

Linear Systems: Word Problems

Examples and Practice Questions (with solutions)

Topics include rate problems, integer questions, mixture, area, ticket sales, and more.

Example: Spencer has 47 coins in his Snoopy bank.
 The coins consist of nickels, dimes, and quarters. And, there are twice as many dimes as nickels.
 If the bank contains \$8.25, how many dimes are inside?

Solving Linear Systems Word Problems:

- 1) Establish variables
- 2) Draw diagram (if applicable)
- 3) Set up system of equations
- 4) Solve
- 5) Check Answers

Establish Variables $D = \#$ of dimes
 $N = \#$ of nickels
 $Q = \#$ of quarters

Set up Equations $2N = D$
 $N + D + Q = 47$
 $.05N + .10D + .25Q = 8.25$

Solve Using substitution:

$$N + (2N) + Q = 47$$

$$.05N + .10(2N) + .25Q = 8.25$$

Then, rewrite:

$$3N + Q = 47 \qquad 75N + 25Q = 1175$$

$$25N + 25Q = 825 \qquad \underline{25N + 25Q = 825}$$

$$50N = 350$$

$$N = 7$$

therefore, $D = 14...$

$$7 + 14 + Q = 47$$

$$Q = 26$$

Check Answers

7 nickels
 14 dimes total coins: 47 ✓
 26 quarters

7 nickels "twice as many ✓
 14 dimes dimes as nickels"

7 nickels: 35 cents
 14 dimes: \$1.40 \$8.25 total ✓
 26 quarters: \$6.50

Example A bottling plant has 2 machines.
 Machine A can fill 1400 bottles per hour.
 Machine B can fill 2200 bottles per hour.
 Machine A begins running at 8:30.
 After maintenance, Machine B is turned on at 10:00.
 What time will they fill 19,200 bottles?
 How many bottles did Machine A fill?

Solve the system of equations..

Rate and work problems:
 Using table and system of equations:

- 1) Establish variables
- 2) Set up table
- 3) Set up system of equations
- 4) Solve
- 5) Check Answers

	rate (bottles/hour)	time (hours)	bottles
Machine A	1400	$t + 1.5$	B
Machine B	2200	t	$19,200 - B$

Set up Equations Since distance (d) = rate x time,
 $1400(t + 1.5) = B$
 $2200(t) = 19,200 - B$

Solve $1400t + 2100 = B$
 $2200t + B = 19,200$
 (substitution)
 $2200t + (1400t + 2100) = 19,200$
 $3600t = 17,100$
 $t = 4.75$
 then, $1400(4.75 + 1.5) = B$
 $B = 8750$

Then, answer the question...

Machine A fills 8750 bottles

(Machine B fills 10,450 bottles)

$t = 4.75$ or 4 hours 45 minutes

So, the bottles will be finished at 10:00 + 4hr 45min = 2:45 pm

Linear Systems Application: Rate Problems

Here are 3 types of rate applications: "chase", "away", and "toward".

$$d = rt$$

$$\text{distance} = (\text{rate})(\text{time})$$

Example: Jack goes up a hill at a rate of 4 miles per hour. Three hours later, Jill goes up the hill at a rate of 6 miles per hour. How long will it take Jill to catch up to Jack? How far up the hill did they go?

	rate (miles/hour)	time (hours)	distance
Jack	4	t	d
Jill	6	t - 3	d



since Jill will gain 2 miles/hour, it should take 6 hours to close the 12 mile gap...

"Chase"

$$d_{\text{jack}} = 4t$$

$$d_{\text{jill}} = 6(t - 3)$$

$$d_{\text{jack}} = d_{\text{jill}}$$

$$4t = 6(t - 3)$$

$$4t = 6t - 18$$

$$t = 9$$

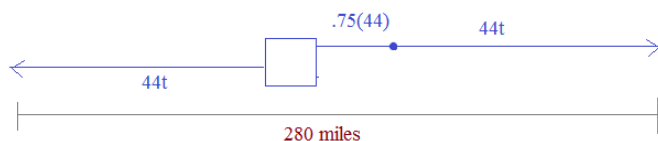
So, Jack will spend 9 hours on the hill
Jill will spend 6 hours on the hill...

And, each of them went 36 miles up the hill!

Example: At Union Station, Train A departs southbound at 3:00pm. At 3:45 Train B departs northbound. If each travels at 44 miles per hour, when will they be 280 miles apart?

	rate (miles/hour)	time (hours)	distance
Train A	44	t + .75	d
Train B	44	t	280 - d

45 minutes = .75 hours



The trains will be 280 miles apart approximately 2 hours 48 minutes after Train B departs...

$$3:45 + 2:48 = 6:33\text{PM}$$

"Away"

distance = rate x time

together, Train A + Train B = 280

$$d_A = 44(t + .75)$$

$$44(t + .75) + 44(t) = 280$$

$$d_B = 44(t)$$

$$44t + 33 + 44t = 280$$

$$88t = 247$$

$$t = 2.807 \text{ hours}$$

Convert to minutes: 2 hours +

$$.807 \text{ hour} \times 60 \frac{\text{minutes}}{\text{hour}} = 48 \text{ minutes (approx)}$$

Quick check: $44 \times 2.807 = 123.5$ miles

$$44 \times 3.557 = 156.5 \text{ miles} \quad 280 \text{ total}$$

Example: Tom and Jerry live 10 miles apart. If Tom rides his bike at 12 miles per hour toward Jerry's house. And, Jerry runs at 7 miles per hour toward Tom's house, How long will it take them to meet?

	rate (miles/hour)	time (hours)	distance
Tom	12	t	d
Jerry	7	t	10 - d

Tom's distance $d = (12 \text{ miles/hour})t$

Jerry's distance $(10 - d) = (7 \text{ miles/hour})t$

Tom's distance + Jerry's distance = total distance

$$12t + 7t = 10 \text{ miles}$$

"Toward"

$$(19 \text{ miles/hour})t = 10 \text{ miles}$$

$$t = 10/19 \text{ hour or } 31.58 \text{ minutes}$$

$$\text{or } 31 \text{ minutes } 35 \text{ seconds}$$

- 6) A cell phone company charges \$15 for the first 1000 text messages.
Then, it charges 10 cents for each additional text.
If the bill is \$48.50, how many text messages were sent?
- 7) A tropical drink uses twice as much pineapple juice as it does coconut juice.
How much coconut juice is used in a 16 ounce drink?
- 8) The length of a rectangle is twice its width.
If the area is 98 square feet, what is the perimeter?
- 9) I have 47 quarters, dimes, and nickels in my pocket.
If I have twice as many quarters as nickels, and a total of \$7.45,
how many quarters do I have?
- 10) The school play is charging \$8 for adults, \$4 for children, and \$5 for seniors.
At Friday's performance, $\frac{1}{2}$ as many seniors as adults attended.
And, twice as many children as adults attended the show.
If the show grossed \$592, how many seniors attended?

- 11) John paid \$14.63 for three bags of chips and four sodas. At the same store, Alice paid \$16.03 for two bags of chips and five sodas. How much does a bag of chips cost?
(Note: there is no sales tax.)

- 12) The length of a rectangle is twice its width.
If the perimeter is 16 feet, what is the area?

- 13) Given a 2-digit number. The sum of the digits is 12.
If you reverse the digits, then the number decreases by 54.
What is the number?

- 14) Call card A costs \$1.50 and 25 cents per minute. And,
call card B costs \$2.75 and charges 15 cents per minute.

When is buying card B a better deal?

- 15) Jack currently has \$75 in a bank account and saves \$10 per week.
Jill has \$125 in her account, and she is saving \$2 per day.
When will they have the same amounts in their accounts?

- 16) Lance has \$100 in a savings account and adds \$30 per month.
Katie has \$300 in an account and withdraws \$20 per month.


When will their accounts have the same amount?

- 17) Carnations cost \$2.15 per stem. Roses cost \$8.25 per stem.
If you have 90 dollars to spend and need 20 stems, how many of each can you buy?

- 18) Josh spilled punch on his homework!

Question 4 had a solution of (6, 3)...

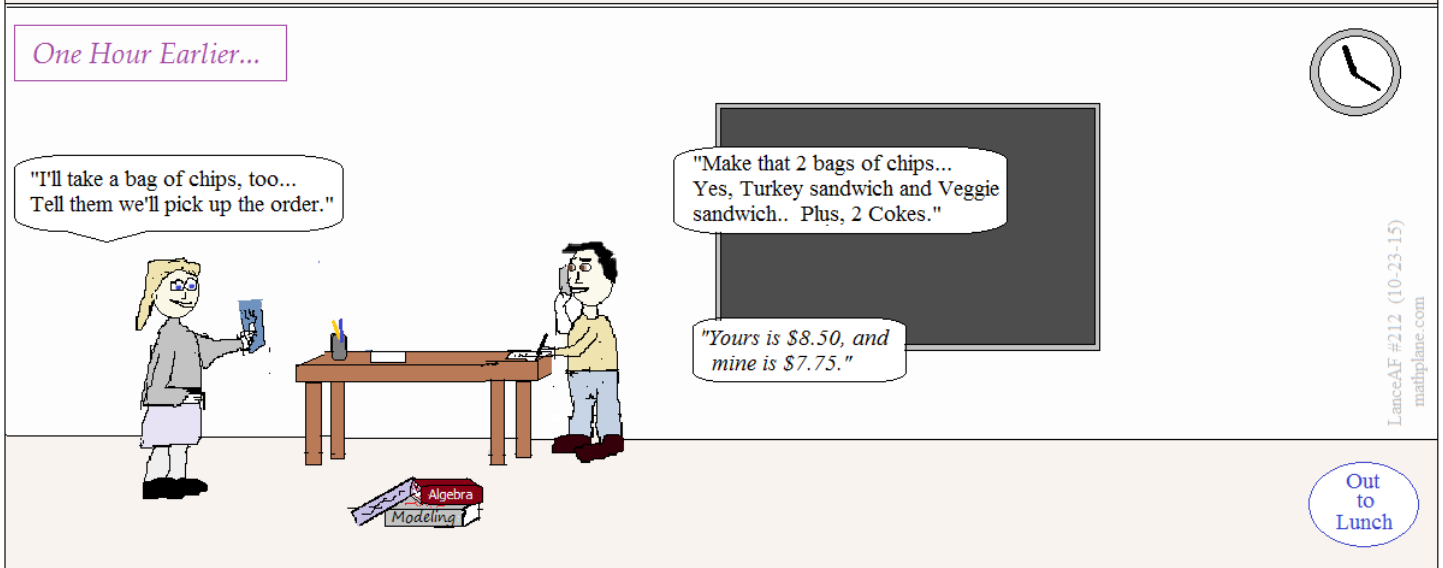
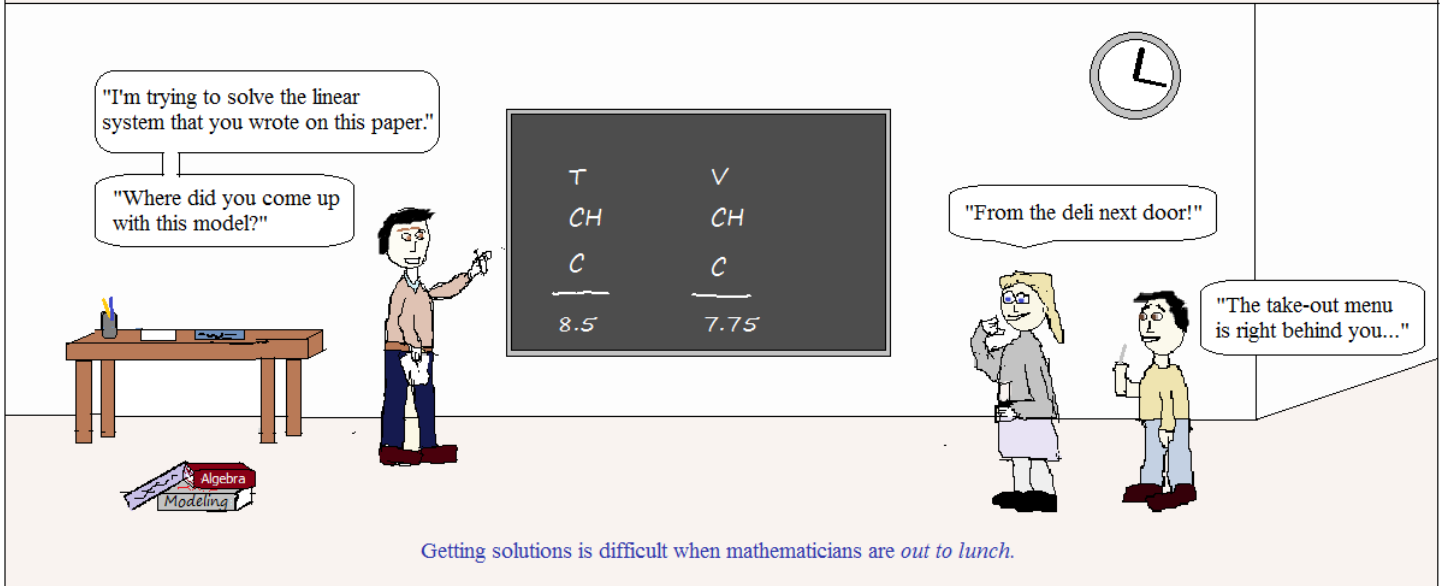
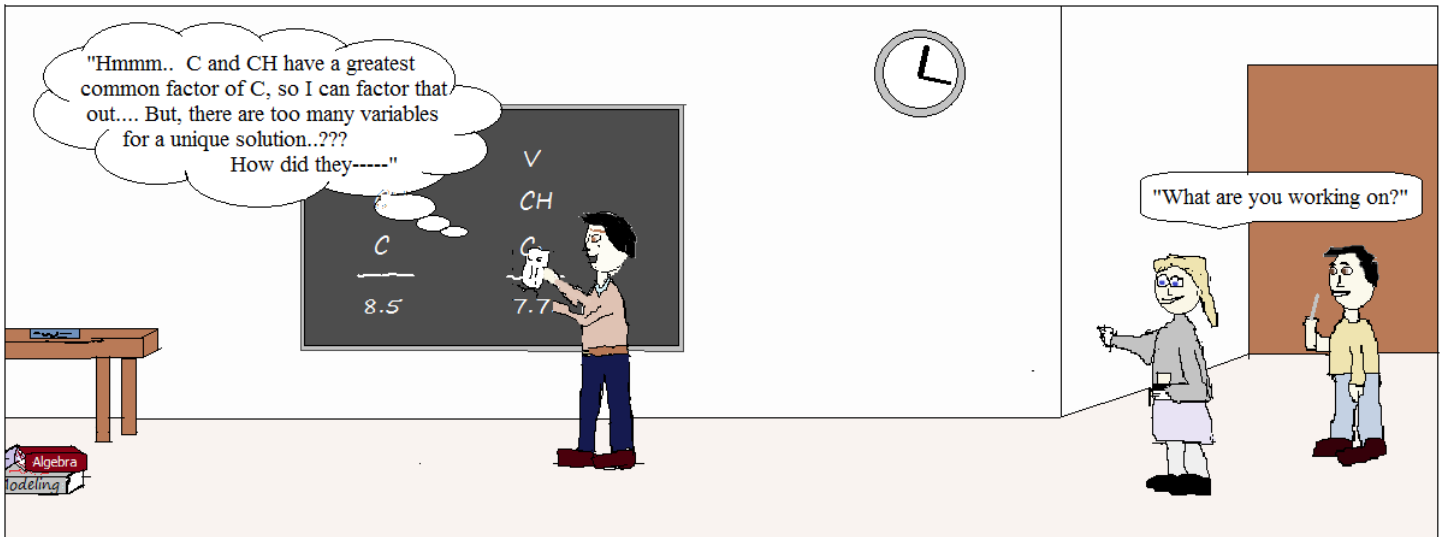
What is a possible equation in the question?

$$\begin{cases} 2x + 3y = 21 \\ 5x \end{cases}$$


- 19) I'm thinking of a 2-digit number. The sum of the digits is 7.
And, if you reverse the digits, the number decreases by 9.
What is the number?

- 20) Challenge question: solve the system *in terms of m*

$$\begin{cases} x + y = m \\ ym + x - 1 = 0 \end{cases}$$



- 1) The sum of two integers is 724.
If their difference is 24, what are the two integers?

"The Integer Question"

The two integers are A and B.

$$A + B = 724$$

Using elimination/combination method:

$$2A + 0B = 748$$

$$A - B = 24$$

$$A = 374... \text{ then, } B = 350$$

350 and 374

- 2) Katie has \$4 in her piggy bank, filled with nickels and dimes.
If there are 47 coins in the piggy bank, how many of them are dimes?

"The Coin Question"

Let D = # of dimes
N = # of nickels

Using substitution: $D = 47 - N$

Since $N = 14$

$$D + N = 47$$

two equations, two unknowns

$$.10(47 - N) + .05(N) = 4$$

and $D + N = 47$,

$$.10(D) + .05(N) = \$4$$

$$4.7 - .10N + .05N = 4$$

$D = 33$

$$-.05N = -.7$$

$$N = 14$$

33 Dimes

- 3) A farmer has a field full of chickens and sheep.
When viewed from above, you can see 65 heads.
And, when viewed from below, you can see 220 legs.
How many chickens are in the field?

"The Animal Feet and Head Question"

Let C = # of chickens

S = # of sheep

chickens have 2 legs
sheep have 4 legs

$$1(C) + 1(S) = 65 \text{ heads}$$

$$2(C) + 4(S) = 220 \text{ legs}$$

Using elimination/combination:

$$-1C + -1S = -65$$

$$2C + 4S = 220$$

$$-2C - 2S = -130$$

$$2S = 90$$

$$S = 45$$

If there are 45 sheep,
then there are

$$C + 45 = 65$$

20 Chickens

- 4) An airplane flies from A-town to B-town in 6 hours.
Then, the next day, it flies from B-town to A-town in 5 hours.
If the airplane's speed is 500 miles per hour, what is the speed of the wind?
How far apart are A and B?

"Wind Speed"

distance = rate x time

distance A to B: $(\text{rate}_{\text{plane}} - \text{rate}_{\text{wind}}) \times 6 \text{ hours}$

distance B to A: $(\text{rate}_{\text{plane}} + \text{rate}_{\text{wind}}) \times 5 \text{ hours}$

Since the distances are the same, we set the equations equal to each other....

$$(P - W) \times 6 = (P + W) \times 5$$

$$6(500 - W) = 5(500 + W)$$

$$3000 - 6W = 2500 + 5W$$

$$500 = 11W$$

$$W = 45.45 \text{ miles/hour}$$

Wind speed is $45 \frac{5}{11}$ miles per hour

Then, since distance = rate(time),

distance = $(500 \text{ m/h} - 45.45 \text{ m/h})(6 \text{ hours})$

$$A \text{ to B} = 2727.27 \text{ miles...}$$

Distance = 2727.27 miles

check: distance from B to A:

$$(500 \text{ m/h} + 45.45 \text{ m/h})(5 \text{ hours}) = 2727.27 \checkmark$$

- 5) Tickets to a math performance cost \$8 for adults and \$3 for children.
If the show sold twice as many children's tickets as adult tickets,
and it raised \$602, how many adults attended the performance?

"Tickets Question"

Let A = # of adults

C = # of children

Since "twice as many children's tickets as adult tickets",

$C = 2A$ (ex: if 10 adult tickets, then 20 children's tickets...)

$$\$8(\text{adults}) + \$3(\text{children}) = \$602$$

$$8A + 3C = 602$$

Using substitution:

$$8A + 3(2A) = 602$$

$$14A = 602$$

$$A = 43$$

43 Adults

Quick Check:

$$43 \text{ adults} \times \$8 = \$344$$

$$86 \text{ children} \times \$3 = \$258$$

total: \$602 ✓

- 6) A cell phone company charges \$15 for the first 1000 text messages. Then, it charges 10 cents for each additional text.

If the bill is \$48.50, how many text messages were sent?

Let $T = \#$ of additional texts

$$\$15 + \$.10(T) = \$48.50$$

$$.10(T) = \$33.50$$

$$T = 335$$

So, the total texts = first 1000 + additional texts

$$= 1000 + 335$$

$$= 1335$$

1335 total text messages

- 7) A tropical drink uses twice as much pineapple juice as it does coconut juice. How much coconut juice is used in a 16 ounce drink?

$P =$ amount of pineapple juice

$C =$ amount of coconut juice

total juice: $P + C = 16$ ounces

portions: $P = 2C$

ex: If you used 10 oz of coconut, then you'd use 20 oz of pineapple.

$$C = 16 - P$$

$$P = 2C$$

$$C = 16 - (2C)$$

$$3C = 16$$

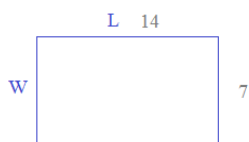
$$C = 16/3$$

and

$$P = 32/3$$

$5\frac{1}{3}$ ounces of coconut juice

- 8) The length of a rectangle is twice its width. If the area is 98 square feet, what is the perimeter?



Let $L =$ length

$W =$ width

$$LW = 98 \text{ square feet}$$

$$L = 2W$$

Using substitution:

$$(2W)W = 98 \text{ square feet}$$

$$2W^2 = 98$$

$$W = 7 \quad \text{then, } L = 14$$

$$\text{Perimeter} = 2L + 2W$$

therefore, perimeter is

$$2(14) + 2(7) = 42$$

perimeter is 42 feet

- 9) I have 47 quarters, dimes, and nickels in my pocket. If I have twice as many quarters as nickels, and a total of \$7.45, how many quarters do I have?

Let $Q = \#$ of quarters

$D = \#$ of dimes

$N = \#$ of nickels

since there are 3 unknown variables, we seek 3 equations...

$$.25Q + .10D + .05N = 7.45$$

$$Q = 2N$$

$$Q + D + N = 47$$

For ease, get rid of the decimals:

$$25Q + 10D + 5N = 745$$

then, use substitution:

$$25(2N) + 10D + 5N = 745$$

$$(2N) + D + N = 47$$

then, elimination:

$$55N + 10D = 745$$

$$- 30N + 10D = 470$$

$$55N + 10D = 745$$

$$3N + D = 47$$

$$30N + 10D = 470$$

$$25N = 275 \quad N = 11$$

22 Quarters

If $N = 11$, then $Q = 22$

Also, $D = 14$

because $Q + D + N = 47$

Quick check:

22 quarters: 5.50

11 nickels: .55

14 dimes: 1.40

total: \$745 ✓

- 10) The school play is charging \$8 for adults, \$4 for children, and \$5 for seniors. At Friday's performance, 1/2 as many seniors as adults attended. And, twice as many children as adults attended the show. If the show grossed \$592, how many seniors attended?

Let $A = \#$ of adults

$C = \#$ of children

$S = \#$ of seniors

to find the 3 variables, we seek 3 equations...

"1/2 as many seniors as adults"

$$S = (1/2)A$$

"twice as many children as adults"

$$C = (2)A$$

"show grossed \$592"

$$\$8(A) + \$4(C) + \$5(S) = \$592$$

Using Substitution:

$$8(A) + 4(2A) + 5(.5A) = 592$$

$$18.5A = 592$$

$$A = 32$$

Since $A = 32$, $C = 64$, and $S = 16$

$$\text{Check: } \$8(32) + \$4(64) + \$5(16) = \$256 + \$256 + \$80 = \$592 \quad \checkmark$$

twice as many children as adults ✓

1/2 as many seniors as adults ✓

16 Seniors attended

SOLUTIONS

11) John paid \$14.63 for three bags of chips and four sodas. At the same store, Alice paid \$16.03 for two bags of chips and five sodas. How much does a bag of chips cost? (Note: there is no sales tax.)

Let C = # of chips	$3C + 4S = 14.63$	$300C + 400S = 1463$
S = # of sodas	$2C + 5S = 16.03$	$200C + 500S = 1603$
	two equations, two unknowns	
	(for ease, multiply by 100 to get rid of fractions)	$-600C - 800S = -2926$
		$600C + 1500S = 4809$
		$700S = 1883$

$$S = 2.69$$

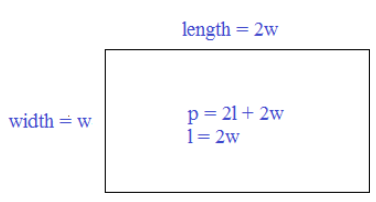
$$2C + 5(2.69) = 16.03$$

$$2C + 13.45 = 16.03$$

$$C = 1.29$$

Chips cost \$1.29
Sodas cost \$2.69

12) The length of a rectangle is twice its width. If the perimeter is 16 feet, what is the area?



perimeter = 2(length) + 2(width)

$$16 = 2(2w) + 2w$$

$$16 = 6w$$

$$w = \frac{8}{3}$$

therefore, length is $\frac{16}{3}$

the area is $\frac{128}{9}$

area is $14.\overline{22}$ square units

13) Given a 2-digit number. The sum of the digits is 12. If you reverse the digits, then the number decreases by 54. What is the number?

XY is the number...

"sum of digits is 12"

$$X + Y = 12$$

2-digit number reversed 2-digit number

$$(10X + Y) - 54 = 10Y + X$$

$$X = 12 - Y$$

$$9X - 9Y = 54$$

Using substitution:

$$9(12 - Y) - 9Y = 54$$

$$108 - 9Y - 9Y = 54$$

"Integer identification question"

93

quick check:

93 and 39 (digits add up to 12)
difference between numbers: 54

14) Call card A costs \$1.50 and 25 cents per minute. And, call card B costs \$2.75 and charges 15 cents per minute.

When is buying card B a better deal?

$A(m) = \$1.50 + \$.25m$

cost of card A where m is minutes used..

$B(m) = \$2.75 + \$.15m$

cost of card B where m is minutes used..

If m = 0, then card A costs \$1.50
card B costs \$2.75
(A is a better deal...)

Set A(m) = B(m)
(the break-even point)

$$1.50 + .25m = 2.75 + .15m$$

$$.10m = 1.25$$

"Break even question"

Since 12 minutes 30 seconds is the break-even,

12 minutes: A is a better deal.
13 minutes: B is a better deal!

15) Jack currently has \$75 in a bank account and saves \$10 per week. Jill has \$125 in her account, and she is saving \$2 per day. When will they have the same amounts in their accounts?

Jack's account: $A(w) = \$75 + \$10w$ where w = # of weeks

Jill's account: $B(d) = \$125 + \$2d$ where d = # of days

Convert to common units: $\$2/\text{day} = \$14/\text{week}$

$A(w) = \$75 + \$10w$

$B(w) = \$125 + \$14w$

Determine when accounts are equal:

$$\$75 + \$10w = \$125 + \$14w$$

$$-\$4w = \$50$$

$$w = -12.5$$

But, w cannot be negative!

Jack and Jill will never have the same amounts in their accounts

SOLUTIONS

- 16) Lance has \$100 in a savings account and adds \$30 per month.
Katie has \$300 in an account and withdraws \$20 per month.

When will their accounts have the same amount?

Lance's Account $B = \$100 + \$30m$

B = Balance
m = number of months

$$\$200 = \$50m$$

$$m = 4 \text{ months...}$$

Katie's Account $B = \$300 - \$20m$

- 17) Carnations cost \$2.15 per stem. Roses cost \$8.25 per stem.
If you have 90 dollars to spend and need 20 stems, how many of each can you buy?

$$\begin{cases} C + R = 20 \\ 2.15C + 8.25R = 90 \\ 2.15C + 2.15R = 43 \end{cases} \rightarrow 6.10R = 47$$

$$R = 7.70 \text{ roses}$$

$$C + R = 20$$

$$C + 7.70 = 20$$

$$C = 12.3 \text{ carnations..}$$

However, since you cannot buy "partial flower stems",

we must adjust the total... If we buy 8 roses and 12 carnations,

$$8 \times \$8.25 = \$66.00$$

$$12 \times \$2.15 = \$25.80$$

But, \$91.80 is more than I have...

If we buy 7 roses and 13 carnations,

$$7 \times \$8.25 = \$57.75$$

$$13 \times \$2.15 = \$27.95$$

And, I can afford \$85.70

- 18) Josh spilled punch on his homework!

Question 4 had a solution of (6, 3)...

What is a possible equation in the question?

$$\begin{cases} 2x + 3y = 21 \\ 5x \end{cases}$$

Possible answer might be $5x - y = 27$

(or, any equation where (6, 3) fits...)

- 19) I'm thinking of a 2-digit number. The sum of the digits is 7.
And, if you reverse the digits, the number decreases by 9.
What is the number?

Let A = the first digit
B = the second digit

$$A + B = 7$$

$$10A + B = \text{the number}$$

$$10B + A = \text{the number when the digits are reversed}$$

$$A + B = 7$$

$$10A + B = 10B + A + 9$$

$$A = 7 - B$$

$$9A - 9B = 9$$

Using substitution: $9(7 - B) - 9B = 9$

$$63 - 9B - 9B = 9$$

$$B = 3 \quad A = 4$$

the number is 43

- 20) Challenge question: solve the system in terms of m

$$\begin{cases} x + y = m \\ ym + x - 1 = 0 \end{cases}$$

$$\begin{array}{r} x + y = m \\ - \quad x + ym = 1 \\ \hline y - ym = m - 1 \\ y(1 - m) = m - 1 \end{array}$$

$$y = \frac{m - 1}{1 - m}$$

$$x + \frac{m - 1}{1 - m} = m$$

$$x = m - \frac{m - 1}{1 - m}$$

check: $ym + x - 1 = 0$

$$\frac{m - 1}{1 - m} \cdot m + m - \frac{m - 1}{1 - m} - 1 = 0$$

$$\frac{m^2 - m}{1 - m} + \frac{m(1 - m)}{1 - m} - \frac{m - 1}{1 - m} = 1$$

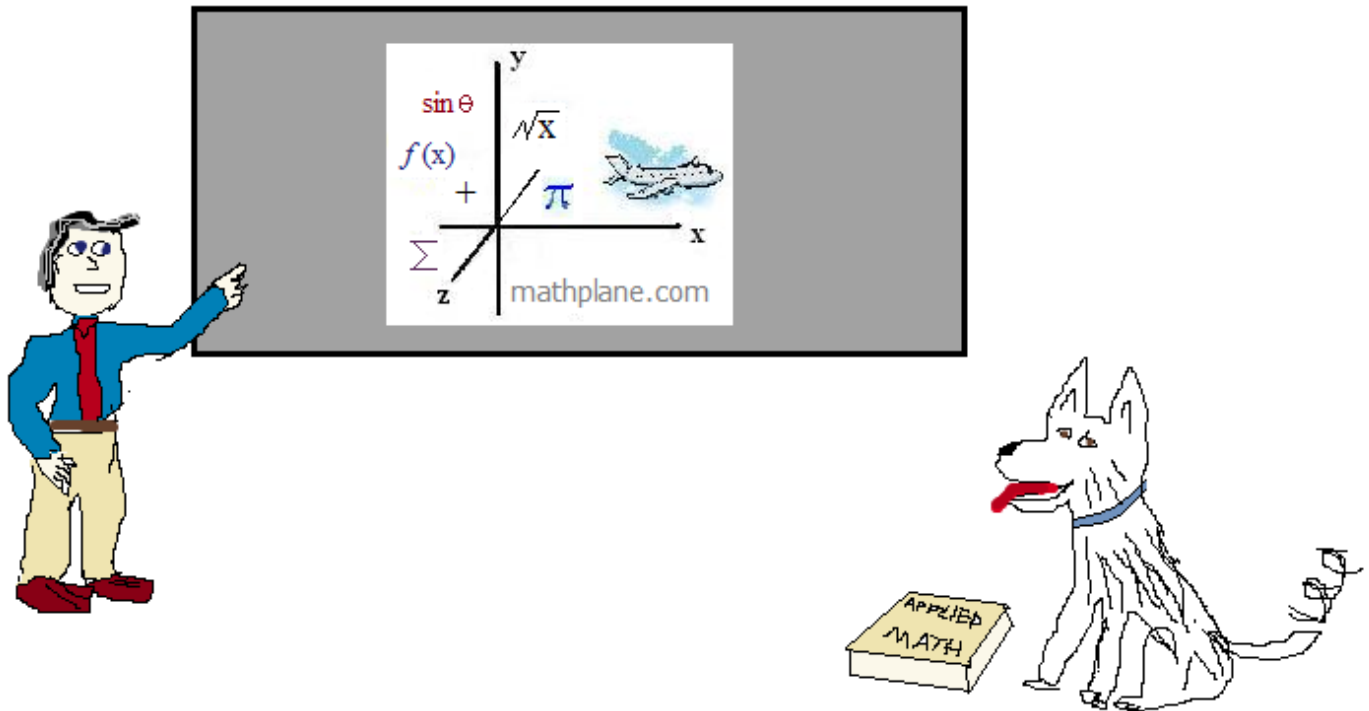
$$\frac{-m + 1}{1 - m} \leftrightarrow -\frac{m - 1}{1 - m} = 1$$

$1 = 1 \checkmark$

Thanks for visiting. (Hope it helps!)

If you have questions, suggestions, or requests, let us know.

Cheers



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