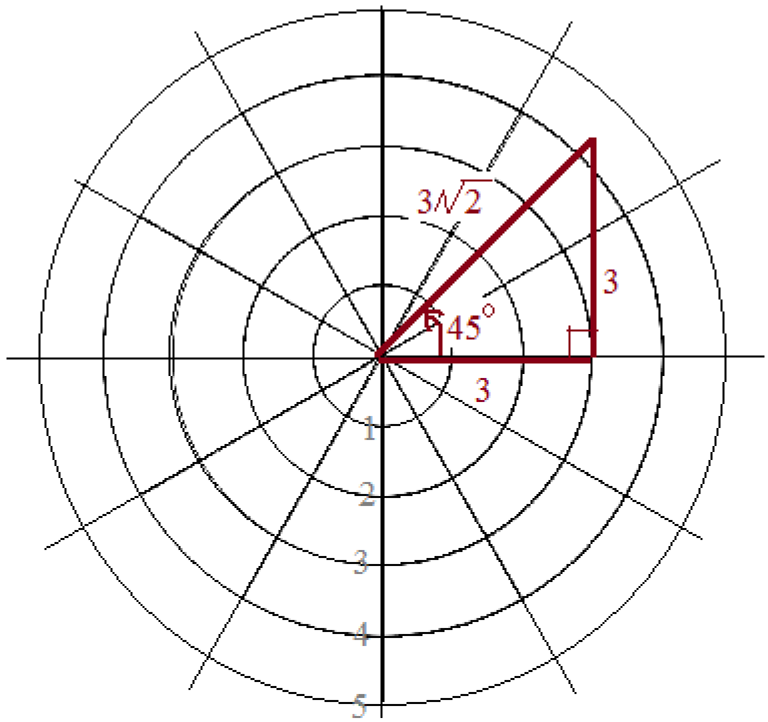
Algebra II/Trigonometry

Working with Polar and Rectangular Coordinates

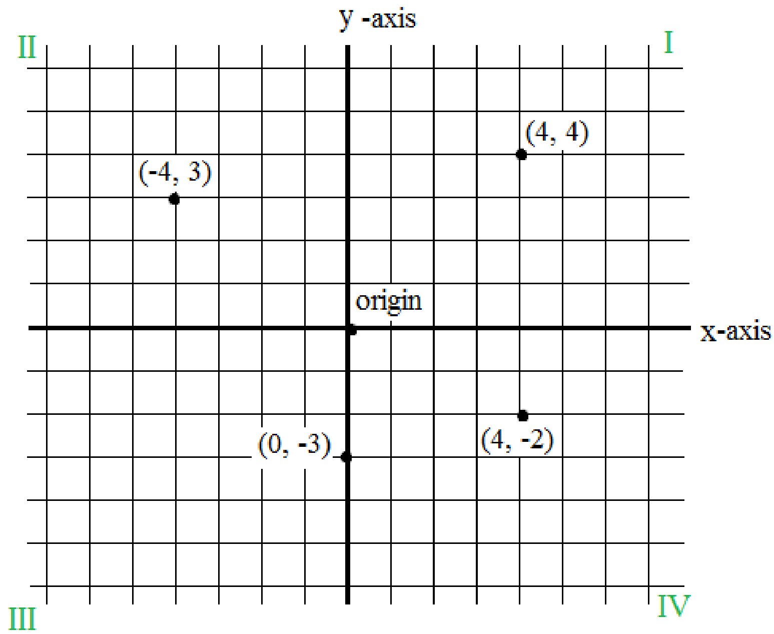


Brief Notes, Examples, and Practice Quiz (and Solutions)

Mathplane.com

Different Planes can be a Pain!

The Cartesian Plane

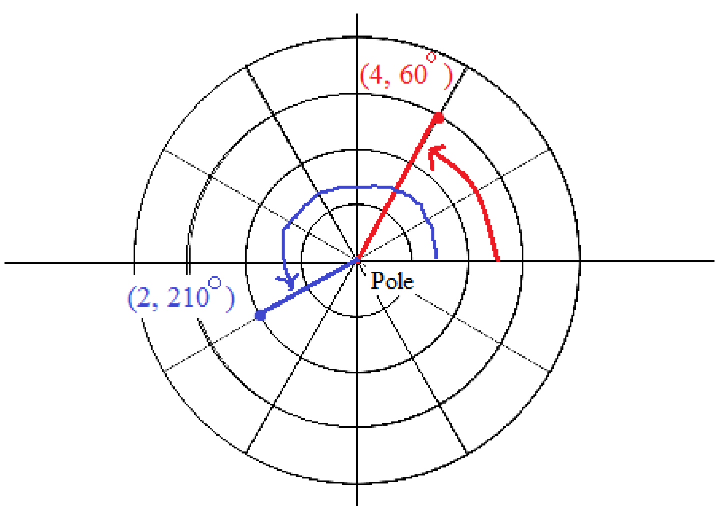


Origin: (0, 0)

Quadrants I, II, III, and IV

Points: (x, y)

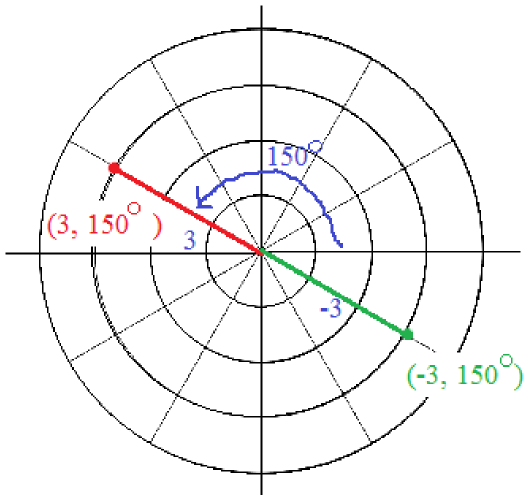
Polar Coordinate System (Plane)



"Origin" or "Pole" : $(0, \theta)$

Points: (r, θ)

Polar Coordinate System (continued)

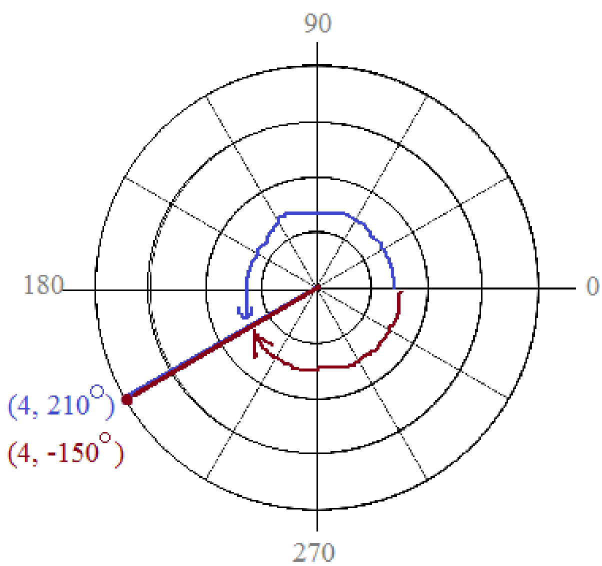


Note the difference!

(3, 150°)

vs.

(-3, 150°)



Note the similarity!

(4, 210°)

and

(4, -150°)

Note: Consider all the coterminal angles and (-r)

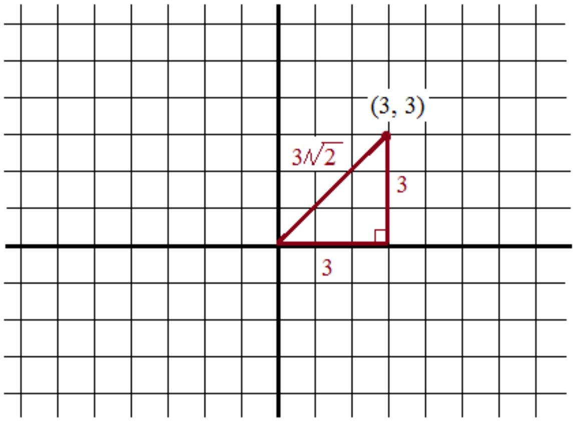
Example: (4, 210°)

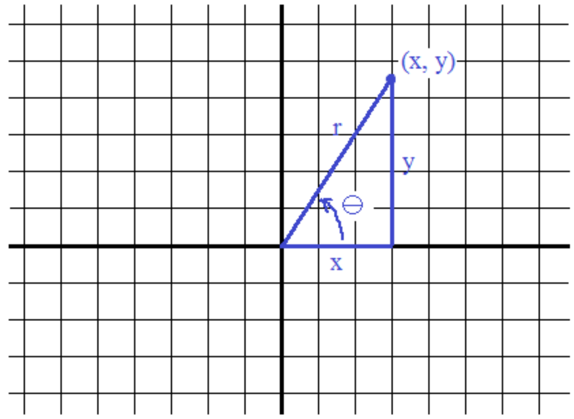
$(4, 210^\circ) = (4, 360^\circ n + 210^\circ)$

$= (-4, 360^\circ n + 30^\circ)$

(n is any integer)

Comparing Rectangular and Polar Coordinates



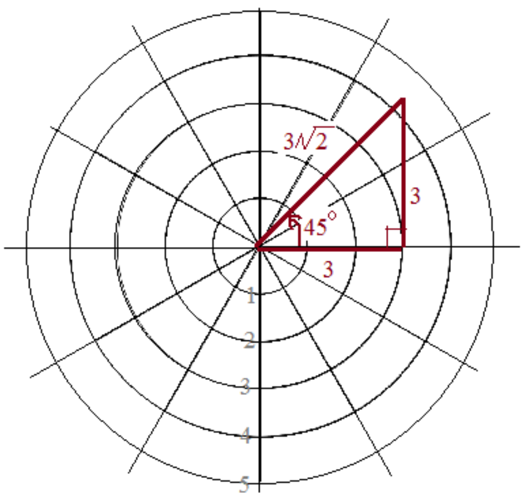


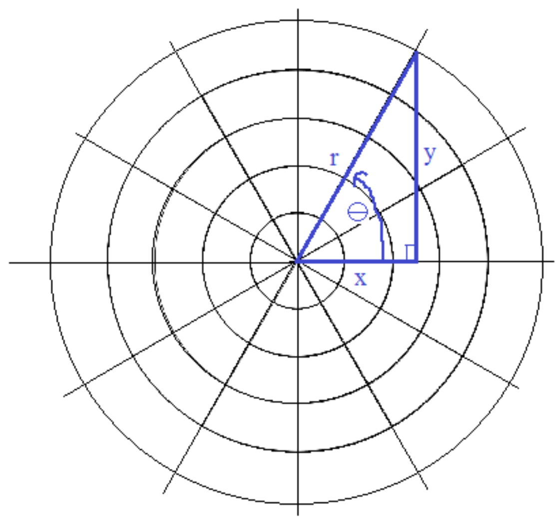
Rectangular: (3, 3)

Polar: $(3\sqrt{2}, 45^\circ)$

Rectangular Coordinates: (x, y)

Polar Coordinates: (r, θ)





Important Implications: To convert from Rectangular to Polar coordinates,

$$\sin\theta = \frac{y}{r} \qquad \cos\theta = \frac{x}{r} \qquad x^2 + y^2 = r^2$$

Or, to convert from polar to rectangular,

$$x = r\cos\theta \qquad y = r\sin\theta$$

Polar Coordinates vs. Rectangular Coordinates

Example: Convert rectangular coordinates (3, 7) into polar coordinates

$x = r\cos\theta$

$y = r\sin\theta$

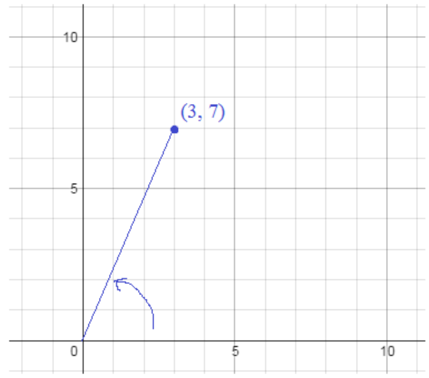
$x^2 + y^2 = r^2$

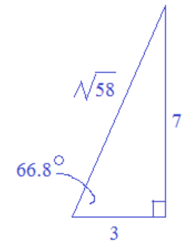
$9 + 49 = 58 \qquad r = \sqrt{58}$

$\tan\theta = \dfrac{y}{x} = \dfrac{7}{3}$

$\theta = 66.8^\circ$

$(r, \theta) = (\sqrt{58}, 66.8^\circ)$

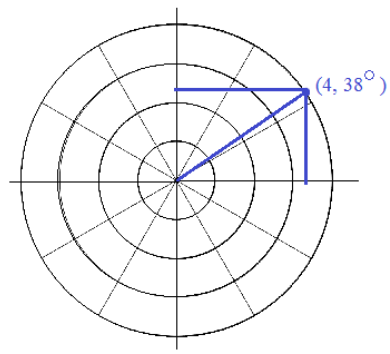


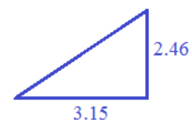


Example: Convert polar coordinates $(4, 38^\circ)$ into rectangular coordinates

$x = r\cos\theta \qquad x = 4\cos(38^\circ) \qquad x = 3.15$

$y = r\sin\theta \qquad y = 4\sin(38^\circ) \qquad y = 2.46$





Example: Change (-6, 11) into polar coordinates

$x = r\cos\theta$

$y = r\sin\theta$

$(r, \theta) = (\sqrt{157}, 118.7^\circ)$

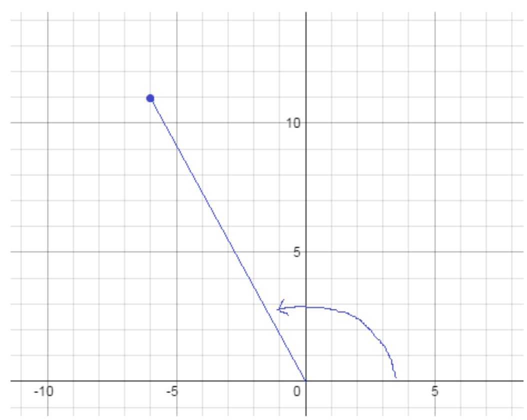
$x^2 + y^2 = r^2$

$(-6)^2 + (11)^2 = r^2$

$36 + 121 = 157 \qquad r = \sqrt{157}$

$\tan\theta = \dfrac{y}{x} = \dfrac{11}{-6} = -61.3^\circ$ and, since it is in Quadrant II,

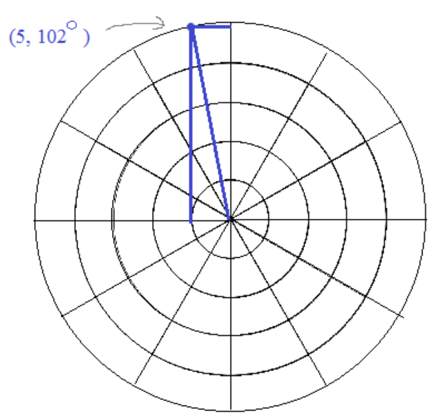
$\theta = 118.7^\circ$



Example: Express $(5, 102^\circ)$ as rectangular coordinates

$x = r\cos\theta \qquad x = 5\cos(102^\circ) \qquad x = -1.04$

$y = r\sin\theta \qquad y = 5\sin(102^\circ) \qquad y = 4.89$

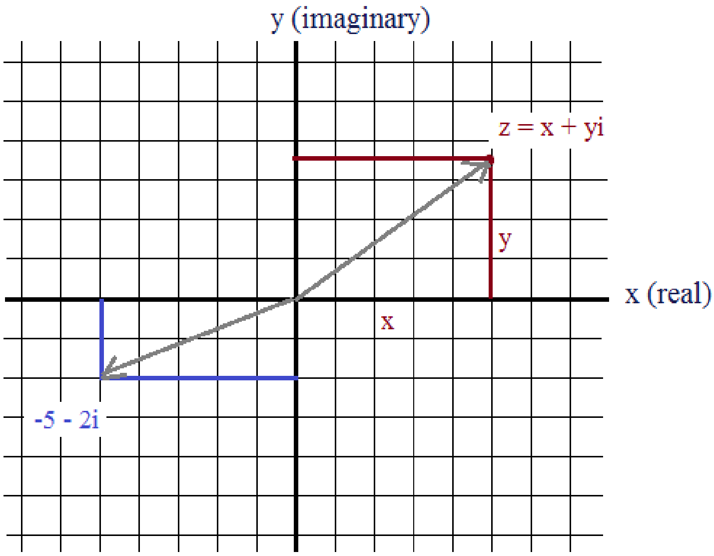


Imaginary Plane of Complex Numbers

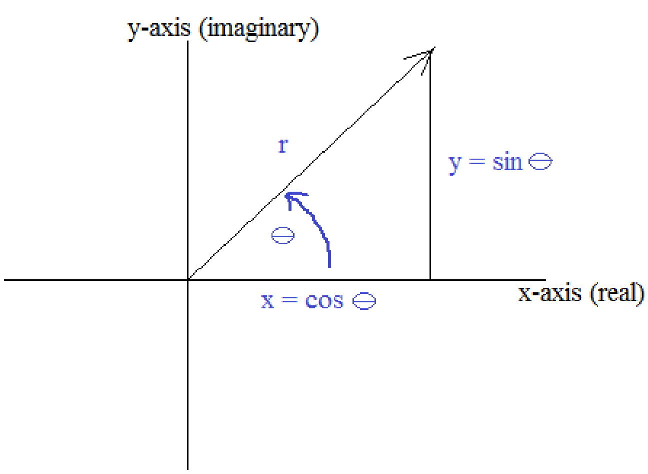
"Imaginary" Number: $i = \sqrt{-1}$

$i^2 = -1$

"Complex" Number: $z = x + yi$ or $z = (x, y)$
where x is the real component
y is the imaginary component



Rectangular Form: $x + yi$



Polar Form:

$z = r(\cos\theta + i \sin\theta)$

or

$r\text{Cis}\theta$

Complex Numbers: Polar and Rectangular
Random Notes and Formulas

$r(\cos\theta + i\sin\theta) \Longrightarrow r\,\mathrm{Cis}\,\theta$

$$Z_1 Z_2 = r_1 r_2 \,\mathrm{Cis}(\theta_1 + \theta_2)$$

$$\frac{Z_1}{Z_2} = \frac{r_1}{r_2}\,\mathrm{Cis}\,(\theta_1 - \theta_2)$$

Polar Form (r, θ)

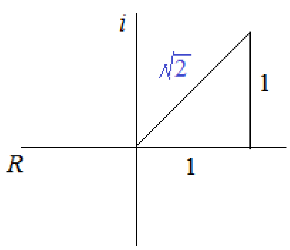
Rectangular Form (x, y)

(Complex) Polar Form $r\mathrm{Cis}\,\theta$

(Complex) Rectangular Form $a + bi$

Converting rectangular to polar using a graph:

Examples: Convert $1 + i$ into polar

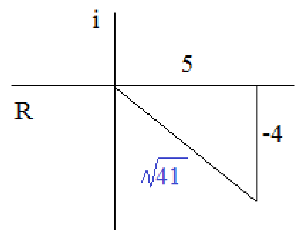


$a = 1$ (real term)
$b = 1$ (imaginary term)

45-45-90 triangle

$r = \sqrt{2}$
$\theta = 45$

Convert $5 - 4i$ into polar



$a = 5$
$b = -4$

$r = \sqrt{41}$
(pythagorean theorem)

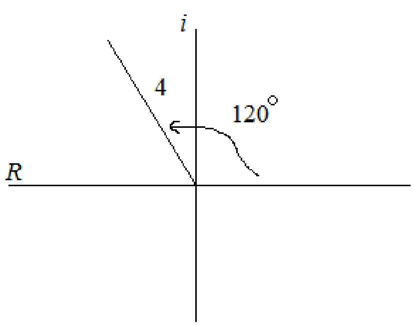
$\sqrt{41}\,\mathrm{Cis}(321.35^\circ)$

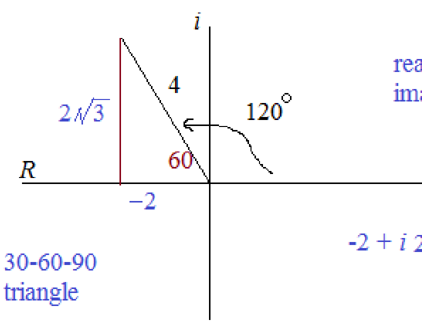
$\cos\theta = \frac{5}{\sqrt{41}} = 38.65^\circ$

$360 - 38.65 = 321.35$

Converting Polar to Rectangular using the graph:

Example: Convert $4\mathrm{Cis}120^\circ$ into Rectangular





real term $a = -2$
imaginary term $b = 2\sqrt{3}$

$-2 + i\,2\sqrt{3}$

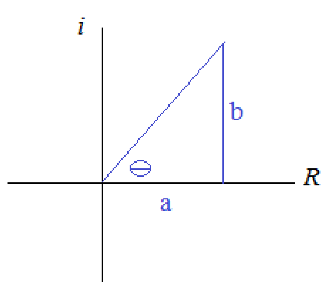
30-60-90
triangle

For $Z = a + bi$

$|Z|$ is 'magnitude' of Z (length of r) $= \sqrt{a^2 + b^2}$

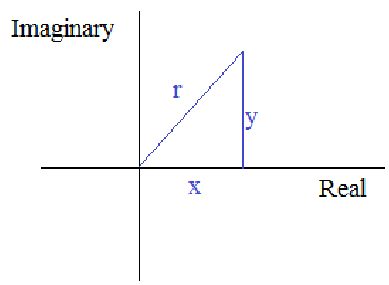
$a = r\cos\theta$

$b = r\sin\theta$



Also, expressed as $Z = x + iy$

(complex Plane or Argand Diagram)



CIS and Component Vectors

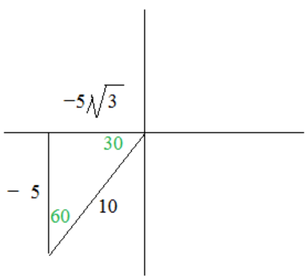
Example: $z_1 = -5\sqrt{3} - 5i$ Find $z_1 z_2$ and $\frac{z_1}{z_2}$ Identify $|z_1|$ and $|z_2|$

$z_2 = 2\sqrt{3} + 2i$

Method 1: Using Cis

$z_1 = -5\sqrt{3} - 5i$ $z_2 = 2\sqrt{3} + 2i$

10Cis(210)



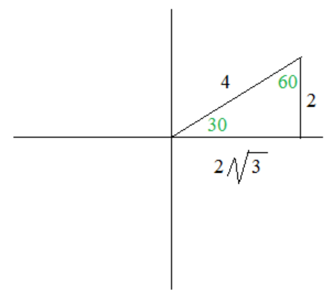
$x = r\cos\theta$

$-5\sqrt{3} = 10\cos\theta$

$\cos\theta = \frac{-5\sqrt{3}}{2}$

$\theta = 210^\circ$ (quadrant III)

4Cis(30)



$y = r\sin\theta$

$2 = 4\sin\theta$

$\sin\theta = \frac{2}{4}$

$\theta = 30^\circ$ (quadrant I)

$z_1 z_2 = 10\text{Cis}(210) \cdot 4\text{Cis}(30) = 40\text{Cis}(240)$

$\frac{z_1}{z_2} = 10\text{Cis}(210) / 4\text{Cis}(30) = \frac{5}{2}\text{Cis}(180)$

Method 2: Using component vector

$z_1 = -5\sqrt{3} - 5i$ $z_2 = 2\sqrt{3} + 2i$

$z_1 z_2 = (-5\sqrt{3} - 5i)(2\sqrt{3} + 2i)$ FOIL

$-30 - 10\sqrt{3}\,i - 10\sqrt{3}\,i + 10$

$-20 - 20\sqrt{3}\,i$

40Cis(240)

$\frac{z_1}{z_2} = \frac{-5\sqrt{3} - 5i}{2\sqrt{3} + 2i} \cdot \frac{(2\sqrt{3} - 2i)}{(2\sqrt{3} - 2i)}$

$\frac{-30 + 10\sqrt{3}\,i - 10\sqrt{3}\,i - 10}{12 + 4}$

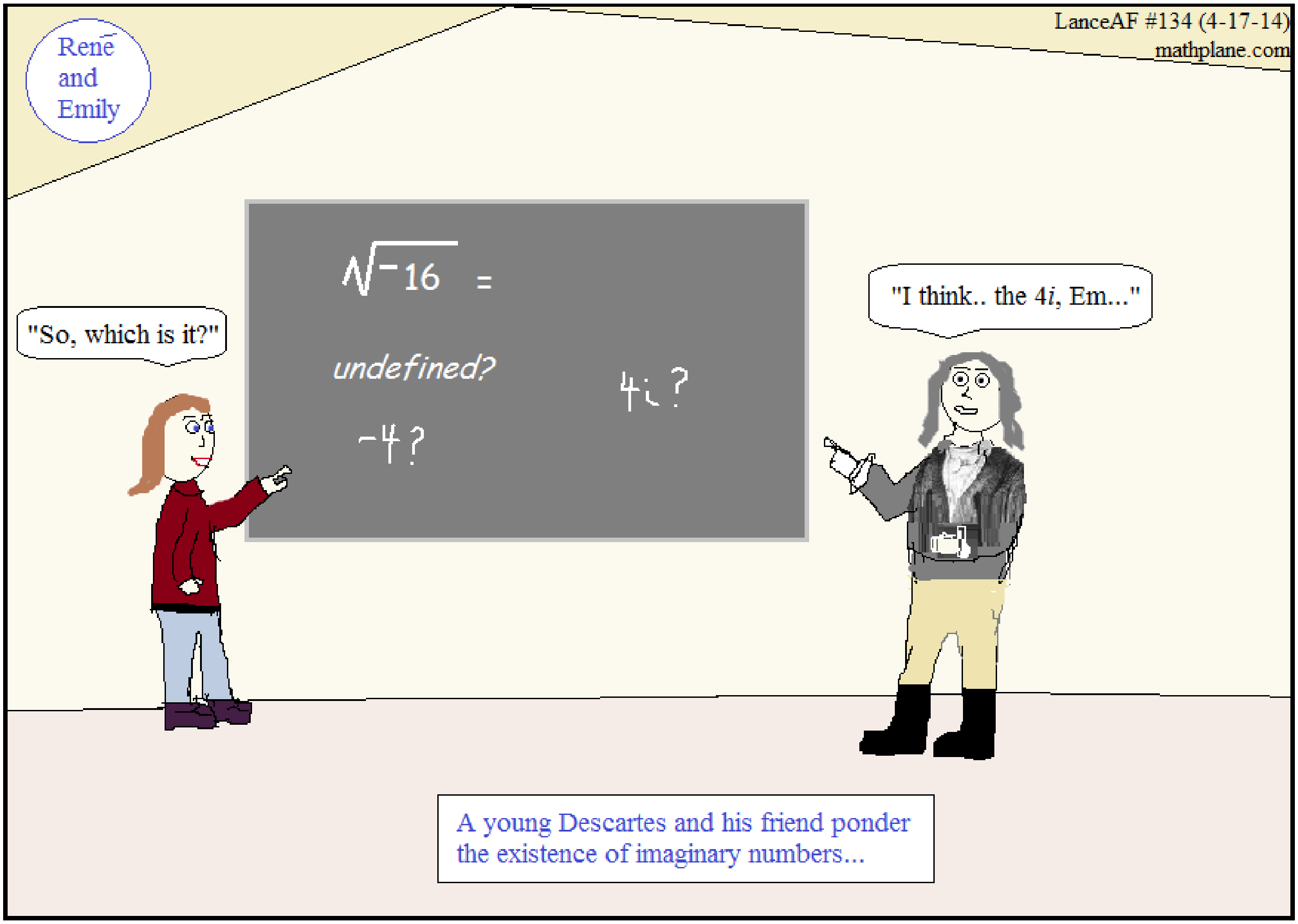
$\frac{-40 + 0i}{16} = \frac{-5}{2} + 0i$

$\frac{5}{2}\text{Cis}(180)$

$z_1 = -5\sqrt{3} - 5i \implies |z_1| = \sqrt{(-5\sqrt{3})^2 + (-5)^2} = \sqrt{75 + 25} = 10$

$z_2 = 2\sqrt{3} + 2i \implies |z_2| = \sqrt{(2\sqrt{3})^2 + (2)^2} = \sqrt{12 + 4} = 4$

Note: These are the same measures as each hypotenuse in the above graphs

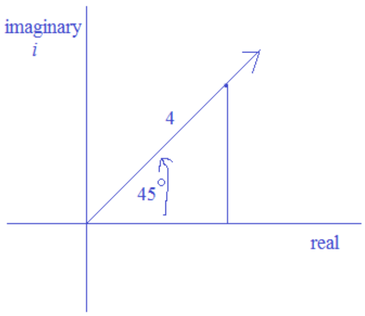


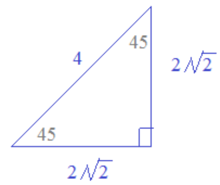
More Examples-→

Examples: Convert to rectangular coordinates and a + b*i* complex form

$4cis45^\circ$

We know the radius is 4 units
and the angle is 45 degrees





Using the formulas:

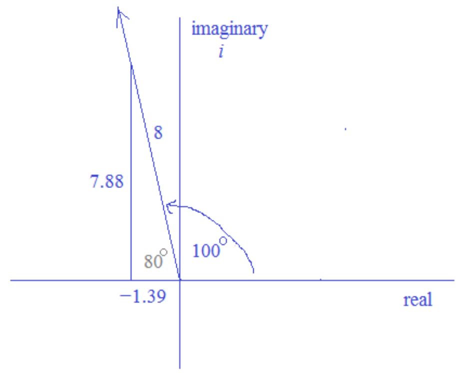
$y = r\sin\theta$ $\qquad x = r\cos\theta$

$y = 4\sin(45^\circ)$ $\qquad x = 4\cos(45^\circ)$

$= 2\sqrt{2}$ $\qquad = 2\sqrt{2}$

$(2\sqrt{2},\ 2\sqrt{2})$ $\qquad 2\sqrt{2} + 2\sqrt{2}\,i$

$8cis100^\circ$



Quadrant II

Using the formulas:

$y = r\sin\theta$ $\qquad x = r\cos\theta$

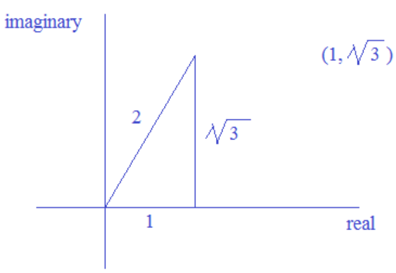
$y = 8\sin(100^\circ)$ $\qquad x = 8\cos(100^\circ)$

$= 7.88$ $\qquad x = -1.39$

$(7.88, -1.39)$ $\qquad -1.39 + 7.88i$

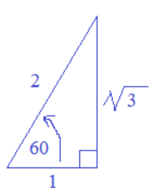
Examples: Identify the related coordinates and convert to polar form $rcis\ \theta$

$1 + i\sqrt{3}$



$(1, \sqrt{3})$

30-60-90
right
triangle

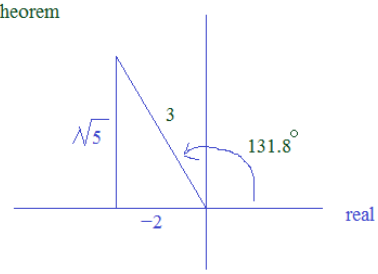


$2cis60^\circ$

$-2 + i\sqrt{5}$

Pythagorean Theorem
radius is 3

$(-2, \sqrt{5})$



$x = r\cos\theta$

$-2 = 3\cos\theta$

$\theta = \cos^{-1}(-2/3)$

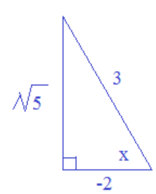
$y = r\sin\theta$

$\sqrt{5} = 3\sin\theta$

$\theta = \sin^{-1}(\sqrt{5}/3)$

in Quadrant II

131.8 degrees



$\cos x = \dfrac{2}{3}$

$x = 48.2^\circ$

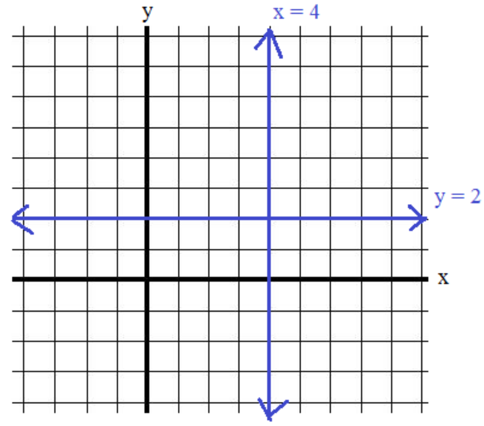
reference angle

$3cis131.8^\circ$

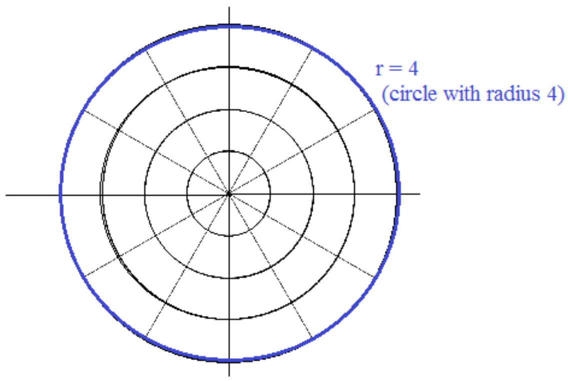
$x = r\cos\theta \qquad y = r\sin\theta$

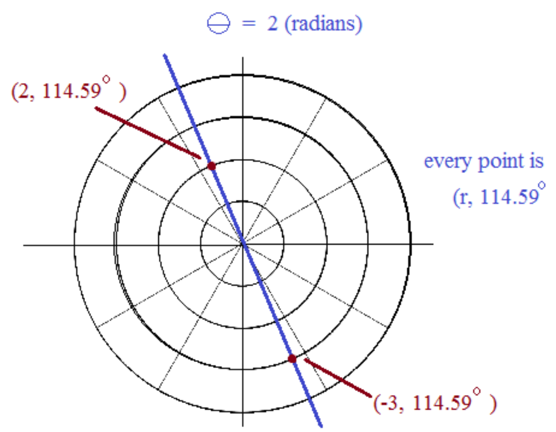
$x^2 + y^2 = r^2$

Example: On a rectangular xy-plane, graph x = 4 and y = 2



On a polar coordinate (Argand) plane, graph r = 4 and θ = 2





Example: For the line y = 2, what is the equation in polar coordinates?

$\sin\theta = \dfrac{y}{r}$

$y = r \sin\theta$

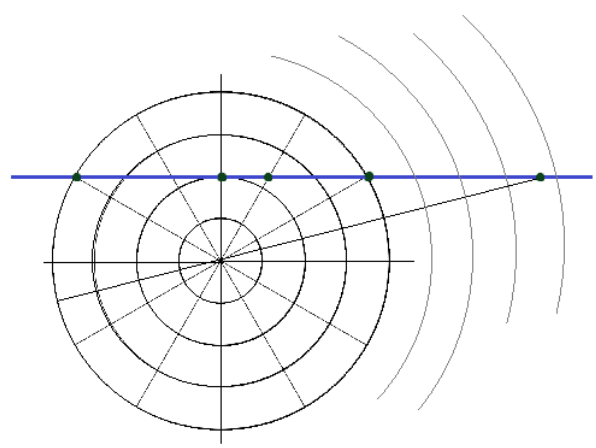
substitue y = 2

$2 = r \sin\theta$

$r = \dfrac{2}{\sin\theta}$

$\boxed{r = 2\csc\theta}$

θ	$\csc\theta$	r
30	2	4
60	$\frac{2}{\sqrt{3}}$	$\frac{4}{\sqrt{3}}$
90	1	2
150	2	4
195	-3.86	-7.72



Example: Find $(5\text{cis}30^\circ)(2\text{cis}60^\circ)$

Method 1: Use the formula

$(5)(2)\text{cis}(30^\circ + 60^\circ) = 10\text{cis}90^\circ$ ← $\boxed{Z_1 Z_2 = r_1 r_2 \text{Cis}(\theta_1 + \theta_2)}$

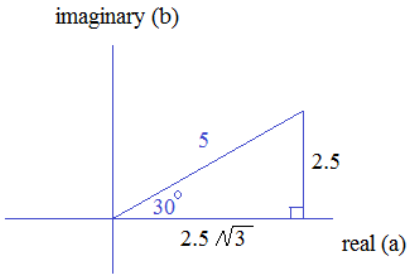
Method 2: Change to complex number form and solve

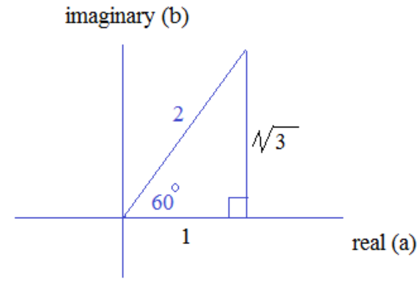
$(5\text{cis}30^\circ) = 2.5\sqrt{3} + 2.5i$

a + b*i*

$(2\text{cis}60^\circ) = 1 + \sqrt{3}\,i$

a + b*i*





$(5\text{cis}30^\circ)(2\text{cis}60^\circ) = (2.5\sqrt{3} + 2.5i)(1 + \sqrt{3}\,i)$

FOIL

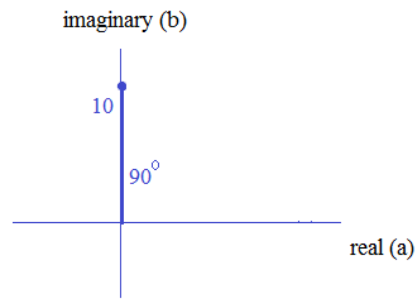
$2.5\sqrt{3} + 7.5i + 2.5i + 2.5\sqrt{3}\,i^2$

$2.5\sqrt{3} + 10i - 2.5\sqrt{3}$

$0 + 10i$

change back to polar form

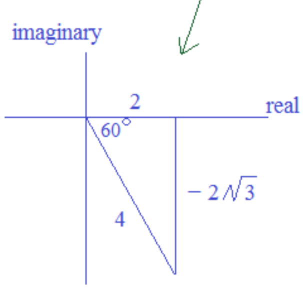
$10\text{cis}90^\circ$



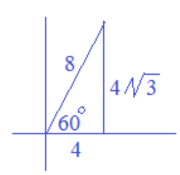
Example: $\left(8\text{cis}\left(\frac{\pi}{3}\right)\right)\left(\frac{1}{2}\text{cis}\left(\frac{-2\pi}{3}\right)\right)$

$\boxed{r_1 r_2 \text{Cis}(\theta_1 + \theta_2)}$

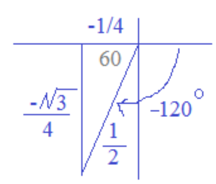
$(8)\left(\frac{1}{2}\right) cis\left(\frac{\pi}{3} + \frac{-2\pi}{3}\right) = \boxed{4\text{cis}\left(\frac{-\pi}{3}\right)}$



$8\text{cis}60^\circ$ ---> $4 + 4\sqrt{3}\,i$



$\frac{1}{2}\text{cis}(-120^\circ)$ ---> $\frac{-1}{4} - \frac{\sqrt{3}}{4}i$



$(4 + 4\sqrt{3}\,i)\left(\frac{-1}{4} - \frac{\sqrt{3}}{4}i\right)$

$-1 - \sqrt{3}\,i - \sqrt{3}\,i - 3i^2$

$\boxed{2 - 2\sqrt{3}\,i}$

Example: Convert $r = -5\sin\theta$

$r = -5\,\frac{y}{r}$

$r^2 = -5y$

$x^2 + y^2 = -5y$

$x^2 + y^2 + 5y = 0$ (circle)

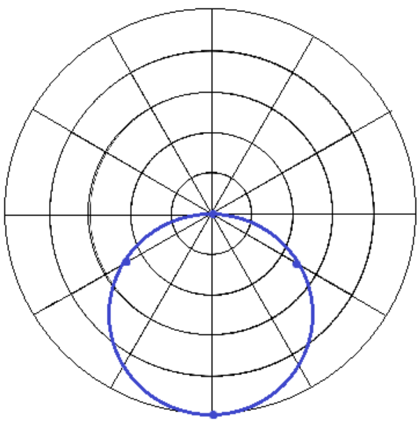
$y = r\sin\theta \longrightarrow \sin\theta = \frac{y}{r}$

$x^2 + y^2 = r^2$

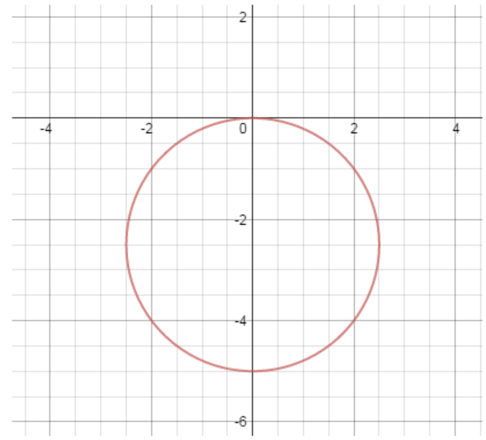
$x^2 + y^2 + 5y = 0$ complete the square

$x^2 + y^2 + 5y + \frac{25}{4} = 0 + \frac{25}{4}$

$x^2 + (y + \frac{5}{2})^2 = \frac{25}{4}$ standard form of circle with center $(0, -5/2)$



$r = -5\sin\theta$



Example: Where do $r = 1 + \sin\theta$ and $r = 2\sin\theta$ intersect?

Solve by substitution:

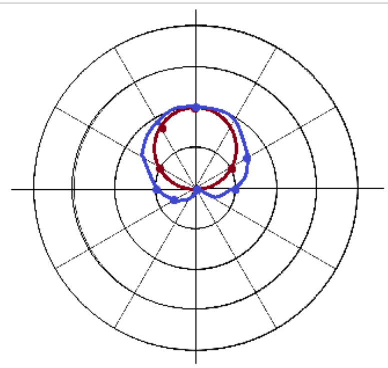
$1 + \sin\theta = 2\sin\theta$

$1 = \sin\theta$

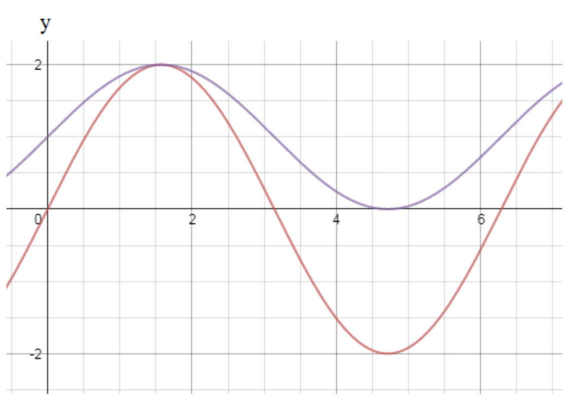
$\theta = 90^\circ$

NOTE: The graphs intersect at $(2, 90^\circ)$

They also pass through the origin, but at different times!



$r = 1 + \sin\theta$ $r = 2\sin\theta$



θ	$\sin\theta$	$r = 1 + \sin\theta$	θ	$r = 2\sin\theta$
0	0	1	0	0
30	1/2	3/2	30	1
90	1	2	90	2
120	$\sqrt{3}/2$	1.86	120	1.73
180	0	1	180	0
210	-1/2	1/2	210	-1
270	-1	0	270	-2
330	-1/2	1/2	330	-1

How many "petals" are on the (polar coordinate) graph of $r = -3\sin 5\theta$?

Polar Coordinate Graph

The graph of $y = -3\sin 5x$ is periodic
with maximum and minimum values of 3 and -3

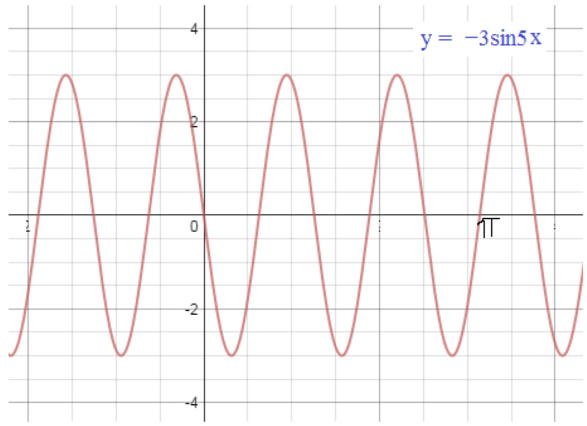
Therefore, to determine the "tips" of the petals, solve

$3 = -3\sin 5\theta$

and

$-3 = -3\sin 5\theta$

(i.e. find values of θ where r = 3 or -3)



$3 = -3\sin 5\theta$

$-1 = \sin 5\theta$

Let $5\theta = A$

$-1 = \sin A$

$A = \frac{3\pi}{2}, \frac{7\pi}{2}, \frac{11\pi}{2}, \frac{15\pi}{2}, \frac{19\pi}{2} \ldots$

5θ

therefore,

$\theta = \frac{3\pi}{10}, \frac{7\pi}{10}, \frac{11\pi}{10}, \frac{15\pi}{10}, \frac{19\pi}{10} \ldots$

$-3 = -3\sin 5\theta$

$1 = \sin 5\theta$

Let $5\theta = A$

$1 = \sin A$

$A = \frac{\pi}{2}, \frac{5\pi}{2}, \frac{9\pi}{2}, \frac{13\pi}{2}, \frac{17\pi}{2} \ldots$

5θ

therefore,

$\theta = \frac{\pi}{10}, \frac{5\pi}{10}, \frac{9\pi}{10}, \frac{13\pi}{10}, \frac{17\pi}{10} \ldots$

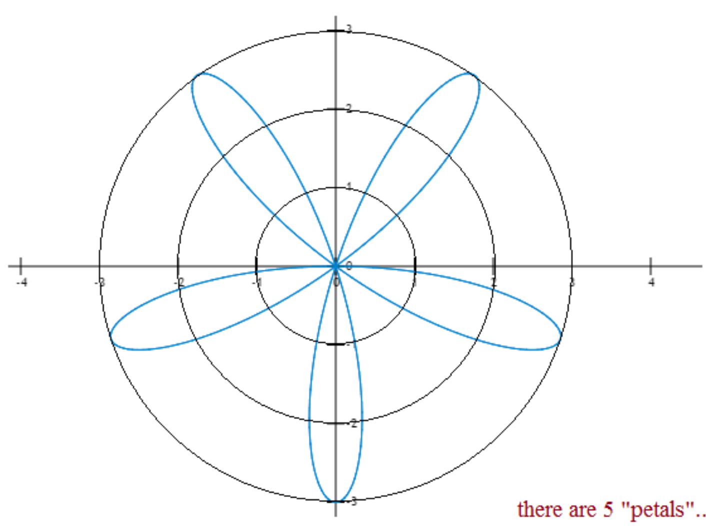
These values of θ are where the tips of the petals occur...
Now, we have to determine which ones overlap....

$r = -3\sin 5\theta$

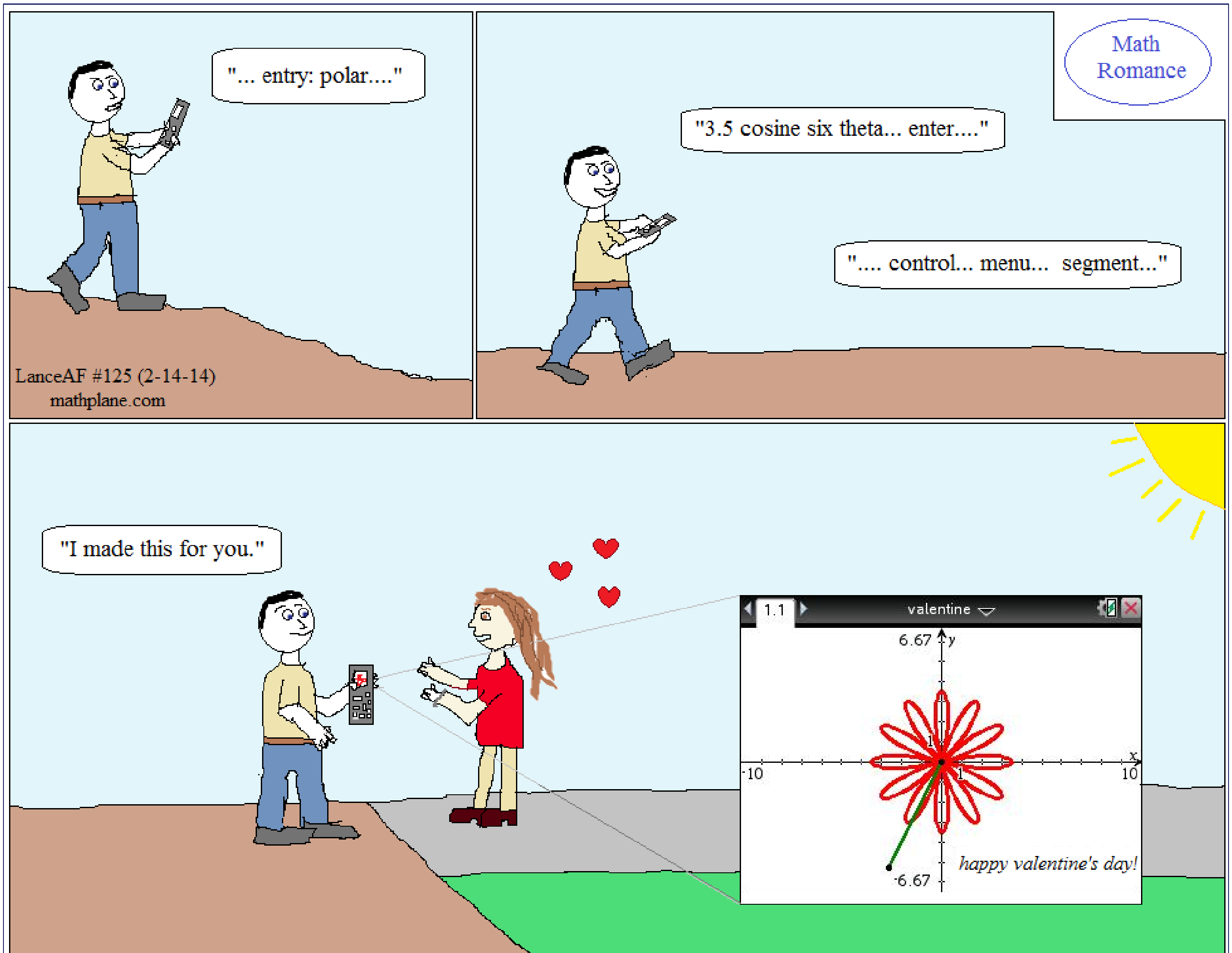
(r, θ)

$(3, \frac{3\pi}{10})$	$(-3, \frac{\pi}{10})$
$(3, \frac{7\pi}{10})$	$(-3, \frac{\pi}{2})$
$(3, \frac{11\pi}{10})$	$(-3, \frac{9\pi}{10})$
$(3, \frac{3\pi}{2})$	$(-3, \frac{13\pi}{10})$
$(3, \frac{19\pi}{10})$	$(-3, \frac{17\pi}{10})$

Same points!



there are 5 "petals"...



A Valentine's Day Flower that lasts forever... (as long as you recharge the batteries!)

Practice Quiz-→

Polar and Rectangular Quick Quiz

I. Convert the following:

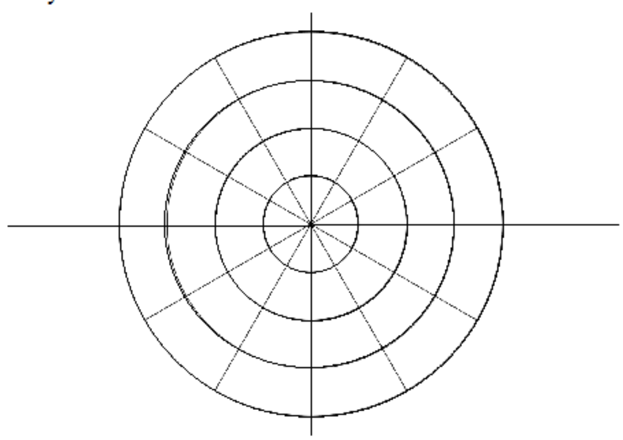
1) Rectangular to Polar

A) $(3, 3)$ B) $(0, -2)$ C) $(-1, \sqrt{3})$

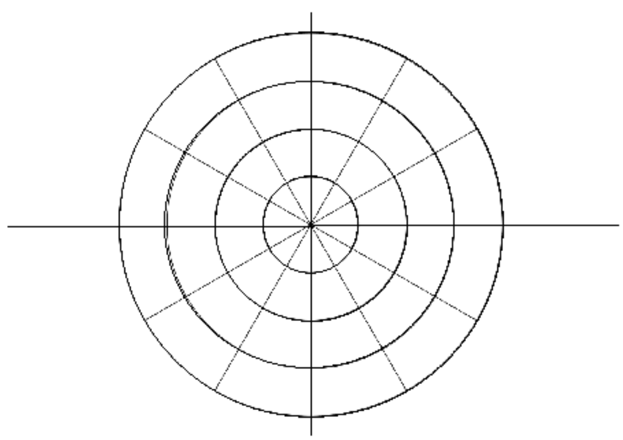
2) Polar to Rectangular

A) $(6, 90^\circ)$ B) $(8, \pi)$ C) $(-2, 60^\circ)$

II. Plot $(3, 120^\circ)$ on the graph. Identify two other coordinates that have the same location.



III. Sketch $r = 1 + \sin\theta$ Give the rectangular equation.



Polar and Rectangular Quick Quiz (continued)

IV: Complex Numbers

1) $Z_1 = 3 - i$ $\qquad Z_2 = 4 + 4i$

A) Express Z_1 and Z_2 in polar form

B) Find $Z_1 Z_2$

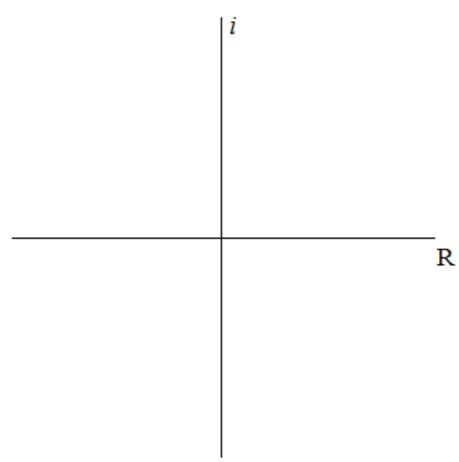
C) Determine $|Z_1|$ and $|Z_2|$

2) $Z = 2\text{Cis}120^\circ$

A) Find Z^2

B) Find Z^5

C) Express the answers in A) and B) in Complex form; and, graph.



Polar and Rectangular Quick Quiz (continued)

Express each product in polar and rectangular form.

A) $(2\text{Cis}115^\circ)(3\text{Cis}65^\circ)$

B) $(8\text{Cis}60^\circ)(\frac{1}{2}\text{Cis}(-120^\circ))$

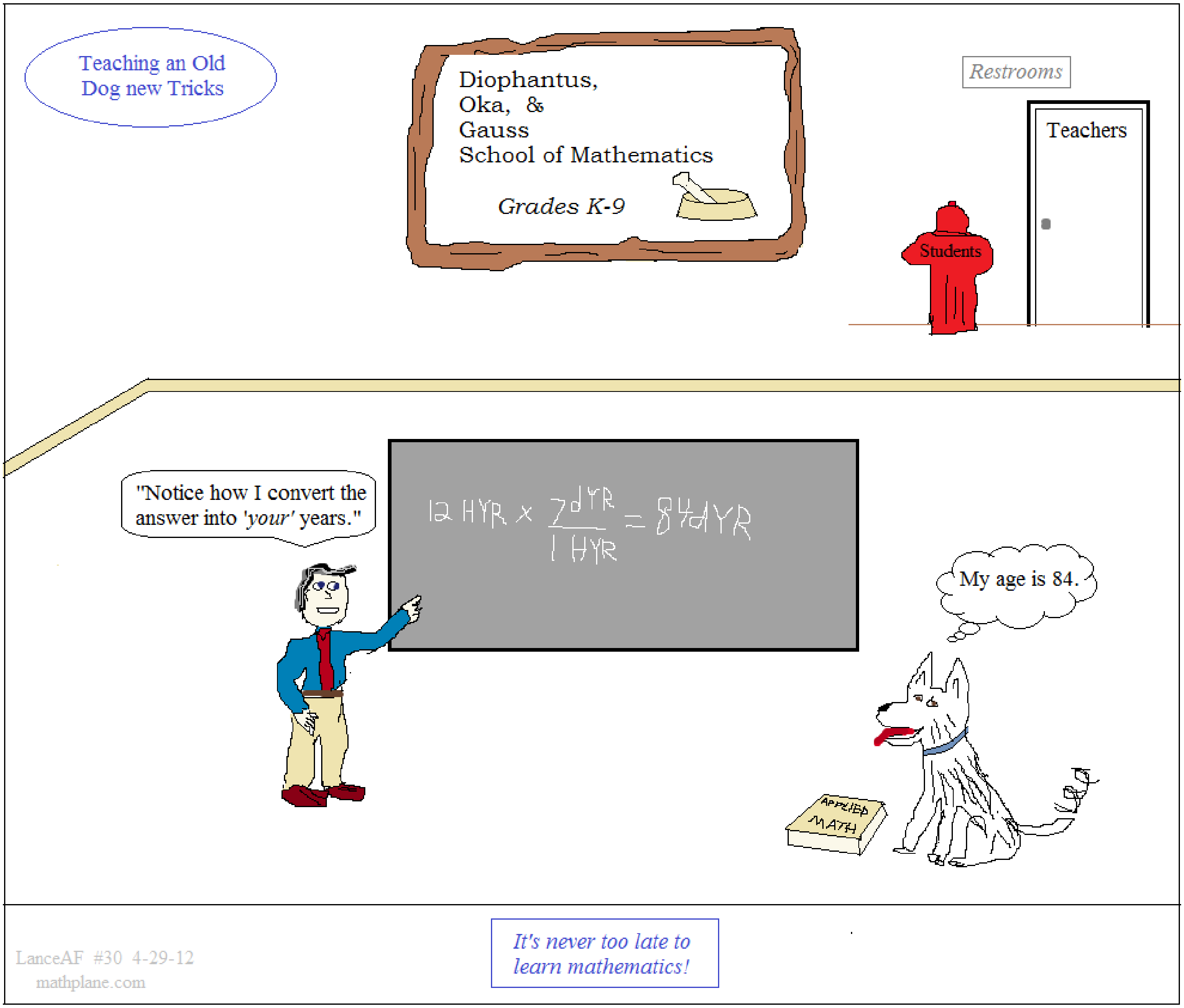
V. Compute using 2 methods — Verify solutions from A) and B) are equivalent!

A) $(1 - i\sqrt{3})(1 - i\sqrt{3})$

B) Convert to polar form (CIS) and solve.

A) $\dfrac{6\text{Cis}30^\circ}{3\text{Cis}150^\circ}$

B) Convert to Complex/Rectangular Form a + b*i*.
then, divide to confirm the answer in A)



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"Notice how I convert the answer into 'your' years."
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Solutions -→

Polar and Rectangular Quick Quiz

SOLUTIONS

I. Convert the following:

1) Rectangular to Polar

A) (3, 3)

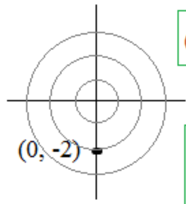
$x^2 + y^2 = r^2$ $\quad \tan\theta = \frac{y}{x}$

$9 + 9 = r^2$ $\quad = \frac{3}{3}$

$r = 3\sqrt{3}$ $\quad \theta = 45^\circ$

$\boxed{(3\sqrt{3}, 45^\circ)}$

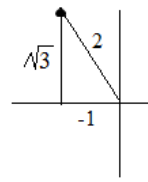
B) $(0, -2)$



$\boxed{(2, 270^\circ)}$

also, $(-2, 90^\circ)$

C) $(-1, \sqrt{3})$



(30-60-90 triangle)

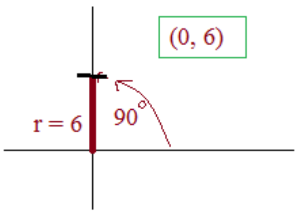
$\boxed{(2, 120^\circ)}$

$x = r\cos\theta$

$(-1) = (2)\cos 120^\circ$ ✓

2) Polar to Rectangular

A) $(6, 90^\circ)$



$\boxed{(0, 6)}$

B) $(8, \pi)$

$x = r\cos\theta$

$x = 8(-1) = -8$

$y = r\sin\theta$

$y = 8(0) = 0$

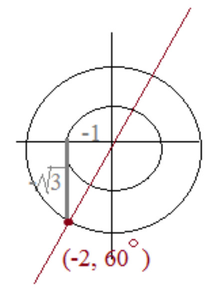
$\boxed{(-8, 0)}$

C) $(-2, 60^\circ)$

$x = -2\cos 60$
$= -2(1/2) = -1$

$y = -2\sin 60$
$= -2(\sqrt{3}/2) = -\sqrt{3}$

$\boxed{(-1, -\sqrt{3})}$



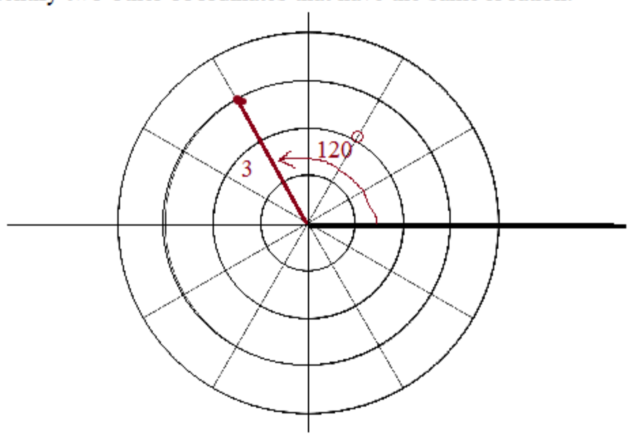
II. Plot $(3, 120^\circ)$ on the graph. Identify two other coordinates that have the same location.

$(3, 480^\circ)$

$(-3, -60^\circ)$

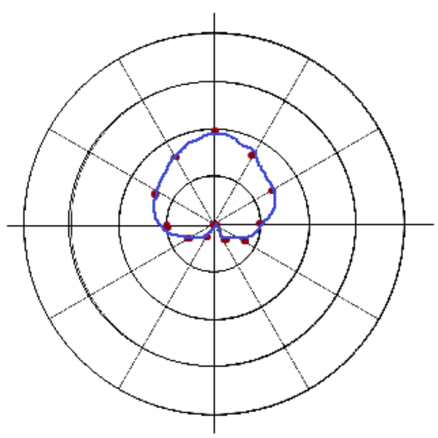
$(3, -240^\circ)$

are 3 possibilities....



III. Sketch $r = 1 + \sin\theta$ Give the rectangular equation.

θ	r
0	1
30	3/2
60	$(2+\sqrt{3})/2$
90	2
120	$(2+\sqrt{3})/2$
150	3/2
180	1
210	1/2
240	$(2-\sqrt{3})/2$
270	0
330	1/2
360	1



$\sin\theta = \frac{y}{r}$

$r^2 = x^2 + y^2$

$r = \sqrt{x^2 + y^2}$

(substitute and simplify)

$r = 1 + \frac{y}{r}$

$r^2 = r + y$

$\boxed{x^2 + y^2 = \sqrt{x^2 + y^2} + y}$

Polar and Rectangular Quick Quiz (continued)

SOLUTIONS

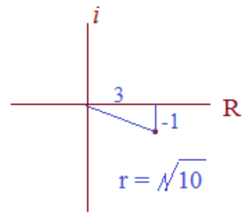
IV: Complex Numbers

1) $Z_1 = 3 - i$ $\quad Z_2 = 4 + 4i$

A) Express Z_1 and Z_2 in polar form

$Z = \sqrt{10}\text{Cis}(341.6^\circ)$

$Z = 4\sqrt{2}\text{Cis}45^\circ$

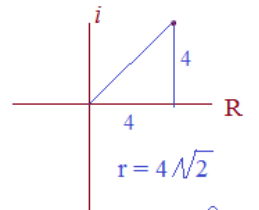


$\tan\theta = -1/3$

$\theta = -18.4^\circ$

$= 341.6^\circ$

(45-45-90 triangle)



$\theta = 45^\circ$

B) Find $Z_1 Z_2$

$4\sqrt{20}\text{Cis}(386.6^\circ) =$

$\boxed{8\sqrt{5}\text{Cis}(26.6^\circ)}$

C) Determine $|Z_1|$ and $|Z_2|$

$|Z_1| = \sqrt{3^2 + (-1)^2} = \boxed{\sqrt{10}}$ $\qquad |Z_2| = \boxed{4\sqrt{2}}$

2) Z = 2Cis120

A) Find Z^2

$(2\text{Cis}120^\circ)(2\text{Cis}120^\circ) = (2\text{x}2)\text{Cis}(120+120) = \boxed{4\text{Cis}240^\circ}$

B) Find Z^5

$2^5\ \text{Cis}(5 \text{ x } 120) = 32\text{Cis}(600^\circ) = \boxed{32\text{Cis}240^\circ}$

C) Express the answers in A) and B) in Complex form; and, graph.

2Cis120

$= (-1, -\sqrt{3})$

$r = 4$

$x = r\cos 240$

$x = 4\,(-1/2) = -2$

$y = r\sin 240$

$y = 4\,(-\sqrt{3}/2) = -2\sqrt{3}$

$\boxed{(-2, -2\sqrt{3})}$

$r = 32$

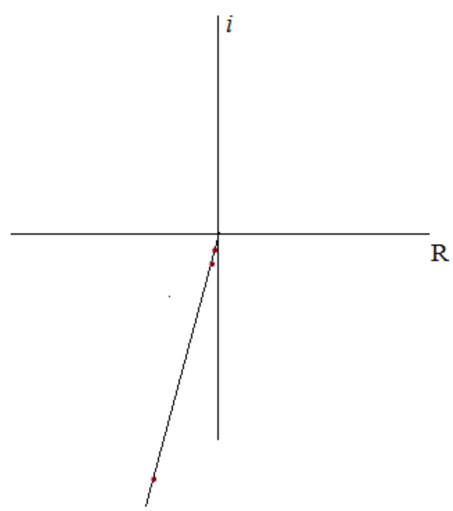
$x = 32\cos 240$

$x = 32(-1/2) = -16$

$y = 32\sin 240$

$y = 32\,(-\sqrt{3}/2) = -16\sqrt{3}$

$\boxed{(-16, -16\sqrt{3})}$



Polar and Rectangular Quick Quiz (continued)

SOLUTIONS

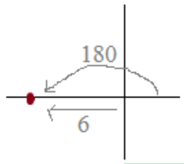
Express each product in polar and rectangular form.

$Z_1 Z_2 = r_1 r_2 \text{Cis}(\theta_1 + \theta_2)$

A) $(2\text{Cis}115^\circ)(3\text{Cis}65^\circ)$

$2 \cdot 3 \text{ Cis}(115 + 65) =$

$\boxed{6\text{Cis}(180^\circ)}$ (Polar)



$-6 + 0i = \boxed{-6}$ (rectangular)

B) $(8\text{Cis}60^\circ)(\frac{1}{2}\text{Cis}(-120^\circ))$

$8 \cdot \frac{1}{2} \text{ Cis}(60 + (-120)) =$

$4\text{Cis}(-60) =$

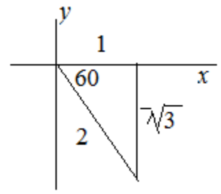
$\boxed{4\text{Cis}(300^\circ)}$

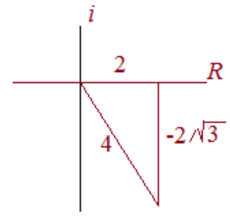
(note: 300 is the coterminal angle of -60 that is between 0 and 360)

$4(\cos 300 + i\sin 300) =$

$4(\frac{1}{2} - i\,\frac{\sqrt{3}}{2})$

$\boxed{2 - i\,2\sqrt{3}}$





V. Compute using 2 methods — Verify solutions from A) and B) are equivalent!

A) $(1 - i\sqrt{3})(1 - i\sqrt{3})$

"FOIL" $1 - i\sqrt{3} - i\sqrt{3} + i^2(3)$

$1 - 2i\sqrt{3} - 3$

$\boxed{-2 - i\,2\sqrt{3}}$

B) Convert to polar form (CIS) and solve.

$1 - i\sqrt{3}$

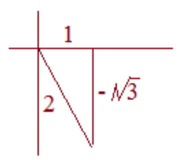
$r = \sqrt{(1)^2 + (-\sqrt{3})^2} = 2$

$\theta = 300^\circ$

$\tan\theta = \frac{y}{x}$

$\tan\theta = \frac{\sqrt{3}}{1}$

$\boxed{1 - i\sqrt{3} = 2\text{Cis}300^\circ}$



$(2\text{CIS}300)(2\text{CIS}300) =$

$4\text{CIS}600^\circ = -360^\circ$

$\boxed{4\text{Cis}240^\circ}$

$4(\cos 240 + i\sin 240)$

$4(-1/2 - i\sqrt{3}/2)$

$\boxed{-2 - i\,2\sqrt{3}}$

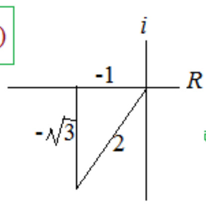
A) $\dfrac{6\text{Cis}30^\circ}{3\text{Cis}150^\circ}$

$\frac{Z_1}{Z_2} = \frac{r_1}{r_2} \text{CIS}(\theta_1 - \theta_2)$

$= \frac{6}{3} \text{CIS}(30 - 150)$

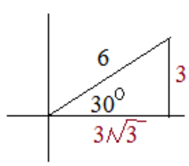
$= 2\text{Cis}(-120)$

$\boxed{= 2\text{Cis}(240^\circ)}$



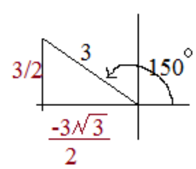
B) Convert to Complex/Rectangular Form a + b*i*. then, divide to confirm the answer in A)

$6\text{Cis}30 =$



$3\sqrt{3} + 3i$

$3\text{Cis}150 =$



$\frac{-3\sqrt{3}}{2} + \frac{3i}{2}$

$\frac{1}{2}(-3\sqrt{3} + 3i)$

$\dfrac{3\sqrt{3} + 3i}{\frac{1}{2}(-3\sqrt{3} + 3i)}$ multiply by $\frac{2}{2}$

$\dfrac{6\sqrt{3} + 6i}{(-3\sqrt{3} + 3i)} \cdot \dfrac{(-3\sqrt{3} - 3i)}{(-3\sqrt{3} - 3i)}$ mult. by $\frac{\text{conjugate}}{\text{conjugate}}$

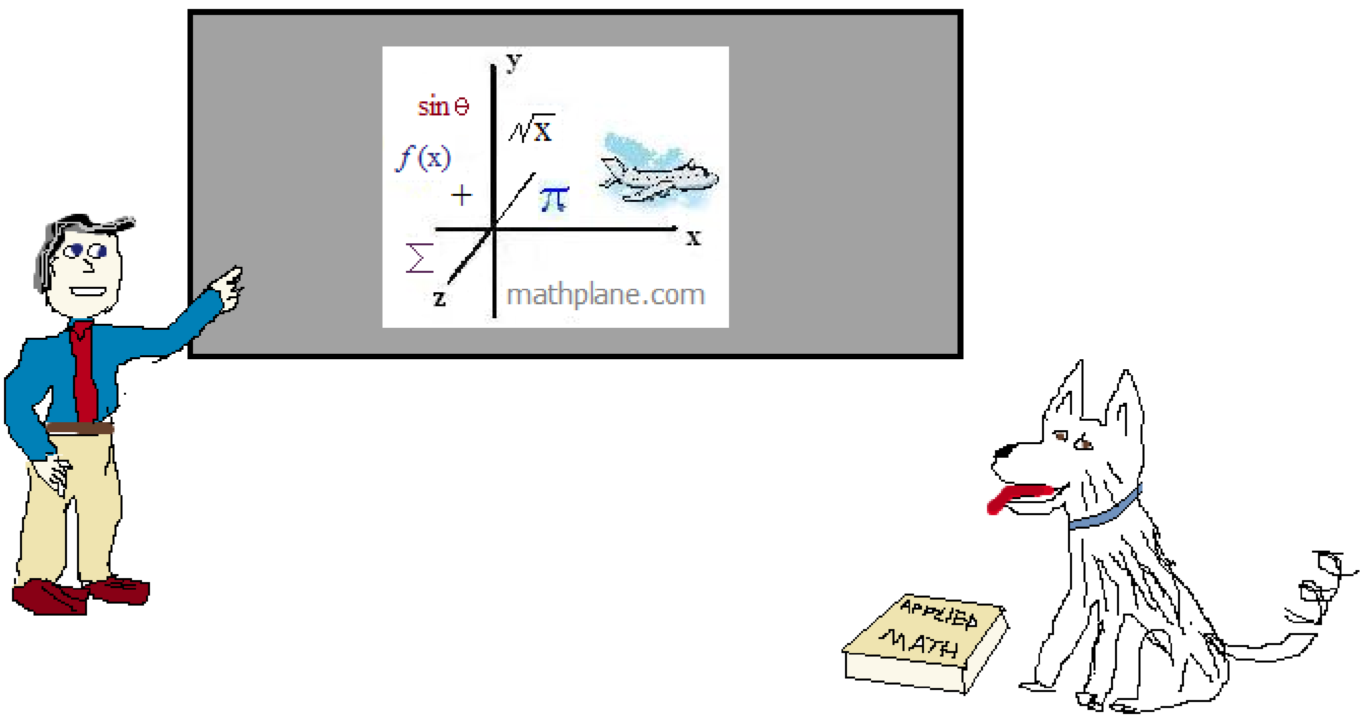
$\dfrac{-54 + 18 - 18\sqrt{3}\,i - 18\sqrt{3}\,i}{27 + 9}$

$\boxed{-1 - \sqrt{3}\,i}$

Thanks for downloading this packet. (Hope it helps!)

If you have questions, suggestions, or feedback, let us know.

Cheers



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