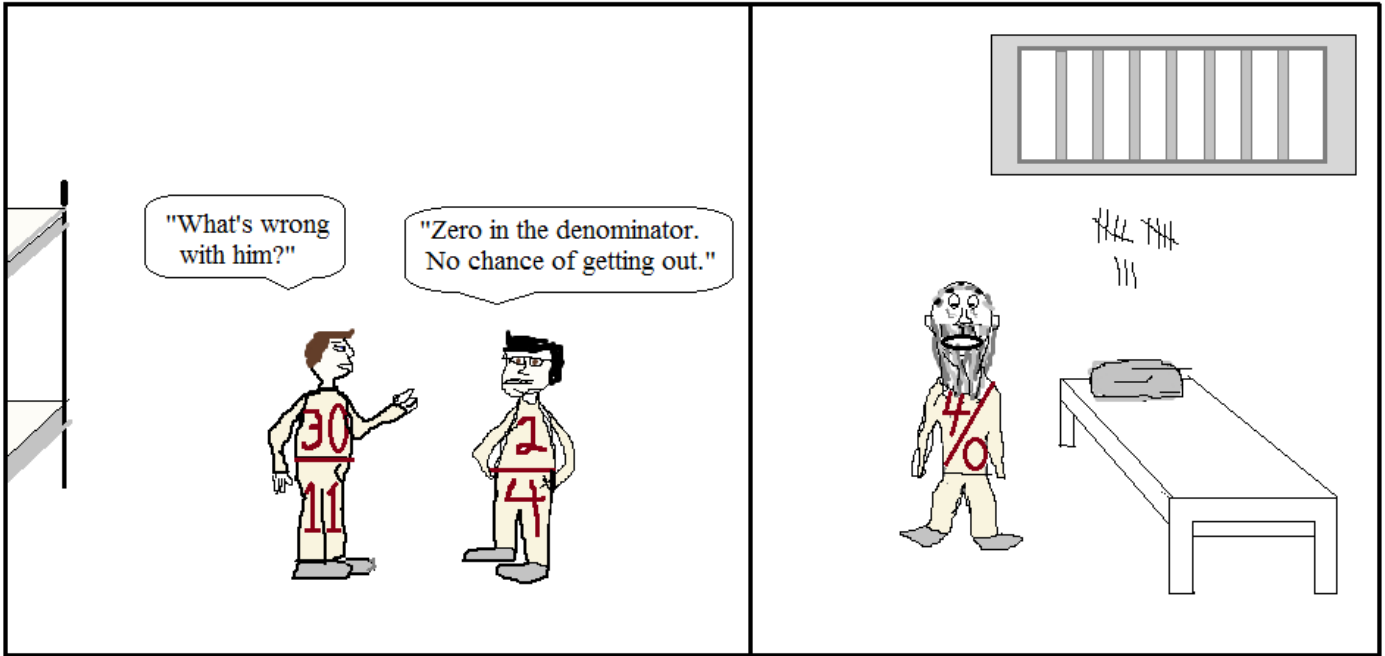
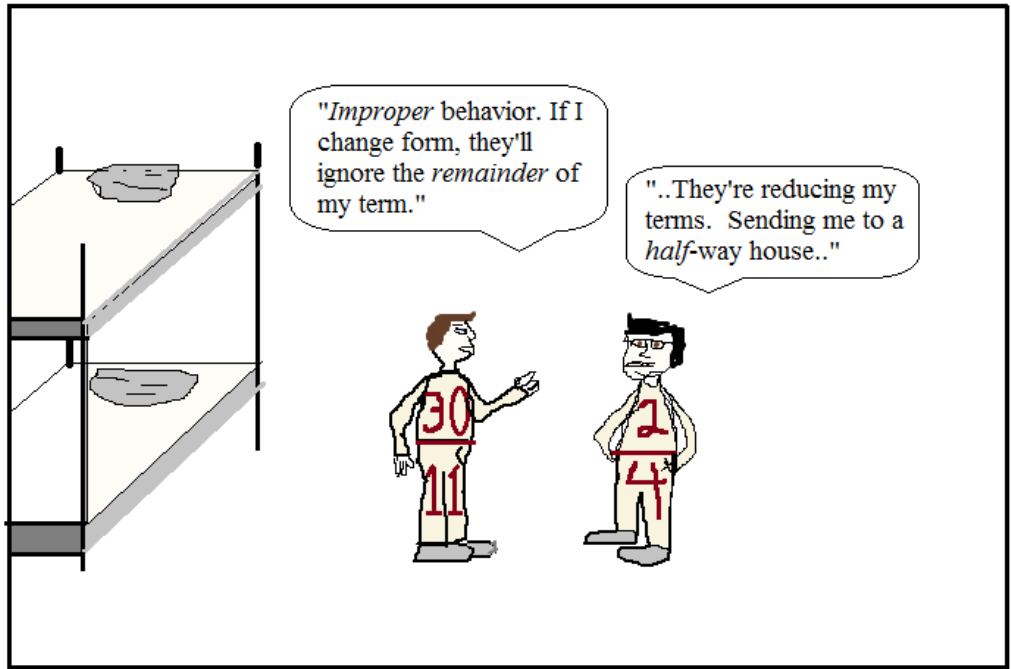


# Fractions

Notes, Examples, and Exercises (with solutions)

*Topics include common denominator, mixed numbers, improper fractions, and more.*

in-Fractions



Math Problems: Fractions
--------------------------

## I. Addition

a)  $\frac{3}{8} + \frac{1}{48} =$

b)  $\frac{1}{2} + \frac{1}{3} + \frac{1}{6} =$

c)  $\frac{3}{27} + \frac{2}{6} =$

d)  $\frac{10}{25} + \frac{1}{5} =$

e)  $3 + \frac{2}{7} + \frac{3}{14} =$

## II. Subtraction

a)  $\frac{1}{2} - \frac{1}{8} =$

b)  $\frac{1}{8} - \frac{1}{2} =$

c)  $\frac{5}{8} - \frac{27}{56} =$

d)  $\frac{3}{4} - \frac{4}{9} =$

e)  $1 - \frac{47}{99} =$

Math Problems: Fractions
--------------------------

## III. Multiplication

a)  $4 \cdot \frac{7}{8} =$

b)  $\frac{-3}{8} \cdot \frac{5}{6} =$

c)  $\frac{2}{9} \cdot \frac{3}{28} =$

d)  $\frac{14}{17} \cdot \frac{85}{96} =$

e)  $\frac{-47}{48} \cdot \frac{96}{94} =$

## IV. Division

a)  $3 \div \frac{3}{4} =$

b)  $\frac{2}{5} \div \frac{4}{15} =$

c)  $0 \div \frac{9}{10} =$

d)  $\frac{1}{6} \div \frac{11}{60} =$

e)  $\frac{27}{63} \div \frac{1}{7} =$

*Hidden Message*

Clue: "What fractions can be?"

Solve and answer the thirteen questions below. Then, translate numbers to letters to reveal the hidden message.

Letter Key:

1	2	3	4	5	6	7	8	9	0
A	I	L	M	N	O	P	R	S	T

1) Find X:  $\frac{3}{X} = \frac{1}{3}$

→ \_\_\_\_\_

2) Express  $3\frac{1}{4}$  as an improper fraction.

$\frac{13}{4}$   
 → \_\_\_\_\_

3) If  $m = n \neq 0$ , then what is  $\frac{m}{n}$  ?

→ \_\_\_\_\_

4)  $\frac{5}{7} + \frac{7}{5} = \frac{74}{\square 5}$

→ \_\_\_\_\_

5)  $2/3 \times 9/2 =$

→ \_\_\_\_\_

6) Write  $\frac{29}{4}$  as a mixed number.

$\frac{1}{4}$  → \_\_\_\_\_

7)  $\frac{1}{2} - \frac{1}{3} = \frac{1}{\square}$

→ \_\_\_\_\_

8) What is the least common denominator of  $1, \frac{1}{2}, \frac{1}{4},$  and  $\frac{1}{8}$  ?

→ \_\_\_\_\_

9)  $\frac{(17-17)}{(1.232 + 323)} =$

→ \_\_\_\_\_

10)  $\frac{\text{number of 'a's in the instructions box above}}{\text{number of 'd's in the instructions box above}} =$

→ \_\_\_\_\_

11) What is the reciprocal of  $\frac{1}{6}$  ?

→ \_\_\_\_\_

12)  $\frac{1}{.2} =$

→ \_\_\_\_\_

13)  $3 \div \frac{1}{3} =$

→ \_\_\_\_\_

Pennsylvania  
Middle School

"If I spend 10 hours per day in a coffin, how many hours per year, do I avoid the sunlight?"



rates and units

$$\frac{10 \text{ hours}}{1 \text{ day}} \times \frac{365 \text{ days}}{1 \text{ year}} =$$

Announcements

10/31/15  
Career Day  
Mortician  
Grave Digger

Guest speaker:  
Herman  
Munster



"Excuse me, Mr. Dracula. What about leap years?!?"



"When will we ever use this in real life?"



"I know. This class bites."



Zzzz...



Why are his office hours 2:00 - 6:00 AM?

Algebra 13

Horror-ible  
Teacher

A boring teacher can suck the life out of you!



L. Friedman #213 (10-30-15)  
mathplane.com

ANSWERS-→

## I. Addition

a)  $\frac{3}{8} + \frac{1}{48} = \frac{18}{48} + \frac{1}{48} = \frac{19}{48}$

b)  $\frac{1}{2} + \frac{1}{3} + \frac{1}{6} = \frac{3}{6} + \frac{2}{6} + \frac{1}{6} = 1$

c)  $\frac{3}{27} + \frac{2}{6} = \frac{1}{9} + \frac{1}{3} = \frac{1}{9} + \frac{3}{9} = \frac{4}{9}$

d)  $\frac{10}{25} + \frac{1}{5} = \frac{2}{5} + \frac{1}{5} = \frac{3}{5}$

e)  $3 + \frac{2}{7} + \frac{3}{14} = 3 + \frac{4}{14} + \frac{3}{14} = 3\frac{1}{2}$

## II. Subtraction

a)  $\frac{1}{2} - \frac{1}{8} = \frac{4}{8} - \frac{1}{8} = \frac{3}{8}$

b)  $\frac{1}{8} - \frac{1}{2} = \frac{1}{8} - \frac{4}{8} = -\frac{3}{8}$

) (opposites)

c)  $\frac{5}{8} - \frac{27}{56} = \frac{35}{56} - \frac{27}{56} = \frac{8}{56} = \frac{1}{7}$

d)  $\frac{3}{4} - \frac{4}{9} = \frac{27}{36} - \frac{16}{36} = \frac{11}{36}$

e)  $1 - \frac{47}{99} = \frac{99}{99} - \frac{47}{99} = \frac{52}{99}$

## III. Multiplication

$$a) 4 \cdot \frac{7}{8} = \frac{4}{1} \cdot \frac{7}{8} = \frac{28}{8} = \frac{7}{2}$$

$$b) \frac{-3}{8} \cdot \frac{5}{6} = \frac{-15}{48} = \frac{-5}{16}$$

$$c) \frac{\overset{1}{\cancel{2}}}{\underset{3}{\cancel{9}}} \cdot \frac{\overset{1}{\cancel{3}}}{\underset{14}{\cancel{28}}} = \frac{1}{42}$$

$$d) \frac{\overset{1}{\cancel{14}}}{\underset{1}{\cancel{17}}} \cdot \frac{\overset{5}{\cancel{85}}}{\underset{48}{\cancel{96}}} = \frac{\overset{7}{\cancel{14}}}{1} \cdot \frac{5}{\cancel{96}} = \frac{35}{48}$$

$$e) \frac{\overset{-1}{\cancel{47}}}{\underset{1}{\cancel{48}}} \cdot \frac{\overset{2}{\cancel{96}}}{\underset{2}{\cancel{94}}} = -1$$

## IV. Division

(to divide fractions, simply  
"invert and multiply" )

$$a) 3 \div \frac{3}{4} = \frac{3}{1} \cdot \frac{4}{3} = 4$$

$$b) \frac{2}{5} \div \frac{4}{15} = \frac{\overset{1}{\cancel{2}}}{5} \cdot \frac{\overset{3}{\cancel{15}}}{\underset{2}{\cancel{4}}} = \frac{3}{2}$$

$$c) 0 \div \frac{9}{10} = 0$$

$$d) \frac{1}{6} \div \frac{11}{60} = \frac{1}{\underset{1}{\cancel{6}}} \cdot \frac{\overset{10}{\cancel{60}}}{11} = \frac{10}{11}$$

$$e) \frac{27}{63} \div \frac{1}{7} = \frac{\overset{3}{\cancel{27}}}{\underset{1}{\cancel{63}}} \cdot \frac{7}{1} = 3$$



**Hidden Message**

Clue: "What fractions can be?"

Solve and answer the thirteen questions below. Then, translate numbers to letters to reveal the hidden message.

**Letter Key:**

1	2	3	4	5	6	7	8	9	0
A	I	L	M	N	O	P	R	S	T

**SOLUTIONS**

1) Find X:  $\frac{3}{X} = \frac{1}{3}$  (cross multiply)  $3 \cdot 3 = 1 \cdot X$   $X = 9$

2) Express  $3\frac{1}{4}$  as an improper fraction.  $(4 \times 3) + 1 = 13$   $\frac{13}{4}$

3) If  $m = n \neq 0$ , then what is  $\frac{m}{n}$ ?  $\frac{m}{n} = 1$  if  $m=n$  and  $\neq 0$

4)  $\frac{5}{7} + \frac{7}{5} = \frac{74}{35}$   $\frac{25}{35} + \frac{49}{35} = \frac{74}{35}$

5)  $2/3 \times 9/2 = \frac{2}{3} \times \frac{9}{2} = 9/3 = 3$

6) Write  $\frac{29}{4}$  as a mixed number. 29 divided by 4 is 7 with a remainder of 1...  $7$  and  $1/4$

7)  $\frac{1}{2} - \frac{1}{3} = \frac{1}{6}$   $\frac{3}{6} - \frac{2}{6} = \frac{1}{6}$

8) What is the least common denominator of  $1, \frac{1}{2}, \frac{1}{4},$  and  $\frac{1}{8}$ ?  $\frac{8}{8}, \frac{4}{8}, \frac{2}{8}, \frac{1}{8}$

9)  $\frac{(17-17)}{(1.232 + 323)} = \frac{0}{?} = 0$

10)  $\frac{\text{number of 'a's in the instructions box above}}{\text{number of 'd's in the instructions box above}} = \frac{6}{3} = 2$   
(a's and d's are outlined above)

11) What is the reciprocal of  $\frac{1}{6}$ ?  $\frac{1}{\frac{1}{6}} = 1 \times \frac{6}{1} = 6$

12)  $\frac{1}{.2} =$  (change .2 into 1/5)  $1/(1/5) = 5$

13)  $3 \div \frac{1}{3} =$  (invert and multiply)  $3 \times \frac{3}{1} = 9$

What fractions can be?  
"Small Portions"

9 → S

$\frac{13}{4}$  → M

1 → A

3 → L

3 → L

7  $\frac{1}{4}$  → P

6 → O

8 → R

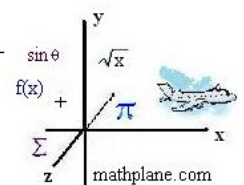
0 → T

2 → I

6 → O

5 → N

9 → S



Mixed Number (or, Mixed Fraction)

What is it? A number written as a whole number combined with a proper fraction

Examples:

$5 \frac{2}{3}$

Yes.

5 is whole number

$\frac{2}{3}$  is a proper fraction

~~$6 \frac{7}{4}$~~

No

6 is a whole number, but  $\frac{7}{4}$  is not a proper fraction...

$-3 \frac{23}{24}$

Not exactly

-3 is not a whole number..  
But, written as

$(-1) \cdot 3 \frac{23}{24}$

3 is a whole number and  $\frac{23}{24}$  is a proper fraction...

The value of a mixed number = whole + fraction

Examples:  $3 \frac{5}{8} = 3 + \frac{5}{8}$

$$11 \frac{14}{15} = 11 + \frac{14}{15}$$

whole
proper fraction

Improper Fraction

What is it? A fraction where the numerator is greater than the denominator.

Examples:

$\frac{7}{4}$

Yes

$7 > 4$

~~$\frac{5}{9}$~~

No

~~$5 < 9$~~

(this is a proper fraction)

$\frac{-8}{5}$

????

rewritten as  $(-1) \frac{8}{5}$

$8 > 5$ , so this is an improper fraction

Converting Improper Fraction into Mixed Number

Example:  $\frac{23}{4}$

How many times can 4 go into 23?

$23 \div 4 = 5$  with remainder 3

$5 \frac{3}{4}$

Notice, converting improper fraction into mixed number is simply "the numerator divided by the denominator", where the remainder is expressed as a fraction...

Converting Mixed Number into Improper Fraction

Example:  $7 \frac{2}{5}$

Remember,  $7 \frac{2}{5} = 7 + \frac{2}{5} \Rightarrow \frac{35}{5} + \frac{2}{5} = \frac{37}{5}$

common denominator

\*\*\*SHORTCUT: "Multiply, then add.. Over the denominator.." Or, "Denominator times whole plus numerator, over the denominator"

$$\begin{array}{l}
 + \\
 \begin{array}{r}
 7 \\
 \times 5 \\
 \hline
 35 \\
 + 2 \\
 \hline
 37
 \end{array} \\
 \times
 \end{array}$$

$7 \times 5 = 35$   
 $35 + 2 = 37 \Rightarrow \frac{37}{5}$

$$\begin{array}{c}
 \text{numerator} \\
 | \\
 \text{whole} - 7 \frac{2}{5} \\
 | \\
 \text{denominator}
 \end{array}
 \qquad
 \frac{5 \times 7 + 2}{5} = \frac{37}{5}$$

Mixed Number Operations

Multiplication: "Convert to improper fractions" then, "multiply and simplify"

Examples:

$$2\frac{3}{4} \times 1\frac{2}{5}$$

convert to improper fractions

$$\frac{11}{4} \times \frac{7}{5}$$

multiply (and simplify)

$$\frac{77}{20} \text{ or } 3\frac{17}{20}$$

$$5\frac{2}{3} \cdot 4\frac{1}{5}$$

convert to improper fractions

$$\frac{17}{3} \cdot \frac{21}{5}$$

cross cancel the 21 and 3.

$$\frac{17}{1} \cdot \frac{7}{5}$$

multiply and simplify

$$\frac{119}{5} \text{ or } 23\frac{4}{5}$$

Division: "Convert to improper fractions" then, "invert & multiply and simplify"

Examples:

$$7\frac{2}{3} \div 2\frac{1}{4}$$

convert to improper fractions

$$\frac{23}{3} \div \frac{9}{4}$$

invert and multiply

$$\frac{23}{3} \times \frac{4}{9}$$

$$\frac{92}{27} \text{ or } 3\frac{11}{27}$$

$$4\frac{1}{3} \div 9\frac{3}{4}$$

convert to improper fractions

$$\frac{13}{3} \div \frac{39}{4}$$

invert and multiply (cross cancel)

$$\frac{13}{3} \times \frac{4}{39}$$

$$\frac{1}{3} \times \frac{4}{3} = \frac{4}{9}$$

Addition: "Add the individual parts" then, "simplify"

OR "Convert to improper fractions" then, "add and simplify"

Example:

$$6\frac{2}{7} + 5\frac{5}{6}$$

$$6 + \frac{2}{7} + 5 + \frac{5}{6}$$

$$11 + \frac{47}{42} = 12\frac{5}{42}$$

sum of whole numbers      sum of fractions

OR

$$6\frac{2}{7} + 5\frac{5}{6}$$

$$\frac{44}{7} + \frac{35}{6}$$

$$\frac{264}{42} + \frac{245}{42} = \frac{509}{42} = 12\frac{5}{42}$$

NOTE: this second approach involves larger numbers, so it can be a bit more difficult..

Subtraction and "borrowing" ( Sometimes the mixed number being subtracted has a larger proper fraction ... )

Example:  $10\frac{1}{8} - 7\frac{1}{4}$

Approach 1: subtract whole numbers, subtract fractions, then add results...

$$10 - 7 = 3 \text{ difference of whole numbers}$$

$$\frac{1}{8} - \frac{1}{4} = -\frac{1}{8} \text{ difference of the fractions}$$

$$3 + (-\frac{1}{8}) = 2\frac{7}{8}$$

Approach 2: Convert to improper fractions

$$\frac{81}{8} - \frac{29}{4} =$$

$$\frac{81}{8} - \frac{58}{8} = \frac{23}{8}$$

$$= 2\frac{7}{8}$$

Approach 3: "Borrowing"

$$10\frac{1}{8} - 7\frac{2}{8}$$

since  $2/8 > 1/8$ , we'll 'borrow' from the whole number 10.. (i.e. change 10 into  $9 + 8/8$ )

$$9\frac{9}{8} - 7\frac{2}{8}$$

subtract whole numbers      subtract fractions

$$9 - 7 = 2 \quad \frac{9}{8} - \frac{2}{8} = \frac{7}{8}$$

$$= 2\frac{7}{8}$$

## Adding and Subtracting Mixed Fractions

**Method 1:**  
 Add whole numbers  
 Add fractions  
 Combine

*Example:*  $3\frac{2}{3} + 5\frac{3}{4} =$

Step 1: Add the whole numbers

$$3 + 5 = 8$$

Step 2: Common Denominators?

$$\frac{2}{3} \quad \frac{3}{4}$$

↓            ↓

12 is the least common denominator

Step 3: Add the fractions

$$\frac{8}{12} + \frac{9}{12} = \frac{17}{12}$$

↓

$$\begin{array}{r} 12 \overline{) 17} \\ - 12 \\ \hline 5 \end{array}$$

$$\begin{array}{r} 1 \text{ R}5 \\ 12 \overline{) 17} \\ - 12 \\ \hline 5 \end{array}$$

Step 4: Change improper fraction?

Step 5: Combine

$$8 + 1\frac{5}{12} = 9\frac{5}{12}$$

**Method 2:**  
 Convert to improper fractions  
 Add/Subtract  
 Convert back to mixed fraction

*Example:*  $5\frac{1}{7} - 3\frac{1}{5} =$

Step 1: Convert to improper fractions

$$5\frac{1}{7} - 3\frac{1}{5} =$$

$$7 \times 5 + 1 = 36 \quad 5 \times 3 + 1 = 16$$

Step 2: Common Denominators?

$$\frac{36}{7} - \frac{16}{5} =$$

↓            ↓

35 is the least common denominator

Step 3: Subtract

$$\frac{180}{35} - \frac{112}{35} = \frac{68}{35}$$

Step 4: Change improper fraction

$$1\frac{33}{35}$$

$$\begin{array}{r} 1 \text{ R}33 \\ 35 \overline{) 68} \\ - 35 \\ \hline 33 \end{array}$$

Mixed Numbers/Mixed Fractions Quiz

1) Circle the mixed numbers. Underline the improper fractions.

$\frac{7}{2}$

$5\frac{6}{53}$

$5\frac{10}{7}$

$\frac{3}{11}$

$-4\frac{11}{12}$

"fourteen fourths"

"seven and five sixths"

"five sevenths"

2) Convert to mixed numbers

$\frac{7}{4} =$

$\frac{21}{5} =$

$-\frac{71}{6} =$

$\frac{100}{8} =$

"sixteen fifths" equals

3) Convert to improper fractions

$3\frac{1}{3} =$

$-7\frac{5}{8} =$

$2\frac{21}{23} =$

"four and five ninths" equals

"eight and a third" equals

4) Addition/Subtraction

$6\frac{1}{2} + 11\frac{1}{2} =$

$5\frac{1}{6} + 8\frac{1}{3} =$

"fourteen and a half plus six and an eighth" equals

$10\frac{1}{8} - 7\frac{1}{4} =$

5) Multiplication/Division

$2\frac{1}{5} \times 5\frac{1}{2} =$

$7\frac{2}{9} \cdot 3\frac{3}{5} =$

$9\frac{1}{7} \div 4\frac{2}{3} =$

"four and a half times seven and a fourth"

Mixed Numbers/Mixed Fractions Quiz

SOLUTIONS

1) Circle the mixed numbers. Underline the improper fractions.

$\underline{\frac{7}{2}}$      $5 \frac{6}{53}$      $5 \frac{10}{7}$      $\frac{3}{11}$      $-4 \frac{11}{12}$     "fourteen fourths"    "seven and five sixths"    "five sevenths"  
 (not exactly)     $-1 \times 4 \frac{11}{12}$      $\frac{5}{7}$

2) Convert to mixed numbers

$\frac{7}{4} = 1 \frac{3}{4}$      $\frac{21}{5} = 4 \frac{1}{5}$      $-\frac{71}{6} = -11 \frac{5}{6}$      $\frac{100}{8} = 12 \frac{4}{8} = 12 \frac{1}{2}$     "sixteen fifths" equals  $\frac{16}{5} = 3 \frac{1}{5}$   
 100 divided by 8 equals 12 with a remainder of 4    "three and one fifth"

3) Convert to improper fractions

$3 \frac{1}{3} = 3 \times 3 + 1 = 10$ , so  $\frac{10}{3}$      $-7 \frac{5}{8} = (-1)7 \frac{5}{8}$ ,  $8 \times 7 + 5 = 61$ , then, multiply by -1, so  $-\frac{61}{8}$      $2 \frac{21}{23} = 23 \times 2 + 21 = 67$ , so  $\frac{67}{23}$     "four and five ninths" equals  $4 \frac{5}{9} = \frac{9 \times 4 + 5}{9} \rightarrow \frac{41}{9}$  "forty-one ninths"    "eight and a third" equals  $8 \frac{1}{3} = \frac{3 \times 8 + 1}{3} \rightarrow \frac{25}{3}$  "twenty-five thirds"

4) Addition/Subtraction

$6 \frac{1}{2} + 11 \frac{1}{2} = 17 + 1 = 18$  (add the parts)    OR  $\frac{13}{2} + \frac{23}{2} = \frac{36}{2} = 18$  (add improper fractions)    "fourteen and a half plus six and an eighth" equals  $14 \frac{1}{2} + 6 \frac{1}{8} = 14 + 6 = 20$      $\frac{1}{2} + \frac{1}{8} = \frac{5}{8}$  "twenty and five eighths"     $10 \frac{1}{8} - 7 \frac{1}{4} = \frac{81}{8} - \frac{29}{4} = \frac{81}{8} - \frac{58}{8} = \frac{23}{8} = 2 \frac{7}{8}$      $10 - 7 = 3$      $\frac{1}{8} - \frac{1}{4} = -\frac{1}{8}$      $3 - \frac{1}{8} = 2 \frac{7}{8}$     OR  $9 \frac{9}{8} - 7 \frac{2}{8} = 2 \frac{7}{8}$

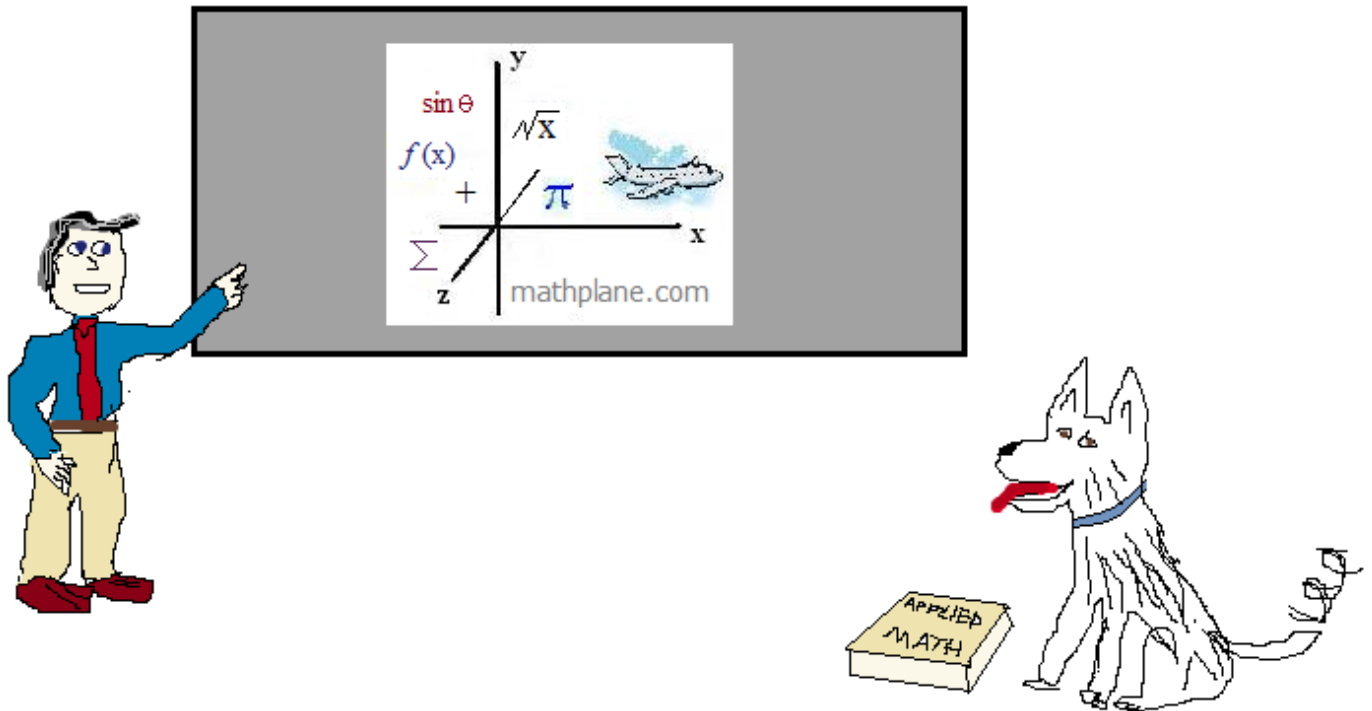
5) Multiplication/Division

$2 \frac{1}{5} \times 5 \frac{1}{2} = \frac{11}{5} \times \frac{11}{2} = \frac{121}{10}$     or  $12 \frac{1}{10}$      $2.2 \times 5.5 = 12.1$      $7 \frac{2}{9} \cdot 3 \frac{3}{5} = \frac{65}{9} \cdot \frac{18}{5} = \frac{13 \cancel{65} \cdot \cancel{18} 2}{1 \cdot \cancel{9} \cdot \cancel{5} 1} = 26$  ("cross cancelling")     $9 \frac{1}{7} \div 4 \frac{2}{3} = \frac{64}{7} \div \frac{14}{3} = \frac{64}{7} \times \frac{3}{14} = \frac{32}{7} \times \frac{3}{7} = \frac{96}{49} = 1 \frac{47}{49}$     "four and a half times seven and a fourth"     $4 \frac{1}{2} \cdot 7 \frac{1}{4} = \frac{9}{2} \cdot \frac{29}{4} = \frac{261}{8} = 32 \frac{5}{8}$  "Thirty-two and five eighths"

Thanks for visiting. (Hope it helps!)

If you have questions, suggestions, or requests, then let us know

Cheers



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