Implicit Differentiation

Notes, examples, applications, and practice test (with solutions)

Topics include logarithms, inverse trig, tangent lines, graphing, related rates, and more.

Implicit Differentiation Notes and Examples

Explicit vs. Implicit Form:

Equations involving 2 variables are generally expressed in *explicit form*

$$y = f(x)$$

In other words, one of the two variables is explicitly given in terms of the other.

$y = 3x + 5$
$d = (.05t)^2 + 20t - 7$
$s = \sqrt{r + 1}$

(explicit form: put in the input variable, and easily get the other)

Equations where relationships are not given explicitly are in *implicit form*.

$2x - y = 4$
$xy = 1$
$x^3 + 2xy + y^2 = 0$

(implicit form: the relationship between x and y isn't easily seen)

Sometimes it is possible to change the form from implicit to explicit...

$$2x - y = 4 \longrightarrow y = 2x - 4$$

$$xy = 1 \longrightarrow y = \frac{1}{x}$$

.... But, other times it is very difficult or impossible to express in explicit form.

$$x^2 + 2xy + y^2 = 0 \longrightarrow y = ?$$

So, to find the derivative, implicit differentiation is an easier approach.

Implicit Differentiation:

Method:

1) Take derivatives
2) When taking derivative of y, insert $\frac{dy}{dx}$ (or y')
3) Solve for $\frac{dy}{dx}$ (or y')

Implicit Differentiation Example:

$$x^2 - 2y^3 + 4x = 2$$

$$2x - 6y^2\frac{dy}{dx} + 4 = 0$$

$$\frac{dy}{dx} = \frac{-2x - 4}{-6y^2}$$

$$\frac{dy}{dx} = \frac{x + 2}{3y^2}$$

Example: Find the derivative with respect to x of

$$x^2 + 2xy + y^2 = 0$$

$$2x + 2(1y + 1y'x) + 2yy' = 0$$

$$2x + 2y + 2xy' + 2yy' = 0$$

$$2x + 2y = -2xy' - 2yy'$$

$$x + y = y'(-x - y)$$

$$y' = \frac{x + y}{-(x + y)} = -1$$

Verifying Implicit Differentiation: An Example

Find the derivative of $x^2 + y^2 = 25$

Implicit Differentiation:

$x^2 + y^2 = 25$

Take derivatives, inserting y' next to derivatives of y..

$2x + 2yy' = 0$

$2yy' = -2x$

Solve for y'

$y' = \frac{-x}{y}$

> Notice that implicit differentiation used fewer steps and easier equations!

Explicit Differentiation:

$x^2 + y^2 = 25$

Change to implicit form

$y^2 = 25 - x^2$

$y = \pm\sqrt{25 - x^2}$

Find derivative (using power rule)

$y = (25 - x^2)^{\frac{1}{2}}$

$y' = \frac{1}{2}(25 - x^2)^{\frac{-1}{2}}(-2x)$

Simplify the result.

(note: we could substitute the denominator for y)

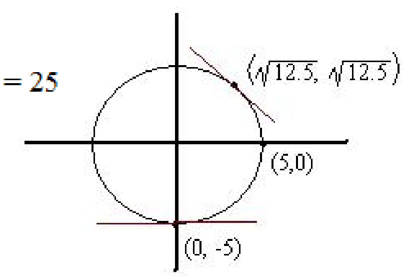
$y' = \frac{-x}{(25 - x^2)^{\frac{1}{2}}} = \frac{-x}{y}$

Then, for $y = -\sqrt{25 - x^2}$ $\quad y' = \frac{-x}{y}$

$y' = (-1)\frac{1}{2}(25 - x^2)^{\frac{-1}{2}}(-2x)$

$y' = \frac{x}{(25 - x^2)^{\frac{1}{2}}} = \frac{x}{-y}$ <u>Same solution</u>

Graph: $x^2 + y^2 = 25$



slope (rate of change) at

$(0, -5)$: $\frac{-(0)}{(5)} = 0$

$(5, 0)$: $\frac{-(5)}{(0)} =$ undefined

$(\sqrt{12.5}, \sqrt{12.5})$: $\frac{-\sqrt{12.5}}{\sqrt{12.5}} = -1$

Implicit differentiation notes and examples

Example: Given $x^3 + xy + y^2 = 8$ Find $\frac{dy}{dx}$

Derivative uses [product rule]
Inserting $\frac{dy}{dx}$ (or y')

$$3x^2 + \left[1 \cdot y + x \cdot 1\left(\frac{dy}{dx}\right)\right] + 2y\left(\frac{dy}{dx}\right) = 0$$

Simplify

$$3x^2 + y + x\left(\frac{dy}{dx}\right) + 2y\left(\frac{dy}{dx}\right) = 0$$

("Move everything without a dy/dx to the other side")

$$x\left(\frac{dy}{dx}\right) + 2y\left(\frac{dy}{dx}\right) = -3x^2 - y$$

("Factor out dy/dx")

$$\left(\frac{dy}{dx}\right)x + 2y = -3x^2 - y$$

Divide to finish

$$\left(\frac{dy}{dx}\right) = \frac{-3x^2 - y}{x + 2y}$$

Example: Given $x^3 - 2x^2y + 3xy^2 = 38$
Evaluate the derivative at (2,3)

Use implicit differentiation

$$3x^2 - 2\,[2xy + 1 \cdot y'\,x^2] + 3\,[1 \cdot y^2 + 2y\,y'\,x] = 0$$

Simplify

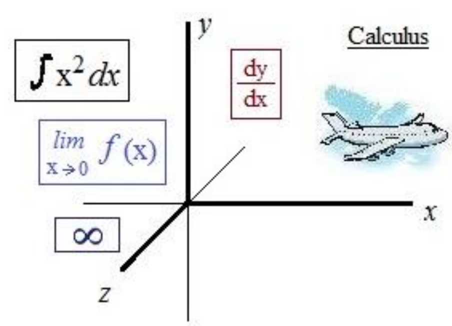
$$3x^2 - 4xy - 2y'x^2 + 3y^2 + 6y\,y'\,x = 0$$

$$6y\,y'\,x - 2y'x^2 = -3x^2 + 4xy - 3y^2$$

$$\boxed{y' = \frac{-3x^2 + 4xy - 3y^2}{6xy - 2x^2}}$$

Plug in (2, 3)

$$\frac{-3(4) + 4(6) - 3(9)}{6(6) - 2(4)} = \frac{-15}{28}$$



Implicit Differentiation and Tangent Lines

Find the equation of the line tangent to $x^2 + xy - y^2 = 1$ @ (2, 3)

To find the equation of a line, we need the slope and a point.
The point is given: (2, 3)
And, the slope is the *instantaneous rate of change (IROC)* at the given point.
Use implicit differentiation to find the *IROC*

$$2x + (1)y + x(1)\frac{dy}{dx} - 2y\frac{dy}{dx} = 0$$

$$x\frac{dy}{dx} - 2y\frac{dy}{dx} = -2x - y$$

$$\frac{dy}{dx}(x - 2y) = -2x - y$$

$$\frac{dy}{dx} = \frac{-2x - y}{(x - 2y)}$$

then, the IROC at (2, 3) is

$$\frac{-2(2) - (3)}{(2) - 2(3)} = \frac{-7}{-4}$$

So, the slope is $\frac{7}{4}$

$$(y - 3) = \frac{7}{4}(x - 2)$$

$$\boxed{y = \frac{7}{4}x - \frac{1}{2}}$$

Find the equation of the line tangent to $2xy + \pi \sin y = 2\pi$ @ $(1, \frac{\pi}{2})$

Use implicit differentiation to find the IROC, which is the slope of the tangent lines.

$$2[(1)y + x(1)\frac{dy}{dx}] + \pi(\cos y)\frac{dy}{dx} = 0$$

$$2y + 2x\frac{dy}{dx} + \pi(\cos y)\frac{dy}{dx} = 0$$

$$\frac{dy}{dx}(2x + \pi(\cos y)) = -2y$$

$$\frac{dy}{dx} = \frac{-2y}{2x + \pi(\cos y)}$$

then, the slope is

$$\frac{-2(\frac{\pi}{2})}{2(1) + \pi(\cos\frac{\pi}{2})} = \frac{-\pi}{2 + 0}$$

and, the equation of the tangent line is:

$$(y - \frac{\pi}{2}) = -\frac{\pi}{2}(x - 1)$$

$$\boxed{y = -\frac{\pi}{2}x + \pi}$$

Implicit differentiation to find slope

Example: Find slope of $x^2 + y^2 = 13$ at $(-2, 3)$ and at $(3, 2)$

Find y':

$$2x + 2yy' = 0$$

$$2yy' = -2x$$

$$y' = \frac{-2x}{2y}$$

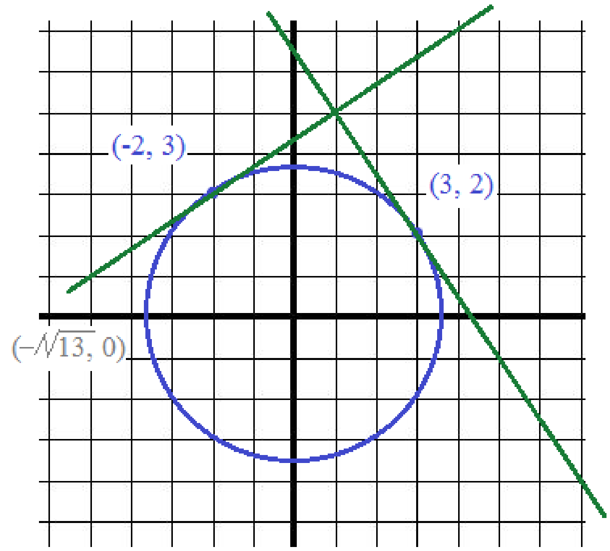
$$y' = \frac{-x}{y}$$

Then, to find slope at (-2, 3):

$$y' = \frac{-(-2)}{(3)} = \frac{2}{3}$$

at (3, 2):

$$y' = \frac{-(3)}{(2)}$$



Example: Find lines that are tangent and normal to $x^2 y^2 = 9$ at the point (-1, 3)

Utilize implicit differentiation to find y'

(product rule)

$$2xy^2 + 2yy'x^2 = 0$$

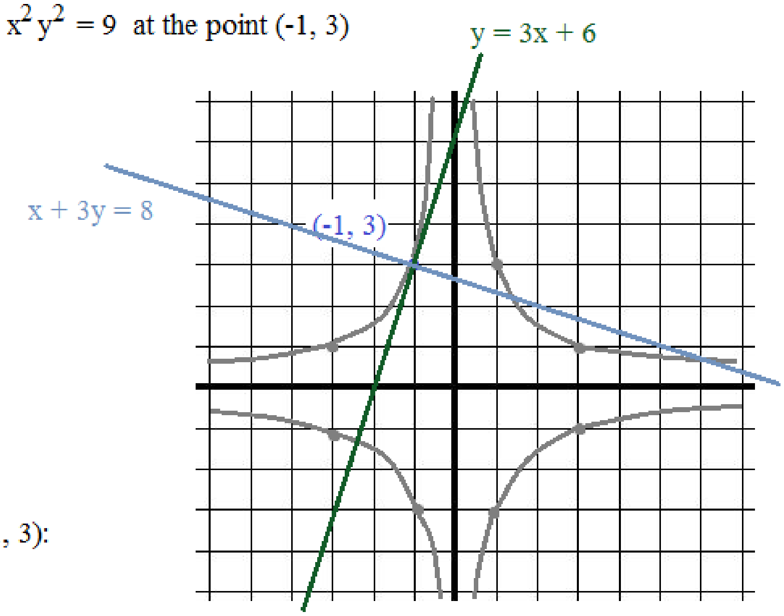
(solve for y')

$$2yy'x^2 = -2xy^2$$

$$y' = \frac{-2xy^2}{2yx^2} = \frac{-y}{x}$$

Instantaneous rate of change (dy/dx) at (-1, 3):

$$\frac{-(3)}{(-1)} = 3$$



Slope of tangent: 3 Equation of tangent line: $(y - 3) = 3(x - (-1))$

$$y = 3x + 6$$

Slope of normal: -1/3 Equation of normal line: $(y - 3) = -1/3(x - (-1))$

$$y = \frac{-1}{3}x + \frac{8}{3} \qquad x + 3y = 8$$

Example: Given the curve $x^2 - xy + y^2 = 16$

Find the coordinate(s) where the tangents are *vertical*:

Implicit Differentiation and Vertical Tangent Lines

SOLUTION:

(If a tangent line is horizontal, then the slope is 0)
If a tangent line is *vertical*, then the slope is undefined!

To find the instantaneous rate of change, find the derivative:

(implicit differentiation)

$$2x - [(1)y + x(1)\frac{dy}{dx}] + 2y\frac{dy}{dx} = 0$$

$$2x - y - x\frac{dy}{dx} + 2y\frac{dy}{dx} = 0$$

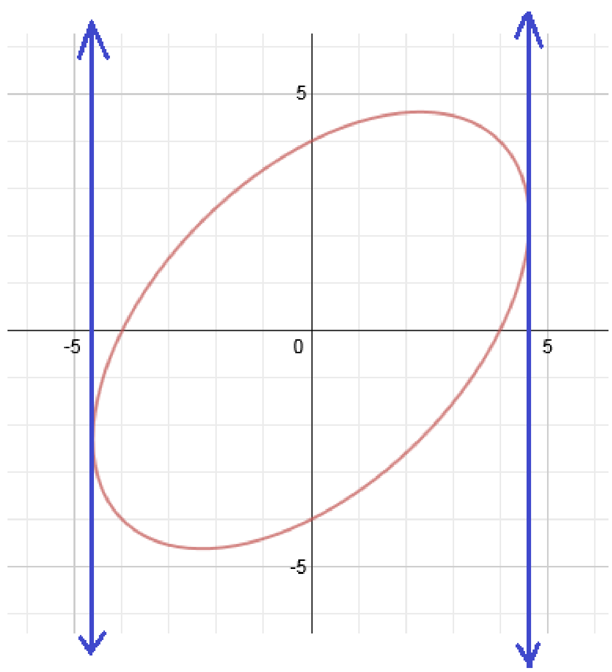
$$\frac{dy}{dx}(2y - x) = y - 2x$$

$$\frac{dy}{dx} = \frac{y-2x}{2y-x}$$

$\frac{y-2x}{2y-x}$ is undefined when the denominator = 0

$$2y - x = 0$$

$$x = 2y$$



Now, find x and y:

$x = 2y$

$x^2 - xy + y^2 = 16$

(substitution)

$$(2y)^2 - (2y)y + y^2 = 16$$

$$4y^2 - 2y^2 + y^2 = 16$$

$$3y^2 = 16$$

$$y = \pm\sqrt{\frac{16}{3}} = \text{approx. } \pm 2.31$$

then, $x = \pm 4.62$ (approx.)

$\boxed{(-4.62, -2.31) \text{ and } (4.62, 2.31)}$

Example: $x^2 - \frac{4}{y^2} = x$

Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$

Since it is a bit of effort to change equation (from implicit to explicit form), we'll use implicit differentiation.

$$x^2 - 4y^{-2} = x$$

$$2x - -8y^{-3}\frac{dy}{dx} = 1$$

$$\frac{8}{y^3}\frac{dy}{dx} = 1 - 2x$$

$$\boxed{\frac{dy}{dx} = \frac{y^3(1-2x)}{8}}$$

1st derivative

$$\frac{dy}{dx} = \frac{1}{8}y^3 - \frac{1}{4}xy^3$$

product rule

$$\frac{d^2y}{dx^2} = \frac{3}{8}y^2\frac{dy}{dx} - \frac{1}{4}\left(y^3 + x\cdot 3y^2\frac{dy}{dx}\right)$$

then, substitute $\frac{dy}{dx}$

$$\frac{d^2y}{dx^2} = \frac{3}{8}y^2\left(\frac{y^3(1-2x)}{8}\right) - \frac{1}{4}\left(y^3 + x\cdot 3y^2\cdot\frac{y^3(1-2x)}{8}\right)$$

$$= \frac{3y^5(1-2x)}{64} - \frac{y^3}{4} - \frac{3xy^5(1-2x)}{32}$$

2nd derivative

$$= \frac{3y^5 - 6xy^5 - 16y^3 - 6xy^5 + 12x^2y^5}{64} = \boxed{\frac{3y^5 - 12xy^5 - 16y^3 + 12x^2y^5}{64}}$$

Example: $y = x^{3/5}$ Find the 1st and 2nd derivatives.

Then, use implicit differentiation to verify these 1st and 2nd derivatives: $y^5 = x^3$

First derivative (using the power rule)

$$\frac{dy}{dx} = (3/5)x^{-2/5}$$

Second derivative

$$\frac{d^2y}{dx^2} = (-6/25)x^{-7/5}$$

$$5y^4\frac{dy}{dx} = 3x^2$$

$$\frac{dy}{dx} = \frac{3}{5}\frac{x^2}{y^4}$$

since $y = x^{\frac{3}{5}}$

then $y^4 = x^{\frac{12}{5}}$

$$\boxed{\frac{dy}{dx} = \frac{3}{5}\frac{x^2}{x^{\frac{12}{5}}}}$$

First derivative

We continue to find the 2nd derivative.....

$$\frac{dy}{dx} = \frac{3}{5}\frac{x^2}{y^4} = \boxed{\frac{3}{5}x^2y^{-4}}$$

product rule

$$\frac{d^2y}{dx^2} = \frac{3}{5}\left(2xy^{-4} + -4y^{-5}\frac{dy}{dx}x^2\right)$$

substitute $\frac{dy}{dx}$

$$\frac{d^2y}{dx^2} = \frac{3}{5}\left(\frac{2x}{y^4} - \frac{4x^2}{y^5}\cdot\frac{3}{5}x^2y^{-4}\right)$$

$$= \frac{3}{5}\left(\frac{2xy - 4x^2\frac{3x^2}{5y^4}}{y^5}\right)$$

$$= \frac{3}{5}\left(\frac{2xy - \frac{12x^4}{5y^4}}{y^5}\right)$$

$$\frac{d^2y}{dx^2} = \frac{3}{5}\left(\frac{\frac{10xy^5}{5y^4} - \frac{12x^4}{5y^4}}{y^5}\right)$$

$$= \frac{3}{5}\left(\frac{10xy^5 - 12x^4}{5y^9}\right)$$

$$= \frac{6}{25}x\,\frac{5y^5 - 6x^3}{y^9}$$

Note: $x^3 = y^5$

$y = x^{3/5}$

$$= \frac{6}{25}x\,\frac{5y^5 - 6y^5}{y^9} = -\frac{6}{25}x\,\frac{y^5}{y^9}$$

Second derivative

$$= -\frac{6}{25}x\cdot x^{-12/5} \quad \boxed{= -\frac{6}{25}x^{-7/5}}$$

Inverse

Implicit Differentiation Notes and Examples

Let's compare y' vs x'

derivative of y with respect to x $\frac{dy}{dx}$ vs $\frac{dx}{dy}$ derivative of x with respect to y

Given: $x^3 - xy + y^2 = 4$

Find y' or $\frac{dy}{dx}$

$$3x^2 - [\,(1)y + x(1)\,y'\,] + 2y\,y' = 0$$

$$3x^2 - y - x\,y' + 2y\,y' = 0$$

$$-x\,y' + 2y\,y' = -3x^2 + y$$

$$y'\,(-x + 2y) = -3x^2 + y$$

$$y' = \frac{-3x^2 + y}{(-x + 2y)}$$

Find x' or $\frac{dx}{dy}$

"To find dx/dy, we insert x' whenever taking the derivative of x"

$$3x^2 x' - [\,(1)x'\,y + x(1)\,] + 2y = 0$$

$$3x^2 x' - x'\,y - x + 2y = 0$$

$$3x^2 x' - x'\,y = x - 2y$$

$$x'\,(3x^2 - y) = x - 2y$$

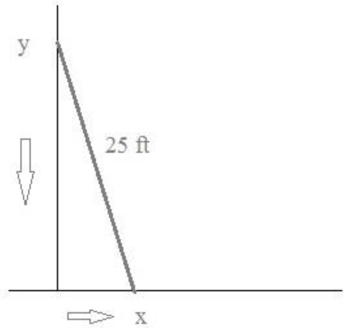
$$x' = \frac{x - 2y}{(3x^2 - y)} = \frac{-x + 2y}{-3x^2 + y}$$

Reciprocals

Implicit Differentiation: Word Problem Examples

1) A 25-foot ladder is leaning against a wall. If the top of the ladder is slipping down the wall at a rate of 2 feet/second, how fast will the bottom be moving away from the wall when the top is 20 feet above the ground?

Step 1: Draw diagram, list variables and formulas



length to bottom of ladder = x
length to top of ladder = y

$x^2 + y^2 = 625 \text{ ft}^2$ (pythagorean theorem)

down the wall at a rate of 2 ft/sec $\quad \frac{dy}{dt} = -2$ ft/sec

(change of y *with respect to time*)

moving away from the wall $\quad \frac{dx}{dt} = ?$

(change of x *with respect to time*)

Important note: <u>we're seeking dx/dt,</u> (the change of x *with respect to time*)..
Simply taking the derivative of $y = \sqrt{625 - x^2}$

$1/2\,(625 - x^2)^{(-1/2)} \cdot (-2x) = \dfrac{-x}{\sqrt{(625 - x^2)}}$

shows us dy/dx, (the change in *y with respect to x*)

Step 2: Set up equation and use implicit differentiation.

$$x^2 + y^2 = 625 \text{ ft}^2$$

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

<u>derivative with respect to time</u>

Substitute and solve:

$$2x\frac{dx}{dt} + 2(20 \text{ ft})(-2 \text{ ft/sec}) = 0$$

$$\boxed{\begin{array}{l} x^2 + y^2 = 625 \text{ ft}^2 \\ (x)^2 + (20 \text{ ft})^2 = 625 \text{ ft}^2 \\ \text{When } y = 20 \text{ ft, } x = 15 \text{ feet} \end{array}}$$

$$2(15 \text{ ft})\frac{dx}{dt} + (-80 \text{ ft}^2/\text{sec}) = 0$$

$$30 \text{ ft}\frac{dx}{dt} = 80 \text{ ft}^2/\text{sec}$$

$$\frac{dx}{dt} = \frac{80 \text{ ft}^{\cancel{2}}/\text{sec}}{30 \cancel{\text{ft}}} = \boxed{2.67 \text{ ft/sec}}$$

Using explicit differentiation & chain rule

$$x = \sqrt{625 - y^2}$$

$$\frac{dx}{dy} = 1/2\,(625 - y^2)^{(-1/2)} \cdot (-2y) = \frac{-y}{\sqrt{(625 - y^2)}}$$

$$\frac{dx}{dt} = \frac{dy}{dt} \cdot \frac{dx}{dy}$$

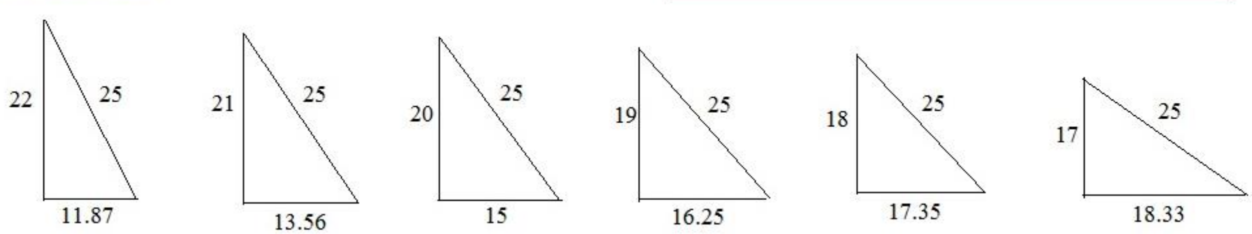
$$-2\,\frac{\text{feet}}{\text{sec}} \cdot \frac{-y}{\sqrt{625 \text{ ft}^2 - y^2}}$$

If y = 20 feet, then

$$\frac{dx}{dt} = -2\,\frac{\text{feet}}{\text{sec}} \cdot \frac{-(20 \text{ feet})}{\sqrt{625 \text{ ft}^2 - 400 \text{ ft}^2}}$$

$$= -2\,\frac{\text{feet}}{\text{sec}} \cdot \frac{-20 \cancel{\text{feet}}}{15 \cancel{\text{feet}}} = \boxed{2.67 \text{ ft/sec}}$$

Step 3: Check answer



From 22 to 20 feet (one second), the ladder moved out 3.13 feet

From 21 to 19 feet (one second), the ladder moved out 2.69 feet...

From 20 to 18 feet (one second) the ladder moved 2.35 feet...

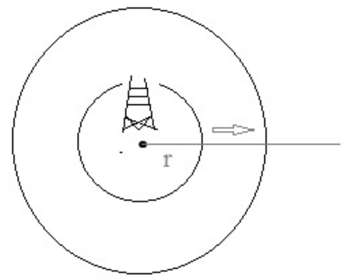
2.67 feet per second is a reasonable answer! ✓

Implicit Differentiation: Word Problem Examples (continued)

2) Oil erupts from a ruptured tanker, spreading in a circle whose area increases at a constant rate of 6 square miles per hour. How fast is the radius of the spill increasing when the area is 9π square miles?

Step 1: Draw a diagram, list variables, and consider formulas

spill area $A = \pi r^2$



"area is increasing at a rate of 6 square miles per hour" $\quad \frac{dA}{dt} = 6 \frac{\text{miles}^2}{\text{hour}}$

"how fast is the radius of the spill increasing?" $\quad \frac{dr}{dt} = ?$

When area is 9π sq. miles, the radius is 3 miles.

Step 2: Implicit differentiation

Take derivative *with respect to t*

$$A = \pi r^2$$

$$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$$

Plug in values and solve

$$6 \frac{\text{miles}^2}{\text{hour}} = 2\pi (3 \text{ miles}) \frac{dr}{dt}$$

$$\frac{dr}{dt} = \frac{6 \text{ miles}^{\cancel{2}}}{\text{hour} \cdot 2\pi (3 \cancel{\text{miles}})} = \boxed{\frac{1}{\pi} \text{ miles/hour}}$$

$\boxed{\text{(or, .318 miles/hour)}}$

When area of spill is 9π square miles, the radius is increasing at .318 miles per hour.

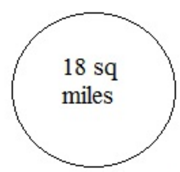
Step 3: Verify Answer

$A = (3.14)r^2$

$12 = (3.14)r^2$

$r = 1.95$ miles

(one hour later)



$18 = (3.14)r^2$

$r = 2.39$ miles

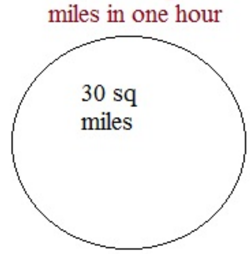
(one hour later)

$24 = (3.14)r^2$

$r = 2.76$ miles

radius changed .33 miles in one hour

(one hour later)



$30 = (3.14)r^2$

$r = 3.09$ miles

9π sq. miles (28.27 sq miles) $\xrightarrow{\text{(one hour)}}$ 9π sq. miles + 6 sq. miles = 34.27 sq. miles

$r = 3$ miles $\qquad r = 3.30$

radius changed .30 miles in one hour

Find dy/dx if $y = x^x$

Answer: $\ln y = x\ln(x)$ logarithm power rule

dy/dx:

$$\frac{1}{y}\frac{dy}{dx} = (1)\ln x + x\frac{1}{x}$$

$$\frac{1}{y}\frac{dy}{dx} = \ln x + 1$$

$$\frac{dy}{dx} = y(\ln x + 1)$$

$$= x^x(\ln x + 1)$$

Implicit Differentiation & Inverse Trig Derivatives

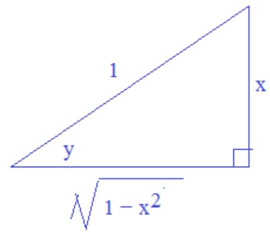
Example: $y = \sin^{-1} x$ What is $\dfrac{dy}{dx}$?

Step 1: Change the inverse trig term

$$\sin y = \sin\left(\sin^{-1} x\right)$$

$$\sin(y) = x$$

Step 2: "Draw the triangle"



$$\text{Sine } y = \frac{\text{opposite}}{\text{hypotenuse}}$$

Pythagorean Theorem

$$a^2 + b^2 = c^2$$

Step 3: Use implicit diffentiation to find *dy*/*dx*

$$\cos(y)\frac{dy}{dx} = 1$$

$$\frac{dy}{dx} = \frac{1}{\cos(y)} \implies \text{using the triangle, } \frac{dy}{dx} = \frac{1}{\sqrt{1 - x^2}}$$

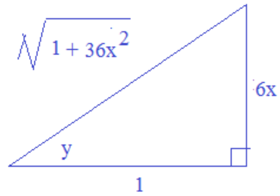
Example: $y = \tan^{-1}(6x)$ Find the derivative.

Step 1: Change the inverse trig term

$$\tan y = \tan\left(\tan^{-1}(6x)\right)$$

$$\tan(y) = 6x$$

Step 2: "Draw the triangle"



$$\text{Tan } y = \frac{\text{opposite}}{\text{adjacent}}$$

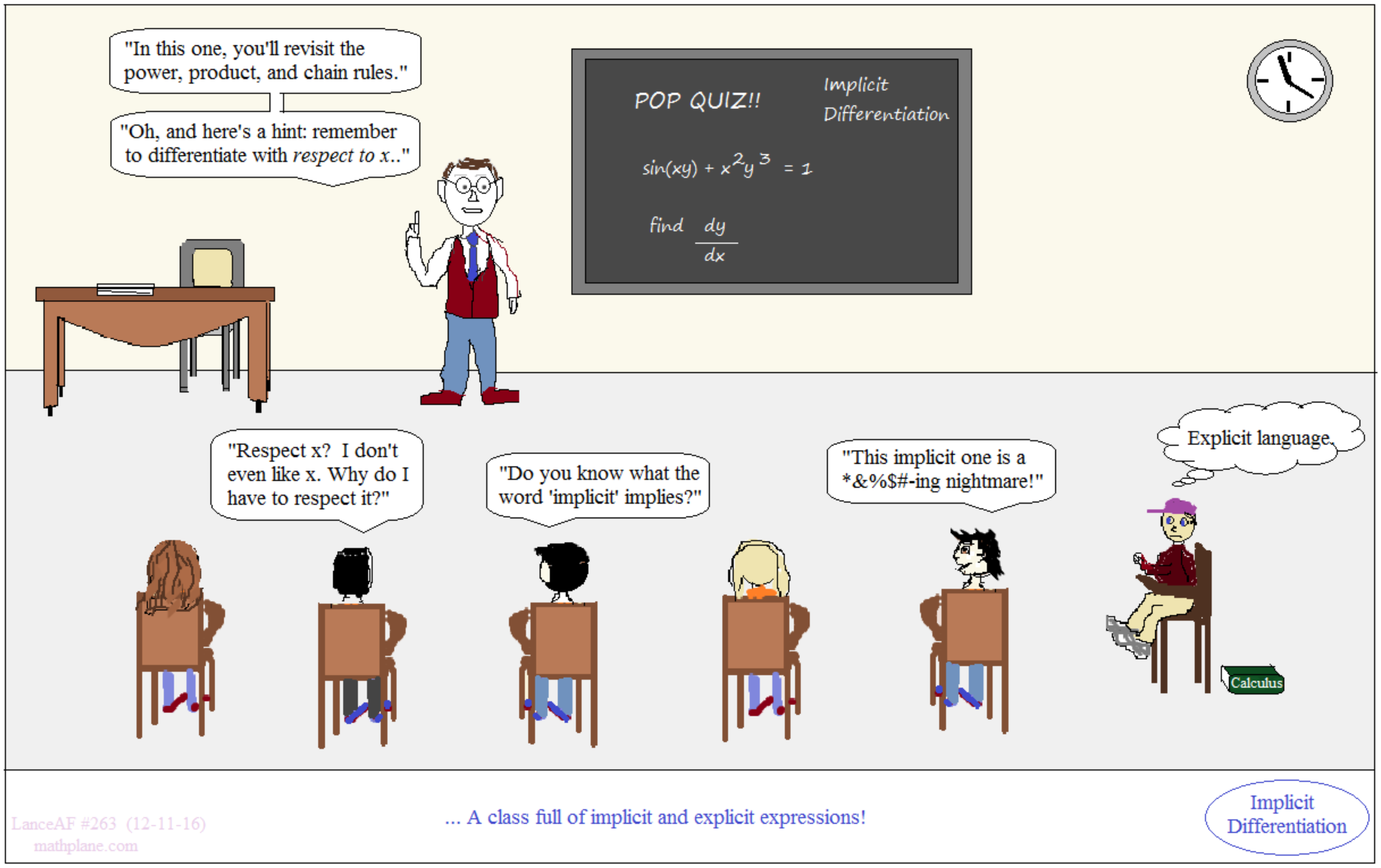
Pythagorean Theorem

$$a^2 + b^2 = c^2$$

Step 3: Use implicit diffentiation to find *dy*/*dx*

$$\sec^2(y)\frac{dy}{dx} = 6$$

$$\frac{dy}{dx} = \frac{6}{\sec^2(y)} \implies \text{using the triangle, } \frac{dy}{dx} = \frac{6}{\left(\sqrt{1 + 36x^2}\right)^2} = \frac{6}{1 + 36x^2}$$



Practice Quiz-→

Implicit Differentiation

1) If $y = \sin(x)\cos(y)$, then @ $(\pi, 0)$ $\frac{dy}{dx} =$

a) -1

b) 0

c) 1

d) π

e) 2π

2) If $x^2 + 2y^2 = 22$, what is the behavior of the graph at $(-2, 3)$

a) increasing, concave up

b) increasing, concave down

c) decreasing, concave up

d) decreasing, concave down

e) increasing, point of inflection

3) A plane flies 6 miles high above the ground.
At the moment, it is directly 10 miles from an airport tower, and it is approaching the tower at a rate of 400 miles per hour.

How fast is the plane traveling?

Implicit Differentiation

SOLUTIONS

Use implicit differentiation to find the derivative (instantaneous rate of change)

1) If $y = \sin(x)\cos(y)$, then @ $(\pi, 0)$ $\frac{dy}{dx} =$

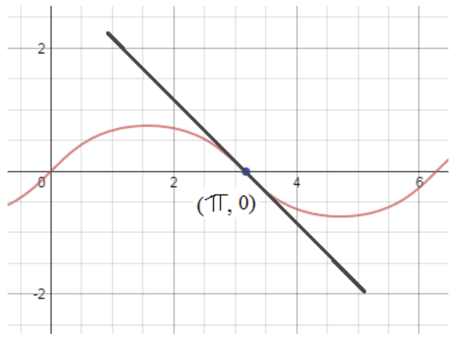
$\boxed{\text{a) } -1}$

b) 0

c) 1

d) π

e) 2π



product rule

$$1 \cdot \frac{dy}{dx} = \cos(x)\cos(y) + \left(-\sin(y)\frac{dy}{dx}\right)\sin(x)$$

to find IROC at point, substitute $(\pi, 0)$

$$\frac{dy}{dx} = \cos(\pi)\cos(0) - \sin(0)\frac{dy}{dx} \cdot \sin(\pi)$$

$$= (-1)(1) + (0)\frac{dy}{dx}(0) = \boxed{-1}$$

2) If $x^2 + 2y^2 = 22$, what is the behavior of the graph at $(-2, 3)$

a) increasing, concave up

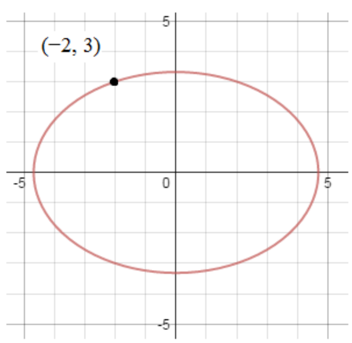
$\boxed{\text{b) increasing, concave down}}$

c) decreasing, concave up

d) decreasing, concave down

e) increasing, point of inflection

Notice, the graph is an ellipse!



To determine increasing or decreasing, find first derivative...

$$2x + 4y\frac{dy}{dx} = 0$$

$$4y\frac{dy}{dx} = -2x \qquad \text{at } (-2, 3) \quad \frac{dy}{dx} = \frac{-(-2)}{2(3)} = \frac{1}{3} > 0$$

$$\frac{dy}{dx} = \frac{-x}{2y}$$

increasing...

To determine concavity, find second derivative...

$$\frac{dy}{dx} = \frac{-x}{2y}$$

Quotient Rule

$$\frac{d}{dx} \cdot \frac{dy}{dx} \longrightarrow \frac{d^2y}{dx^2} = \frac{-1(2y) - 2\frac{dy}{dx}(-x)}{(2y)^2} = \frac{-2y + 2x\frac{dy}{dx}}{4y^2}$$

$$\frac{-y + x\frac{dy}{dx}}{2y^2} = \frac{-y + x\left(\frac{-x}{2y}\right)}{2y^2} \qquad \text{at } (-2, 3) \quad \frac{d^2y}{dx^2} = \frac{-(3) + (-2)\frac{1}{3}}{2(3)^2}$$

$$= \frac{-11/3}{18} < 0$$

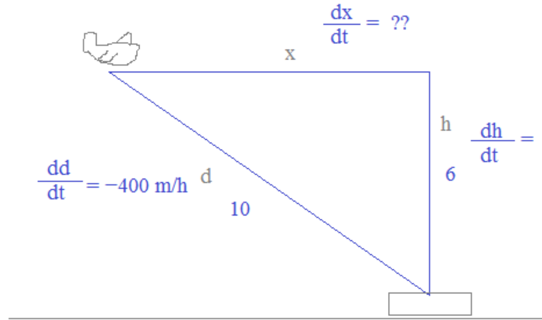
concave down...

3) A plane flies 6 miles high above the ground.
At the moment, it is directly 10 miles from an airport tower, and it is approaching the tower at a rate of 400 miles per hour.

How fast is the plane traveling?

x = horizontal distance
d = direct distance
h = height above ground

$\frac{dx}{dt} = ??$



$\frac{dh}{dt} = 0$

$\frac{dd}{dt} = -400$ m/h

$$\boxed{\frac{dx}{dt} = 500 \text{ m/h}}$$

$$x^2 + h^2 = d^2$$

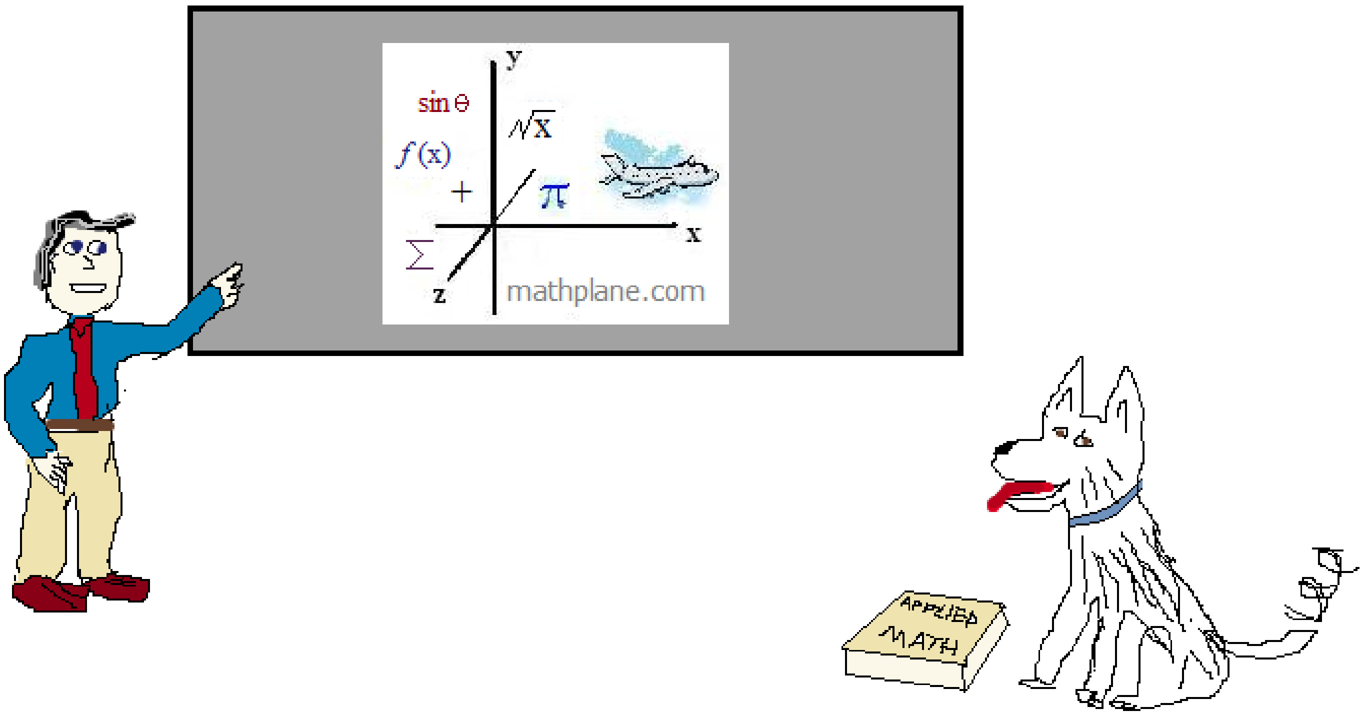
$$2x\frac{dx}{dt} + 2h\frac{dh}{dt} = 2d\frac{dd}{dt}$$

$$16\frac{dx}{dt} + 12(0) = 20(-400)$$

Thanks for visiting. (Hope it helped!)

If you have questions, suggestions, or requests, let us know.

Cheers



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