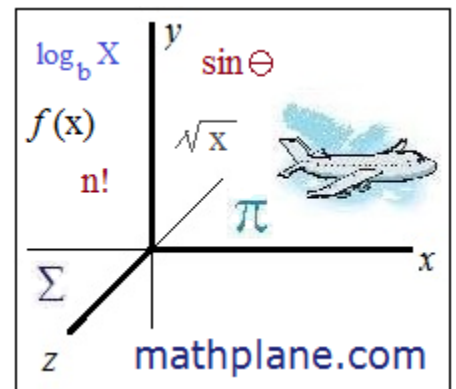# %   Percentages   %

Notes, Examples, and Quiz (w/Solutions)

mathplane.com

Percentages: Notes & Examples

A percentage expresses "a ratio out of 100"

Per - Cent   ---> Per 100   "portion out of 100"   (In latin, *cent*um means 100)
                                                                  century
                                                                  centipede
                                                                  centimeter....

Example: 37 percent

37%   .37 $\frac{37}{100}$ 37 out of every 100

A percentage shows a "portion of something"

25% represents 25 out of 100   or, 50 out of 200

                                     or, 1 out of 4

                                     or, any other equivalent fraction or ratio

Finding X% of a given number:

Set up ratios. Then, solve...

Example: What is 25% of 80?

Set up the ratios: $\frac{25}{100} = \frac{X}{80}$ Part
                                                        Whole

Solve: (cross multiply)

100X = (25)(80)   ("20 out of 80 is the
$\boxed{X = 20}$ same as 25 out of 100")

Shortcut: Change percentage to a decimal. Then, multiply...

Example: What is 35% of 200?

35% ---->   .35

.35 x 200 = $\boxed{70}$

Finding the percentage of one number out of another:

Set up the ratios. Solve.

Example: What percentage of 30 is 12?   Or, "12 is what percentage of 30?"

Set up the ratios: $\dfrac{X}{100} = \dfrac{12}{30}$ — Part/Portion
    ⎯ Whole

Solve for X: $\dfrac{X}{100} = \dfrac{2}{5}$ Reduce the fraction

$5X = 2(100)$ Cross multiply

$X = 40$ **40%**

Alternate method: Convert the ratio into a decimal. Then, convert the decimal into a percentage.
(i.e. move 2 decimal places and add % symbol)

Example: 23 is what percentage of 58?   Or, "what percent of 58 is 23?"

Convert the ratio into a decimal: $\dfrac{23}{58}$ is approximately .39655

Convert into a percentage: .39655 ------> **39.655%**
2 decimal places

Quick check:
20 out of 50 is 40%
23 out of 58 is close to 40%

X% of what number is another number:

Set up ratios. Solve.

Example: 30% of what number is 42?

Set up ratios: $\dfrac{30}{100} = \dfrac{42}{X}$ — Part that's taken
    ⎯ Whole

Solve: $\dfrac{3}{10} = \dfrac{42}{X}$

$3X = 420$ **X = 140**

Shortcut: Set up decimal equation. Solve.

Example: 28% of what number is 20?

.28X = 20

$X = \dfrac{20}{.28} \cong$ **71.43**   (i.e. 28% of 71.43 is approx. 20...)

| Percentages: Notes & Examples | Steps to determine percentage increase/decrease: |

Steps to determine percentage increase/decrease:
1) Find Change
2) Establish "starting point"
3) Solve
4) Check for "reasonableness"

Percentage Increase/Decrease

Example: What is the percentage increase from 20 to 25?

Find change: the increase is 5

"starting point": the starting point is 20

solve: what percentage is 5 out of 20? $\dfrac{5}{20} = .25$ therefore, 25%

Example: What is the percentage decrease from 25 to 20?

Find change: the decrease is 5   (i.e. -5)

"starting point" (basis): the starting point is 25

solve: what percentage is 5 out of 25? $\dfrac{5}{25} = .20$ therefore, 20%

check: 10% of 25 is 2.5... So, 20% of 25 is 5...

***NOTE: In the above examples, the numbers are 20 and 25... And, the change is 5...
But, the percentages are different! (*percentage change depends on the starting point!*)

Example: After a 30% increase, the total is 100. What was the <u>original</u> amount?

Suppose you simply decreased 30% from 100. The result is 70. But, that is NOT correct!
IF you added 30% to 70, the answer is not 100... It is 91...

SOLUTION: original amount + increase = final amount

$\qquad\qquad$ X $\qquad$ + (30% of X) = 100 $\qquad\qquad$ check:

$\qquad\qquad$ X $\qquad$ + .30X = 100 $\qquad\qquad\qquad$ 30% of 76.9 is approx.
$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ 23.1

$\qquad\qquad\qquad\qquad$ 1.3X = 100

$\qquad\qquad\qquad\qquad$ 76.9 + 23.1 = 100 ✓

$\qquad\qquad\qquad$ X = 76.9 (approximately)

Example: A $34 shirt is on sale for 20% off. What is the price of the shirt?

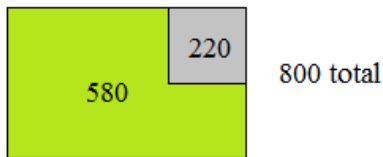$\qquad\qquad$ original amount + increase/decrease = final amount

$\qquad\qquad\qquad$ $34 $\quad$ + $\quad$ (-.20)($34) $\quad$ = final price

$\qquad\qquad\qquad$ $34 $\quad$ − $\quad$ $6.8 $\quad\quad$ = $27.20

Word problems:

Like most word problems, a solid strategy is to draw a picture and/or label variables. Then, construct the formula and solve.

Example: Joe's entire backyard is 800 square feet. If the garden is 220 square feet, what percentage of Joe's backyard is the garden?

580 | 220 | 800 total

$$\text{percentage} = \frac{X \ (percent)}{100} = \frac{220}{800} \quad \text{— "portion" (garden)}$$
$$\text{whole (backyard)}$$

$$\frac{X}{100} = \frac{11}{40}$$

$$40X = 1100 \qquad X = 27.5$$

27.5%

("check for reasonableness": 25% or 1/4 of 800 is 200... So, 220 out of 800 should be a little bit more than 25%)

Example: Sam enjoyed the dinner and service at his favorite restaurant. The final bill was $48. If he left an 18% tip, how much did he spend at the restaurant?

Total cost = dinner bill + tip

  Cost = $48 + (18% of $48)

  $48 + $8.64 = $56.64

.18 x $48 = $8.64

or, $\dfrac{18}{100} = \dfrac{X}{48}$ X = 8.64

Example: <u>After</u> a 35% discount, the cost of a shirt $19.50. What is the *original* price of the shirt (without the discount)?

Original price − discount = Cost of shirt

  X − (35% of X) = $19.50

  X − .35X = $19.50

  .65X = $19.50

  X = $30

35% of X

$$\frac{35}{100} = \frac{discount}{X}$$

$$35X = 100(discount)$$

$$discount = \frac{35X}{100} = .35X$$
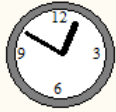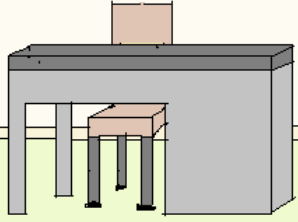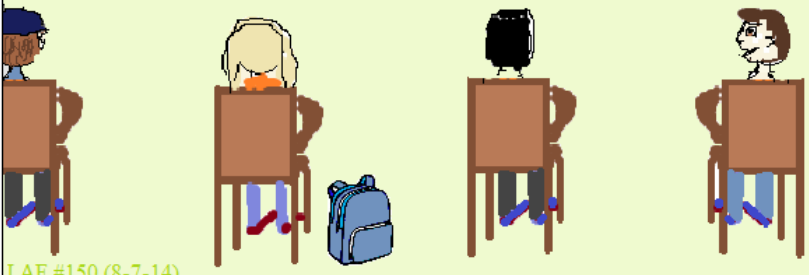
Practice Test (And, Solutions) -→

Percentages Quiz

I. Find X in each expression:

1) 35% of 200 is X

2) 4% of 20 is X

3) 22% of X is 11

4) 55% of X is 100

5) X% of 48 is 6

6) X% of 6 is 48

II. Percentage Increase/Decrease

Determine the percentage increase/decrease:

1) 4 to 6

2) 90 to 108

3) 30 to 70

4) 41 to 38

5) 26.50 to 21.50

6) 9 to 0

Find the result:

7) cut 13% from 200

8) decrease 88 by 25%

9) increase 34 by 20%

10) increase 25 by 500%

III. Word problems/applications

1) The price of the lunch special is $9. If sales tax is 7%, what is the total cost?

2) At the local high school, there are 357 boys and 395 girls. What percentage of the students are boys?

3) The price of gas has gone up from $2.75 to $4.35. What is the percentage increase?

4) If you borrow $500 at an annual interest rate of 6%, how much will you owe after 1 year?

5) After a 20% discount, the price of a men's suit is $185. What was the *original* price of the suit?

***Challenge: Last year, an investment lost 30% of its value.
What *percentage* increase is necessary this year to recover the lost value?

Hidden Message

Clue: "It may be 4%"

Solve/Answer questions below.
Translate numbers to letters.

%   %

%   Letter Key   %

1	2	3	4	5	6	7	8	9	0
A	C	E	I	M	N	O	R	S	T

1) 40% of 5 ☐ → __

2) A bag contains 20 colored marbles (red, blue, or green).
   If 35% are blue, 6 are green, how many are red? ☐ → __

3) 20% of math students will get an 'A'.
   If a class has 14 girls and 11 boys,
   how many students will earn 'A's? ☐ → __

4) An 80% free throw shooter attempts 25 shots.
   How many shots does he expect to miss? ☐ → __

5) 2 out of 50: ☐ % ☐ → __

6) 18% of 217   3☐.06 → __

7) 150% of 6 ☐ → __

8) A matinee cost \$5.40. If the show ordinarily cost 9 dollars,
   what discount (%) did you receive? ☐0% → __

9) Dinner cost \$30 plus sales tax. If tax is 10% and you leave
   \$40, what tip did you leave your waiter? ☐ → __

10) Inside the instructions box above, what percentage of
    the letters are 'a' ? ☐% → __

11) .8% of 1000 ☐ → __

12) A shirt retails for \$28. If you get a 25% discount,
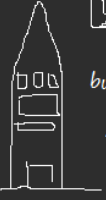   how much does the shirt cost you? \$2☐ → __

13) 30% of produced cars are white. If 1698 cars are white,
    what is the total number manufactured? 566☐ → __

14) A square has an area of 100 sq. feet.
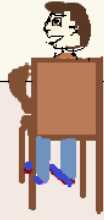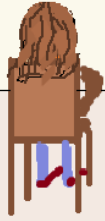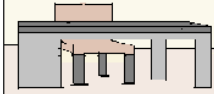   If you reduce the length of each side by 40%, what is the
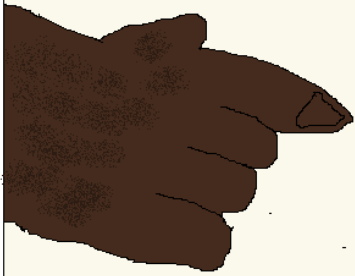   area of the new square? ☐6 ft^2 → __
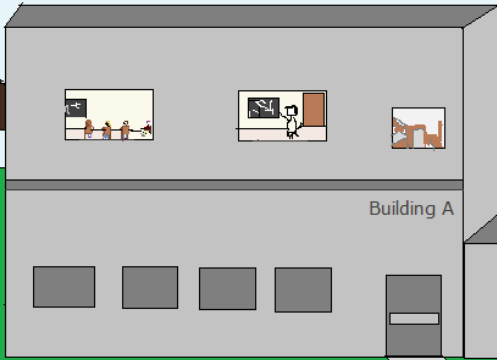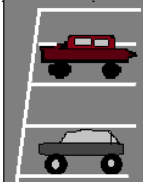
Solutions→

Percentages Quiz

I. Find X in each expression:

1) 35% of 200 is X $\frac{35}{100} = \frac{X}{200}$

.35 x 200 = 70   OR   100X = 35(200)

$X = 70$

2) 4% of 20 is X   note: 40% of 20 is 8
                           then, 4% of 20 is .8

.04 x 20 = .8

3) 22% of X is 11

$\frac{22}{100} = \frac{11}{X}$ ⌐ part
                     ⌐ whole   22X = 1100

$X = 50$

4) 55% of X is 100

$\frac{55}{100} = \frac{100}{X}$ 55X = 10000

$X \cong 181.82$

5) X% of 48 is 6   "6 out of 48 is what %?"

$\frac{X}{100} = \frac{6}{48}$

$\frac{X}{100} = \frac{1}{8}$ $X = 12.5$

6) X% of 6 is 48

$\frac{X}{100} = \frac{48}{6}$

$X = 800$

II. Percentage Increase/Decrease

Determine the percentage increase/decrease:

1) 4 to 6   increase amount: 2   2 out of
              starting amount: 4   4 is 50%

2) 90 to 108   increase: 18   starting basis: 90

18 out of 90 ---> $\frac{18}{90} = .20$ 20%

3) 30 to 70   increase: 40
                starting amount: 30 $\frac{40}{30} = 1.3\overline{3}$

133.3$\overline{3}$%

4) 41 to 38   decrease: 3
                starting point: 41 $\frac{3}{41} \cong .0732$

approx. 7.32%

5) 26.50 to 21.50

decrease: 5 $\frac{5}{26.5} \cong .189$
start: 26.50

approx. 18.9%

6) 9 to 0

taking all away ---> 100%

decrease: 9   starting basis: 9

$\frac{9}{9} = 1$ (move 2 decimal
                    places)

Find the result:

7) cut 13% from 200

.13 x 200 = 26   200 - 26 = 174

8) decrease 88 by 25%

25% of 88 is 22....   88 - 22 = 66 $\frac{25}{100} = \frac{22}{88}$

9) increase 34 by 20%

.20 x 34 = 6.8   then, 34 + 6.8 = 40.8

10) increase 25 by 500%

increase 25 by 100% is +25
so, increase 25 by 500% is 5 x 25 = 125

25 + 125 = 150

III. Word problems/applications   SOLUTIONS

1) The price of the lunch special is $9. If sales tax is 7%, what is the total cost?

      Cost = lunch price + sales tax

       = $9   +   (.07)($9)   = $9.63

2) At the local high school, there are 357 boys and 395 girls. What percentage of the students are boys?

     total population: $357 + 395 = 752$ $\dfrac{X}{100} = \dfrac{357}{752}$ — "portion" $752X = 100(357)$

                                             "whole" $X = 47.47$

3) The price of gas has gone up from $2.75 to $4.35. What is the percentage increase?

     The increase is $1.60 $\dfrac{1.6}{2.75}$ is approx. .582
     The "starting point" (basis) is $2.75

                                 price has gone up about 58.2%

4) If you borrow $500 at an annual interest rate of 6%, how much will you owe after 1 year?

     After 1 year, you will owe $500   +   interest   =

                   $500   +   .06($500)   = $530

5) <u>After</u> a 20% discount, the price of a men's suit is $185. What was the *original* price of the suit?

Notice, we are looking for the *original* price.   Original price − discount   =   final price
So, we need to construct the approriate
formula and variables....   X   −   X(.20)   = $185

      Quick check: 231.25   .80X = $185
             20% of 231.25 is 46.25
             231.25 - 46.25 = 185   X = $231.25

***Challenge: Last year, an investment lost 30% of its value.
               What *percentage* increase is necessary this year to recover the lost value?

Suppose the investment were $100...
   After 1 year, the investment would be worth $70..   *So, what percentage is necessary to*
              $100 − .30(100) = 70$ *increase 70 to 100?*

                                       increase is 30; starting basis is 70

                              $\dfrac{30}{70} \cong .4286$ approximately 42.86%

Hidden Message

Clue: "It may be 4%"

Solve/Answer questions below.
Translate numbers to letters.

Solutions

%	Letter Key	%
1 2 3 4 5 6 7 8 9 0		
A C E I M N O R S T		

1) 40% of 5   .40 x 5 = 2   2 → C

2) A bag contains 20 colored marbles (red, blue, or green). If 35% are blue, 6 are green, how many are red?
   20 total: 7 blue (.35 x 20)
      6 green
      7 red   7 → O

3) 20% of math students will get an 'A'. If a class has 14 girls and 11 boys, how many students will earn 'A's?
   .20 (14 + 11) = 5   5 → M

4) An 80% free throw shooter attempts 25 shots. How many shots does he expect to miss?
   An 80% shooter expects to miss 20%....
     25 x .20 = 5   5 → M

5) 2 out of 50: [4]%
   2 out of 50 is equivalent to 4 out of 100 ---> 4% $\frac{2}{50}$ = .04   4 → I

6) 18% of 217   .18 x 217 = 39.06   3[9].06 → S

7) 150% of 6   100% of 6 is 6... 50% of 6 is 3   150% = 9   1.50 x 6 = 9   9 → S

8) A matinee cost $5.40. If the show ordinarily cost 9 dollars, what discount (%) did you receive?
   original   discount   matinee
   9.00 − (9.00x) = 5.40
     − (9.00x) = −3.60
      x = .40 (40%)   [4]0% → I

9) Dinner cost $30 plus sales tax. If tax is 10% and you leave $40, what tip did you leave your waiter?
   total bill: 30 dinner
      +3 tax
      33 dollars
   If you leave $40, then the tip is $7.   7 → O

10) Inside the instructions box above, what percentage of the letters are 'a' ?
   there are 50 total letters and 3 'a's... (see above)
   3 out of 50 is 6%   6% → N

11) .8% of 1000   .008 x 1000 = 8   8% of 100 is 8 so, .8% of 1000 is 8   8 → R

12) A shirt retails for $28. If you get a 25% discount, how much does the shirt cost you?
   25% of 28 = 7 therefore, shirt costs 28−7 = $21 $2[1] → A

13) 30% of produced cars are white. If 1698 cars are white, what is the total number manufactured?
   $\frac{30}{100}$ = $\frac{1698 \text{ white}}{x \text{ total}}$ cross multiply and solve to get 5660   566[0] → T

14) A square has an area of 100 sq. feet. If you reduce the length of each side by 40%, what is the area of the new square?
   10 − (.40 x 10) = 6
   10   100 sq. ft.   10   6   36 sq ft   6   3[6] ft^2 → E

Thanks for visiting. (Hope it helped!)

If you have questions, suggestions, or requests, let us know!

Cheers
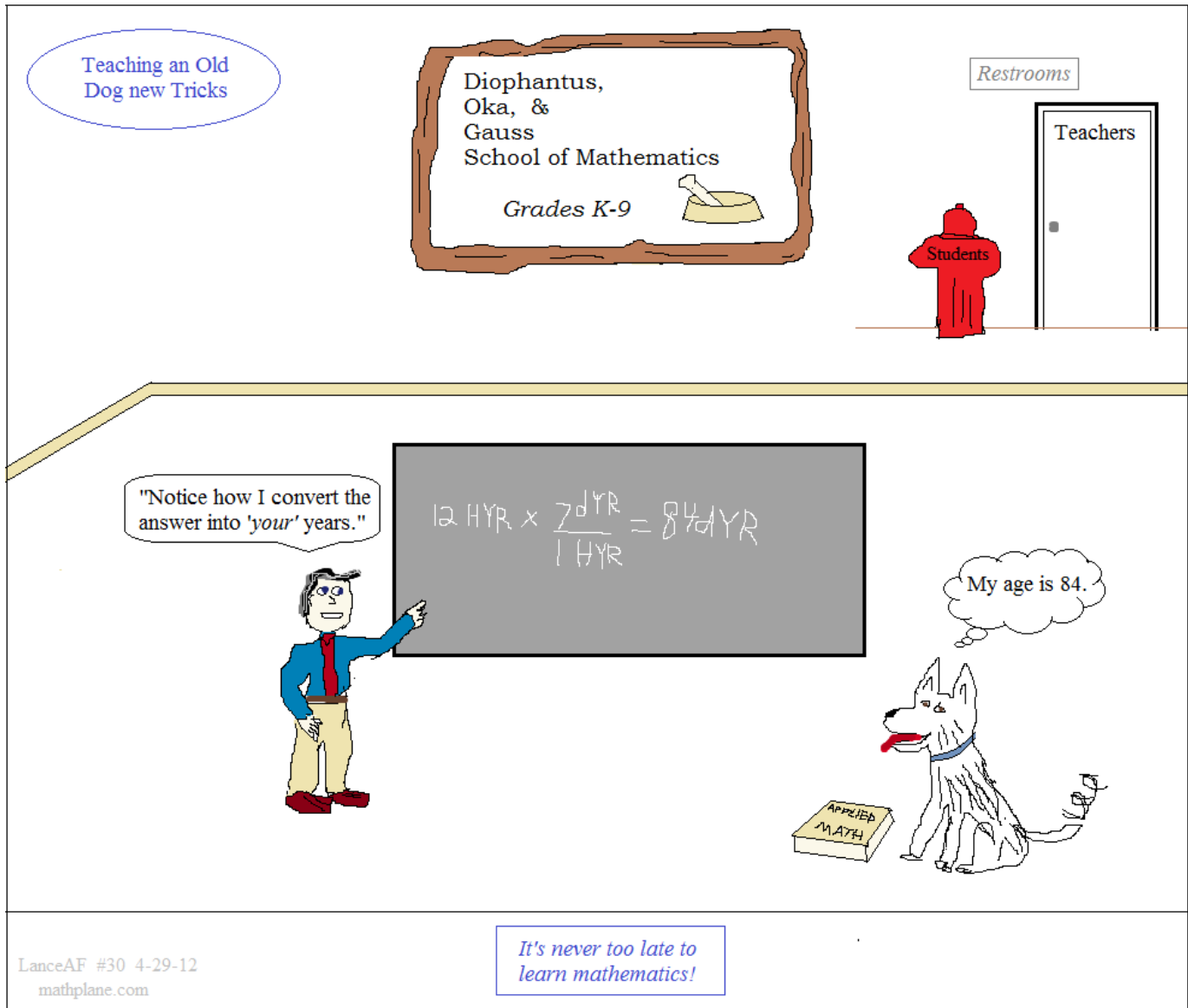
Find more comics and math resources at the Mathplane site, Facebook, Google+, TeachersPayTeachers, TES, and Pinterest

Also, Mathplane *Express* for mobile at Mathplane.ORG

TWO MORE QUESTIONS….

In my backyard, I have a rectangular garden with dimensions 30' x 10' (length x width)..

If I reduce the length by 20%, what percentage should I increase the width
      a) to maintain the same area?
      b) to maintain the same perimeter?
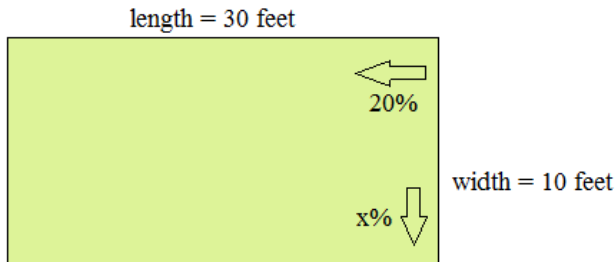
length = 30 feet

20%

width = 10 feet

x%
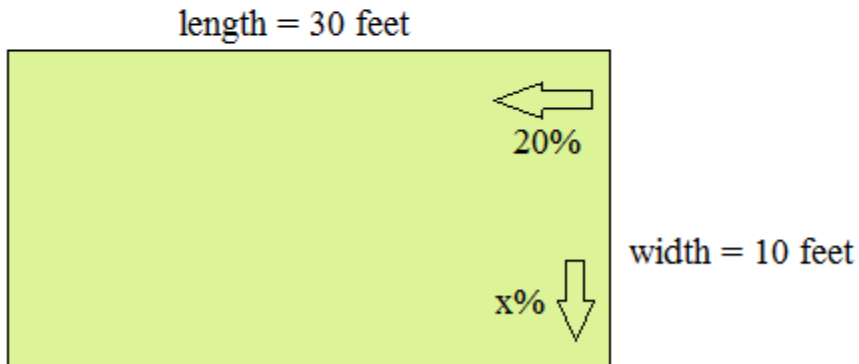
ANSWERS on next page….

In my backyard, I have a rectangular garden with dimensions 30' x 10' (length x width)..

If I reduce the length by 20%, what percentage should I increase the width
    a) to maintain the same area?
    b) to maintain the same perimeter?

length = 30 feet

20%

x%

width = 10 feet

ANSWERS

a) Area....

    Original area is 300 square feet...

    New length is 30' − (.20 x 30') = 24'

    24' x (new width) = 300 square feet

        new width = 12.5'

10 feet ---> 12.5 feet (increase of 2.5 feet)

percentage increase: $\dfrac{2.5 \text{ feet}}{10 \text{ feet}}$ = 25%

b) Perimeter....

    Original perimeter is 80 feet....
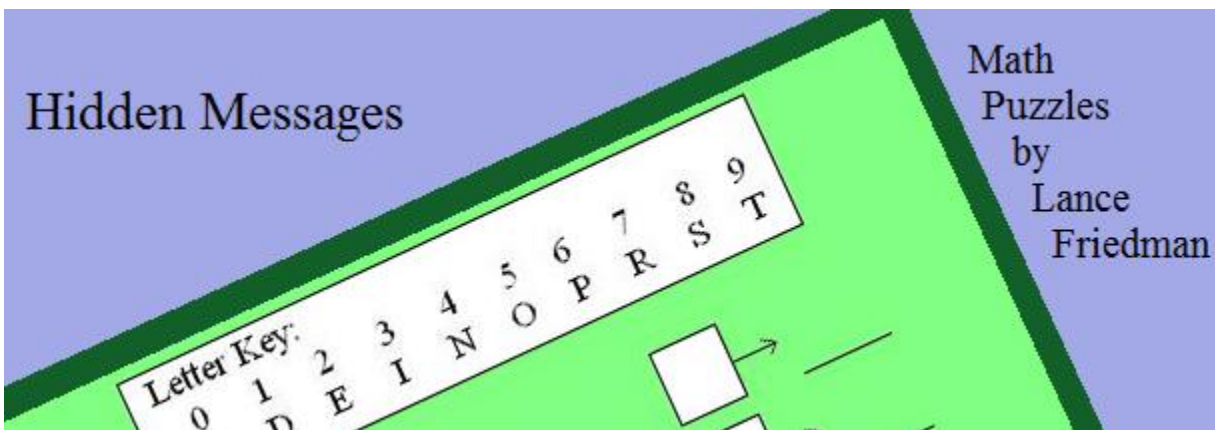
    New length is 24'...

    (2 x 24') + (2 x new width) = 80 feet

        2 x new width = 32 feet

        new width = 16 feet

10 feet ---> 16 feet (increase of 6 feet)

percentage increase: $\dfrac{6 \text{ feet}}{10 \text{ feet}}$ = 60%

******

Hidden Messages

Letter Key:

0	1	2	3	4	5	6	7	8	9
D	E	I	N	O	P	R	S	T	

Math Puzzles by Lance Friedman

Available at mathplane.com