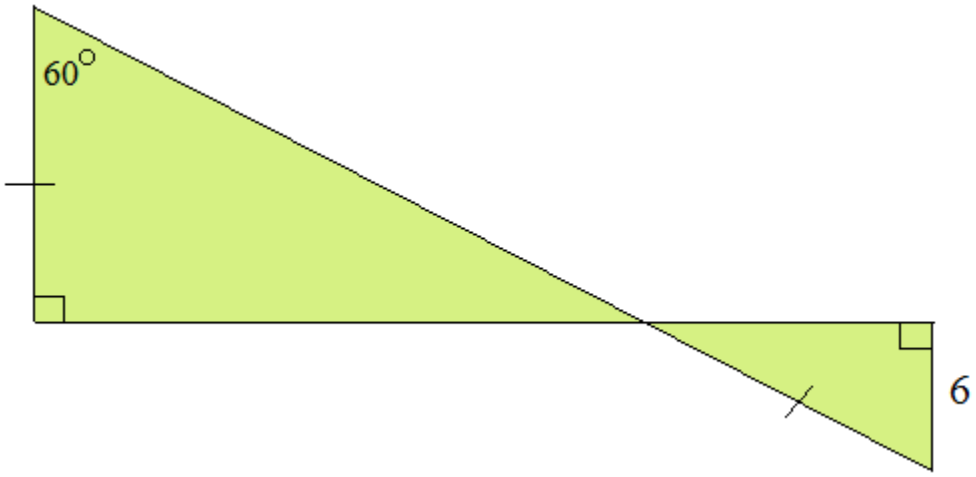
Special Right Triangles Review

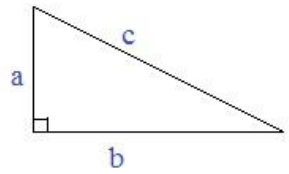
Notes, Examples, Puzzle, and Practice Quiz (with Solutions)



**What is the area inside the green triangles?

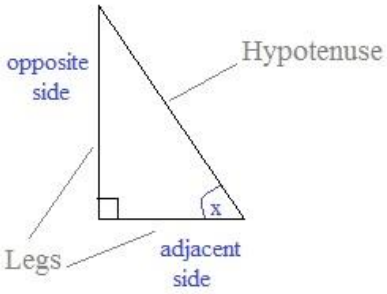
Topics include 30-60-90, 45-45-90, Pythagorean Triples, and more.

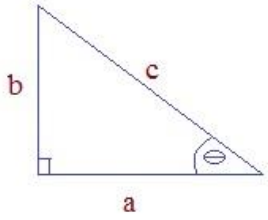
Right Triangle Review Notes:



Pythagorean Theorem

$a^2 + b^2 = c^2$





Trigonometry Relations

$\sin\ominus = \frac{b}{c}$ $\quad \csc\ominus = \frac{c}{b}$

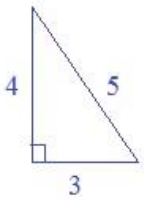
$\cos\ominus = \frac{a}{c}$ $\quad \sec\ominus = \frac{c}{a}$

$\tan\ominus = \frac{b}{a}$ $\quad \cot\ominus = \frac{a}{b}$

Utilizing the Pythagorean Theorem or Trig Identities can determine angle and side measurements of any right triangle. However, "Special Right Triangles" have features that make calculations easy!!

Special Right Triangles:

"Sides"

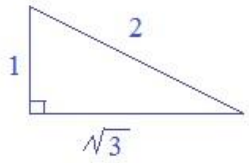


3 - 4 - 5
Right Triangle

Others include: 5 - 12 - 13
7 - 24 - 25
8 - 15 - 17

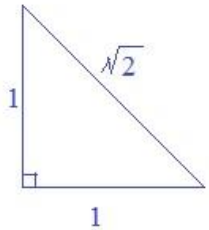
Note:
-- Pythagorean Theorem confirms
$3^2 + 4^2 = 5^2$
-- Any multiple of 3-4-5 will work!
Examples: 30-40-50 or 15-20-25

"Angles:



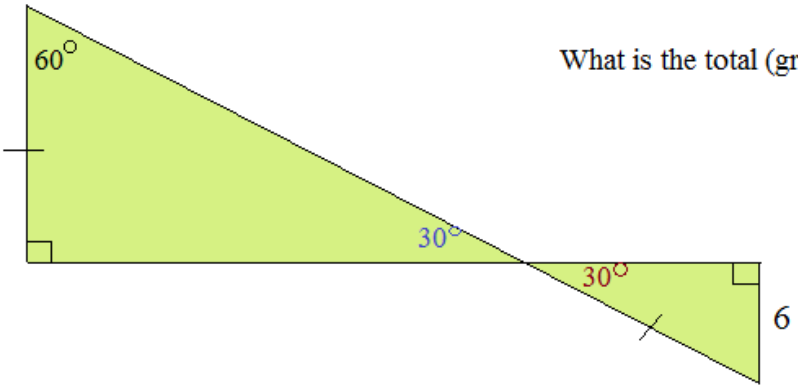
30 - 60 - 90
Right Triangle

Note:
-- Pythagorean Theorem and trig relations confirm
(ex: $\sin 30^\circ = 1/2 = .5$)
-- Any ratio of $1 - \sqrt{3} - 2$ will work.
$\longrightarrow$ $X - \sqrt{3}X - 2X$



45 - 45 - 90
Right Triangle

Note:
-- Pythagorean theorem and trig relations confirm
-- Congruent sides imply congruent (opposite) angles
-- any ratio of $1 - 1 - \sqrt{2}$ will work.
$\longrightarrow$ $X - X - \sqrt{2}X$

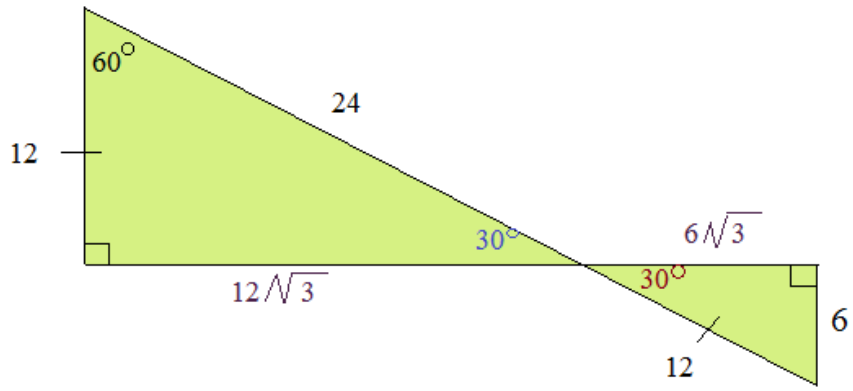


What is the total (green) area inside the 2 triangles?

Since one angle is 60 degrees, the other angle is 30 degrees.. (sum of interior angles of triangle = 180°)
30-60-90 right triangle...

Then, the small angle in the small triangle is 30 degrees. (vertical angles congruent)
So, another 30-60-90 right triangle.

In 30-60-90 right triangle, the hypotenuse = 2(small leg)



Small leg in big triangle is also 12. (congruent segments)

Ratio of sides of 30-60-90 triangles ----> x, x $\sqrt{3}$, 2x

Area of triangle: $\frac{1}{2}$ (base)(height)

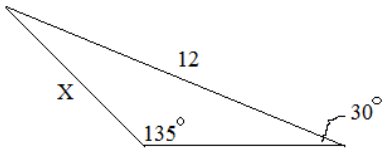
small triangle: $18\sqrt{3}$

large triangle: $72\sqrt{3}$

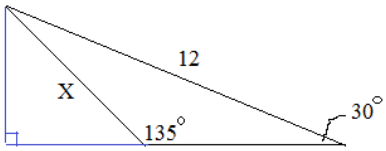
Total green area: $90\sqrt{3} \approx 155.88$

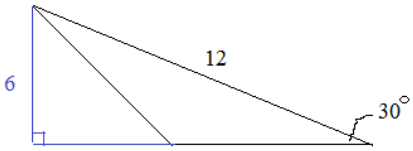
Special Right Triangle Applications

Example: Find X

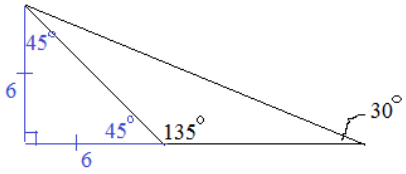


Drop an altitude, creating another triangle....





30-60-90 right triangle: small side is 1/2 of hypotenuse... therefore, side opposite 30 degree angle is 6...



(supplementary angles) We know the left triangle is a 45-45-90 right triangle: hypotenuse is leg $\cdot \sqrt{2}$

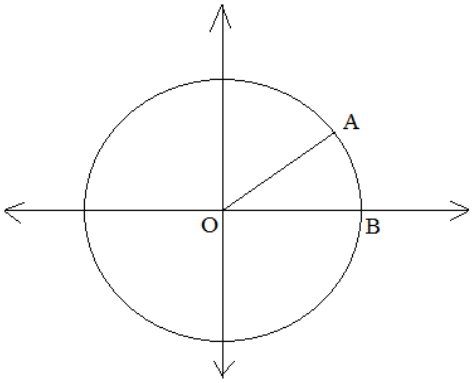
therefore, $X = 6\sqrt{2}$

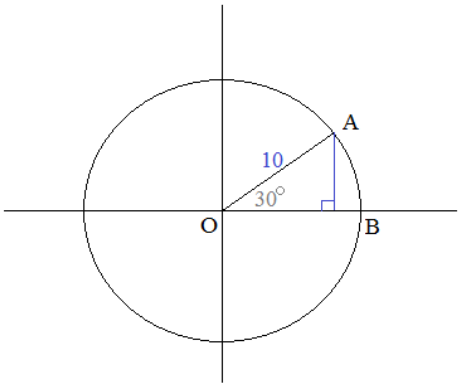
Example: Here is a circle that is centered on the origin. If the radius is 10 and $\angle AOB$ is 30°,

what is the coordinate of B?

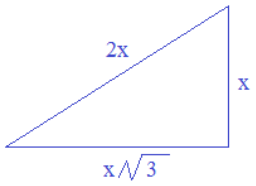
(10, 0)

what is the coordinate of A?





30-60-90 triangle



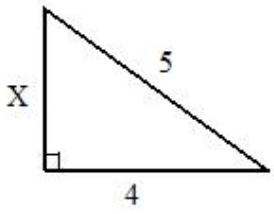
AB = 5

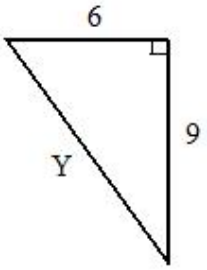
$OB = 5\sqrt{3}$

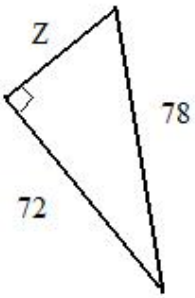
$(5\sqrt{3}, 5)$

Right Triangles: Finding sides and angles (without a calculator!)

1)







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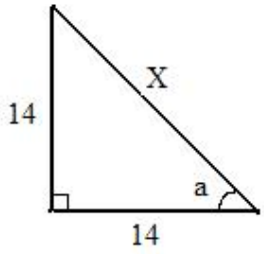

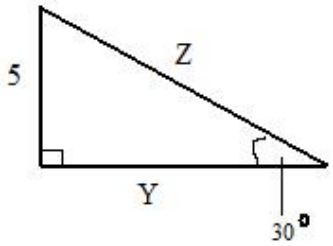

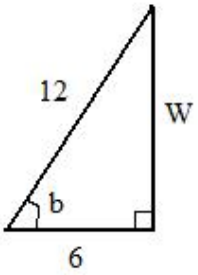

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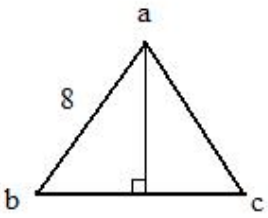
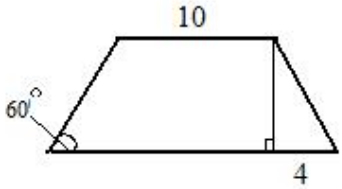
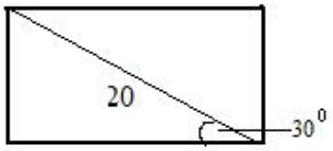


Find the altitude of equilateral △ abc



Find the area of the above isosceles trapezoid

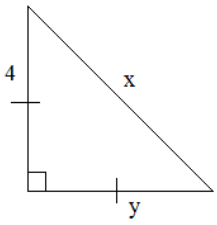


Given the diagonal is 20. Find the area of the above rectangle.

Special Right Triangles

In each triangle, find x and y. (calculator is NOT necessary)

A)



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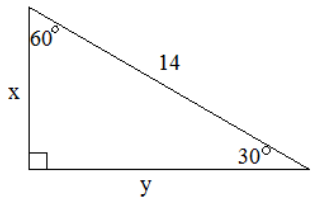
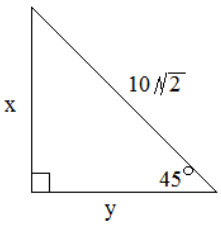
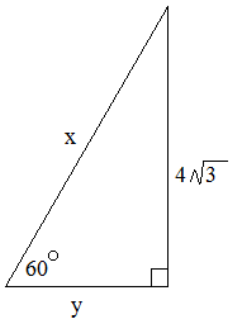


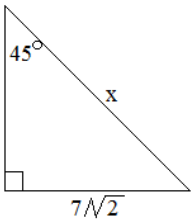
C)



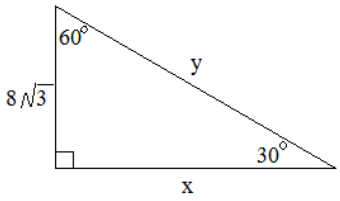
D)



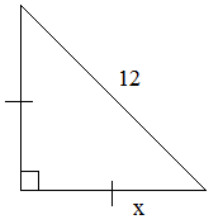
E)



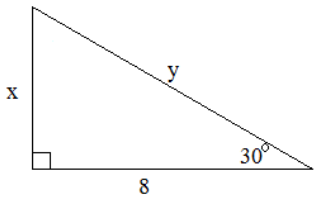
F)



G)



H)



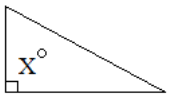
Hidden Message

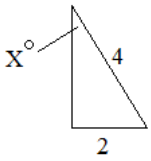
Hint: Why are most of the questions "correct"?

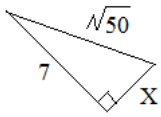
Find the measures of X. Then, convert to letters to discover the hidden message.

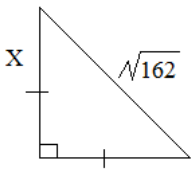
Letter Key:

0	1	2	3	4	5	6	7	8	9
A	E	G	H	I	L	N	R	S	T

1) 

2) 

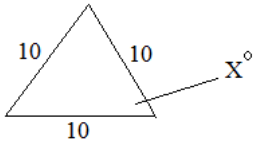
3) 

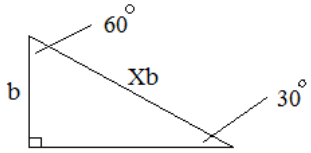
4) 

5) X−24−25 is a special one

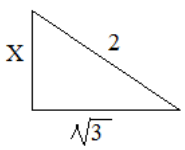
6) 3−X−5 is a special one

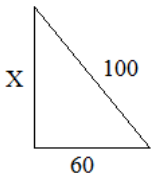
7) Number of obtuse angles in a right triangle?

8) 

9) 

10) X−12−13 is a special one

11) 

12) 

☐ 0 → ____

☐ 0 → ____

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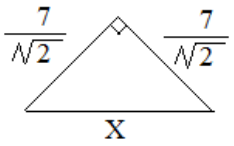
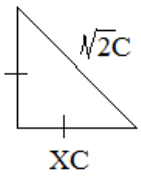
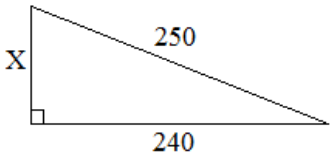
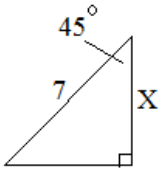
13) Number of right angles in an equilateral triangle?

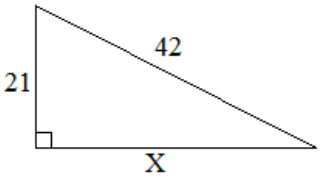
14) 

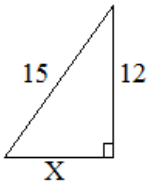
15) 

16) 

17) Number of right angles in a square?

18) 

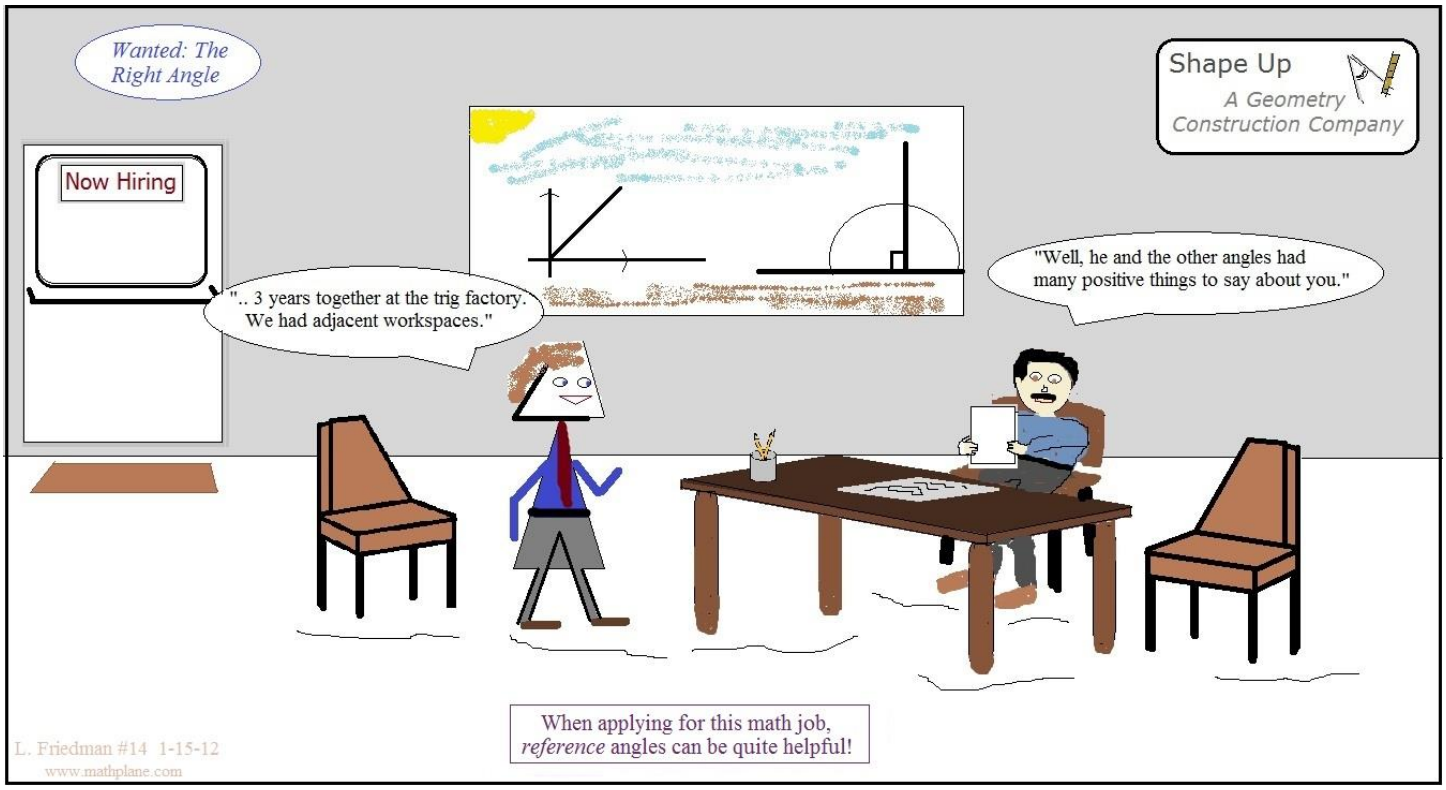
19) 

20) 

Letter Key:									
0	1	2	3	4	5	6	7	8	9
A	E	G	H	I	L	N	R	S	T

Hidden Message

Hint: Why are most of the questions "correct"?


Wanted: The Right Angle
Shape Up
A Geometry Construction Company
Now Hiring
".. 3 years together at the trig factory. We had adjacent workspaces."
"Well, he and the other angles had many positive things to say about you."
When applying for this math job, *reference* angles can be quite helpful!
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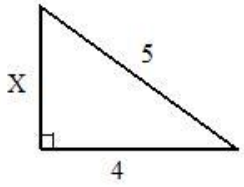
SOLUTIONS -→

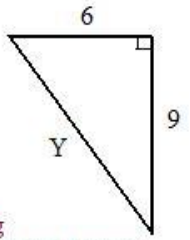
Right Triangles: Finding sides and angles (without a calculator!)

SOLUTIONS

1)

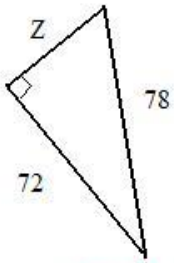


X = 3 because
3-4-5 special triangle



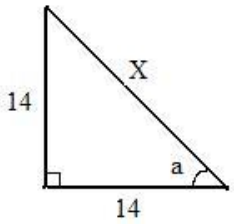
using
pythagorean theorem:

$6^2 + 9^2 = Y^2$
$36 + 81 = Y^2$
$Y = \sqrt{117}$

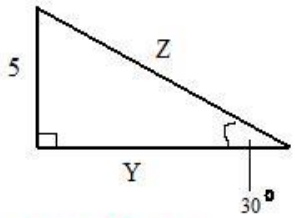


Z = 30 because 30-72-78 is
6 x (5-12-13) triangle OR,
using pythagorean theorem--
$78^2 = 72^2 + Z^2$

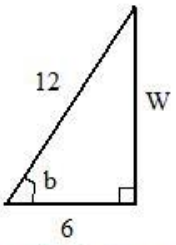
2)



45-45-90 triangle
therefore, $X = 14\sqrt{2}$
Or, $X = \sqrt{196 + 196}$
$= 14\sqrt{2}$

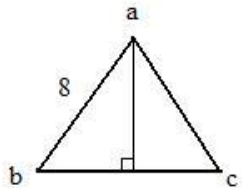


30-60-90 triangle
$Z = 5 \times 2 = 10$
$Y = 5 \times \sqrt{3}$
To check answer, try pythagorean theorem $5^2 + 5\sqrt{3}^2 = 10^2$
$100 = 100$



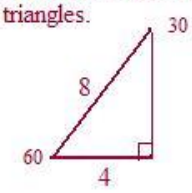
Since the hypoteneuse is 2x one of the legs, we can conclude it is a 30-60-90 triangle. Therefore, angle b is 60 degrees. And, side W is $6\sqrt{3}$

3)

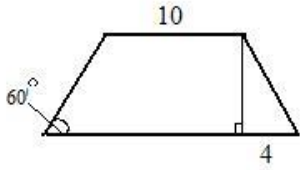


Find the altitude of
equilateral △ abc

An equilateral triangle has angles of 60-60-60.. this is helpful, since the altitude produces two 30-60-90 triangles.

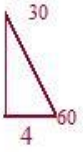


And, altitude is $4\sqrt{3}$

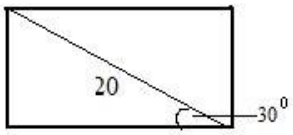


Find the area of the above
isosceles trapezoid

Area Trapezoid = $1/2 (b_1 + b_2) h$
base 1 = 10
base 2 = 4 + 10 + 4 = 18
To finde height, we consider the 30-60-90 right triangle.
---> height = $4\sqrt{3}$

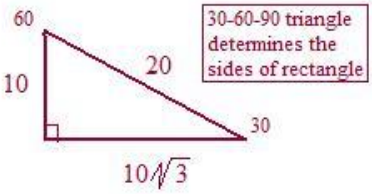


Area of Trapezoid =
$1/2 (10 + 18) 4\sqrt{3} =$
$56\sqrt{3}$



Given the diagonal is 20.
Find the area of the
above rectangle.

We know the diagonal (hypoteneuse) and the angle formed by the diagonal.



$10\sqrt{3}$

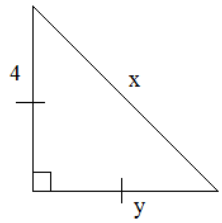
Area of rectangle is length x width =
$10 \times 10\sqrt{3} = 100\sqrt{3}$

Special Right Triangles

SOLUTIONS

In each triangle, find x and y. (calculator is NOT necessary)

A)

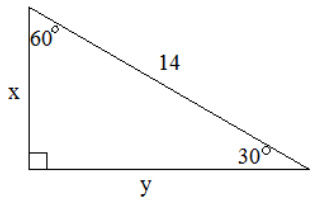


2 congruent legs, so it is a 45-45-90 right triangle...

$y = 4$

$x = 4\sqrt{2}$

B)



30-60-90 right triangle...

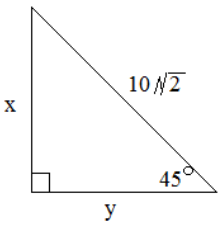
small leg is 1/2 the hypotenuse..

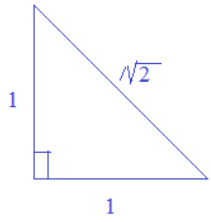
$x = 7$

medium side is small $\cdot \sqrt{3}$

$y = 7\sqrt{3}$

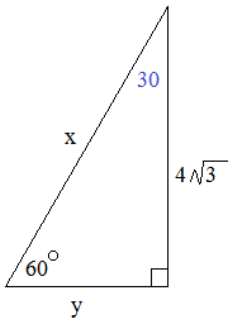
C)

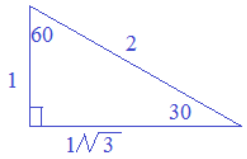




$x = 10$ $y = 10$

D)

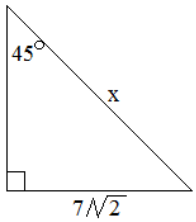




recognizing the ratios of the sides,

$y = 4$ and $x = 8$

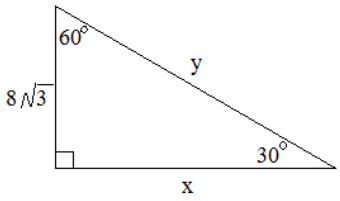
E)



$7\sqrt{2} \cdot \sqrt{2} = 14$

$x = 14$

F)



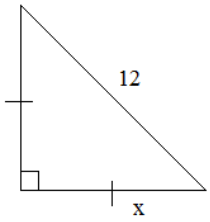
since the small leg is $8\sqrt{3}$,

the big leg is $\sqrt{3} \cdot 8\sqrt{3} = 24 = x$

and, the hypotenuse is

$2 \cdot 8\sqrt{3} = 16\sqrt{3} = y$

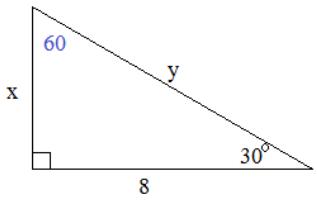
G)



$$\frac{\sqrt{2}}{1} = \frac{12}{x}$$

$$x = \frac{12}{\sqrt{2}} = 6\sqrt{2}$$

H)



$$\frac{8}{x} = \frac{\sqrt{3}}{1}$$

$\sqrt{3}\, x = 8$

$$x = \frac{8}{\sqrt{3}}$$

$$y = 2 \cdot \frac{8}{\sqrt{3}} = \frac{16}{\sqrt{3}}$$

Hidden Message

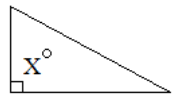
Hint: Why are most of the questions "correct"?

Find the measures of X. Then, convert to letters to discover the hidden message.

Letter Key:

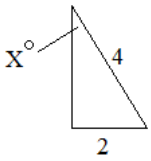
0	1	2	3	4	5	6	7	8	9
A	E	G	H	I	L	N	R	S	T

SOLUTIONS

1) 

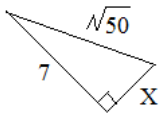
Right angles are 90 degrees.

9 0 → T

2) 

In a 30-60-90 right triangle, the length of the hypotenuse is *twice* the measure of the small side opposite the 30 degree angle.

3 0 → H

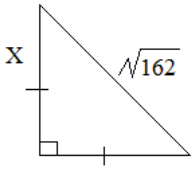
3) 

Use Pythagorean Theorem:

$X^2 + (7)^2 = (\sqrt{50})^2$

X = 1

1 → E

4) 

Since 2 sides are congruent, this is an 45-45-90 triangle.

therefore, $X = \frac{\sqrt{162}}{\sqrt{2}} = 9$

9 → T

5) X−24−25 is a special one

7 - 24 - 25

(Use Pythagorean Theorem to confirm)

7 → R

6) 3−X−5 is a special one

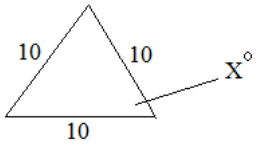
3 - 4 - 5

4 → I

7) Number of obtuse angles in a right triangle?

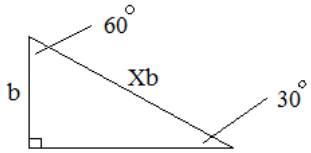
There are NO obtuse angles in a right triangle.

0 → A

8) 

All sides congruent, so it is an equilateral triangle.. And, all angles are 60 degrees.

6 0 → N

9) 

30-60-90 triangle, so hypotenuse is twice the length of the small side.

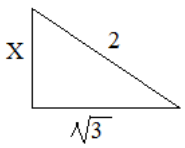
X = 2

2 → G

10) X−12−13 is a special one

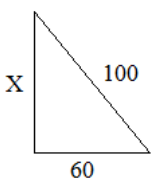
5 - 12 - 13 is a special right triangle

5 → L

11) 

X = 1
(use pythagorean thm or 30-60-90)

1 → E

12) 

3 - 4 - 5 ---> 60 - 80 -100

(multiply by 20)

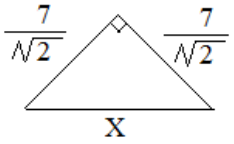
8 0 → S

SOLUTIONS

13) Number of right angles in an equilateral triangle?

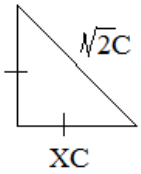
ZERO (all angles are 60 degrees)

0 → A

14) 

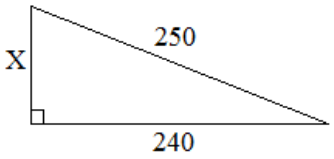
2 sides are congruent ---> 45-45-90
and, hypotenuse is $\sqrt{2}$ • (side) X = 7

7 → R

15) 

45-45-90 right triangle... X = 1

1 → E

16) 

divide by 10: X - 24 - 25
then, X would be 7

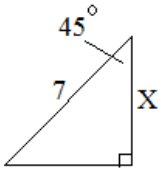
X = 70

7 0 → R

17) Number of right angles in a square?

Four right angles in a square 

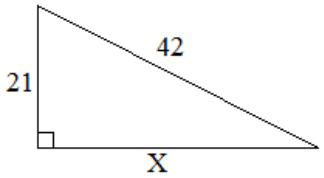
4 → I

18) 

(similar to 1 - 1 - $\sqrt{2}$)

$X = \frac{7}{\sqrt{2}}$

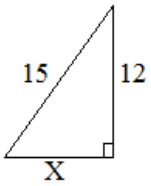
$\frac{7\sqrt{2}}{2}$ → G

19) 

Hypotenuse is 2x the small side..
therefore, X = $21\sqrt{3}$

(similar to 1-$\sqrt{3}$ - 2)

$21\sqrt{3}$ → H

20) 

Use Pythagorean Theorem
(or recognize similarity to 3-4-5)

X = 9 9-12-15

9 → T

Letter Key:									
0	1	2	3	4	5	6	7	8	9
A	E	G	H	I	L	N	R	S	T

Hidden Message

Hint: Why are most of the questions "correct"?

The triangles are "Right"

Thanks for visiting. (Hope it helped!)

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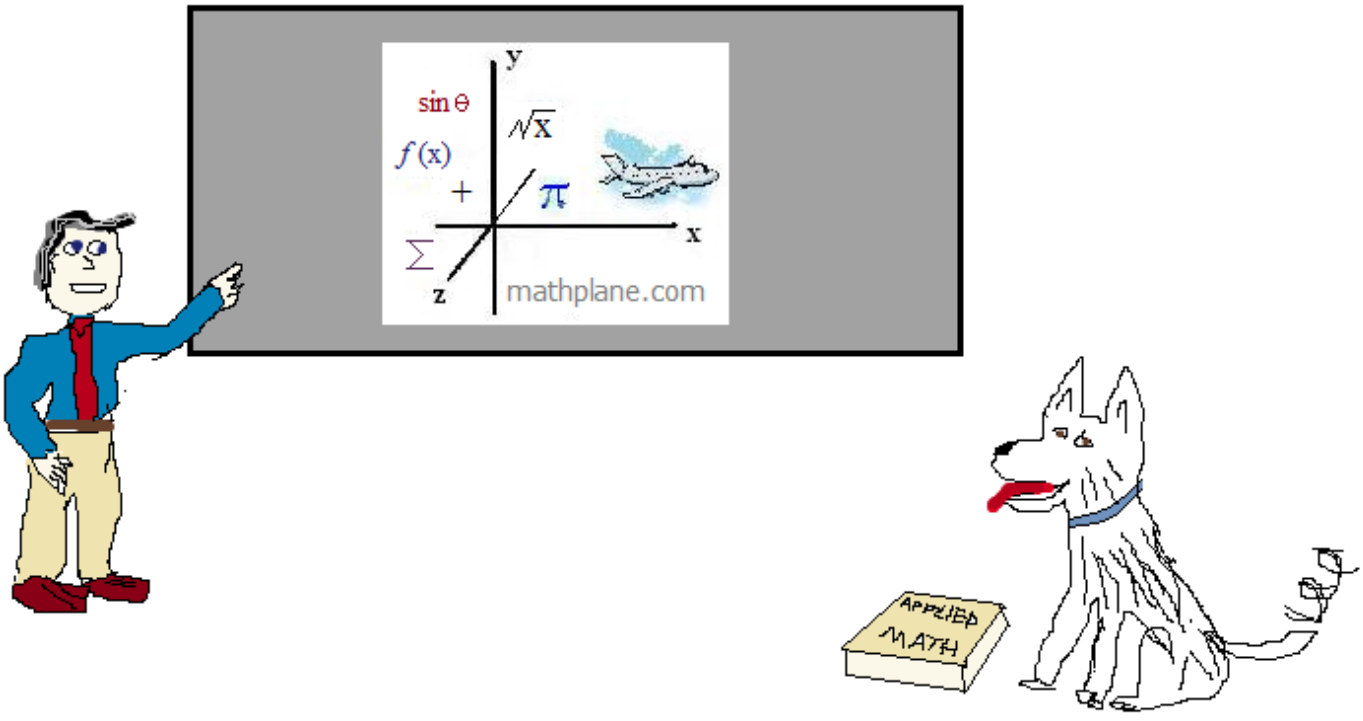
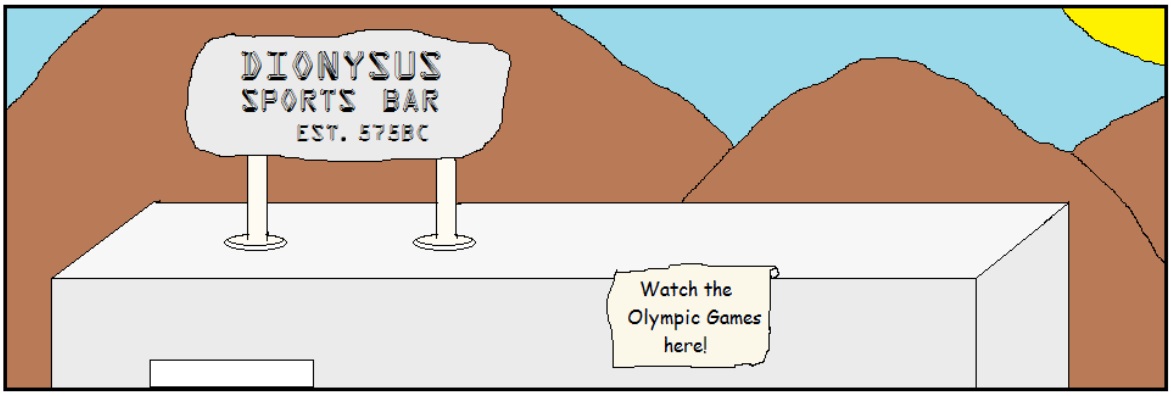
If you have questions, suggestions, or requests, let us know.

Enjoy



Also, at Facebook, Google+, and TeachersPayTeachers

Math Mixer in Ancient Greece



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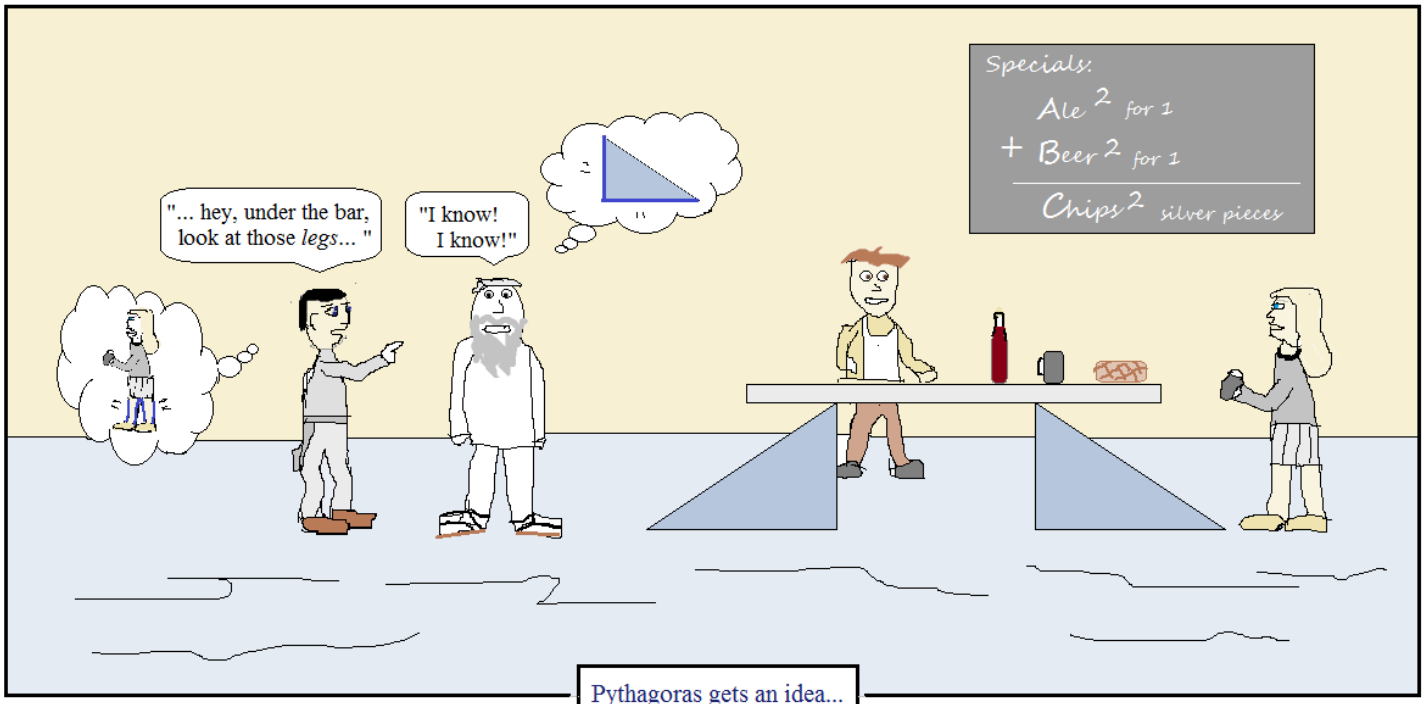

Pythagoras gets an idea...