-intercept:

y-intercept:



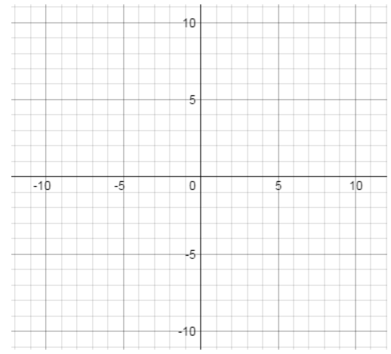
B) $y = \frac{1}{2}x + 4$

Linear Form:

Slope:

x-intercept:

y-intercept:



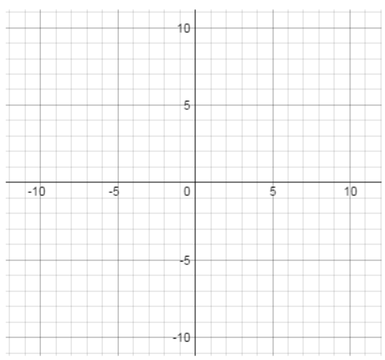
C) $y + 5 = -3(x + 1)$

Linear Form:

Slope:

x-intercept:

y-intercept:



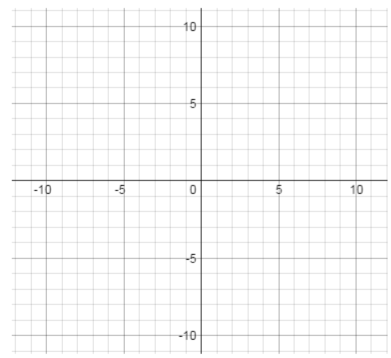
D) $y = .2x - 6$

Linear Form:

Slope:

x-intercept:

y-intercept:



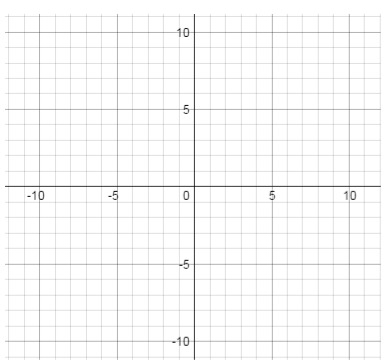
E) $5x - y = 5$

Linear Form:

Slope:

x-intercept:

y-intercept:



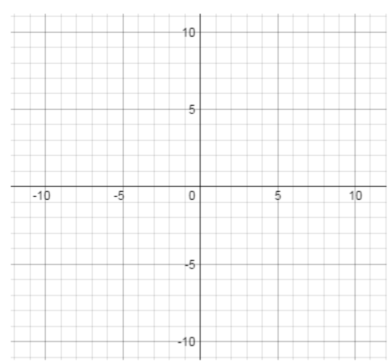
F) $(y - 2) = (x + 5)$

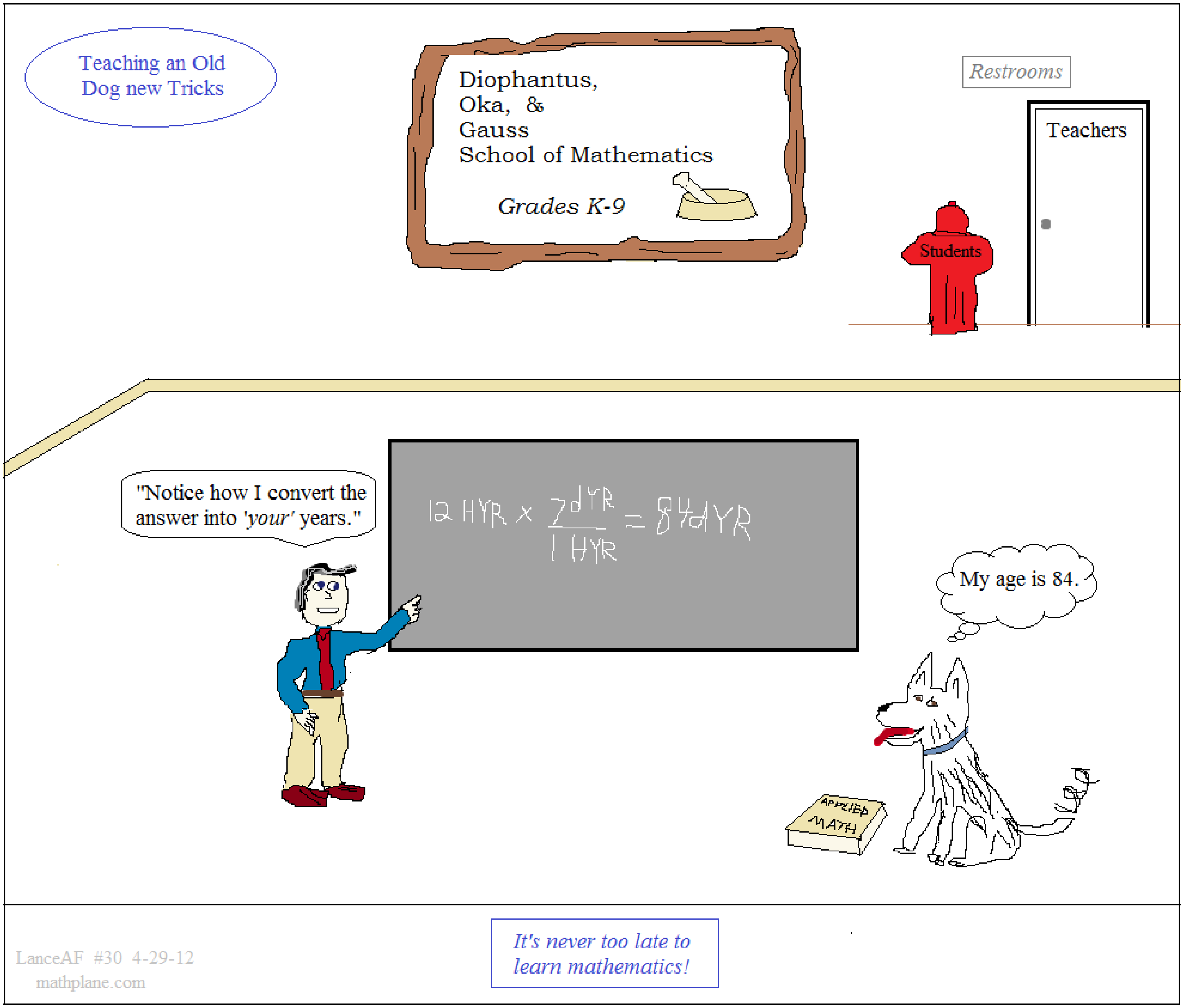
Linear Form:

Slope:

x-intercept:

y-intercept:





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"Notice how I convert the answer into 'your' years."
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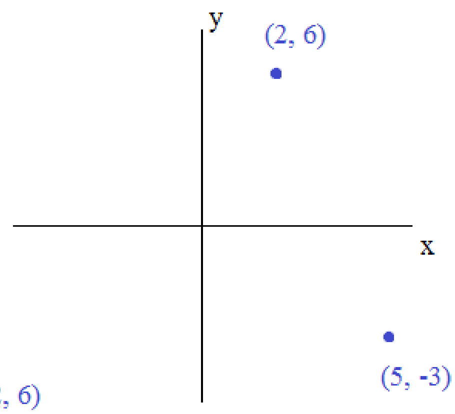
Solutions →

Linear Equations Test

SOLUTIONS

Part 1:

5 points 1) Plot the coordinates (2, 6) (5, -3) on the plane.



10 points 2) What is the slope of a line passing through these two points?

$$m = \frac{y_1 - y_2}{x_1 - x_2} = \frac{6 - (-3)}{2 - 5} = \boxed{-3}$$

15 points 3) Write the equation of this line in

a) Point slope form: $y - y_1 = m(x - x_1)$

using (2, 6)

$$\boxed{y - 6 = -3(x - 2)}$$

b) Slope intercept form:

$y = mx + b$

method 1: rewrite point slope form

$y - 6 = -3x + 6$

$$\boxed{y = -3x + 12}$$

method 2: plug in numbers

since slope $m = -3$ $\Rightarrow$ $y = -3x + b$

then, to find the slope intercept (b), plug in one of the points....

using (2, 6)

$6 = -3(2) + b$

$b = 12$

c) Standard form:

standard form: $ax + by = c$

rearrange other forms: $\boxed{3x + y = 12}$

10 points 4) What are the intercepts? (i.e. x-intercept & y-intercept)

y-intercept is where line 'intercepts y-axis'.. since coordinate is (0, y), simply plug in x = 0

$\boxed{(0\ 12)}$

x-intercept is where line 'intercepts x-axis' since coordinate is (x, 0), simply plug in y = 0

$\boxed{(4, 0)}$

10 points 5) Is the point (10, -17) on this line?

Any point on the line will satisfy the equation!

since line is $3x + y = 12$, we'll substitute (10, -17)

$3(10) + (-17) = 12$

$13 = 12$ False

$\boxed{\text{(10, -17) is NOT on the line}}$

Linear Equations Test (continued)

Assume line l is $y = -3x + 17$

To find equation of a line, you need slope and a point...

15 points 6) Write an equation for the line parallel to l that passes through (-3, 6)

since line is parallel, the slope is −3
and, the given point is (-3, 6)

$$\boxed{\begin{array}{ll}\text{point slope form:} & y - 6 = -3(x + 3) \\ \text{slope intercept form:} & y = -3x - 3\end{array}}$$

10 points 7) Write the equation for a line perpendicular to l that passes through (2, 6)

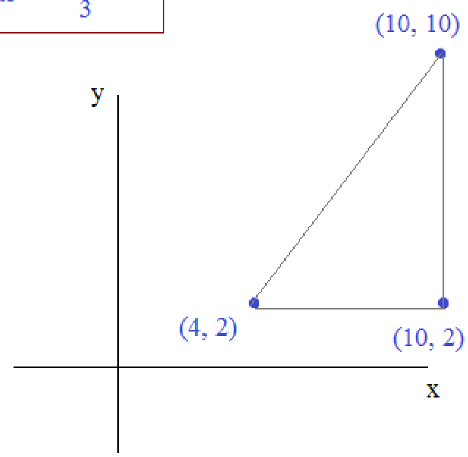
since line is perpendicular, the slope is the opposite reciprocal:

1/3

$$\boxed{\begin{array}{l} y - 6 = \frac{1}{3}(x - 2) \\ y = \frac{1}{3}x + \frac{16}{3}\end{array}}$$

Part 2:

5 points 1) Plot and connect the following points:
(4, 2) (10, 2) (10, 10)



10 points 2) What is the midpoint of (4, 2) and (10, 10)?

midpoint is the halfway point: $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$ $\boxed{(7, 6)}$

10 points 3) What is the distance between (4, 2) and (10, 2)?

$\boxed{\text{6 units}}$

Extra Credit

(10 bonus points) Find the *area* of the triangle formed by the three points (in Part 2).

$$\text{area} = \frac{1}{2}\text{(base)(height)} = (1/2)(6)(8) = \boxed{\text{24 square units}}$$

(10 bonus points) Find the *perimeter* of the triangle formed by the three points.

length of triangle sides: base = 6
side = 8

it's a 6-8-10 right triangle, so hypotenuse is 10

$\boxed{\text{24 units}}$

Part 3: Concepts SOLUTIONS Linear Equations Test

1) What is the sum of the y-intercept and the slope of $4x - 8y = 6$?

a) -2
b) -1/2
c) $\boxed{-1/4}$
d) 2
e) 6

The y-intercept occurs when x = 0...
y-intercept is (0, -3/4)

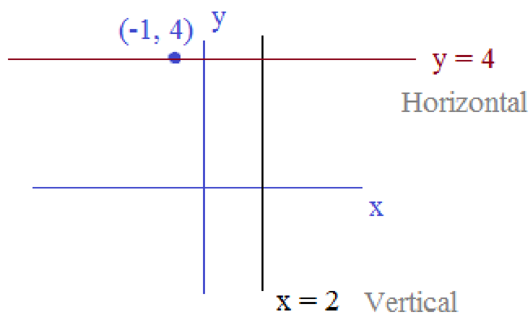
then, to find the slope: $-8y = -4x + 6$
$y = (1/2)x - 3/4$
slope is 1/2

the sum of slope and y-intercept

$1/2 + (-3/4) = -1/4$

2) What is the equation of a line perpendicular to x = 2 and goes through (-1, 4)?

a) y = 2
b) x = -1
c) $\boxed{y = 4}$
d) x = 4
e) y = -1



3) Which equation creates an infinite number of solutions when solved for a system with y = 8x - 9 ?

a) y = 9x - 8 — slope is 9 NO
b) 3y - 24x = -36 — slope is 8, but intercept is -12 NO
c) 4y + 24x = -27 — slope is -6 NO
d) $\boxed{4y - 32x = -36}$ — slope is 8, intercept is -9 YES (this is the same equation)
e) 2y + 16x = -18 — $y + 8x = -9 \longrightarrow y = -8x - 9$ close, but NO

4) If you shifted $y = 3x + 6$ five units to the right, what would the new linear equation be?

a) y = 3x + 11
b) y = 8x + 6
c) y = 3x + 1
d) $\boxed{y = 3x - 9}$
e) y = 8x + 11

Since the entire line is shifted, the slope is the SAME... slope is 3

If the line is shifted 5 units to the right, then presumably, the x-intercept would move 5 units to the right...

original x-intercept is (-2, 0)... Then, new x-intercept is (3, 0)

therefore, equation is $y - 0 = 3(x - 3)$ or $y = 3x - 9$

SOLUTIONS

Linear Equations Test

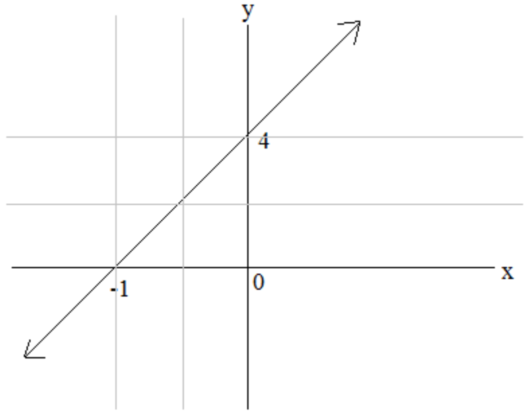
5) Which is the equation of the line?

a) y = -x + 4

b) y = 8x + 4

c) y = x + 4

$\boxed{\text{d) } y = 4x + 4}$



The y-intercept is (0, 4)

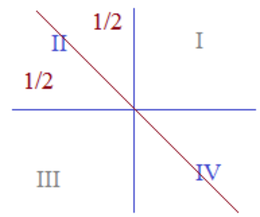
The slope is "rise"/"run"

4/1 = 4

$\boxed{y = 4x + 4}$

6) Write the equation of a line that bisects quadrants II and IV.

$\boxed{\text{Answer: } y = -x}$



7) Find the missing term:

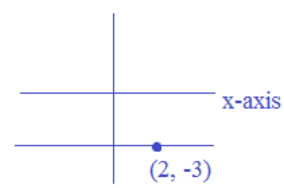
x	y
-12	17
-2	-3
-1	-5
0	$\square$
6	-19

$\boxed{\text{Answer: } -7}$

(slope/rate of change is -2)

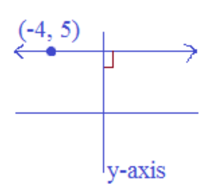
8) What is the equation of a line that is parallel to the x-axis and passes through (2, -3)?

$\boxed{y = -3}$



9) What is the equation of a line that is perpendicular to the y-axis and passes through the (-4, 5)?

$\boxed{y = 5}$



Identify the parts of each linear equation. Then, graph. SOLUTIONS

Linear Equations Exercise

A) $2x + 7y = 14$

Linear Form: Standard Form

Slope: -2/7

x-intercept: (7, 0)

y-intercept: (0, 2)

$7y = -2x + 14$

$y = \frac{-2}{7}x + 2$

slope y-intercept

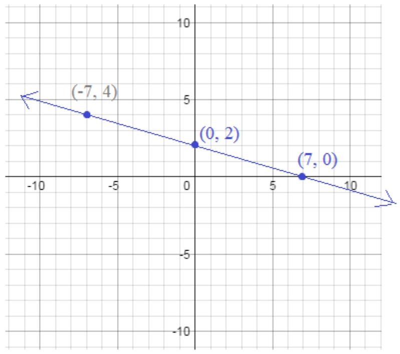
x-intercept: let y = 0

$2x + 7(0) = 14$
$x = 7$

y-intercept: let x = 0

$2(0) + 7y = 14$
$y = 2$

Standard:
$Ax + By = C$



B) $y = \frac{1}{2}x + 4$

Linear Form: Slope intercept Form

Slope: 1/2

x-intercept: (-8, 0)

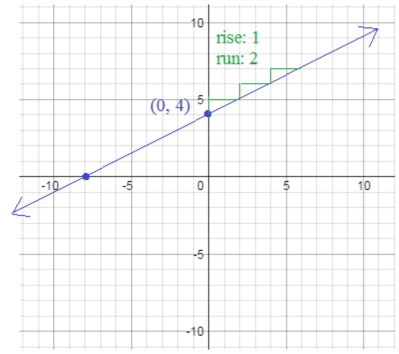
y-intercept: (0, 4)

Slope intercept:
$y = mx + b$
m is slope; b is y-intercept

x-intercept: let y = 0

$0 = \frac{1}{2}x + 4$

$x = -8$



C) $y + 5 = -3(x + 1)$

Linear Form: Point-Slope Form

Slope: -3

x-intercept: (-8/3, 0)

y-intercept: (0, -8)

x-intercept: y = 0

$0 + 5 = -3(x + 1)$

$5 = -3x - 3$

$x = -8/3$

y-intercept: x = 0

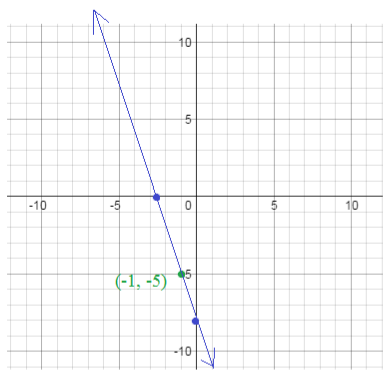
$y + 5 = -3 \quad y = -8$

Point-Slope:

$(y - y_1) = m(x - x_1)$

m is slope;

(x_1, y_1) is a point on the line



D) $y = .2x - 6$

Linear Form: Slope intercept Form

Slope: .2 or 1/5

x-intercept: (30, 0)

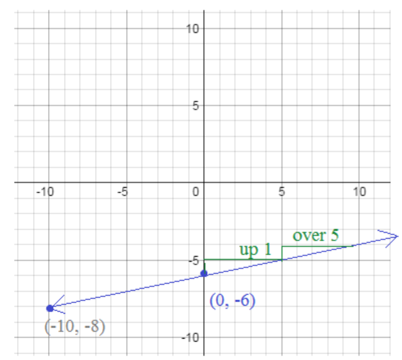
y-intercept: (0. -6)

x-intercept: let y = 0

$0 = .2x - 6$

$6 = .2x$

$60 = 2x \quad x = 30$



E) $5x - y = 5$

Linear Form: Standard Form A = 5 B = -1 C = 5

Slope: 5

x-intercept: (1, 0)

y-intercept: (0, -5)

rewrite in intercept form:

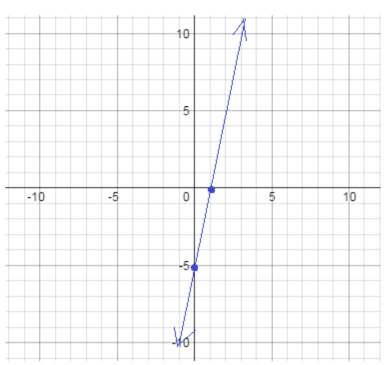
$5x - y = 5$

$-y = -5x + 5$

$y = 5x - 5$

If x = 0, y = -5

If y = 0, x = 1



(-5, 2)

F) $(y - 2) = (x + 5)$

Linear Form: Point Slope Form

Slope: 1

x-intercept: (-7, 0)

y-intercept: (0, 7)

x-intercept:

$(0 - 2) = 1(x + 5)$

$-2 = x + 5$

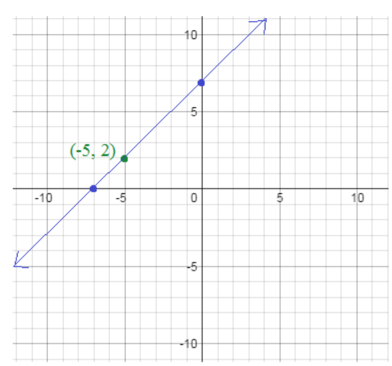
$x = -7$

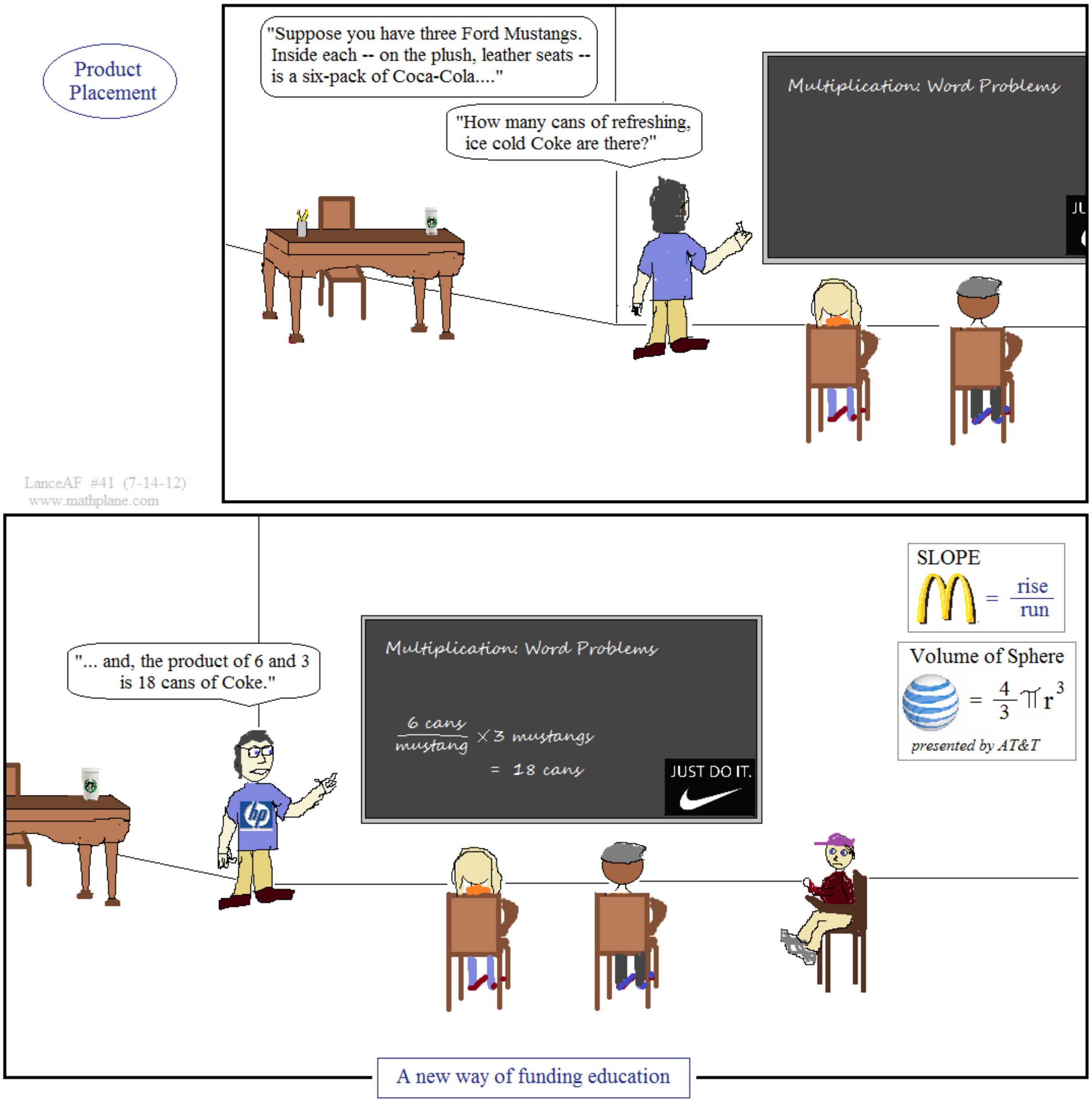
y-intercept:

$(y - 2) = 1(0 + 5)$

$y - 2 = 5$

$y = 7$





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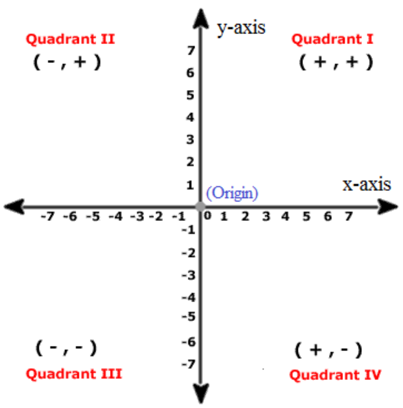
Coordinate Geometry Topics and Notes

I. Coordinate Plane (or Cartesian Plane -- named after mathematician Rene Descarte)



- x-axis and y-axis are perpendicular

$(\boxed{x}, y)$

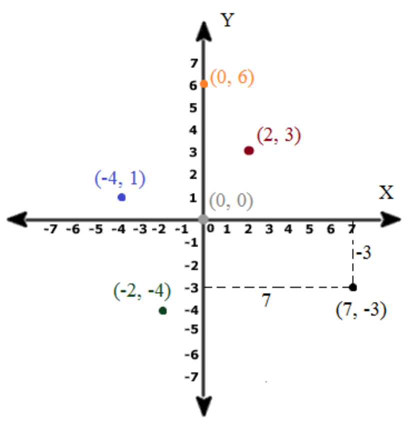
- "Left" of y-axis is negative
  "Right" of y-axis is positive

("left" is negative
"right" is positive)

$(x, \boxed{y})$

- "Above" the x-axis is positive
  "Below" the x-axis is negative

("up" is positive
"down" is negative)



- Each point is an "ordered pair"
- Origin is (0, 0)

The first term in the ordered pair is the x value.
(*horizontal* movement from the origin)

The second term in the ordered pair is the y value.
(*vertical* movement from the origin)

II. Slope

$$\text{Slope } m = \frac{\text{"rise"}}{\text{"run"}} = \frac{\text{vertical change}}{\text{horizontal change}} = \frac{y_1 - y_2}{x_1 - x_2}$$

Examples: B = (4, 3) C = (-2, -2)

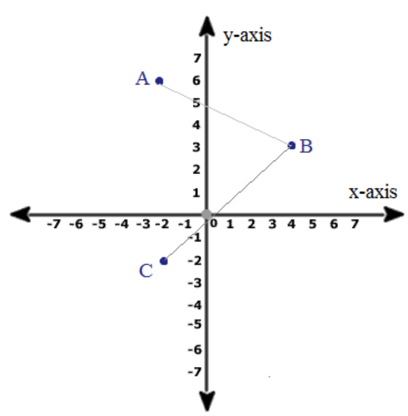
Slope of $\overline{BC} = \frac{3-(-2)}{4-(-2)} = \frac{5}{6}$ ("positive slope goes upward)

A = (-2, 6) B = (4, 3)

Slope of $\overline{AB} = \frac{6-3}{-2-4} = \frac{-1}{2}$ ("negative" slope goes downward)

Also, slope of $\overline{AC} = \frac{6-(-2)}{-2-(-2)} = \frac{8}{0}$ Undefined!

Vertical lines have undefined slope.
Horizontal lines have 0 slope.



Coordinate Geometry Topics and Notes

III. Linear Equations (Review)

Slope Intercept Form

$y = mx + b$

slope: m; y-intercept: b

Point Slope Form

$y - y_1 = m(x - x_1)$

point: (x_1, y_1); slope: m

Standard Form

$Ax + By = C$

where A, B, and C are integers...

note: the y-intercept b is <u>not</u> the same as the B coefficient of y

Horizontal line (form): $y = b$

Vertical line (form): $x = a$

Using Algebra to verify equivalent linear forms:

$m = \dfrac{y_1 - y_2}{x_1 - x_2}$ Begin with definition of slope....

$\dfrac{m}{1} = \dfrac{y_1 - y_2}{x_1 - x_2}$ cross multiply...

$y - y_1 = m(x - x_1)$ *Point Slope Form!

$y - b = m(x - 0)$ substitute y-intercept (0, b)

$y = mx + b$ *Slope Intercept Form!

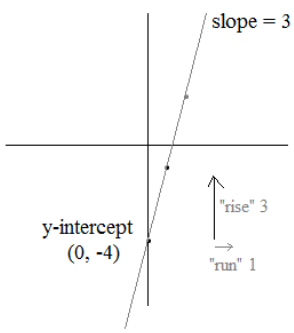
Important

Parallel lines have the *same* slope

Perpendicular lines have *negative reciprocal* slopes

Examples:

1) Graph $y = 3x - 4$



2) Is $2x + 3y = 6$ parallel to $y = \frac{-2}{3}x + 14$?

$y = \frac{-2}{3}x + 14$ slope intercept form; slope = -2/3

$2x + 3y = 6$ (change to intercept form)

$3y = -2x + 6$

$y = \frac{-2}{3}x + 3$ slope intercept form; slope = -2/3

slopes are the same! parallel lines...

3) What is the y-intercept of $4x - 3y = 12$? What is the x-intercept?

The y-intercept is the point where the line crosses the y-axis.. Its coordinate is (0, b)

$4(0) - 3(b) = 12$ (substitute (0, b) into the equation)

$-3b = 12$

$b = -4$

$\boxed{(0, -4)}$

The x-intercept is the point where a line crosses the x-axis.. Its coordinate is (?, 0) (substitute (?, 0) into the equation)

$4(?) + 3(0) = 12$

$4(?) = 12$

$\boxed{(3, 0)}$

4) Write the equation of a line with slope 4 that passes through (3, -1).

point slope form: $y - (-1) = 4(x - 3)$

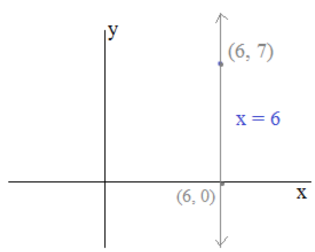
$y + 1 = 4(x - 3)$

slope intercept form: $y + 1 = 4x - 12$

$y = 4x - 13$

standard form: $4x - y = 13$

5) Write the equation of a vertical line passing through (6, 7).



6) Write the equation of a line perpendicular to $y = 3x + 5$ and passing through (2, 4)

The slope of the given line is 3.... therefore, the slope of a perpendicular line is -1/3

So, a line with slope -1/3 passing through (2, 4):

$y - 4 = -1/3(x - 2)$ (pt. slope form)

Parallel Lines and Slope

Parallel lines have the same slope.

Find the equation of a line *parallel* to $x + 2y = 6$ and passing through (3, 7).

Method 1: Find the slope of $x + 2y = 6$

$$2y = -x + 6$$

$$y = \frac{-1}{2}x + 3$$

the slope is $\frac{-1}{2}$

Then, write equation of line in point slope form

slope: -1/2
point: (3, 7)

$$y - 7 = \frac{-1}{2}(x - 3)$$ point slope form

Then, using basic algebra, convert to other forms:

$$y = \frac{-1}{2}x + \frac{3}{2} + 7$$

$$y = \frac{-1}{2}x + \frac{17}{2}$$ slope intercept form

$$2y = -x + 17$$

$$x + 2y = 17$$ standard form

Method 2: Directly substitute point to find new constant

$$x + 2y = 6 \longrightarrow x + 2y = ?$$

(3, 7) $\quad (3) + 2(7) = 17$

$$x + 2y = \cancel{6}$$

$$x + 2y = 17$$ standard form

note: $x + 2y = 6$
and
$x + 2y = 17$
have the same slope!

IV: Midpoint

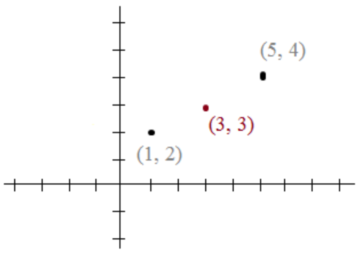
The "half-way point between two locations".
It is equidistant to each point.

The midpoint is similar to the "average"

$$\frac{P_1 + P_2}{2} = \text{Midpoint}$$

The midpoint extends to the Cartesian Plane:

Simply find the midpoint of the X values. And, the midpoint of the Y values.



The midpoint of the X Values:
$$\frac{1+5}{2} = 3$$

The midpoint of the Y Values:
$$\frac{2+4}{2} = 3$$

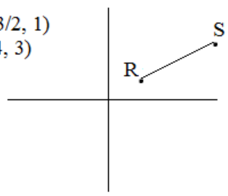
$$\left(\frac{X_1 + X_2}{2}, \frac{Y_1 + Y_2}{2} \right)$$

Midpoint Formula

Examples:

Where does the perpendicular bisector pass through $\overline{RS}$?

R = (3/2, 1)
S = (4, 3)



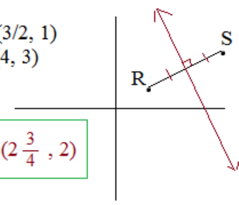
Find the midpoint of $\overline{RS}$:

X coordinate: $\frac{3/2 + 4}{2} = \frac{11/2}{2} = \frac{11}{4}$

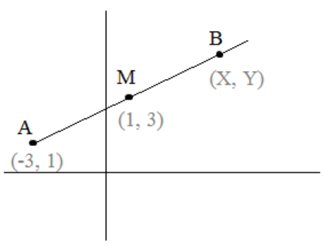
Y coordinate: $\frac{1+3}{2} = 2$

R = (3/2, 1)
S = (4, 3)

$\boxed{M = (2\tfrac{3}{4}, 2)}$



Given AB with midpoint M:
A = (-3, 1) M = (1, 3) What is B?



"Formula" Method

$$\frac{X_A + X_B}{2} = X_M \qquad \frac{Y_A + Y_B}{2} = Y_M$$

$$\frac{-3 + X_B}{2} = 1 \qquad \frac{1 + Y_B}{2} = 3$$

$$X_B = 5 \qquad \boxed{(5, 5)} \qquad Y_B = 5$$

"Travel" Method

Start at the endpoint. Determine how far you "travel" to the midpoint. Then, add the same amount.

A $\quad$ M
$(-3, 1) \longrightarrow (1, 3)$

X value increased 4 units..
Y value increased 2 units..

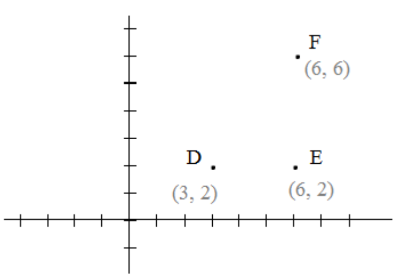
M $\quad$ B
$(1, 3) \longrightarrow (1 + 4, 3 + 2)$

$\boxed{(5, 5)}$

V. Distance

The space between 2 points.
The length of the line segment connecting two points.

Cartesian Plane:



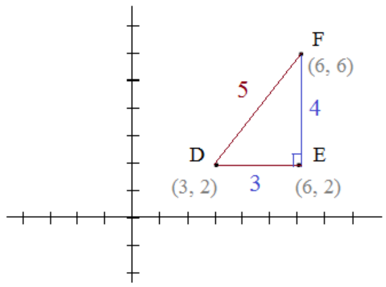
The distance between D and E is 3 units...
(3, 2), (4, 2), (5, 2), and (6, 2)

And, the distance between E and F is 4 units...
(6, 2), (6, 3), (6, 4), (6, 5), (6, 6)

So, what is the distance between D and F?
(And, it is not 7!!)

Pythagorean Theorem

$a^2 + b^2 = c^2$



Notice, in this case, that the points can be vertices of a right triangle..

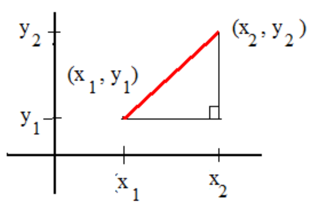
So, $\overline{DE}^2 + \overline{EF}^2 = \overline{DF}^2$

$9 + 16 = 25$

Therefore, the length of $\overline{DF}$
(i.e. distance between D and F)
$= 5$

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

Distance Formula



Find the distance between (-2, 5) and (4, 7).

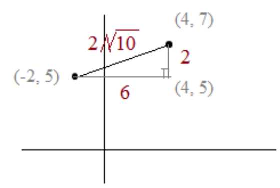
Using Distance Formula:

$$d = \sqrt{(-2-4)^2 + (5-7)^2}$$

$$= \sqrt{(-2-4)^2 + (5-7)^2}$$

$$= \sqrt{36 + 4} = 2\sqrt{10}$$

Using Pythagorean Theorem:

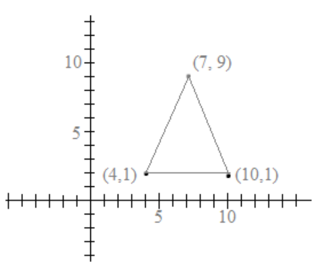


A vertical line drawn from (4, 7) intersects a horizontal line from (-2, 5) at (4, 5).. These form a right triangle!

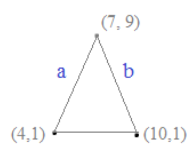
Then, using the pythagorean theorem, the hypotenuse is $2\sqrt{10}$

Examples:

Use coordinate geometry to prove the triangle is isosceles.



Def. of isosceles: triangle with 2 congruent sides.



$$a = \sqrt{(7-4)^2 + (9-1)^2}$$

$$= \sqrt{9 + 64} = \boxed{\sqrt{73}}$$

$$b = \sqrt{(7-10)^2 + (9-1)^2}$$

$$= \sqrt{9 + 64} = \boxed{\sqrt{73}}$$

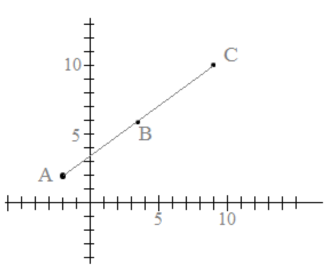
a = b, therefore the triangle is isosceles...

Verify the length of $\overline{AB}$ equals the length of $\overline{BC}$

A = (-2, 2)

B = (3.5, 6)

C = (9, 10)



Method 1: Using Midpoint

Midpoint of $\overline{AC}$

$$\left(\frac{-2+9}{2}, \frac{2+10}{2}\right)$$

$(3.5, 6)$

since B is the midpoint of $\overline{AC}$, $\overline{AB} = \overline{BC}$

Method 2: Using Distance

$$d\overline{AB} = \sqrt{(-2-3.5)^2 + (2-6)^2}$$

$$= \sqrt{30.25 + 16} = \boxed{6.80}$$

$$d\overline{BC} = \sqrt{(3.5-9)^2 + (6-10)^2}$$

$$= \sqrt{30.25 + 16} = \boxed{6.80}$$

$d\overline{AB} = d\overline{BC}$