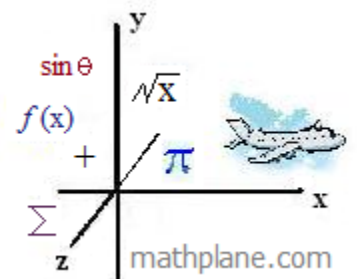# Greatest Common Factor &

Least Common Multiple

Notes, Examples, and Practice Quiz (w/Solutions)

Topics include factor trees, rainbow, GCF, LCM, and more...

Factors:

What are they?   Numbers that are multiplied to get a number.

1, 2, 7, 14   are factors of 14
1, 2, 4, 5, 10, 20 are factors of 20
x, y, 1, 3 are factors of 3xy
1, 41 are the only factors of 41

Note: it is open to debate whether negative numbers can be considered factors; after all, -2 x -6 = 12.. So, are -2 and -6 factors of 12?

Note: Prime numbers have only 2 factors: 1 and the number itself.
1 is a factor of any number.

Common Factors:

What are they?   Factors that are the same for 2 (or more) numbers.

1, 2, 11, 22 are factors of 22
1, 3, 11, 33 are factors of 33

1 & 11 are common factors of 22 and 33

1, 2, 4, 5, 8, 10, 20, 40 are factors of 40
1, 2, 4, 8, 16, x, y   are factors of 16xy

1, 2, 4, & 8 are common factors of 40 and 16xy

Greatest Common Factor:

What is it? The largest number among common factors.

GCF of 22 and 33 is 11
GCF of 40 and 16xy is 8

When do you use it?   To reduce fractions.

$$\frac{24}{60} = \frac{2}{5}$$

Factors of 24:   1, 2, 3, 4, 6, 12, 24
Factors of 60:   1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60

(1, 2, 3, 4, 6, and 12 are common factors.
But, 12 is the greatest common factor!)

Divide numerator and denominator by 12 to reduce fraction to lowest terms.

Multiples:

What are they? Numbers added to themselves.

2, 4, 6, 8, 10...   (multiples of 2)

5, 10, 15, 20...   (multiples of 5)

3xy, 6xy, 9xy, 12xy...   (multiples of 3xy)

> Note: Multiples include zero and negative numbers.
>
> Example: -44, -33, -22, -11, 0, 11, 22, 33, 44...
>   are multiples of 11
>
> ** However, when finding the "least common multiple",
>   search for the smallest positive multiple.

Common Multiples:

What are they? Multiples that are the same for 2 (or more numbers)

3 6 9 12 15 18 21 24 27 30 33 36 39 ...   (multiples of 3)

4 8 12 16 20 24 28 32 36 40 44 ...   (multiples of 4)

12   24   36 ...   (common multiples of 3 and 4)

observation: the common multiples of 3 and 4 are multiples of 12

2 4 6 8 10 12 ...   (multiples of 2)
5 10 15 20 25 ...   (multiples of 5)
10 20 30 40 50 ...   (multiples of 10)

10   20   30 ...   (common multiples of 2, 5, and 10)

observation: 2 and 5 are also factors of 10

Least Common Multiple:

What is it? The lowest (positive) number among common multiples.

6 12 18 24 30 36 42 48 54 60 ... (multiples of 6)
2 4 6 8 10 12   ... 24 26 28 30 ...   (multiples of 2)
5 10 15 20 25 30 35 40 45 ...   (multiples of 5)

Least Common Multiple of 2 and 5?   10

LCM of 2 and 6?   6

LCM of 5 and 6?   30

LCM of 2, 5, and 6?   30

Note: 30 60 90 120 ... are common multiples of 5 and 6;
                 But, the least common multiple is only 30

Definition: "Smallest positive number that is divisible into all of the sets' members, leaving no remainder."

Example: Set A = multiples of 7   (all numbers 7n, where n is an integer)

Set B = multiples of 5   (all numbers 5n, where n is an integer)

Set A = {... -21, -14, -7, 0, 7, 14, 21, 28, 35, 42... }

Set B = {... -10, -5, 0, 5, 10, 15, 20, 25, 30, 35, 40... }

eliminate 0 and negative numbers...

find first common multiple...

When would we use 'least common multiple'?

-----> To find the common denominator of 2 or more fractions.

$$\frac{3}{8} + \frac{2}{5} = \frac{15}{40} + \frac{16}{40} = \frac{31}{40}$$

(40 is the least common multiple of 5 and 8)

multiples of 5:   5 10 15 20 25 30 35 40 45 50 ...

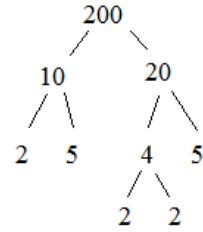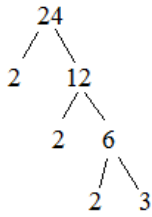multiples of 8:   8 16 24 32 40 48 56 ...

Finding factors: "The rainbow"

Start with the obvious factors: 1 and the number itself...
Then, work your way to the inside by finding factor pairs...

Example: Find the factors of 24

1   24

1 2   12 24

1 2 3   8 12 24

1 2 3 4   6 8 12 24

1 2 3 4 6 8 12 24

Example: Find the factors of 100

1   100

1 2   50 100

(skip 3, because it's not a factor)

1 2 4   25 50 100

1 2 4 5   20 25 50 100

(skip 6, 7, 8, and 9... Not factors)

1 2 4 5 10   20 25 50 100

What is a factor tree?

A branching diagram showing the factors of number.

Examples:

```
        24                    49              200
       / \                   / \             /    \
      2   12                7   7          10      20
         / \                              / \      / \
        2   6                            2   5    4   5
           / \                                   / \
          2   3                                 2   2
```

Note: the end of each branch is a prime number.

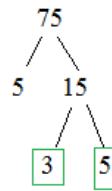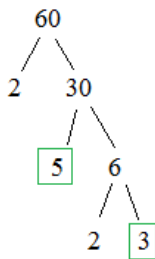Applications:

1) Displaying all prime factors

```
           150
          /   \
        10     15
       / \    / \
      2   5  3   5
```
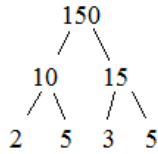
2, 3, and 5 are all prime factors

2) Finding greatest common factor

```
        60                         75
       / \                        /  \
      2   30                     5    15
         / \                          / \
       [5]  6                       [3] [5]
           / \
          2  [3]
```

Since 3 and 5 are common factors, the greatest common factor is 3 x 5 = [15]

Sitting under a factor tree may inspire *numerous* ideas.

The *Nature* of *Mathematics*

L Friedman #25 3-25-12
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QUICK QUIZ-→

GCF, LCM, and Factor Tree Quick Quiz

I. Find the GCF (Greatest Common Factor)

   A) 8 and 20   B)   42 and 91   C) 5 and 25

II. Find the LCM (Least Common Multiple)

   A) 6 and 11   B)   3 and 4   C) 5 and 25

III. Using a factor tree, find all the prime factors.

   A) 42   B) 78   C) 350

IV. Miscellaneous

   A) What is the greatest common factor of 2 prime numbers?

   B) What is the least common multiple of 3, 5, and 7?

   C) List all factors of 120.

I. Find the GCF (Greatest Common Factor)

A) 8 and 20

factors of 8: 1, 2, 4, 8
factors of 20: 1, 2, 4, 5, 10, 20

4

B) 42 and 91

factors of 42: 1, 2, 3, 6, 7, 14, 21, 42
factors of 91: 1, 7, 13, 91

7

C) 5 and 25

factors of 5: 1, 5
factors of 25: 1, 5, 25

5

II. Find the LCM (Least Common Multiple)

A) 6 and 11

multiples of 6: 6, 12, 18, 24, ... 60, 66
multiples of 11: 11, 22, 33, ... 55, 66

66

B) 3 and 4

multiples of 3: 3, 6, 9, 12, 15
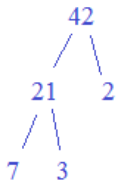multiples of 4: 4, 8, 12, 16, ...

12

C) 5 and 25

multiples of 5: 5, 10, 15, 20, 25, 30
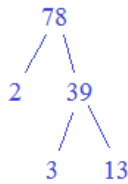multiples of 25: 25, 50, ..

25

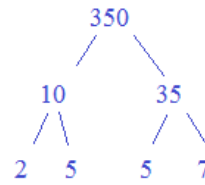## III. Using a factor tree, find all the prime factors.

A) 42

```
      42
     /  \
   21    2
  /  \
 7    3
```

2, 3, and 7

B) 78

```
     78
    /  \
   2    39
       /  \
      3    13
```

2, 3, and 13

C) 350

```
        350
       /   \
      10     35
     / \    / \
    2   5  5   7
```

2, 5, and 7

IV. Miscellaneous

A) What is the greatest common factor of 2 prime numbers?

Since prime numbers have only 2 factors -- 1 and themselves -- the GCF between two primes is 1.

B) What is the least common multiple of 3, 5, and 7?

3, 5, and 7 are prime.. Therefore, the first common multiple will be 3 x 5 x 7 = 105
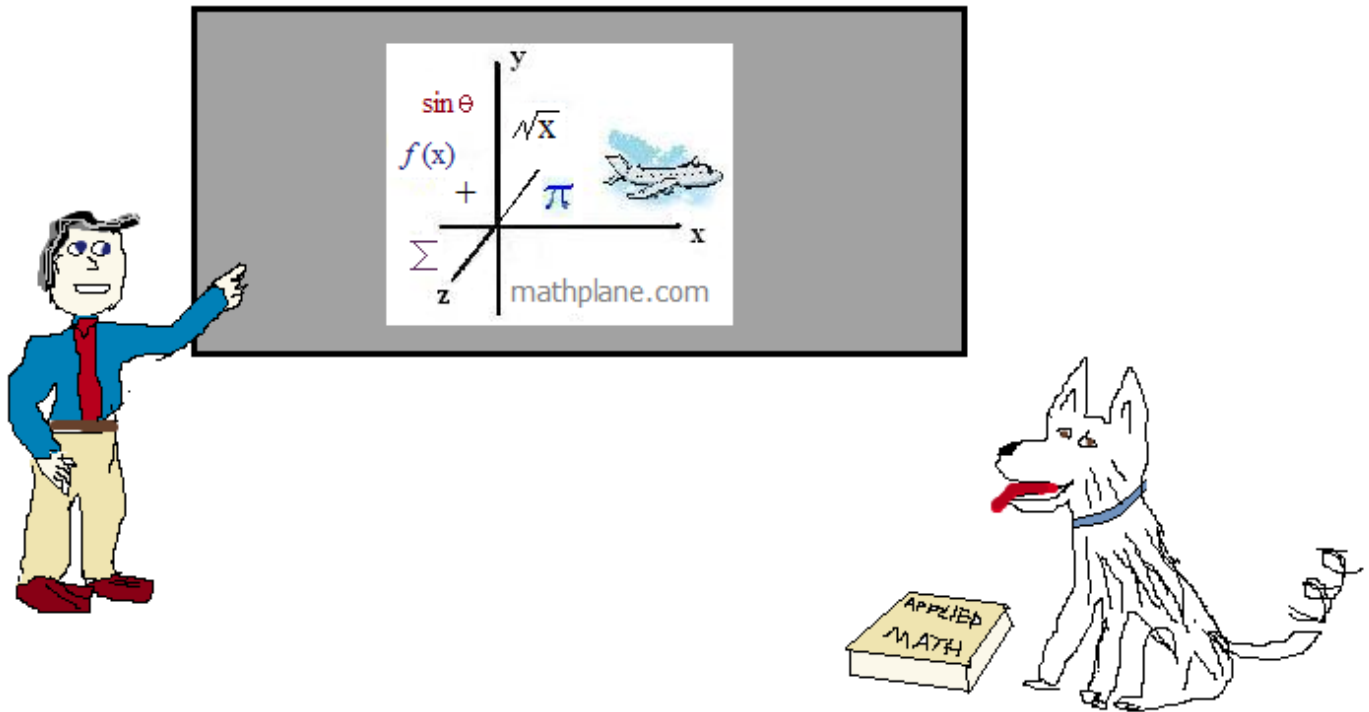
C) List all factors of 120.

1, 2, 3, 4, 5, 6, 8, 10, 12, 15, 20, 24, 30, 40, 60, 120

1 x 120
2 x 60
3 x 40
4 x 30
5 x 24
6 x 20
8 x 15
10 x 12

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Thanks for visiting. (Hope it helped!)

If you have questions, suggestions, or requests, let us know

Cheers

ONE MORE MATH QUESTION:

How many numbers are both multiples of 3 AND factors of 60?

(Answer on next page)

Factors of 60: 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60

Multiples of 3: 3, 6, 9, 12 … 54, 57, 60…

BOTH: 3, 6, 12, 15, 30, 60