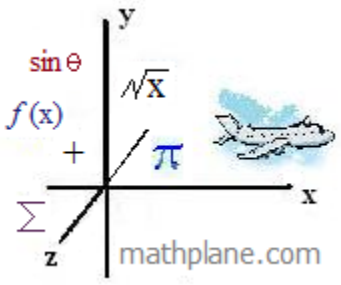
Conversion and Measurement

Notes, examples, and a practice quiz (with Solutions)



Math Conversion Notes

I. Simple conversions

The purpose and goal is to change the units without changing the initial amount.

1) "multiply by 1"
2) cancel units
3) simplify
4) check answer

Example: Convert 5 feet into inches.

12 inches = 1 foot Therefore, $\frac{1 \text{ foot}}{12 \text{ inches}} = \frac{12 \text{ inches}}{1 \text{ foot}} = 1$

$$5 \text{ feet} \times \left\langle \frac{12 \text{ inches}}{1 \text{ feet}} \right\rangle =$$

Note: the amount of 5 feet didn't change, because it was multiplied by 1

$$5 \text{ feet} \times \left\langle \frac{12 \text{ inches}}{1 \text{ feet}} \right\rangle = 60 \text{ inches}$$

Note: cancel the 'feet' units, leaving 'inches' (this is a useful way to check answers)

Example: How many quarters are in $10?

There are 4 quarters in 1 dollar.

$$\frac{4 \text{ quarters}}{1 \text{ dollar}} = 1 = \frac{1 \text{ dollar}}{4 \text{ quarters}}$$

Which fraction should be used?

$$10 \text{ dollars} \cdot \left\langle \frac{4 \text{ quarters}}{1 \text{ dollar}} \right\rangle = \frac{40 \text{ dollars(quarters)}}{\text{dollar}} = 40 \text{ quarters}$$

this answer seems reasonable, AND the units are correct!

or

$$10 \text{ dollars} \cdot \left\langle \frac{1 \text{ dollar}}{4 \text{ quarters}} \right\rangle = \frac{10 \text{ dollars(dollar)}}{4 \text{ quarters}} = \frac{5 \text{ dollar}^2}{2 \text{ quarters}}$$

The units are incorrect; the solution is wrong.

Example: Convert 30 mph into feet/minute.

rewrite 30 mph: $\frac{30 \text{ miles}}{1 \text{ hour}}$

convert hours into minutes: $\frac{30 \text{ miles}}{1 \text{ hour}} \times \left\langle \frac{1 \text{ hour}}{60 \text{ minutes}} \right\rangle = \frac{30 \text{ miles}}{60 \text{ minutes}}$

$$= \frac{1 \text{ miles}}{2 \text{ minutes}}$$

convert miles into feet: $\frac{1 \text{ miles}}{2 \text{ minutes}} \times \left\langle \frac{5280 \text{ feet}}{1 \text{ mile}} \right\rangle = \frac{5280 \text{ feet}}{2 \text{ minutes}} = 2640 \text{ feet/minute}$

Math Conversion Notes

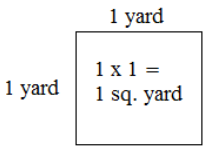
II. Complex conversions

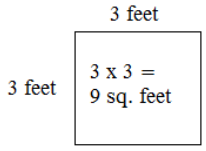
When calculating/converting area and volume, convert, calculate, and keep track of the units!!

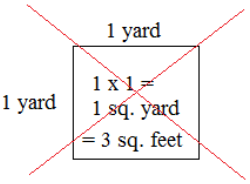
1 yard = 3 feet

1 square yard = 9 square feet

$1\ yd^2 = 9\ feet^2$





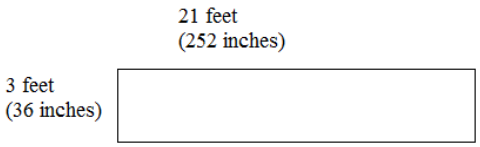


Check the units!

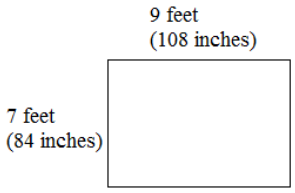
Incorrect

Example: A rectangular backyard is 63 square feet. Convert into square inches.

Method 1: Convert each side to inches; Then, calculate the area.



252 inches x 36 inches = 9072 square inches



108 inches x 84 inches = 9072 square inches

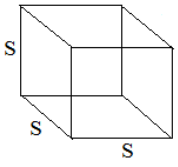
Method 2: Convert the area directly, keeping track of the units.

$$63\ feet^2 \cdot \left\langle \frac{12\ inches}{1\ feet} \right\rangle = \frac{756\ feet^2(inches)}{feet} = 756\ feet(inches)$$

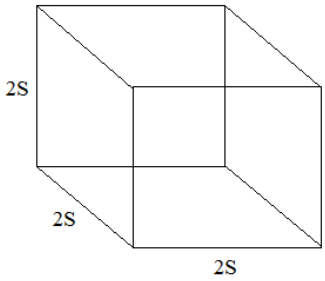
$$= 756\ feet(inches) \cdot \left\langle \frac{12\ inches}{1\ feet} \right\rangle = \frac{9072\ feet(inches)(inches)}{feet} = 9072\ inches^2$$

Example: A cube has sides of length S. What is its volume?

Suppose the length is *doubled*. What is the new volume?



The volume is $S \times S \times S = S^3$



The new volume is $2S \times 2S \times 2S = 8S^3$

**Although we converted each side from S to 2S, the volume did not double. It increased eight-fold!

Conversion Notes and Examples

(Converting one unit of measurement)

"Multiply by 1"

A convenient method is to multiply by the necessary ratio.

Example: How many feet are in 5 miles?

We want to convert miles into feet, so we need a ratio that equates feet to miles.

Since 5280 feet = 1 mile

$$\frac{1 \text{ mile}}{5280 \text{ feet}} = 1 \quad \text{or} \quad \frac{5280 \text{ feet}}{1 \text{ mile}} = 1$$

$$5 \text{ miles} \times \frac{5280 \text{ feet}}{1 \text{ mile}} = 26{,}400 \text{ feet}$$

(Note: the unit "miles" cancel, leaving "feet")

Example: Convert 5 liters into ounces.

1 liter is approximately 33.8 ounces.

Therefore, $\frac{33.8 \text{ ounces}}{1 \text{ liter}} \approx 1$

or $\frac{1 \text{ liter}}{33.8 \text{ ounces}} \approx 1$

$$5 \text{ liters} \times \frac{33.8 \text{ ounces}}{1 \text{ liter}} = 169 \text{ ounces (approx.)}$$

Unit Measures

What is it? An amount, ratio, or proportion expressed in terms of 1 unit

How do you find it? Express the measurement as a fraction. Then, convert to a fraction with denominator 1.

Example. A store charges $24 for 3 boxes of candy. What is the cost per unit?

$$\frac{24 \text{ dollars}}{3 \text{ boxes}} = \frac{? \text{ dollars}}{1 \text{ box}}$$

method 1: "Compare"

$$\frac{24 \text{ dollars}}{3 \text{ boxes}} = \frac{? \text{ dollars}}{1 \text{ box}}$$

divide by 3

$$\frac{24 \text{ dollars}}{3 \text{ boxes}} = \frac{8 \text{ dollars}}{1 \text{ box}}$$

divide by 3

method 2: "Cross Multiply"

$$\frac{24 \text{ dollars}}{3 \text{ boxes}} = \frac{X \text{ dollars}}{1 \text{ box}}$$

(24 dollars)(1 box) = (X dollars)(3 boxes)

24 = 3X

X = 8 8 dollars/box

Conversion Notes and Examples (Converting more than 1 unit of measurement)

Example: Convert 50 yards/minute into feet/second

Beginning and End fractions $\frac{50 \text{ yards}}{1 \text{ minute}}$ $\frac{\text{feet}}{\text{second}}$

"Multiply by 1" $\frac{50 \text{ yards}}{1 \text{ minute}} \times \frac{1 \text{ minute}}{60 \text{ seconds}}$ $\frac{\text{feet}}{\text{second}}$

Simplify terms (cancel measurements/labels and reduce fractions) $\frac{\overset{5}{\cancel{50} \text{ yards}}}{1 \text{ minute}} \times \frac{1 \text{ minute}}{\underset{6}{\cancel{60} \text{ seconds}}}$ $\frac{\text{feet}}{\text{second}}$

$\frac{5 \text{ yards}}{6 \text{ seconds}}$ $\frac{\text{feet}}{\text{second}}$

"Multiply by 1" $\frac{5 \text{ yards}}{6 \text{ seconds}} \times \frac{3 \text{ feet}}{1 \text{ yard}}$ $\frac{\text{feet}}{\text{second}}$

Simplify terms and measurements $\frac{\cancel{5 \text{ yards}}}{\underset{2}{\cancel{6} \text{ seconds}}} \times \frac{\cancel{3 \text{ feet}}^{1}}{1 \cancel{\text{ yard}}}$ $\frac{\text{feet}}{\text{second}}$

$\frac{5 \text{ feet}}{2 \text{ seconds}} = 2.5 \text{ feet/second}$

Use the units of measurement as a guide to check your work!!

Example: Convert 10 kilometers/minute into meters/hour

$\frac{10 \text{ km}}{1 \text{ minute}}$ $\frac{\text{meters}}{\text{hour}}$

$\frac{\overset{1}{\cancel{10} \text{ km}}}{1 \text{ minute}} \times \frac{1 \text{ hour}}{\underset{6}{\cancel{60} \text{ minutes}}}$ $\frac{\text{meters}}{\text{hour}}$ Incorrect

$\frac{1 \text{ km} \cdot \text{hour}}{6 \text{ minutes} \cdot \text{minutes}}$

Since the units are messed up, the ratio was inserted incorrectly!!

Let's try again....

$\frac{10 \text{ km}}{1 \text{ minute}}$ $\frac{\text{meters}}{\text{hour}}$

$\frac{10 \text{ km}}{\cancel{1 \text{ minute}}} \times \frac{60 \cancel{\text{minutes}}}{1 \text{ hour}}$ $\frac{\text{meters}}{\text{hour}}$

$\frac{600 \text{ km}}{1 \text{ hour}} \times \frac{1000 \text{ meters}}{1 \cancel{\text{km}}}$ $\frac{\text{meters}}{\text{hour}}$ Correct

$= \frac{600{,}000 \text{ meters}}{1 \text{ hour}}$

Conversions and Ratios Applications

Example: Jim makes bricks measuring 20 cm x 6 cm x 14 cm.
If the density of one brick is 2000 kg/m^3 ,
what is the weight of 150 bricks?

brick density (ratio) $\frac{2000 \text{ kg}}{1 \text{ m}^3}$

The brick density ratio is using kg and meters... We need to convert the given units into those density ratio units...

$20\text{cm} \times \frac{1 \text{ meter}}{100 \text{ cm}} = .20 \text{ meters}$

20cm x 6cm x 14cm =

.20 meters x .06 meters x .14 meters = .00168 meters^3

$\text{density} = \frac{2000 \text{ kg}}{1 \text{ meter}^3}$ $\quad \frac{2000 \text{ kg}}{1 \text{ meter}^3} \cdot .00168 \text{ meters}^3 = 3.36 \text{ kg}$ (per brick)

$3.36 \frac{\text{kg}}{\text{brick}} \cdot 150 \text{ bricks} =$ **504 kg**

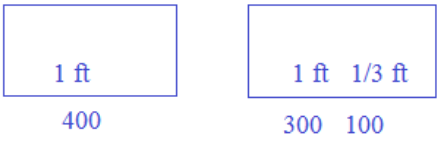
Example: Two architects design floor plans for my house.
The first architect uses a 1:400 scale.
And, the other uses a 1:300 scale.

Which floor plan will be bigger?

Imagine the house were 400 feet long....

$\frac{1}{400} = \frac{1}{300}$

Larger number!



Example: A container weighs 20 grams and has a capacity of 100 cm^3
If the density of water is 1 g/cm^3
what does the container weigh when filled with water?

Reminder: Include the container's 20 grams in the total weight!

$\frac{\text{weight}}{\text{volume}} = \text{density} = \frac{1 \text{ gram}}{1 \text{ cm}^3}$

Then, set up ratio: $\frac{1 \text{ gram}}{1 \text{ cm}^3} = \frac{\text{water weight}}{100 \text{ cm}^3}$

100 grams of water weight + 20 grams (container)

= **120 grams**

Example: *Which is a better deal?*

$2.64 for 16 ounces of juice

OR

$3.30 for 22 ounces of juice

Approach 1: find cost per ounce

$$\frac{\$2.64}{16 \text{ ounces}} = \frac{\$.165}{1 \text{ ounce}}$$

$$\frac{\$3.30}{22 \text{ ounces}} = \frac{\$.15}{1 \text{ ounce}}$$

Since the 22 ounces cost 15 cents per ounce --
(less than 16.5 cents per ounce) -- it's the better deal...

Approach 2: find ounce per dollar

$$\frac{16 \text{ ounces}}{\$2.64} = \frac{6.06 \text{ ounces}}{1 \text{ dollar}}$$

$$\frac{22 \text{ ounces}}{\$3.30} = \frac{6.67 \text{ ounces}}{1 \text{ dollar}}$$

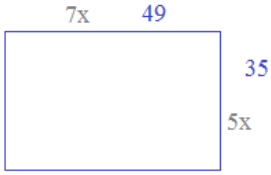
Since the 22 ounces offers *more* juice
per dollar, it's a better deal!

Example: The ratio of width to length in a rectangle is 5:7.
If the width is 35 cm, what is the area of the rectangle?

width: length is 5:7

$$\frac{\text{width}}{\text{length}} = \frac{5}{7} = \frac{35 \text{ cm}}{\text{length}}$$

length = 49 cm



area = length x width

area = 49 cm x 35 cm = $\boxed{1715 \text{ cm}^2}$

Example: Which is the equivalent of .45 hours?

a) 27 minutes

b) 45 minutes

c) 2700 seconds

d) 16,200 seconds

$$.45 \text{ hours} \cdot \frac{60 \text{ minutes}}{1 \text{ hour}} = \boxed{27 \text{ minutes}}$$

$$.45 \text{ hours} \cdot \frac{60 \text{ minutes}}{1 \text{ hour}} \cdot \frac{60 \text{ seconds}}{1 \text{ minute}} = 1620 \text{ seconds}$$

LanceAF (10-27-12)
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By any measure,
Robin, Clark, and Hilda collected a lot of candy that night!

Practice Quiz

Conversion Quiz

I. Quantity conversions

1) 48 ounces = _____ pounds

2) 12 yards = _____ inches

3) 190 meters = _____ kilometers

4) 20 dollars = _____ quarters

5) 14 pints = _____ gallons

II. Unit Measures -- Convert to unit measures

1) $\frac{44 \text{ feet}}{8 \text{ seconds}}$ = _____ ft/second

2) $78 for 20 feet. What is the cost per foot?

3) $4.80 per dozen is equal to $_____ per unit

4) Driving 440 miles in 7 hours 20 minutes is a rate of _____ miles/hour

5) 6-pack of soda is $1.80. Cost is _____ cents/can.

III. Rate Conversions

1) 30 feet/second = _____ yards/second

2) 5 km/hour = _____ meters/second

3) $280/day = _____ dollars/week

4) Six-packs of soda are 4 for $10.00. Cost of each unit is _____ dollars/can

5) Running 10 miles in 1 hour is a rate of _____ minutes/mile

Conversion Quiz

1 meter ≈ 3.3 feet

4 quarters = 1 dollar

4 quarts = 1 gallon

1 liter ≈ 33.8 ounces

1 mile ≈ 1.6 kilometers

IV. Miscellaneous conversions

My car has a 16 gallon tank. If I drove 380 miles, using one tank of gas, what is my miles/gallon?

A recipe requires 6 cups of oil to make 12 dozen cookies.
How many cups of oil are necessary for 24 cookies?

It takes Kate 20 minutes to ride her bike to the ice cream shop. If the shop is 2 miles away, what is her average speed (in miles per hour)?

Which is greater: a six-pack of 12-ounce cans or a 2-liter bottle?

Approximately how many feet are in a 5 kilometer race?

If you save 8 quarters per day, how long will it take to save $10?

In an open road in Europe, I drove over 200 km/hour.
How fast was I driving in miles/hour?

If my car requires 3.3 gallons of oil, how many quarts do I need to buy for a complete oil change?

Approximately how many liters are in 32 ounces?

V. Challenge: Complex conversions (area)

1) How many square inches are in one square foot?

2) How many square feet are in 25 square yards?

Conversion Quiz

Solutions

I. Quantity conversions

1) 48 ounces = 3 pounds

$\frac{48 \text{ ounces}}{1} \times \frac{1 \text{ pound}}{16 \text{ ounces}} = 3 \text{ pounds}$

2) 12 yards = 432 inches

$12 \text{ yards} \cdot \frac{3 \text{ ft}}{1 \text{ yd}} \cdot \frac{12 \text{ inches}}{1 \text{ ft}} = 432 \text{ inches}$

3) 190 meters = .19 kilometers

$\frac{190 \text{ meters}}{1} \times \frac{1 \text{ km}}{1000 \text{ meters}} = .19 \text{ km}$

4) 20 dollars = 80 quarters

20 dollars x (4 quarters/1 dollar) = 80 quarters

5) 14 pints = 1.75 gallons

2 pints = 1 quart and 4 quarts = 1 gallon
so, 8 pints = 1 gallon
or, .125 gallons = pint
14 x .125 = 1.75 gallons

II. Unit Measures -- Convert to unit measures

1) $\frac{44 \text{ feet}}{8 \text{ seconds}}$ = 5.5 ft/second

$\frac{44}{8} = \frac{?}{1}$? = 5.5

2) $78 for 20 feet. What is the cost per foot? $3.90/foot

$\frac{\$78}{20 \text{ ft}} = \frac{X}{1 \text{ ft}}$ $X = \frac{\$78}{20} = \$3.90$

3) $4.80 per dozen is equal to $ 0.40 per unit

$\frac{\$4.80}{12} = \$.40$

4) Driving 440 miles in 7 hours 20 minutes is a rate of 60 miles/hour

$\frac{440 \text{ miles}}{7 \text{ 1/3 hours}} = 60 \text{ miles/hour}$

5) 6-pack of soda is $1.80. Cost is 30 cents/can.

III. Rate Conversions

1) 30 feet/second = 10 yards/second

$\frac{30 \text{ ft}}{1 \text{ sec}} \times \frac{1 \text{ yard}}{3 \text{ feet}} = 10 \text{ yds/sec}$

2) 5 km/hour = 1.39 meters/second

$\frac{5 \text{ km}}{1 \text{ hour}} \cdot \frac{1000 \text{ m}}{1 \text{ km}} = 5000 \text{ m/hr}$ then, $\frac{5000 \text{ meters}}{1 \text{ hour}} \cdot \frac{1 \text{ hour}}{3600 \text{ sec}}$ = 1.39 m/sec

3) $280/day = 1960 dollars/week

$\frac{\$280}{1 \text{ day}} \cdot \frac{7 \text{ days}}{1 \text{ week}} = \$1960 \text{ per week}$

4) Six-packs of soda are 4 for $10.00. Cost of each unit is .416 dollars/can

4 six-packs are 24 cans..
$\frac{\$10}{24 \text{ cans}}$ = approx. 41 cents

5) Running 10 miles in 1 hour is a rate of 6 minutes/mile

$\frac{60 \text{ minutes}}{10 \text{ miles}} = 6 \text{ minutes/mile}$

Conversion Quiz

Solutions

IV. Miscellaneous conversions

My car has a 16 gallon tank. If I drove 380 miles, using one tank of gas, what is my miles/gallon?

$\frac{380 \text{ miles}}{1 \text{ tank}} \cdot \frac{1 \text{ tank}}{16 \text{ g}} = \frac{95 \text{ miles}}{4 \text{ gallon}} = 23.75 \text{ miles/gallon}$

23.75 miles/gallon

A recipe requires 6 cups of oil to make 12 dozen cookies. How many cups of oil are necessary for 24 cookies?

$\frac{6 \text{ cups}}{12 \text{ dozen}} \cdot \frac{1 \text{ dozen}}{12 \text{ cookies}} = \frac{1 \text{ cup}}{24 \text{ cookies}}$

one cup

It takes Kate 20 minutes to ride her bike to the ice cream shop. If the shop is 2 miles away, what is her average speed (in miles per hour)?

$\frac{2 \text{ miles}}{20 \text{ minute}} \cdot \frac{60 \text{ minutes}}{1 \text{ hour}} = 6 \text{ miles/hour}$

6 miles/hour

Which is greater: a six-pack of 12-ounce cans or a 2-liter bottle?

6 pack of 12 ounce cans = 72 ounces 2 liters $\cong$ 67.6 ounces

six-pack is greater

Approximately how many feet are in a 5 kilometer race?

5km = 5000 meters $5000 \text{ m} \cdot \frac{3.3 \text{ feet}}{1 \text{ meter}} \approx 16{,}500 \text{ feet}$

approx. 16,500 feet

If you save 8 quarters per day, how long will it take to save $10?

$\frac{8 \text{ quarters}}{1 \text{ day}} \quad \frac{1 \text{ dollar}}{4 \text{ quarters}} = \frac{2 \text{ dollars}}{1 \text{ day}}$ then, find $\frac{2 \text{ dollars}}{1 \text{ day}} = \frac{10 \text{ dollars}}{X \text{ days}}$

5 Days $X = 5$

In an open road in Europe, I drove over 200 km/hour. How fast was I driving in miles/hour?

$\frac{200 \text{ km}}{1 \text{ hour}} \text{ X } \frac{1 \text{ mile}}{1.6 \text{ km}} \approx 125 \text{ miles/hour}$

approx. 125 miles/hour 1 mile is approx. 1.6 km

If my car requires 3.3 gallons of oil, how many quarts do I need to buy for a complete oil change?

You need to buy 14 quarts in order to complete the oil change

4 quarts = 1 gallon

$3.3 \text{ gallons} \cdot \frac{4 \text{ quarts}}{1 \text{ gallon}} = 13.2 \text{ quarts}$

Approximately how many liters are in 32 ounces?

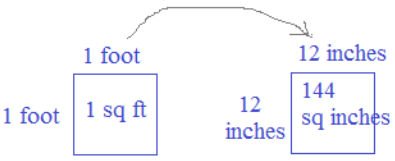
approx. .95 liters

$32 \text{ ounces X } \frac{1 \text{ liter}}{33.8 \text{ ounces}} = .95 \text{ liters (approximately)}$

V. Challenge: Complex conversions (area)

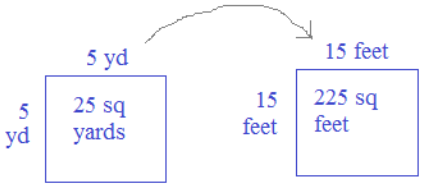
1) How many square inches are in one square foot?

144 square inches



2) How many square feet are in 25 square yards?

225 square feet



1 meter $\approx$ 3.3 feet

4 quarters = 1 dollar

4 quarts = 1 gallon

1 liter $\approx$ 33.8 ounces

1 mile $\approx$ 1.6 kilometers

Thanks for visiting the site. (Hope it helped!)

If you have questions, suggestions, or requests, let us know.

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ONE MORE QUESTION:

6-pack

or

2-liter bottle?

Which has more?

(Answer on the next page)

6-pack of 12oz cans
vs.
2-liter bottle

1 liter = 33.814 fluid ounces
(approximately)

ANSWER:

cans: 6 x 12oz = 72 ounces

bottle: 2 liters x $\left\langle \frac{33.814 \text{ ounces}}{1 \text{ liter}} \right\rangle$ = 67.62 ounces (approximately)

Therefore, a 6-pack of cans has more soda than a 2-liter bottle!