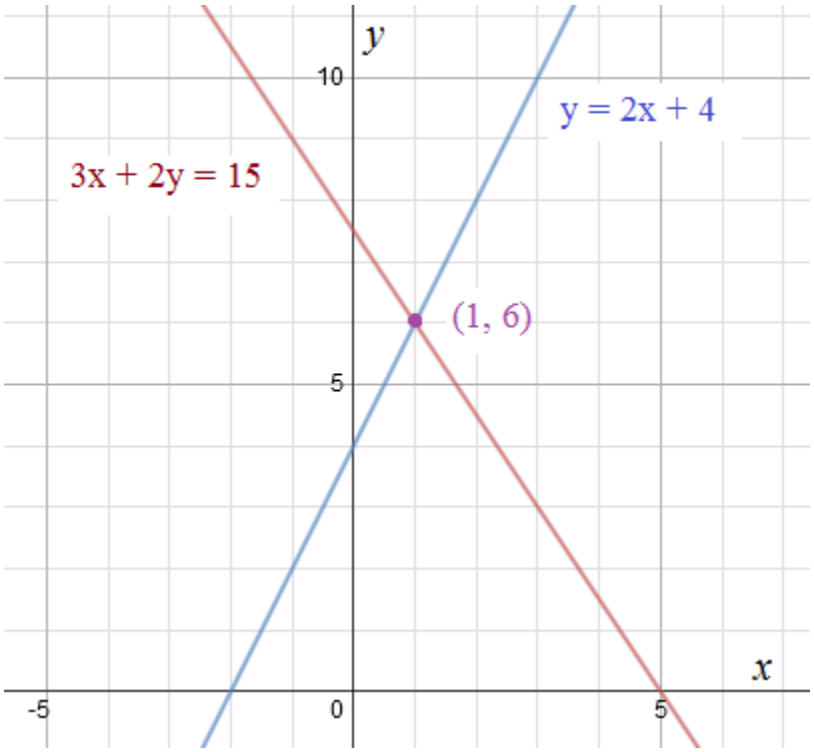
Linear Systems

Examples and Practice Tests (and, solutions)

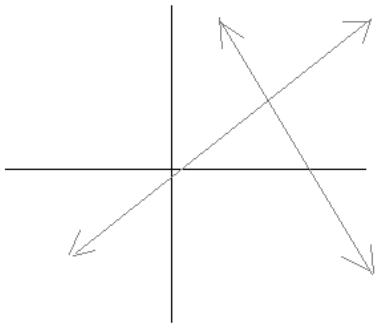


Topics include solving, graphing, elimination and substitution methods, word problems, classifying systems, 3 variables, and more.

Classify the following linear systems:
(consistent/inconsistent - independent/dependent)

Linear systems: Consistent or Inconsistent?

a)



Consistent and independent

one solution

b) $2x + 3y = -10$
$-4x - 6y = 20$

Consistent and dependent
(same lines)

infinite number of solutions

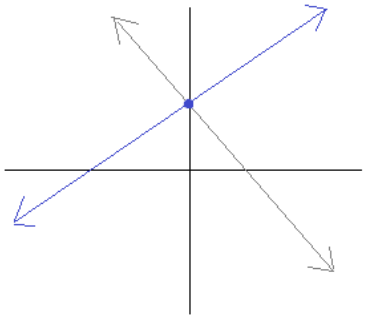
c) $y = 3x + 7$
$y = 3x - 7$

Inconsistent
(parallel lines)

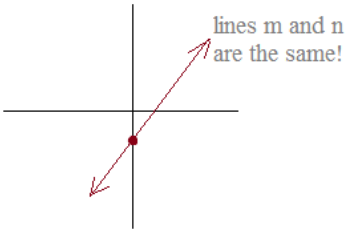
no solutions

Can you draw consistent, independent linear system where the lines have the same y-intercept?

Yes!



Note: a consistent, dependent linear system would obviously have a common y-intercept...



Inconsistent linear system: parallel lines

Consistent linear system: intersecting lines

Dependent ---> all points

Independent ---> one point of intersection

Six Ways to solve a linear system: $2x + 3y = 12$, $y = x - 11$

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1) Elimination/Combination Method

(Write equations in standard form)

$2x + 3y = 12$

$x - y = 11$

(Choose variable to eliminate, and if necessary, change equation(s))

$2x + 3y = 12$

$3x - 3y = 33$

(Combine equations and eliminate variable)

$5x + 0y = 45$

$x = 9$

Since $x = 9$, then $y = (9) - 11$, so $y = -2$

2) Substitution Method

$2x + 3y = 12$

$y = x - 11$

Substitute second equation into the first....

$2x + 3(x - 11) = 12$

$2x + 3x - 33 = 12$

$5x = 45$

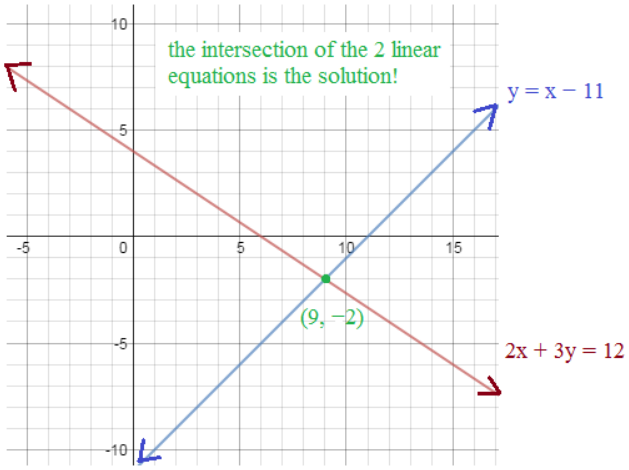
$x = 9$

Since $x = 9$,

$2(9) + 3y = 12$

so, $y = -2$

3) Graphing



4) Matrix $AX = B$ then, $X = A^{-1}B$

$2x + 3y = 12$

$x - y = 11$

Place coefficients and solutions into matrices

$$\begin{bmatrix} 2 & 3 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 12 \\ 11 \end{bmatrix}$$

A X B

Use the inverse of A...

$$\begin{bmatrix} 1/5 & 3/5 \\ 1/5 & -2/5 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1/5 & 3/5 \\ 1/5 & -2/5 \end{bmatrix} \begin{bmatrix} 12 \\ 11 \end{bmatrix}$$

A^{-1} A X A^{-1} B

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 45/5 \\ -10/5 \end{bmatrix}$$

Identity Matrix I X

5) Augmented Matrix

Place coefficients and solutions into 2x3 augmented matrix...

$$\left[\begin{array}{cc|c} 2 & 3 & 12 \\ 1 & -1 & 11 \end{array}\right]$$ switch rows..

$$\left[\begin{array}{cc|c} 1 & -1 & 11 \\ 2 & 3 & 12 \end{array}\right]$$ R1 x (-2), then add to R2

$$\left[\begin{array}{cc|c} 1 & -1 & 11 \\ 0 & 5 & -10 \end{array}\right]$$ R2 x (1/5)

Reduced Row Echelon Form (RREF) displays the solutions for x and y

$$\left[\begin{array}{cc|c} 1 & -1 & 11 \\ 0 & 1 & -2 \end{array}\right]$$ add R2 to R1 $\Rightarrow$ $$\left[\begin{array}{cc|c} 1 & 0 & 9 \\ 0 & 1 & -2 \end{array}\right]$$

6) Cramer's Rule (Using Determinants)

$2x + 3y = 12$

$1x - 1y = 11$

$$D = \begin{vmatrix} 2 & 3 \\ 1 & -1 \end{vmatrix} \quad D_x = \begin{vmatrix} 12 & 3 \\ 11 & -1 \end{vmatrix} \quad D_y = \begin{vmatrix} 2 & 12 \\ 1 & 11 \end{vmatrix}$$

$-2 - 3 = -5$ $\quad -12 - 33 = -45$ $\quad 22 - 12 = 10$

$$x = \frac{D_x}{D} = 9 \qquad y = \frac{D_y}{D} = -2$$

Linear Systems: Word Problem Applications

Example: 34 kids and 6 adult chaperones are going to the amusement park.
They can take cars and/or vans.
Each van seats 7, and each car seats 5.
If all 6 adults will drive, how many will go in each vehicle?

Step 1: Set Variables V = # of vans
C = # of cars

Step 2: Set up equations/constraints

(Number of riders) $7V + 5C = (34 + 6)$ (Van and car drivers) $V + C = 6$

7 in each van; 5 in each car; 40 riders

Step 3: Solve (we have 2 equations and 2 unknowns)

use combination method:

$7V + 5C = 40$ and $V + C = 6$ ⇒ $7V + 5C = 40$ and $-5V - 5C = -30$ ⇒ $2V = 10$

$V = 5$ $C = 1$

The group will take 5 vans and 1 car...

Example: An orchestra has a string to wind ratio of 9:4...
If there are 91 total instruments, how many of each are there?

Step 1: Set up variables Let S = # of string instruments
W = # of wind instruments

Step 2: Set up equations "91 total instruments" $S + W = 91$

"string to wind ratio of 9:4" $9W = 4S$ or $S = \frac{9}{4}W$ (ex: if there are 40 W, then there are 90 S

Step 3: Solve $S + W = 91$ $S = \frac{9}{4}W$

(since we have 2 equations and 2 unknowns, we have a system....)

Use substitution method: $\left(\frac{9}{4}W\right) + W = 91$

$\frac{13}{4}W = 91$

$\frac{4}{13} \cdot \frac{13}{4}W = 91 \cdot \frac{4}{13}$ $W = 28$ so, $S = 63$

The orchestra has 28 wind instruments and 63 string instruments.

Example: The math guy spends an afternoon rowing up and down a river.
In the morning, when he rowed with the current, he traveled 24 miles in 3 hours.
In the afternoon, when he rowed against the current, he went 16 miles in 4 hours.
What is the speed of the current?

Step 1: Figure out the variables Let C = speed of the current Let R = speed of rower)

Step 2: Set equations distance = rate x time

(with the current) 24 miles = (R + C)(3 hours)

(against the current) 16 miles = (R − C)(4 hours)

Step 3: Solve

$8\ \frac{\text{miles}}{\text{hour}} = R + C$

$4\ \frac{\text{miles}}{\text{hour}} = R - C$

combine equations ⇒ $12\ \frac{\text{miles}}{\text{hour}} = 2R + 0C$

R = 6 miles/hour

The math guy is rowing at a speed of 6 miles per hour... So, the speed of the current is 2 miles per hour

Linear Systems

Example: Solve the system:

$$4x + 9y = 8$$
$$8x + 6z = -1$$
$$6y + 6z = -1$$

Rewrite the equations:

$$4x + 9y + 0z = 8$$
$$8x + 0y + 6z = -1$$
$$0x + 6y + 6z = -1$$

Combine 2nd and 3rd equations:

$$8x + 0y + 6z = -1$$
$$0x + 6y + 6z = -1$$

(-1)

$$-8x + 0y - 6z = 1$$
$$0x + 6y + 6z = -1 \longrightarrow -8x + 6y = 0$$

then, combine the outcome with the 1st equation:

$$-8x + 6y = 0$$
$$4x + 9y + 0z = 8$$

(2)

$$8x + 18y = 16 \longrightarrow 24y = 16$$
$$y = 2/3$$

Use substitution to get remaining terms:

$$-8x + 6y = 0$$
$$-8x + 6(2/3) = 0$$
$$4 = 8x$$
$$x = 1/2$$

$$8x + 6z = -1$$
$$8(1/2) + 6z = -1$$
$$6z = -5$$
$$z = -5/6$$

Example: An automobile gets 36 miles per gallon in the city, and 46 miles per gallon on the highway. With a 13-gallon gas tank, this automobile travelled 526 miles. How many gallons were used driving in the city?

Step 1: Establish variables (and make a grid)

x = number of gallons

y = number of miles

	City	Highway
Fuel	x	(13 − x)
Rate	36 m/g	46 m/g
Distance	y	(526 − y)

Step 2: Construct system

$$y = 36\,\frac{\text{miles}}{\text{gallon}}\ (\text{x gallons})$$

$$(526 - y) = 46\,\frac{\text{miles}}{\text{gallon}}\ (13 - x)(\text{gallons})$$

(2 equations, 2 unknowns)

Step 3: Solve (using substitution)

$$(526 - (36x)) = 46(13 - x)$$

$$526 - 598 = 36x - 46x$$

$$-72 = -10x$$

$$x = 7.2 \text{ gallons}$$

Step 4: Check answer

city: 7.2 gallons x 36m/g = 259.2 miles

highway: 5.8 gallons x 46m/g = 266.8 miles

total miles = 526 miles

Three Variable Linear Systems

Example: Solve the system of linear equations

$$2x + 3y - z = 12$$
$$3x - 4y + z = -9$$
$$x + 5y + z = 7$$

Step 1: Recognize the efficient approach...

In this case, it seems the elimination method is easiest.. (get rid of the z's first)

Step 2: Solve

Combine 1st and 2nd equations:

$$\begin{array}{r} 2x + 3y - z = 12 \\ 3x - 4y + z = -9 \\ \hline 5x - y = 3 \end{array}$$

Combine 1st and 3rd equations:

$$\begin{array}{r} 2x + 3y - z = 12 \\ x + 5y + z = 7 \\ \hline 3x + 8y = 19 \end{array}$$

Then, solve the 2 x 2 linear system....

$$5x - y = 3 \Rightarrow 40x - 8y = 24$$
$$3x + 8y = 19 \Rightarrow \underline{3x + 8y = 19}$$
$$43x = 43$$

$x = 1$

so, $y = 2$

If $x = 1$ and $y = 2$, then $z = -4$

Step 3: Check solutions..

(1, 2, -4) Plug into ALL 3 EQUATIONS!

$2x + 3y - z = 12 \Rightarrow 2 + 6 - (-4) = 12$ ✓

$3x - 4y + z = -9 \Rightarrow 3 - 8 + (-4) = -9$ ✓

$x + 5y + z = 7 \Rightarrow 1 + 10 + (-4) = 7$ ✓

Example: Solve the system of linear equations

$$2x + 4y - 7z = 15$$
$$3y + z = 10$$
$$-6x + 2z = -28$$

Step 1: Recognize the efficient approach...

In this case, the substitution method seems most efficient...

Step 2: Using the middle equation, we can solve for z....

$3y + z = 10 \Rightarrow z = 10 - 3y$

Then, substitute into the 3rd equation...

$-6x + 2z = -28$

$-6x + 2(10 - 3y) = -28$

$-6x + 20 - 6y = -28$

$-6x - 6y = -48$

and, into the 1st equation...

$2x + 4y - 7z = 15$

$2x + 4y - 7(10 - 3y) = 15$

$2x + 4y - 70 + 21y = 15$

$2x + 25y = 85$

Now combine the results:

$$-6x - 6y = -48 \Rightarrow -2x - 2y = -16$$
$$2x + 25y = 85 \Rightarrow \underline{2x + 25y = 85}$$
$$23y = 69$$

$y = 3$.... then, $x = 5$

and, $z = 1$

Step 3: Check the answer...

(5, 3, 1)

$2x + 4y - 7z = 15$ $\quad 10 + 12 - 7 = 15$ ✓

$3y + z = 10$ $\quad 9 + 1 = 10$ ✓

$-6x + 2z = -28$ $\quad -30 + 2 = -28$ ✓

3 variable - 2 equation systems

Example: In the following system, solve:

a) in terms of x

b) in terms of y

c) in terms of z

$2x - 3y + z = 11$

$5x + y - 2z = 8$

Using "elimination" method, get rid of the y terms...

$2x - 3y + z = 11$

$5x + y - 2z = 8$

$15x + 3y - 6z = 24$

$17x - 5z = 35$

$17x = 35 + 5z$

$x = \frac{35}{17} + \frac{5}{17}z$ x in terms of z

$17x - 35 = 5z$

z in terms of x $\frac{17}{5}x - 7 = z$

Using "elimination" method, get rid of the x terms...

$2x - 3y + z = 11$

$5x + y - 2z = 8$

$-10x + 15y - 5z = -55$

$10x + 2y - 4z = 16$

$17y - 9z = -39$

$17y = -39 + 9z$

$y = \frac{-39}{17} + \frac{9}{17}z$ y in terms of z

$17y + 39 = 9z$

z in terms of y $\frac{17}{9}y + \frac{39}{9} = z$

Then, using "elimination", get rid of the z's ...
This will find x in terms of y and y in terms of x....

OR, use "substitution" with the above equations...

z in terms of y $\boxed{\frac{17}{9}y + \frac{39}{9}} = z$

x in terms of z $x = \frac{35}{17} + \frac{5}{17}z$

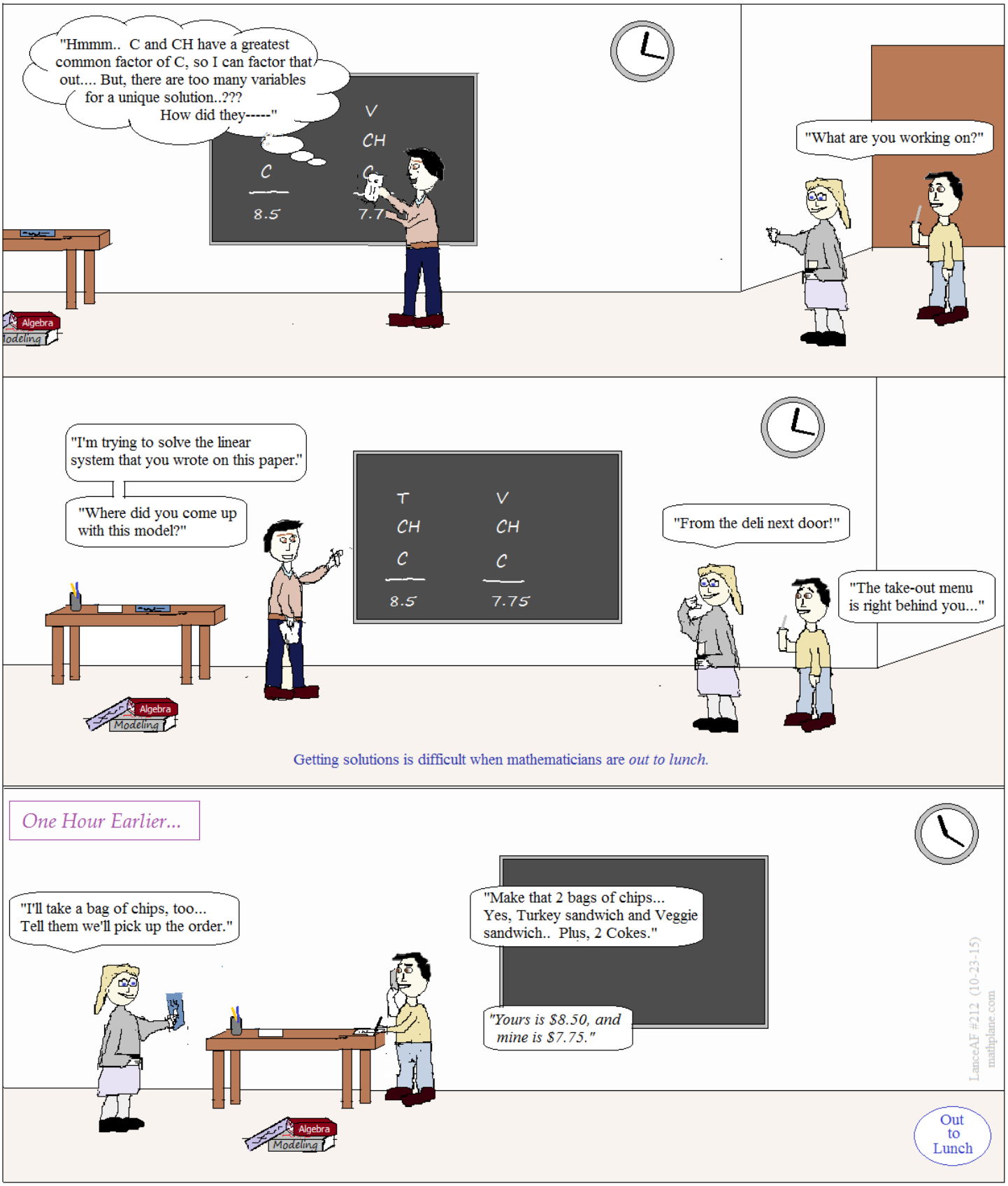
$x = \frac{35}{17} + \frac{5}{17}\left(\boxed{\frac{17}{9}y + \frac{39}{9}}\right)$

$x = \frac{35}{17} + \frac{5}{9}y + \frac{65}{51}$

x in terms of y $x = \frac{5}{9}y + \frac{170}{51}$

$x - \frac{170}{51} = \frac{5}{9}y$

y in terms of x $\frac{9}{5}x - 6 = y$



Practice Quizzes-→

Solve the following linear systems (using elimination/combination or substitution method)

Systems of Linear Equations

1) $y = 3x + 8$

$y = \frac{-1}{2}x + 15$

2) $y = -2x - 7$

$y = 4x + 11$

3) $y = 5x + 12$

$(y - 4) = 6(x + 1)$

4) $y = -4x + 17$

$(y + 2) = \frac{1}{3}(x - 8)$

5) $2x + 7y = 25$

$3x - y = 3$

6) $2x - 3y = 9$

$y = \frac{2}{3}x - 3$

7) $.3x - .5y = 1$

$y = .2x + 6$

8) $2x + y = 7$

$x = 4$

9) $-3x + 6y = 12$

$\frac{1}{2}x - y = 8$

Solve the following linear systems (using elimination/combination or substitution method)

Systems of Linear Equations

10) $y + 5 = -(6 - x)$

$x = -y + 19$

11) $\frac{1}{2}x - 5y = 9$

$\frac{1}{4}x + 6y = -4$

12) $x + y = 2$

$y = -2(x - 5) + 1$

13) $y - 6 = .2(x + 10)$

$y + 1 = .5(x + 15)$

14) $x = 2y - 12$

$(y - 6) = 4x - 7$

Convert each equation into *slope intercept form*

a) $2x + 6y = 12$

b) $y + 3 = 2(x + 8)$

c) $x + \frac{1}{2}y = 5$

Convert each equation into *standard form*

a) $y = 3x + 5$

b) $y + 1 = \frac{1}{2}(x - 6)$

c) $.2x + .7y = 11$

d) $y = -(x + 7) + 8$

Solve the following linear systems (using elimination/combination or substitution method)

Systems of Linear Equations

SOLUTIONS

mathplane.com

1) $y = 3x + 8$

$y = \frac{-1}{2}x + 15$

substitution: $3x + 8 = \frac{-1}{2}x + 15$

for ease, let's get rid of fractions by multiplying both sides by 2

$6x + 16 = -x + 30$

$7x = 14$ $\quad y = 3(2) + 8$

$x = 2$ $\quad y = 14$

(2, 14)

**to check: plug (2, 4) into the other equation...

$14 = \frac{-1}{2}(2) + 15$

$14 = -1 + 15$

2) $y = -2x - 7$

$y = 4x + 11$

substitution: $-2x - 7 = 4x + 11$

$-18 = 6x$

$x = -3$

$y = 4(-3) + 11 = -1$

(-3, -1)

elimination: $y = -2x - 7$

$-(y = 4x + 11)$

$0 = -6x - 18$

$18 = -6x$ $\quad y = -2(-3) - 7$

$x = -3$ $\quad y = -1$

3) $y = 5x + 12$

$(y - 4) = 6(x + 1)$

substitution: put "y" into 2nd equation

$((5x + 12) - 4) = 6(x + 1)$

$5x + 8 = 6x + 6$

$2 = x$

If $x = 2$, then $y = 5(2) + 12$

$y = 22$

(2, 22)

To check: plug solution into the other equation...

$(y - 4) = 6(x + 1)$

$(22 - 4) = 6(2 + 1)$

$18 = 18$

4) $y = -4x + 17$

$(y + 2) = \frac{1}{3}(x - 8)$

substitution: put the "y" into the 2nd equation

$(-4x + 17) + 2) = \frac{1}{3}(x - 8)$

$-4x + 19 = \frac{1}{3}x - \frac{8}{3}$

for ease, multiply by 3 to get rid of fractons

$-12x + 57 = x - 8$

$65 = 13x$ $\quad y = -4(5) + 17$

$x = 5$ $\quad y = -3$

(5, -3)

5) $2x + 7y = 25$

$3x - y = 3$

elimination: (easier because equations are in standard form)

$2x + 7y = 25$

$7(3x - y = 3)$

$2x + 7y = 25$

$+ 21x - 7y = 21$

$23x = 46$

$x = 2$

then, $y = 3$

(2, 3)

6) $2x - 3y = 9$

$y = \frac{2}{3}x - 3$

substitution:

$2x - 3(\frac{2}{3}x - 3) = 9$

$2x - 2x + 9 = 9$

$0 = 0$

elimination:

infinite solutions

$2x - 3y = 9$

$y = \frac{2}{3}x - 3$

$3y = 2x - 9$

SAME LINES!

$9 = 2x - 3y$

7) $.3x - .5y = 1$

$y = .2x + 6$

For ease, let's multiply both equations by 10

$3x - 5y = 10$

$10y = 2x + 60$

elimination method:

$(3x - 5y = 10)(2)$

$-2x + 10y = 60$

$6x - 10y = 20$

$4x = 80$

$x = 20$ then, $y = 10$

(20, 10)

8) $2x + y = 7$

$x = 4$

substitution:

Just place second equation into first!

$2(4) + y = 7$

$y = -1$ and, of course $x = 4$

(4, -1)

9) $-3x + 6y = 12$

$\frac{1}{2}x - y = 8$

elimination:

first, multiply second equation by 6...

$-3x + 6y = 12$

$3x - 6y = 48$

$0 + 0 = 60$

PARALLEL LINES!!

No real Solution

Solve the following linear systems (using elimination/combination or substitution method)

SOLUTIONS

Systems of Linear Equations

mathplane.com

10) $y + 5 = -(6 - x)$

$x = -y + 19$

substitution:

$y + 5 = -(6 - (-y + 19))$

$y + 5 = -(-13 + y)$

$2y = 8$

$y = 4$

If $y = 4$, then $x = -(4) + 19$

$x = 15$

(15, 4)

To check, plug solution into BOTH equations:

$(4) + 5 = -(6 - (15))$

$9 = 9$ ✓

11) $\frac{1}{2}x - 5y = 9$

$\frac{1}{4}x + 6y = -4$

elimination:

For ease, I'll get rid of the fractions..
(multiply 1st by 2; multiply 2nd by 4)

$x - 10y = 18$

$-\ x + 24y = -16$

$-34y = 34$

$y = -1$

if $y = -1$, then using substitution, we can see $x = 8$

(8, -1)

12) $x + y = 2$

$y = -2(x - 5) + 1$

substitution:

since y is isolated in the second equation, we'll substitute it into the first...

$x + -2(x - 5) + 1 = 2$

$x - 2x + 10 + 1 = 2$

$9 = x$

so, $y = -7$

(9, -7)

To check: plug solution into 2nd equation

$(-7) = -2((9) - 5) + 1$

$-7 = -2(4) + 1$

$-7 = -7$ ✓

13) $y - 6 = .2(x + 10)$

$y + 1 = .5(x + 15)$

First, multiply both equations by 10

$10y - 60 = 2x + 20$

$10y + 10 = 5x + 75$

Since y coefficient are the same, we'll use elimination:

$10y - 60 = 2x + 20$

$-\ (10y + 10 = 5x + 75)$

$0y - 70 = -3x - 55$

$3x = 15$

$x = 5$

then, with substitution, $y = 9$

(5, 9)

14) $x = 2y - 12$

$(y - 6) = 4x - 7$

since x is by itself in the 1st equation...

substitution:

$(y - 6) = 4(2y - 12) - 7$

$y - 6 = 8y - 48 - 7$

$49 = 7y$

$y = 7$ then, $x = 2$

(2, 7)

Convert each equation into *slope intercept form*

$y = mx + b$

a) $2x + 6y = 12$

$6y = 12 - 2x$

$y = -\frac{1}{3}x + 2$

b) $y + 3 = 2(x + 8)$

$y + 3 = 2x + 16$

$y = 2x + 13$

c) $x + \frac{1}{2}y = 5$

$\frac{1}{2}y = -x + 5$

$y = -2x + 10$

Convert each equation into *standard form*

$Ax + By = C$ where A is positive integer (and B and C are integers)

a) $y = 3x + 5$

$-3x + y = 5$

$3x - y = -5$

b) $y + 1 = \frac{1}{2}(x - 6)$

$2y + 2 = (x - 6)$

$-x + 2y = -8$

$x - 2y = 8$

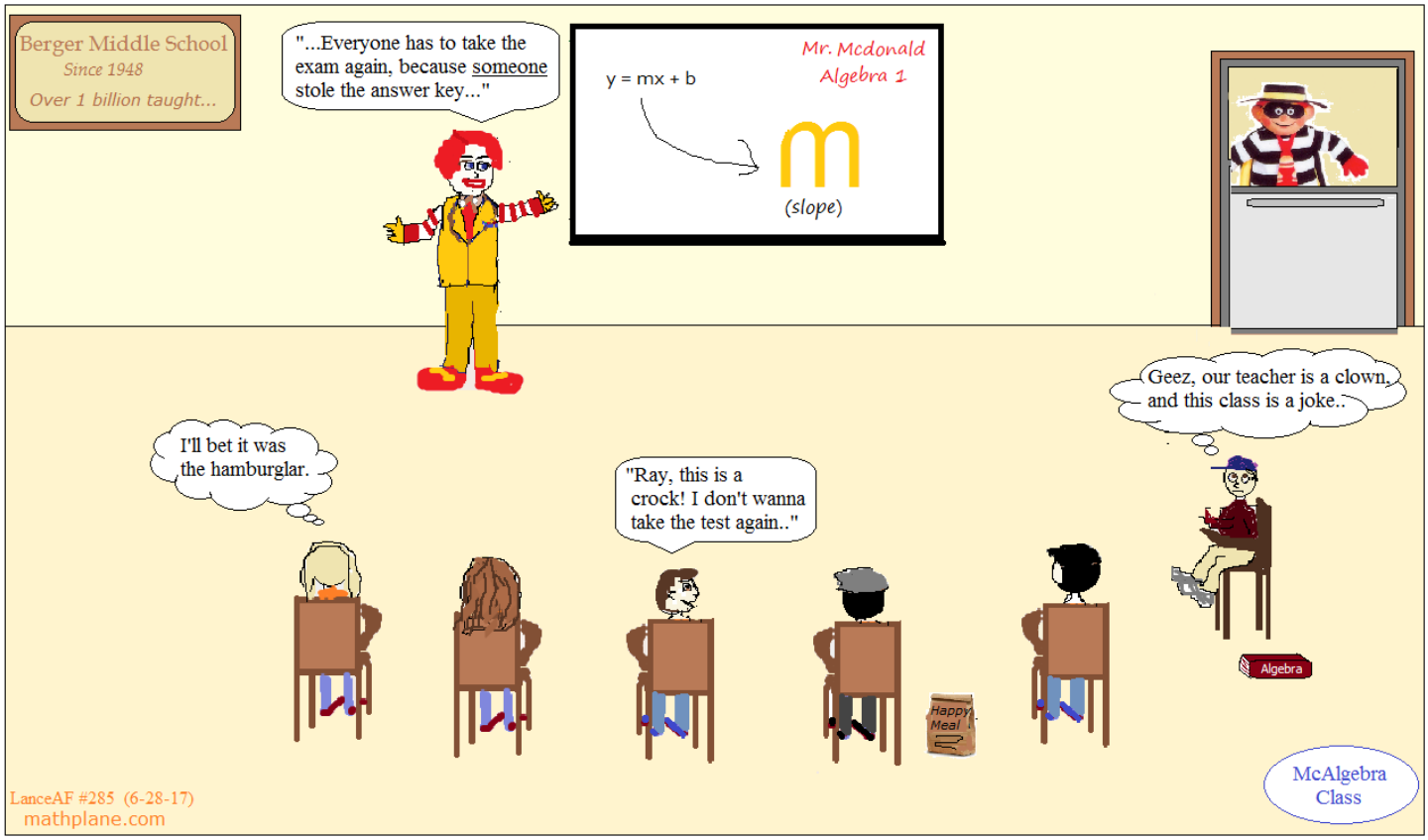
c) $.2x + .7y = 11$

$2x + 7y = 110$

d) $y = -(x + 7) + 8$

$y = -x - 7 + 8$

$x + y = 1$

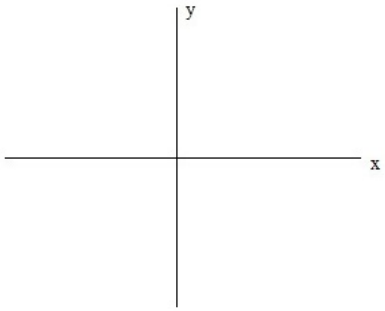


More Practice questions-→

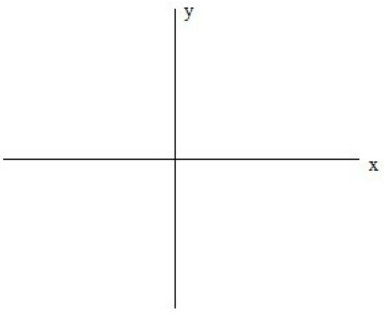
Linear Systems Quick Quiz

Solve and Graph the following Systems:

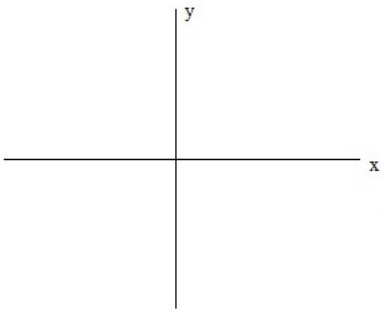
1) $3x + y = 9$
$y = -2x + 4$



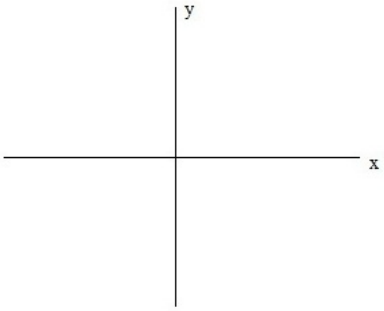
2) $y = 6$
$2x - 3y = 4$



3) $y = \frac{1}{2}x - \frac{1}{4}$
$2x + 4y = 1$



4) $x = 5$
$y = 6$



Answer the following:

Linear Systems Quick Quiz

1) Jim bought 65 cupcakes and cookies for his birthday party. Each cupcake cost $1 and each cookies cost 75 cents. If he paid $57.50 for the treats, how many of each did he buy?

2) A high school play has 2 freshmen, 5 sophomores, and 11 juniors. If 1/3 of the cast is composed of seniors, how many seniors are in the play?

3) A movie theater charges 9 dollars for adults, 5 dollars for kids, and 3 dollars for seniors. Last month, the theater sold 9,500 tickets and generated $57,920. If the theater sold twice as many tickets to kids as seniors, how many of each ticket did the theater sell?

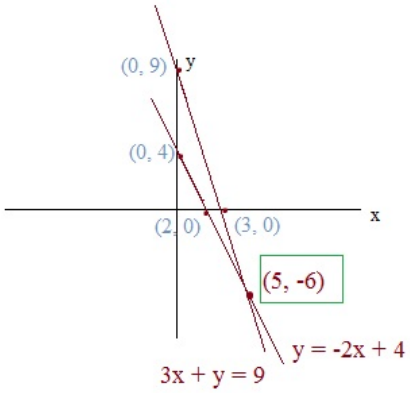
Linear Systems Quick Quiz

SOLUTIONS

Solve and Graph the following Systems:

1) $3x + y = 9$
$y = -2x + 4$

solution is (5, -6)



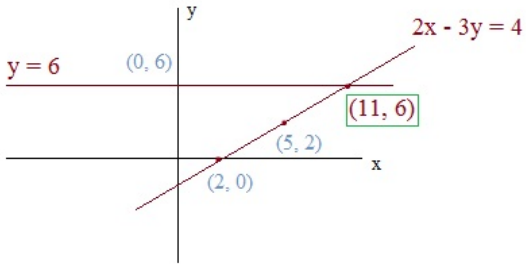
(substitution method)

$3x + (-2x + 4) = 9$ substitute 2nd equation into 1st.
$x = 5$ solve.

$3(5) + y = 9$ place x value into one of the equations to get y.
$y = -6$

check: $(-6) = -2(5) + 4$ check solution in other equation.
$-6 = -6$ ✓

2) $y = 6$
$2x - 3y = 4$



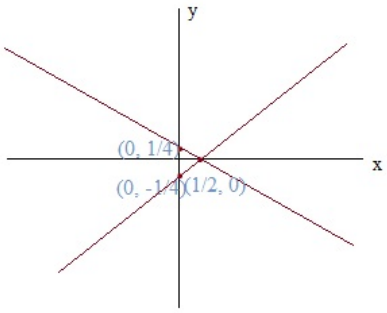
(substitution/combine the equations)

$2x - 3(6) = 4$
$2x = 22$
$x = 11$

solution is (11, 6)

and, $y = 6$

3) $y = \frac{1}{2}x - \frac{1}{4}$ equation 1
$2x + 4y = 1$ equation 2



(combination/elimination method)

$4y = 2x - 1$ 1
$-2x + 4y = -1$ 1 Re-write equation 2. Then, combine with equation 1.
$2x + 4y = 1$ 2

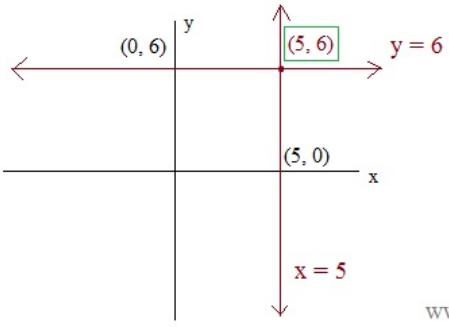
$8y = 0$ 1 + 2

intersection at (1/2, 0)

$y = 0$ solution

$2x + 4(0) = 1$ place y = 0 into the second equation to get x
$2x = 1$
$x = 1/2$

4) $x = 5$
$y = 6$



Graph it first, and you'll see the solution!

The lines intersect at (5, 6)

Answer the following: SOLUTIONS

Linear Systems Quick Quiz

1) Jim bought 65 cupcakes and cookies for his birthday party. Each cupcake cost $1 and each cookies cost 75 cents. If he paid $57.50 for the treats, how many of each did he buy?

Let CP = number of cupcakes
CK = number of cookies

$CP + CK = 65$

$\$1(CP) + \$.75(CK) = \$57.50$

Using substitution method:

$CP = 65 - CK$

then,

$\$1(65 - CK) + \$.75(CK) = \$57.50$

$65 - 1CK + .75CK = 57.50$

$-.25CK = -7.50$

$CK = 30$

and, $CP = 35$

30 cookies
35 cupcakes

quick check: 30 cookies will cost $22.50; and, 35 cupcakes will cost $35.00; total: $57.50 ✓

2) A high school play has 2 freshmen, 5 sophomores, and 11 juniors. If 1/3 of the cast is composed of seniors, how many seniors are in the play?

$2 + 5 + 11 = 18$ non seniors

seniors + non seniors = total cast

$S + 18 = C$

$1/3(C) = S$

S = # of seniors
C = # of cast members

$1/3(C) + 18 = C$
$2/3C = 18$
$2C = 54$
$C = 27$

$S = 9$

3) A movie theater charges 9 dollars for adults, 5 dollars for kids, and 3 dollars for seniors. Last month, the theater sold 9,500 tickets and generated $57,920. If the theater sold twice as many tickets to kids as seniors, how many of each ticket did the theater sell?

A = # of adult tickets
K = # of kid tickets
S = # of senior tickets

1) $A + K + S = 9500$
2) $\$9(A) + \$5(K) + \$3(S) = \$57,920$
3) $K = 2S$

3 equations, 3 unknowns...

Using substitution:

$A + (2S) + S = 9500$ 3rd equation into 1st equation

$9A + 5(2S) + 3S = 57920$ 3rd equation into 2nd equation

Then, combine these two equations:

$A + 3S = 9500$

$9A + 13S = 57,920$

$-9A - 27S = -85,500$

$-14S = -27,580$

$S = 1970$

Since $K = 2S$,

$K = 2(1970)$

$K = 3940$

And, since

$A + K + S = 9500$

$A + 3940 + 1970 = 9500$

$A = 3590$

Quick Check:

$3590 + 3940 + 1970 = 9500$ ✓

Seniors: 1970
Kids : 3940

twice as many kids ✓

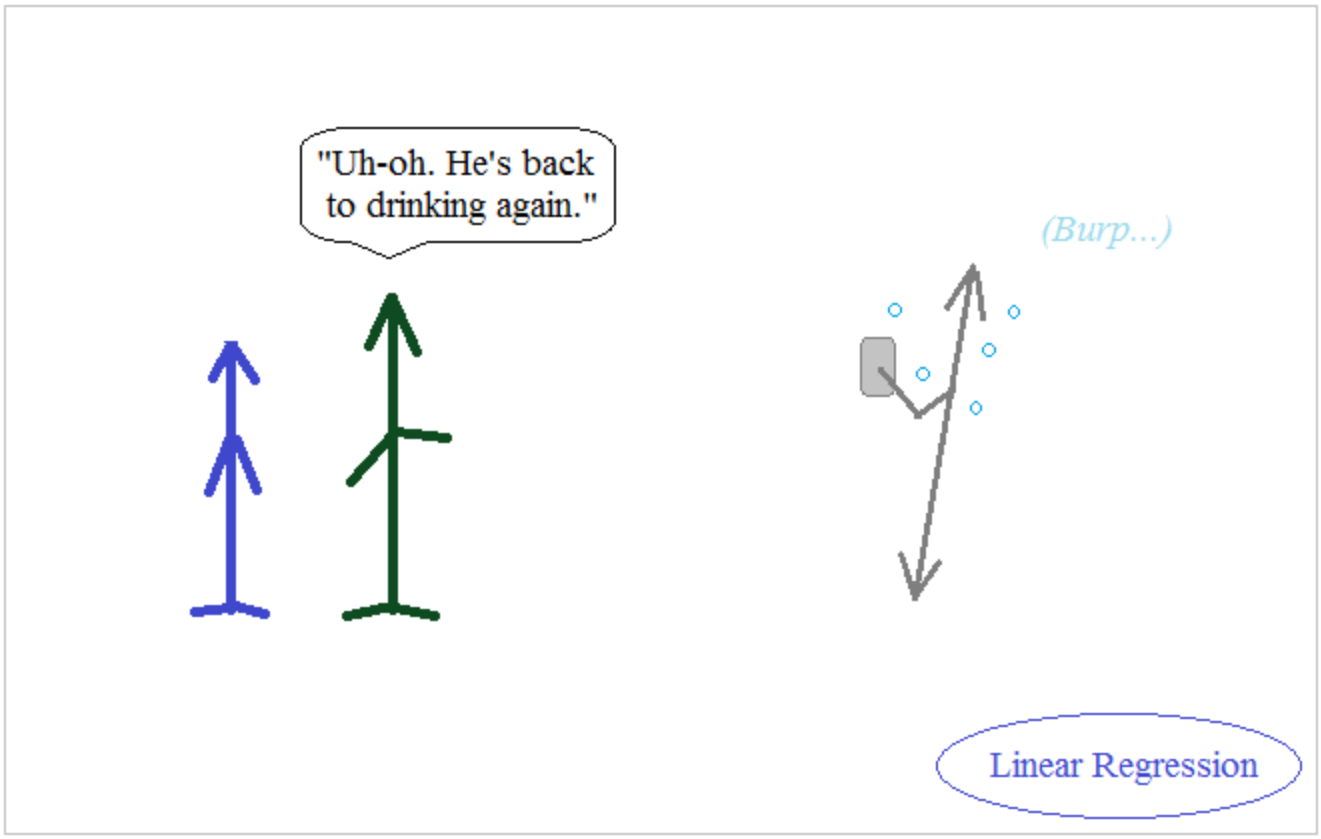
$\$9 \times 3590$ tix $= \$32,310$

$\$5 \times 3940$ tix $= \$19,700$

$\$3 \times 1970$ tix $= \$5,910$

total sales: $57,920 ✓

No longer on the straight and narrow?



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Another Practice Test→

Linear Systems Test 2

Part I: Solving Systems

Use Substitution (and show your work)

1) $y = 3x + 10$
$2x + 3y = -3$

2) $y = 2x - 4$
$3x - y = 9$

Use Elimination (Combination) Method (and show your work)

3) $3x + 7y = 1$
$6x - 5y = -17$

4) $x + 3y = 6$
$3x - y = -12$

Use Any Method

5) $y = 4$
$3x + 5y = 8$

6) $\frac{2}{3}x - y = 4$
$y = 2x - 12$

7) $y = -3x + 10$
$3x + y = 15$

Part II: Graphing

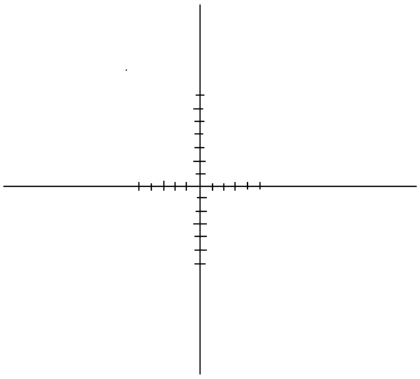
Graph the following: $3x + 5y = 15$

What is the x-intercept?

y-intercept?

slope?

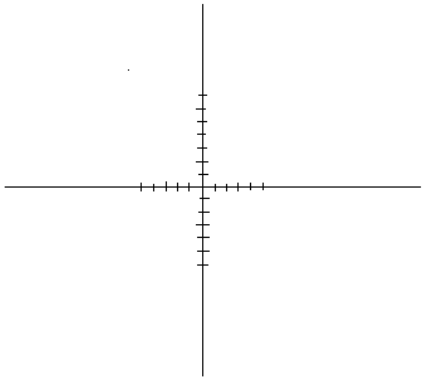
Is (20, -8) a point on this line?



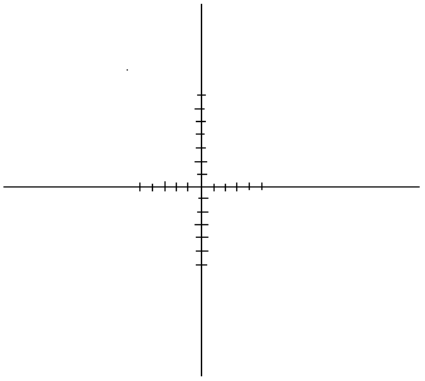
Part III: Graph and Solve

Graph each system. Then, identify the solutions on the graphs.

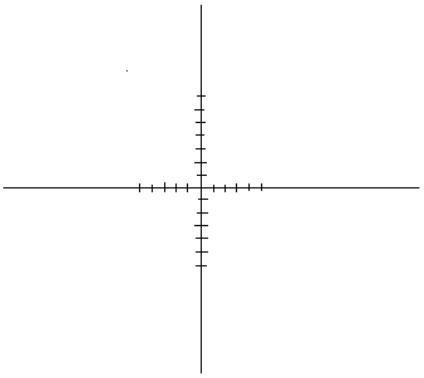
$3x + 2y = 15$
$y = 2x + 4$



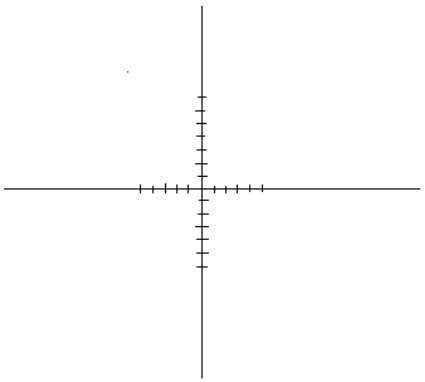
$y < 2x + 4$
$2x - y \leq 4$



$y \leq 3x + 5$
$6x + 2y > -6$



$y = -5$
$x - 6y = 13$

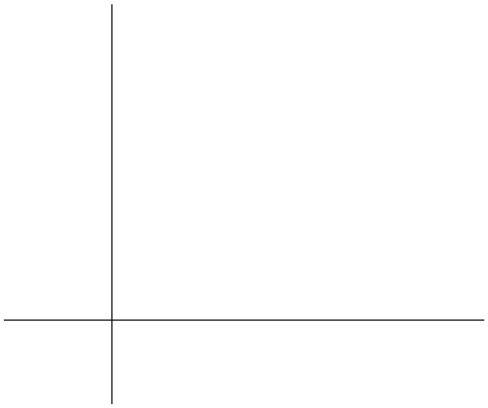


Part IV: Word Problems

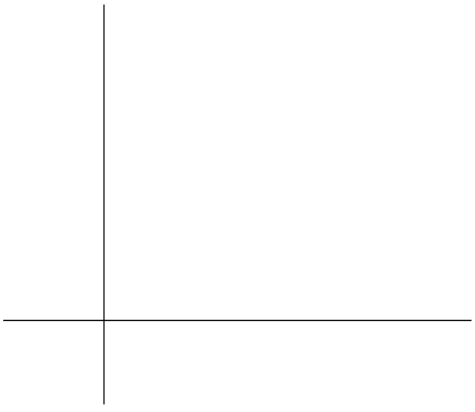
Solve the Linear Systems. (Label the variables and show your work.)

1) A movie theater charges $2.50 for kids and $4.00 for adults. Last Friday, 260 people attended the show. If the theater collected $782, how many of the viewers were adults?

2) At the movie, Lance wants to buy popcorn and candy for himself and four friends.
Popcorn cost $2 and Candy cost $1.
If he wants to spend less than $20 and needs to get at least one treat per person, graph a system that describes all the possible combinations of popcorn and candy he can buy.



3) There is a cafe next to the movie theater. The daily costs for the cafe are $200 plus $2 per order. If each customer pays $5 per order, how many daily customers does the cafe need to make a profit?
(Show your solutions algebraically AND graphically)



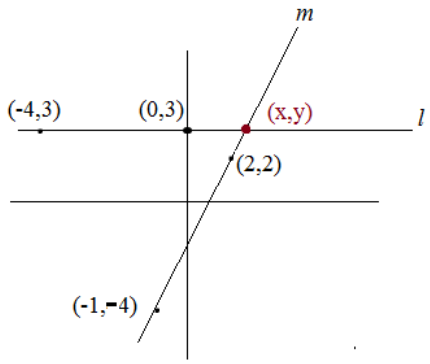
Part V: Miscellaneous Concepts

1) Describe the linear system and solve.

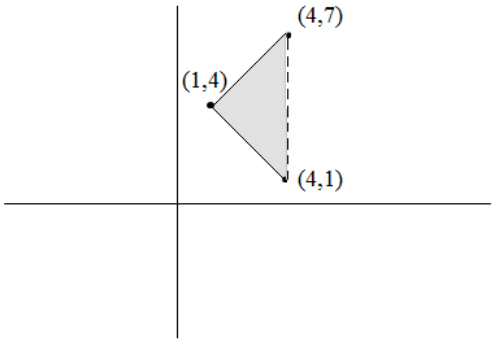
l:

m:

(x, y) = ?



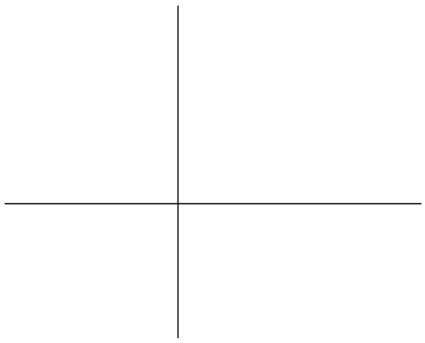
2) Describe the system:

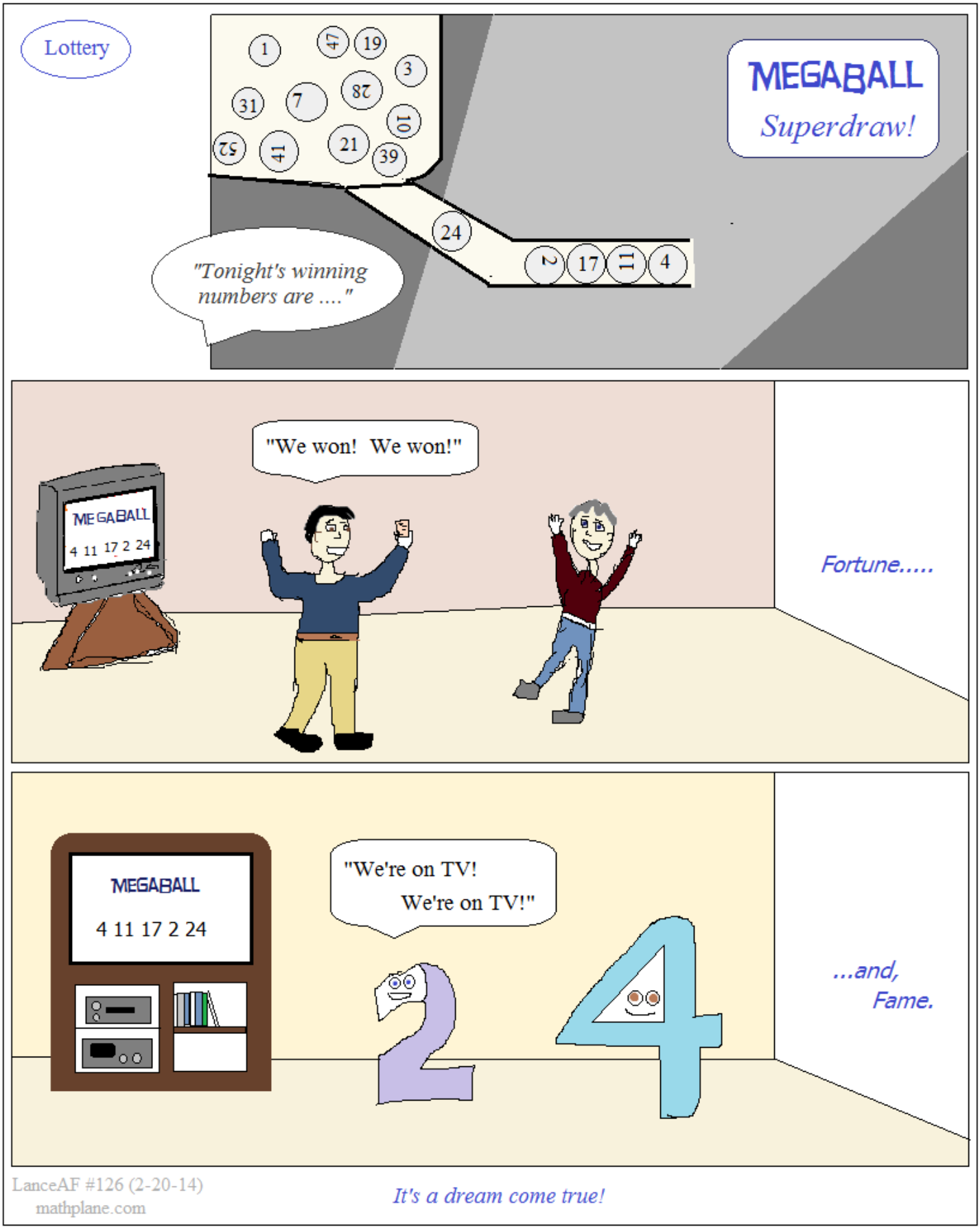


3) Graph and write the linear equation (in standard form):

The x-intercept is (8, 0)

y-intercept is (0, 5)




Lottery
1
47
19
3
31
7
28
10
52
41
21
39
24
2
17
11
4
MEGABALL
Superdraw!
"Tonight's winning numbers are ...."
MEGABALL
4 11 17 2 24
"We won! We won!"
Fortune.....
MEGABALL
4 11 17 2 24
"We're on TV!
We're on TV!"
...and,
Fame.
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It's a dream come true!

Linear Systems Test 2

Solutions

Part I: Solving Systems

Use Substitution (and show your work)

1) $y = 3x + 10$
$2x + 3y = -3$

(substitute y into 2nd equation) $2x + 3(3x + 10) = -3$
$2x + 9x + 30 = -3$
$11x = -33$
$x = -3$

(put x into 1st equation) $y = 3(-3) + 10$
$y = 1$

(check solution!) $2(-3) + 3(1) = -3$
$-6 + 3 = -3$
$-3 = -3$ ✓

(-3, 1)

2) $y = 2x - 4$
$3x - y = 9$

(substitute y into 2nd equation) $3x - (2x - 4) = 9$
$3x - 2x + 4 = 9$
$x = 5$

(put x into 2nd equation) $3(5) - y = 9$
$15 - y = 9$
$y = 6$

(check solutions) $(6) = 2(5) - 4$
$6 = 10 - 4$
$6 = 6$ ✓

$3(5) - (6) = 9$
$15 - 6 = 9$
$9 = 9$ ✓

(5, 6)

Use Elimination (Combination) Method (and show your work)

3) $3x + 7y = 1$
$6x - 5y = -17$

(multiply top by -2) $-6x - 14y = -2$

(combine equations) $-6x - 14y = -2$
$6x - 5y = -17$
$-19y = -19$
$y = 1$

(plug y into top equation) $3x + 7(1) = 1$
$3x = -6$
$x = -2$

(Check bottom equation) $6(-2) - 5(1) = -17$
$-12 - 5 = -17$
$-17 = -17$ ✓

(-2, 1)

4) $x + 3y = 6$
$3x - y = -12$

(multiply bottom by 3) $9x - 3y = -36$

(combine equations) $x + 3y = 6$
$9x - 3y = -36$
$10x = -30$
$x = -3$

(plug x into top equation) $(-3) + 3y = 6$
$3y = 9$
$y = 3$

(check bottom equation) $3(-3) - (3) = -12$
$-9 - 3 = -12$
$-12 = -12$ ✓

(-3, 3)

Use Any Method

5) $y = 4$
$3x + 5y = 8$

(easily plug top equation into bottom)

$3x + 5(4) = 8$
$3x + 20 = 8$
$x = -4$

and, obviously $y = 4$

(-4, 4)

6) $\frac{2}{3}x - y = 4$
$y = 2x - 12$

(rewrite top equation) $y = \frac{2}{3}x - 4$

(set equations equal to each other/substituting y) $\frac{2}{3}x - 4 = 2x - 12$
$8 = \frac{4}{3}x$
$x = 6$

(plug x into top) $\frac{2}{3}(6) - y = 4$
$4 - y = 4$
$y = 0$

(6, 0)

7) $y = -3x + 10$
$3x + y = 15$

(rewrite bottom equation)
$y = -3x + 15$

(compare equations!)
$y = -3x + 15$
$y = -3x + 10$

Same slope, different intercepts!
Parallel lines

NO SOLUTION

Part II: Graphing

Graph the following: $3x + 5y = 15$

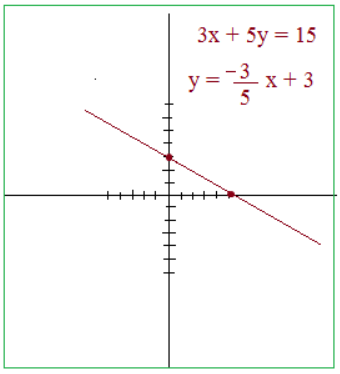
What is the x-intercept? (5, 0) $3x + 5(0) = 15$, $x = 5$

y-intercept? (0, 3) $3(0) + 5y = 15$, $y = 3$

slope? $\frac{-3}{5}$ $m = \frac{0 - 3}{5 - 0} = \frac{-3}{5}$

Is (20, -8) a point on this line? NO

(plug in the point!)
$3(20) + 5(-8) = 15$
$60 - 40 = 15$
$20 \neq 15$



Part III: Graph and Solve

Solutions

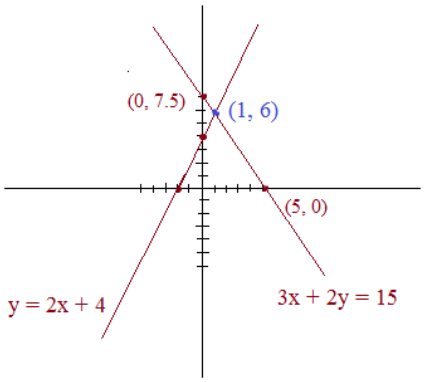
Graph each system. Then, identify the solutions on the graphs.

$3x + 2y = 15$
$y = 2x + 4$

Use substitution method to verify solution:

$3x + 2(2x + 4) = 15$
$3x + 4x + 8 = 15$
$7x = 7$
$x = 1$
$3(1) + 2y = 15$
$3 + 2y = 15$
$2y = 12$
$y = 6$

(1, 6)

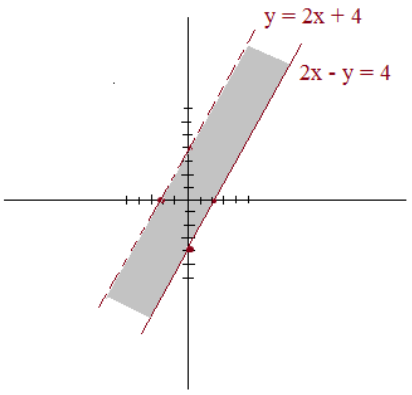


$y < 2x + 4$
$2x - y \leq 4$

draw the line $y = 2x + 4$
since it is $<$, it is a slashed line..
then, test (0, 0)
$(0) < 2(0) + 4$
$0 < 4$ yes.
Region below the line that includes (0, 0) is shaded!

Notice, these are parallel lines!

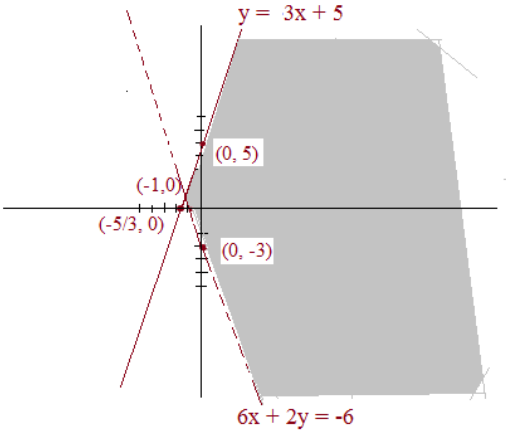
Then,
draw line $2x - y = 4$
or $y = 2x - 4$
since it is $\leq$, it is a solid line.
then, test (0, 0)
$2(0) - (0) \leq 4$ yes.
Region above the line that includes (0, 0) is shaded..



$y \leq 3x + 5$
$6x + 2y > -6$

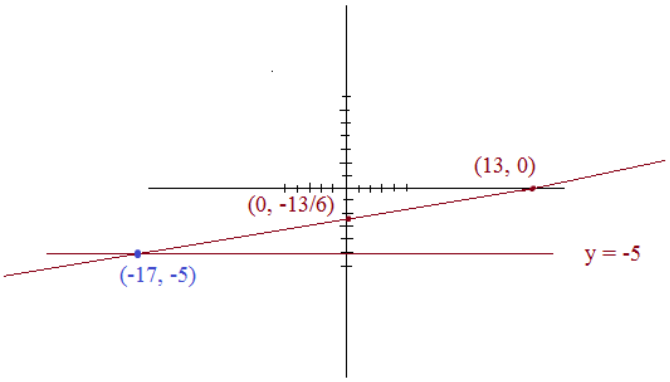
First, graph the top equation by identifying the y-intercept and x-intercept. Then, draw a line that goes through both. (since it is $\leq$, the line is solid)
then, test (0, 0)
$0 \leq 0 + 5$ yes! The area under the line may be shaded.

Then, graph the second equation by drawing line through intercepts. Then, the line is dashed (because it is >)
Test (0, 0):
$6(0) + 2(0) > -6$
$0 > -6$ yes... Area above the dashed line may be shaded.



$y = -5$ (horizontal line)
$x - 6y = 13$

$x - 6(-5) = 13$
$x + 30 = 13$
$x = -17$



Part IV: Word Problems

Solutions

Solve the Linear Systems. (Label the variables and show your work.)

1) A movie theater charges $2.50 for kids and $4.00 for adults. Last Friday, 260 people attended the show. If the theater collected $782, how many of the viewers were adults?

Let K = # of kids $2.5 per kid
A = # of adults $4.0 per adult

$2.5K + $4.0A = $782
or
2.5K + 4A = 782

A + K = 260

Use elimination method to find A and K:

$$\begin{array}{r} 2.5K + 4A = 782 \\ -\ (4K + 4A = 1040) \\ \hline -1.5K = -258 \end{array}$$

K = 172

A + 172 = 260
A = 88

(Check Solution)

$2.50 x 172 = $430

$4.00 x 88 = $352

$782 total!

** Now, answer the question: How many viewers were adults? 88

2) At the movie, Lance wants to buy popcorn and candy <u>for himself and four friends</u>.
Popcorn cost $2 and Candy cost $1.
If he wants to spend less than $20 and needs to get at least one treat per person, graph a system that describes all the possible combinations of popcorn and candy he can buy.

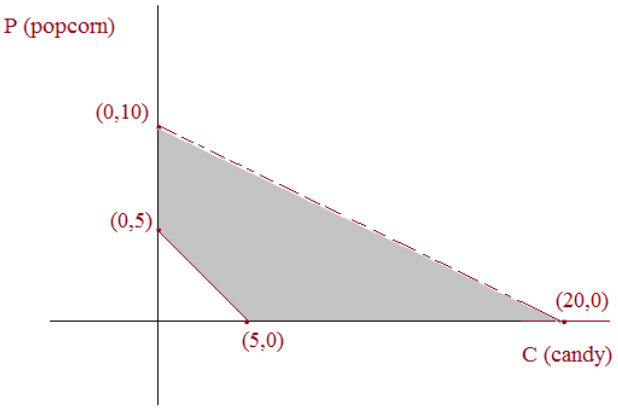
Let P = # of Popcorn
C = # of Candy

Price of candy = $1
Price of popcorn = $2

$1C + $2P < $20 (cost constraint)

$P + C \geq 5$ (quantity constraint)

Any combination of popcorn and candy in the gray region would satisfy the cost constraint (< $20) and satisfy the quantity constraint (everyone gets at least one treat).



3) There is a cafe next to the movie theater. The daily costs for the cafe are $200 plus $2 per order. If each customer pays $5 per order, how many daily customers does the cafe need to make a profit?
(Show your solutions algebraically AND graphically)

Let X = # of customers

Cafe Costs = $200 + $2X

Cafe Revenues = $5X

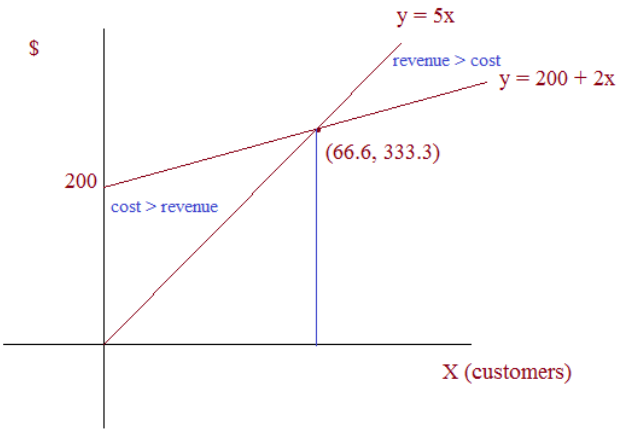
Let's find where revenue > cost...

C(x) = 200 + 2x
R(x) = 5x

Where does revenue = cost?

5x = 200 + 2x
3x = 200
x = 66.6

Since you can't have "fractional customers", the cafe must have 67 or more customers to make a profit.



Part V: Miscellaneous Concepts

Solutions

1) Describe the linear system and solve.

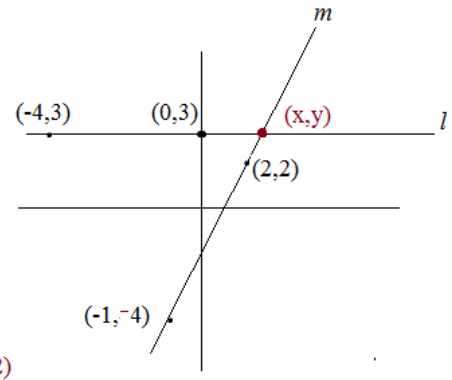
(horizontal line)

l: $y = 3$

m: slope is $6/3 = 2$

so line in point slope form is:

$(y - 2) = 2(x - 2)$



(x, y) = ?

l and *m* intersect at $(3 - 2) = 2(x - 2)$

$1 = 2x - 4$

$x = 5/2$

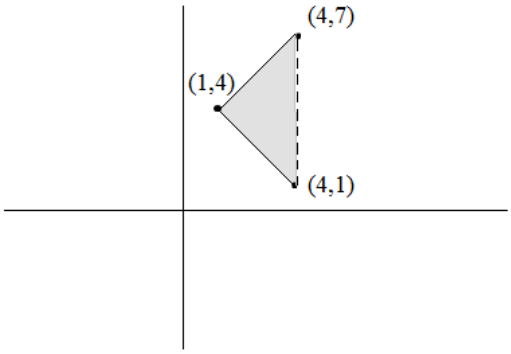
$\left(\frac{5}{2}, 3\right)$

2) Describe the system:

$x < 4$

$y \leq x + 3$

$y \geq -x + 5$



3) Graph and write the linear equation (in standard form):

The x-intercept is (8, 0)

y-intercept is (0, 5)

find slope:

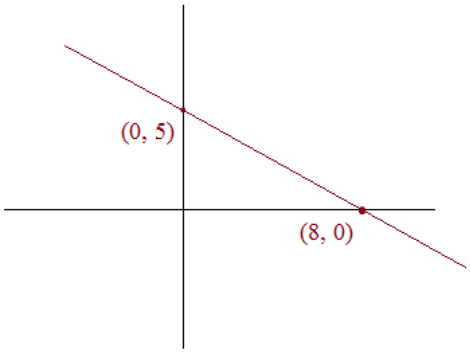
$m = \frac{5 - 0}{0 - 8} = \frac{-5}{8}$

$y - 0 = \frac{-5}{8}(x - 8)$

$y = \frac{-5}{8}x + 5$

$8y = -5x + 40$

$5x + 8y = 40$

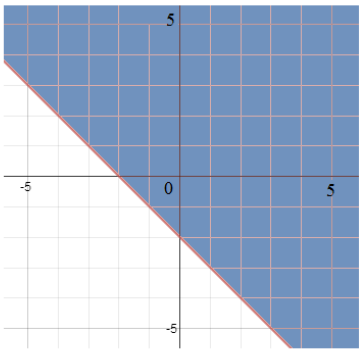


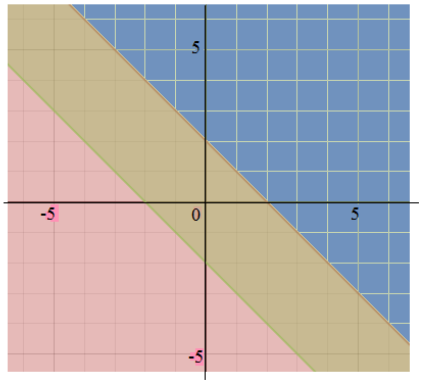
Example: Graph $|x + y| \leq 2$

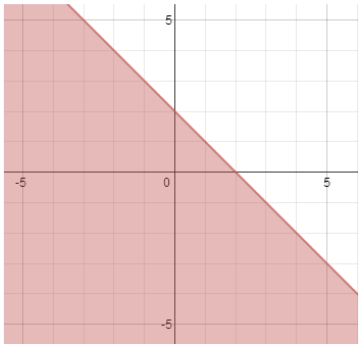
Graph $x + y \leq 2$

Graph $x + y \geq -2$

The intersection is the solution..



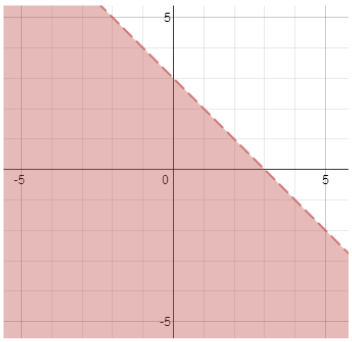




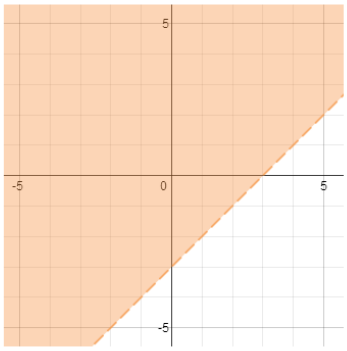
Example: Graph $|x| + |y| < 3$

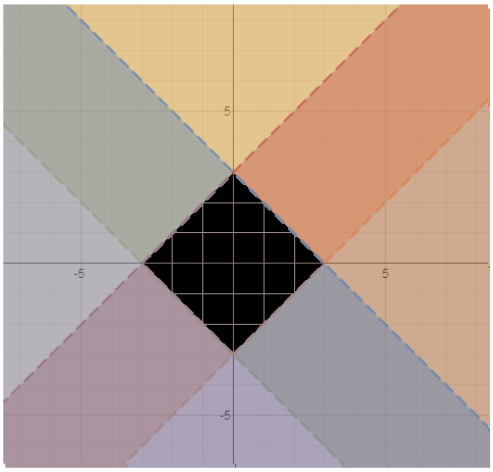
Graph all 4 possibilities...

$x + y < 3$

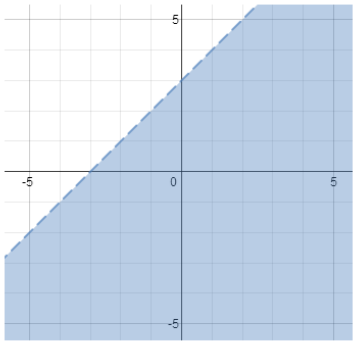


$x + (-y) < 3$

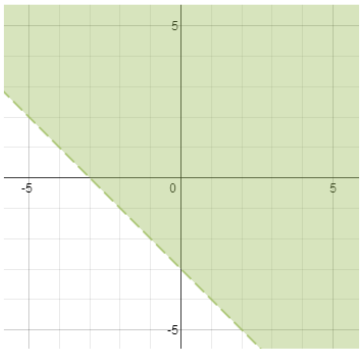




$(-x) + y < 3$



$(-x) + (-y) < 3$



NOTE: To check answer, test points in each region..

(0, 0): $|0| + |0| < 3$ ✓

(5, 0): $|5| + |0| < 3$ ✗

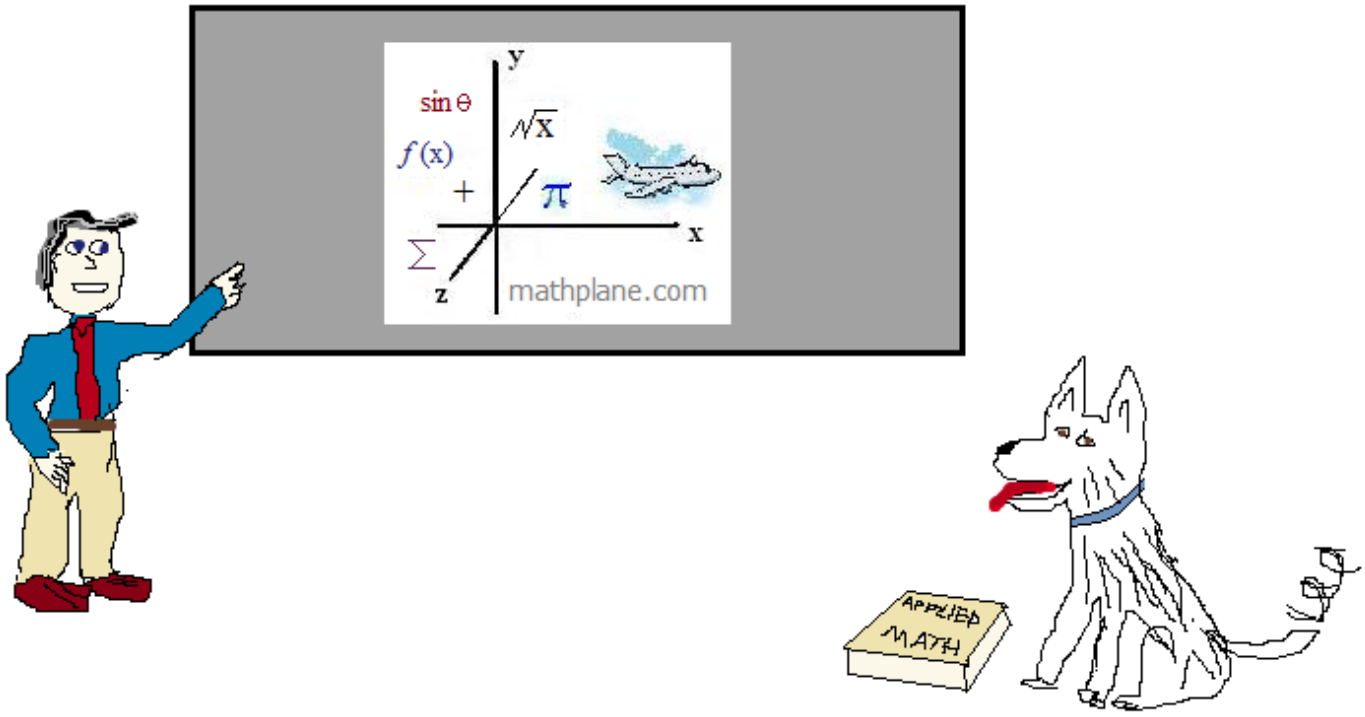
(-1, -1): $|-1| + |-1| < 3$ ✓

(-3, 4): $|-3| + |4| < 3$ ✗

Thanks for visiting. (Hope it helps!)

If you have questions, suggestions, or requests, let us know.

Good luck!



Also, at Facebook, Google+, TeachersPayTeachers, and Pinterest

And, Mathplane *Express* for mobile at Mathplane.ORG

One more question:

Can you graph this linear system?

$y < 2x + 4$

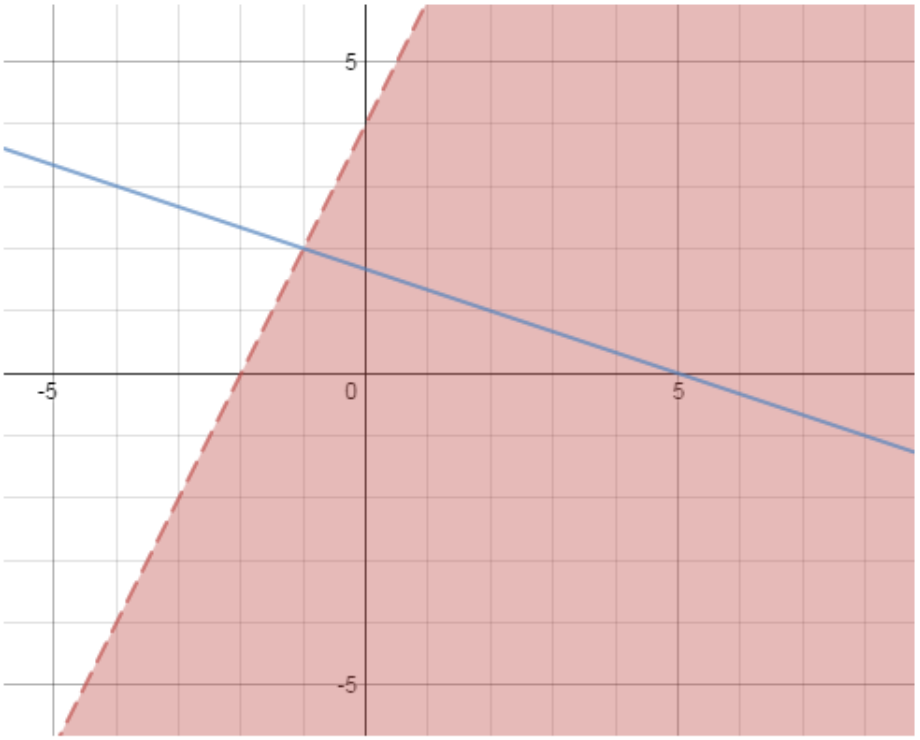
$x + 3y = 5$

Solution on next page-→

Graph the following system: $y < 2x + 4$

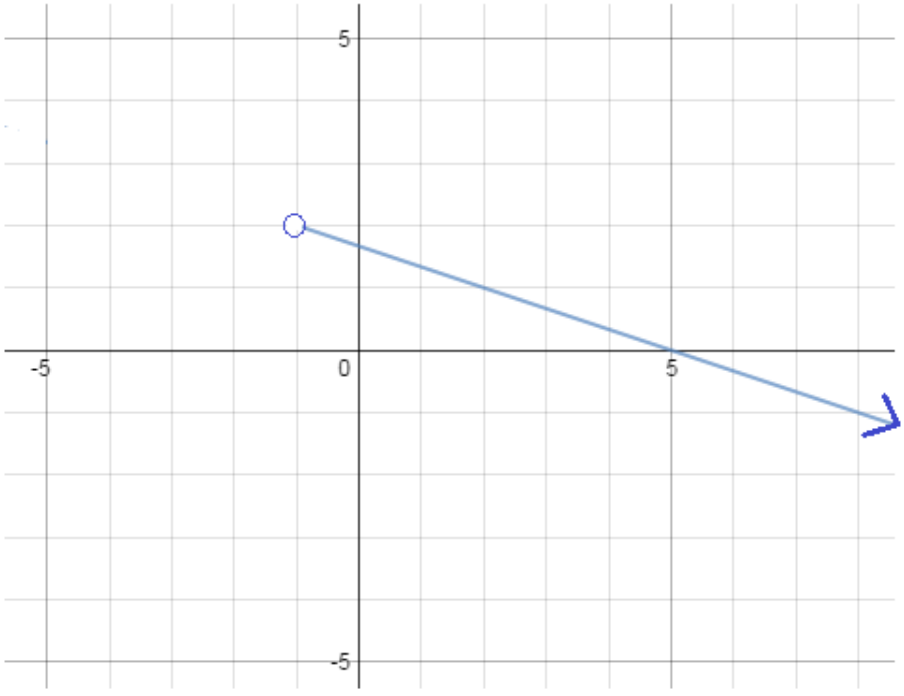
$x + 3y = 5$

$y < 2x + 4$



$x + 3y = 5$

The solution set must satisfy both equations!



The solution set is $x + 3y = 5$ on the interval $(-1, \infty)$