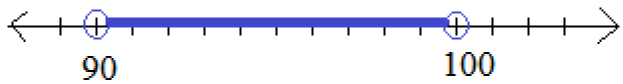
Absolute Value and Inequalities



Notes, strategies, examples, and practice test (w/solutions)

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Absolute Value Quick Review

1) Fill in the blank with $<$, $>$, or $=$

$-(-5)$ _____ $|-5|$

(-5) _____ $|-5|$

$|4-2|$ _____ $|2-4|$

$-2|4|$ _____ $-4|2|$

$|7-(-9)|$ _____ $7-|9|$

$|-3|$ _____ 2

$-|2|$ _____ $-(-2)$

$|-4|$ _____ -4

2) $|-5+3|-|-8| =$

a) -6
b) 0
c) 6
d) 10
e) 16

3) $\dfrac{|-8|}{4} - \dfrac{|14|}{|2|} =$

$\dfrac{|-9|}{-3} - \dfrac{-9}{|-3|} =$

$-|8| - |-10| =$

$-|-8| - |10| =$

$|-8| - |-10| =$

Answers-→

1) Fill in the blank with $<, >,$ or $=$

SOLUTIONS

Absolute Value Quick Review

$-(-5)$ $\underline{\ =\ }$ $|-5|$ $\quad 5 = 5$

(-5) $\underline{\ <\ }$ $|-5|$ $\quad -5 < 5$

$|4-2|$ $\underline{\ =\ }$ $|2-4|$ $\quad 2 = 2$

$-2|4|$ $\underline{\ =\ }$ $-4|2|$ $\quad -8 = -8$

$|7-(-9)|$ $\underline{\ >\ }$ $7-|9|$ $\quad 16 > -2$

$|-3|$ $\underline{\ >\ }$ 2 $\quad 3 > 2$

$-|2|$ $\underline{\ <\ }$ $-(-2)$ $\quad -2 < 2$

$|-4|$ $\underline{\ >\ }$ -4 $\quad 4 > -4$

2) $|-5+3| - |-8| =$

$\boxed{\text{a) } -6}$

b) 0

c) 6

d) 10

e) 16

$|-2| - 8 =$

$2 - 8 = -6$

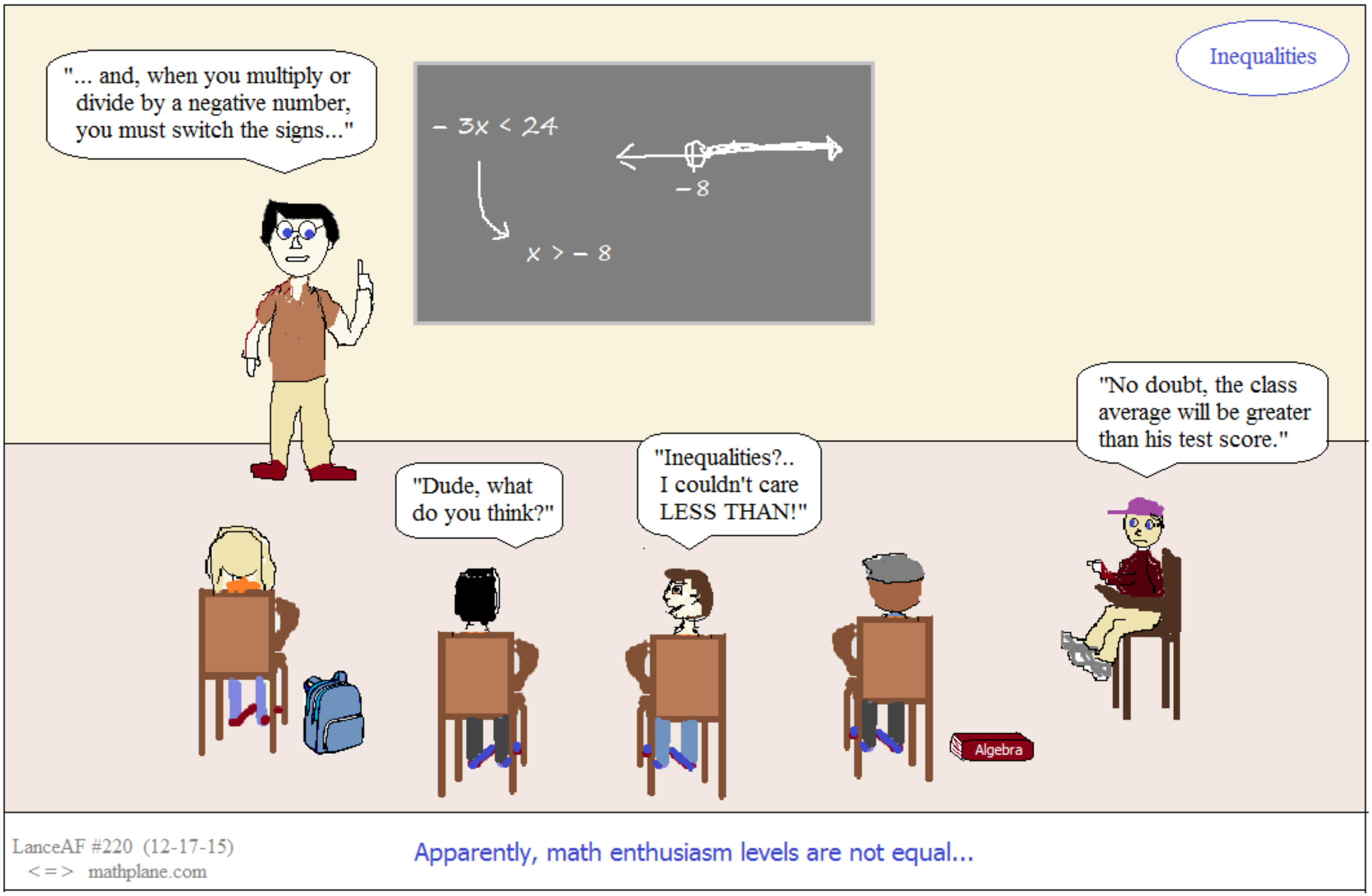
3) $\dfrac{|-8|}{4} - \dfrac{|14|}{|2|} = \dfrac{8}{4} - \dfrac{14}{2} = \boxed{-5}$

$\dfrac{|-9|}{-3} - \dfrac{-9}{|-3|} = \dfrac{9}{-3} - \dfrac{-9}{3} = -3-(-3) = \boxed{0}$

$-|8| - |-10| = -8 - 10 = \boxed{-18}$

$-|-8| - |10| = -8 - 10 = \boxed{-18}$

$|-8| - |-10| = 8 - 10 = \boxed{-2}$



Notes and Examples-→

Solving Inequalities

Solve: Determine "Critical Values"

Graph: "Open Circles" or "Closed Circles"

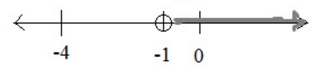
Check Answer: Test regions (Plug solutions into <u>original equation</u>)

Example: $3x + 10 > -2x + 5$

(solve) $3x + 10 = -2x + 5$
$5x = -5$
$x = -1$

(graph)

open circle, because solution does not include -1



(check answer)

$3(-1) + 10 = -2(-1) + 5$
$7 = 7$ ✓

(test regions)

Test Left: Try -4

$3(-4) + 10 > -2(-4) + 5$
$-2 \not> 13$

NO

Test Right: Try 0

$3(0) + 10 > -2(0) + 5$
$10 > 5$

YES

Example: $3x > 6$ OR $-\frac{3}{2}(x + 2) \geq 4$

(solve) $3x = 6$
$x = 2$

$-\frac{3}{2}(x + 2) = 4$

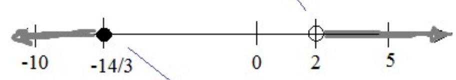
$(x + 2) = 4\left(\frac{-2}{3}\right)$

OR $x + 2 = -8/3$

$x = -14/3$

(graph)

open circle because > does not include 2



closed circle because $\geq$ includes -14/3

(check critical points)

$3(2) = 6$ ✓

$-3/2\,(-14/3 + 2) = 42/6 - 3 = 24/6$
$= 4$ ✓

(test regions)

Test Left: Try -10

$3(-10) > 6$? No

$-3/2\,(-10 + 2) \geq 4$?
$12 > 4$ Yes

(Since -10 satisfies one of the "or" equations, the left side is a solution)

Test Middle: Try 0

$3(0) > 6$? No

$-3/2\,(0 + 2) \geq 4$?
$-4/3 \not\geq 4$ No

(Since 0 satisfies neither equation, the middle is not a solution)

Test Right: Try 5

$3(5) > 6$? Yes

$-3/2\,(5 + 2) \geq 4$?
$-21/2 \geq 4$ No

(Although 5 does not satisfy the second equation, it works in the first one. Therefore, the right region is a solution)

Example: $4y < 8$ AND $3y + 5 \geq y - 13$

(solve: find critical values)

$4y = 8$
$y = 2$

$3y + 5 = y - 13$
$2y = -18$
$y = -9$

(graph: "open and closed" circles -- test regions)

("closed circle") ("open circle")

(Quick check)

$4(2) = 8$ ✓

$3(-9) + 5 = (-9) - 13$ $-22 = -22$ ✓

Test Left with -10: $4(-10) < 8$? Yes
$3(-10) + 5 \geq -10 - 13$? No

Test Middle with 0: $4(0) < 8$? Yes
$3(0) + 5 \geq 0 - 13$? Yes

Test Right with 5: $4(5) < 8$? No
$3(5) + 5 \geq 5 - 13$? Yes

Since this is an 'AND' problem, must satisfy BOTH equations. Therefore, only the middle region works..

Solving Absolute Value Equations

Steps:

1) Isolate Absolute Value
2) Solve for 'Positive Answer'
3) Solve for 'Negative Answer'
4) Check Solutions! (Plug into original equation)

Example: $|3x + 12| = 24$

(the absolute value is 'isolated' already)

"Positive" Answer: $3x + 12 = 24$

$3x = 12$

$\boxed{x = 4}$

"Negative" Answer: $3x + 12 = -24$

$3x = -36$

$\boxed{x = -12}$

Check solutions: $|3(4) + 12| = 24$

$24 = 24$ ✓

$|3(-12) + 12| = 24$

$|-24| = 24$

$24 = 24$ ✓

Example: $2|x - 5| + 6 = 16$

Isolate the absolute value part $2|x - 5| + 6 = 16$

$2|x - 5| = 10$

$|x - 5| = 5$

"Positive" Answer: $x - 5 = 5$

$\boxed{x = 10}$

"Negative" Answer: $x - 5 = -5$

$\boxed{x = 0}$

Check Solutions: $2|(10) - 5| + 6 = 16$

$2 \cdot 5 + 6 = 16$

$16 = 16$ ✓

$2|(0) - 5| + 6 = 16$

$2 \cdot 5 + 6 = 16$

$16 = 16$ ✓

Example: $|3x + 4| + 6 = 2$

Isolate the absolute Value: $|3x + 4| + 6 = 2$

$|3x + 4| = -4$

"Positive" Answer: $3x + 4 = -4$

$3x = -8$

$x = -8/3$

"Negative" Answer: $3x + 4 = 4$

$3x = 0$

$x = 0$

No Solutions!
(absolute value result can never be negative)

Check Solutions!

$|3(-8/3) + 4| + 6 = 2$

$|-4| + 6 = 2$

$10 \neq 2$ ✗

$|3(0) + 4| + 6 = 2$

$4 + 6 = 2$

$10 \neq 2$ ✗

Absolute Value Equations: How many solutions?

Steps for solving absolute value equations:

1) Isolate the absolute value
2) "Split into negative and positive equations"
3) Solve
4) Check your answer(s)!

Two solutions:

$$|5x + 4| - 10 = -5$$

(isolate the absolute value)

$$|5x + 4| = 5$$

('positive equation')

$$5x + 4 = 5$$
$$5x = 1$$
$$x = \frac{1}{5}$$

('negative equation')

$$5x + 4 = -5$$
$$5x = -9$$
$$x = \frac{-9}{5}$$

(Check)

$$|5(1/5) + 4| - 10 = -5$$
$$5 - 10 = -5$$
$$-5 = -5$$

$$|5(-9/5) + 4| - 10 = -5$$
$$|-5| - 10 = -5$$
$$5 - 10 = -5$$

One solution:

$$2|x + 3| + 8 = 8$$
$$2|x + 3| = 0$$
$$|x + 3| = 0$$
$$x + 3 = 0$$
$$x = -3$$

$$2|(-3) + 3| + 8 = 8$$
$$2|0| + 8 = 8$$
$$8 = 8$$

After isolating the absolute value,

for $|ax + b| = c$

if $c > 0$, then 2 solutions
$c = 0$, then 1 solution
$c < 0$, then no solutions

No solutions:

$$5|3x + 2| + 20 = 10$$

(isolate the absolute value)

$$5|3x + 2| = -10$$
$$|3x + 2| = -2$$

Absolute value output is never negative!

('positive equation')

$$3x + 2 = -2$$
$$3x = 4$$
$$x = 4/3$$

$$3x + 2 = 2$$
$$3x = 0$$
$$x = 0$$

('negative equation')

$$5|3(4/3) + 2| + 20 = 10$$
$$5|6| + 20 = 10$$
$$30 + 20 = 10$$

$$5|3(0) + 2| + 20 = 10$$
$$5|2| + 20 = 10$$
$$30 = 10$$

Solving Absolute Value/Inequality Equations

Steps: 1) Solve absolute value equation
(for both 'negative' and 'positive' values)
Determine "critical points"

2) Graph --- "open circles" or "closed circles"?

3) Test regions and Check answers
(plugging points into the ORIGINAL equation)

Example: $|3x + 7| < 10$

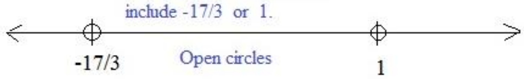
(solve) $|3x + 7| = 10$

$3x + 7 = 10$
$3x = 3$
$x = 1$

$3x + 7 = -10$
$3x = -17$
$x = -17/3$

(graph)

Since it is <, solutions don't include -17/3 or 1.



(test regions)

Left area: try -8
$|3(-8) + 7| < 10$?
$17 < 10$
NO

Middle area: try 0
$|3(0) + 7| < 10$?
$7 < 10$
YES

Right area: try 5
$|3(5) + 7| < 10$?
$15 < 10$
NO

(check answers -- "critical points")

$|3(-17/3) + 7| = |-17 + 7| = 10$ ✓

$|3(1) + 7| = |3 + 7| = 10$ ✓

Example: $3|x - 7| + 4 \geq 10$

(solve)

Isolate the absolute value terms.
Then, solve equal to 'positive answer' and equal to 'negative answer'

$3|x - 7| + 4 = 10$

$3|x - 7| = 6$

$|x - 7| = 2$

'positive' $x - 7 = 2$
$x = 9$

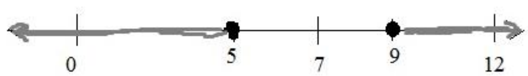
'negative' $x - 7 = -2$
$x = 5$

(graph)

"critical values" are
$x = 5$ and $x = 9$

"closed circles" because solution includes 5 and 9

(test regions)



Use:

0 (left) $3|(0) - 7| + 4 = 25 \geq 10$ YES

7 (middle) $3|(7) - 7| + 4 = 4 \not\geq 10$ NO

12 (right) $3|(12) - 7| + 4 = 19 \geq 10$ YES

(check answers)

$3|(5) - 7| + 4 = 10$ ✓

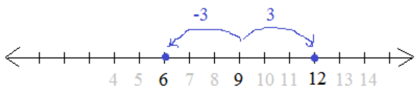
$3|(9) - 7| + 4 = 10$ ✓

(**Remember: plug values into original equation)

"Working Backwards": Determining the absolute value inequality

I. Absolute Value can represent distance on a number line

Example 1:



$|X - 9| = 3$

"6 is three units to the left of 9" $6 - 9 = -3$

"12 is three units to the right of 9" $12 - 9 = 3$

The absolute value of both is 3, and the distance from 9 is 3.

"The distance from 9 (in either direction) is 3"

$X = 12 \quad X = 6$

Example 2:

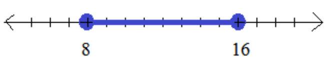
This number line graph represents a set of numbers within a distance.

The middle of the graph is the midpoint: 2 $X - 2$ ("distance from 2")

and, the distance to each endpoint is 5 units $|X - 2| < 5$ ("All numbers X less than 5 units from 2")

Note: You can plug in values from the number line to verify the equation!

II. Identify the equation using absolute value inequalities



Step 1: Identify the midpoint (central value)

The midpoint of 8 and 16: $(8 + 16)/2 = 12$

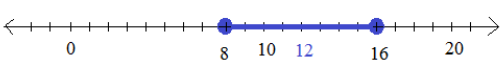
Step 2: Determine the distance to each endpoint

The distance to each endpoint is 4 units.

Step 3: Construct the inequality

$|X - 12| \leq 4$ "The distance between X and 12 must be less than or equal to 4"

Step 4: Check your work!



Plug in values to test endpoints and regions.

Endpoints: $|8 - 12| = 4$ $|16 - 12| = 4$

Regions: $|0 - 12| \nless 4$ $|10 - 12| < 4$ $|20 - 12| \nless 4$

III. Word Problem Applications

Example 1: The length of a local football field must be between 90 and 100 yards. Write an *absolute value inequality* to represent all possible lengths.

Obviously, we could write the equation $90 < L < 100$ where L is possible lengths.
But, the question asks for an absolute value inequality....

Step 1: Find midpoint. 95 yards

Step 2: Determine distance/length to each endpoint. ± 5 yards

Step 3: Construct the inequality $|L - 95| < 5$

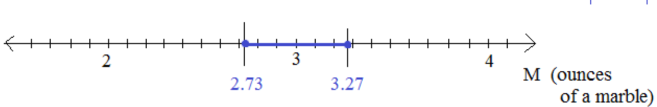
Step 4: Check answers! (construct a number line and check values)

note: in this example, "between 90 and 100" did not include 90 and 100. There may be different interpretations.

Example 2: A marble manufacturer sets a machine to produce 3 ounce marbles. The company allows a tolerance of .27 ounces for any marble it sells. Write an absolute value inequality to represent the acceptable marbles. Then, graph on a number line.

The target value of marbles is 3 (ounces).. $M - 3$

The variation (tolerance) allowed is $\pm$ .27 (ounces) $|M - 3| \leq .27$



"Unusual Absolute Value Solutions"

1) $|3x - 6| - 48 > -48$

$$|3x - 6| > 0$$

$$3x - 6 = 0$$

$$x = 2$$

All Real Numbers <u>except</u> 2

2) $|2x - 4| - 10 \geq -13$

$$|2x - 4| \geq -3$$

All Real Numbers

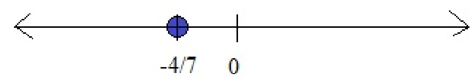
3) $14 + |7x + 4| \leq 14$

$$|7x + 4| \leq 0$$

$$7x + 4 = 0$$

$$x = -4/7$$

Only ONE solution



4) $2 + |.33756x + \sqrt{37}| < -4$

$$|.33756x + \sqrt{37}| < -6$$

ϕ No Solutions

$$|\text{"anything"}| \not< -6$$

"Slope of the Absolute Value Line"

Additional Absolute Value Concepts

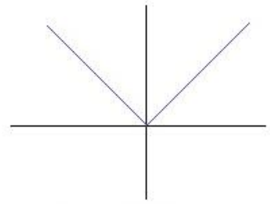
$y = a|x - h| + k$ is the standard form where (h, k) is the vertex

Assume the vertex is on the origin (0, 0).
then,

$y = a|x|$

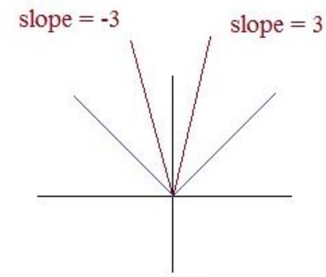
where a is the amount the parent function is "multiplied" or "stretched".

(see graphs and tables)



Parent function: $y = |x|$

x	y
-2	2
-1	1
0	0
1	1
2	2



$y = 3|x|$

x	y
-2	6
-1	3
0	0
1	3
2	6

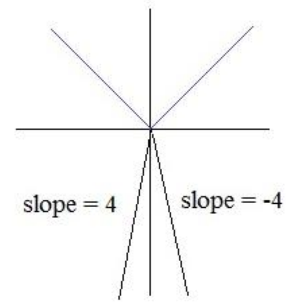
Also, 'a' indicates the slope:

> a is the slope of the absolute value line on the right side of the vertex.
> And,
> -a is the slope of the absolute value line on the left side of the vertex.

If $a < 0$, then the graph 'faces down'
Slope on the left side of the vertex will be positive. And, slope on the right side of the vertex will be negative.

$y = -4|x|$

x	y
-2	-8
-1	-4
0	0
1	-4
2	-8



How many solutions would the equation $a|x + b| + c = d$ have if

a) $a < 0,\ c = d$?

b) $a < 0,\ c > d$?

c) $a > 0,\ c > d$?

First, subtract c from both sides: $a|x + b| = d - c$

a) If $c = d$, then $d - c = 0$ $\quad a|x + b| = 0$

$|x + b| = 0$ (one solution!)

x must equal $-b$

b) If $c > d$, then $a|x + b| =$ 'negative number'
and
if $a < 0$, then $-|x + b| =$ 'negative number' (two solutions!)

Divide both sides by $-$: $|x + b| =$ 'negative'/'negative' = 'positive'

c) If $c > d$, then $a|x + b| =$ 'negative number'
and
if $a > 0$, then $a|x + b|$ must be positive.. (NO solutions!)

Examples:

$3|x + 7| + 8 = 8$
$3|x + 7| = 0$
$|x + 7| = 0$
$x = -7$

$-4|x + 4| + 12 = 8$
$-4|x + 4| = -4$
$|x + 4| = 1$
$x = -3$ or -5

$6|x + 2| + 10 = 4$
$6|x + 2| = -6$
$|x + 2| = -1$
Impossible!

Variables inside and outside the absolute value sign

A) $5|4 - 3x| = 10x - 30$

Isolate the absolute value: $|4 - 3x| = 2x - 6$

Split the equations: $4 - 3x = 2x - 6$ or $4 - 3x = -(2x - 6)$

$10 = 5x$ $\qquad$ $4 - 3x = 6 - 2x$

Solve: $x = 2$ $\qquad$ $x = -2$

Check for extraneous: $5|4 - 3(2)| = 10(2) - 30$ $\qquad$ $5|4 - 3(-2)| = 10(-2) - 30$

$5|-2| = 20 - 30$ $\qquad$ $5|10| = -20 - 30$

$10 = -10$? NO $\qquad$ $50 = -50$? NO

No solutions

B) $\frac{1}{3}|3x + 7| = 6x + 12$

$|3x + 7| = 18x + 36$

Isolate the absolute value:

$3x + 7 = 18x + 36$ or $3x + 7 = -(18x + 36)$

Split the equations: $-29 = 15x$ $\qquad$ $21x = -43$

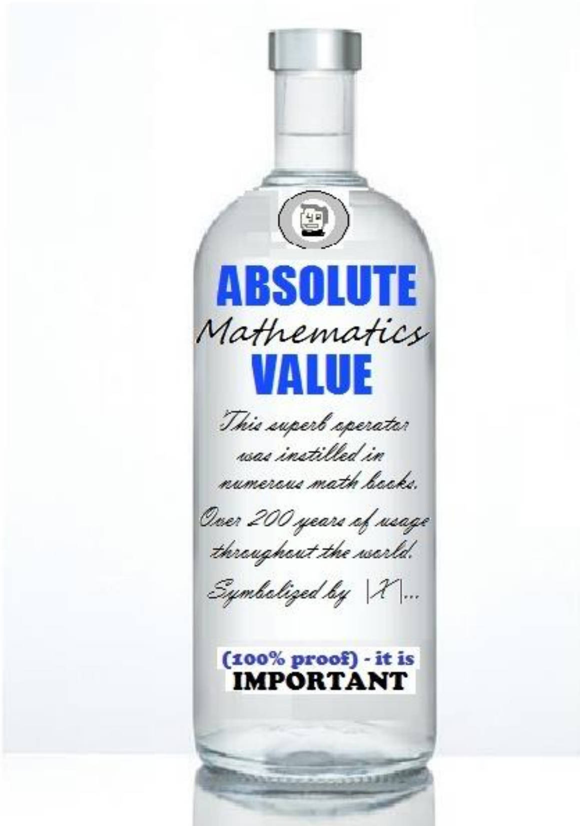
$\boxed{x = -29/15}$ $\qquad$ $x = \cancel{-43/21}$

Solve:

Check for extraneous: $\frac{1}{3}|3(-29/15) + 7| = 6(-29/15) + 12$ $\qquad$ $\frac{1}{3}|3(-43/21) + 7| = 6(-43/21) + 12$

$\frac{1}{3}|-29/5 + 35/5| = -58/5 + 60/5$ $\qquad$ positive answer $\qquad$ negative answer

$2/5 = 2/5$ ✓ $\qquad$ Extraneous!



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Absolutely, the best there is...

Serve with sub-zero temperatures..

Mix with any ingredient to bring out a positive result..

Remember to drink and calculate responsibly.

Practice Test and Solutions-→

Solve the following:

Absolute Value Equations

a) $|x + 2| = 14$

b) $|2x + 6| = 4x$

c) $|x + 5| = -x + 3$

d) $|x - 2| = |5 + x|$

e) $2|x + 1| = 5|x + 1| - 12$

f) $\dfrac{2|x + 7|}{5} = 12$

g) $|3x + 9| = x + 1$

h) $3 - 10|x - 5| = -17$

i) $|x + 6| = -|2x + 10| + 20$

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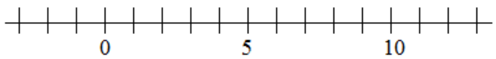
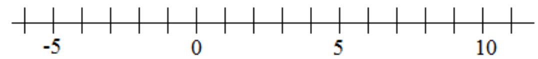
Quiz: Absolute Value, Inequalities, and the Number Line

I. Graph the following on a number line

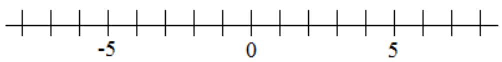
a) $x > 5$



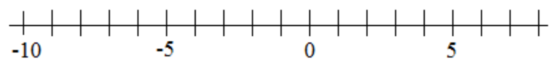
b) $2x \leq 4$



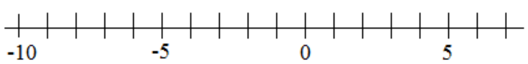
c) $3x + 2 < 9$



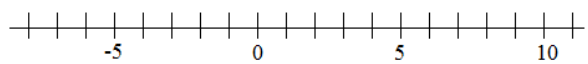
d) $x = -4$



e) $-3x > 6$

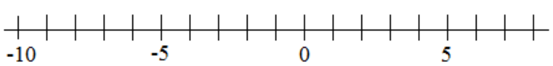


f) $-5 < x \leq 1$

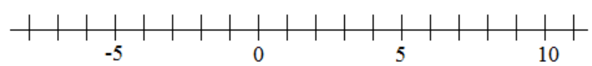


II. Absolute Values: Solve and graph the following

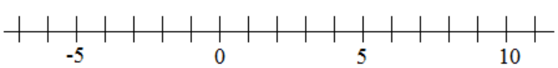
a) $|x + 3| < 6$



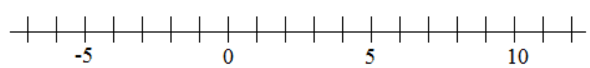
b) $|3x| < 18$



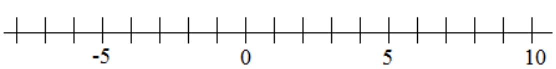
c) $2|x + 1| \geq 6$



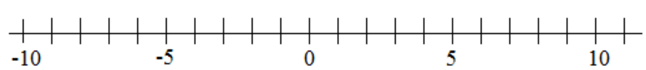
d) $|2x| + 8 < 4$



e) $|x - 3| + 4 \geq 9$

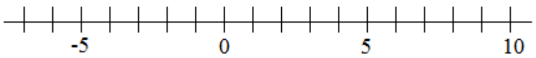


f) $|x + 7| = 1$

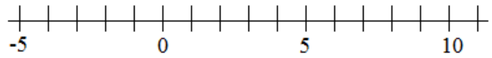


III. Solve and Graph

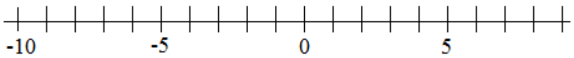
a) $x > 4$ or $x < 0$



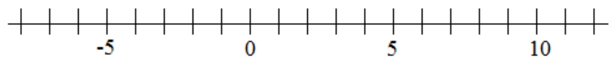
b) $x > 5$ and $x \leq 9$



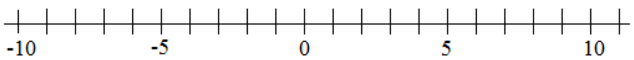
c) $|x + 4| > 0$



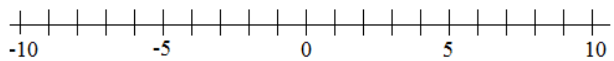
d) $2|x + 1| + 6 > 10$



e) $-|x + 3| > -5$

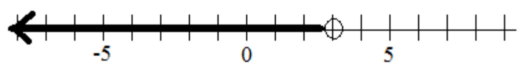


f) $|x + 6| \leq 0$

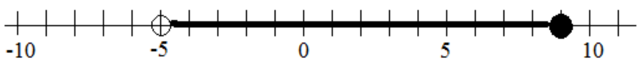


IV: Describe each graph with an equation

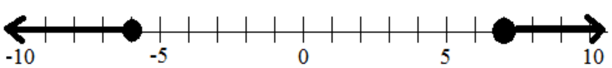
a)



b)



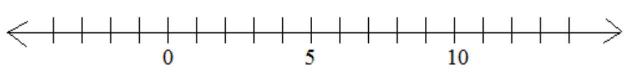
c)



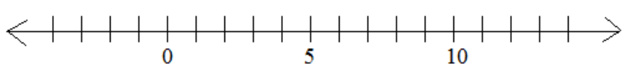
BONUS:

1) Graph the following on a number line:

a) $x \neq 6$

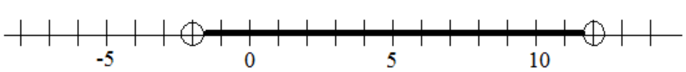


b) $x < 5$ or $x \geq -3$



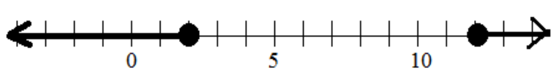
2) Use <u>Absolute Value Inequalities</u> to describe the following number line graphs:

a)



$-2 < x < 12$

b)



$x \leq 2$ or $x \geq 12$

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Solve the following:

SOLUTIONS

Absolute Value Equations

a) $|x + 2| = 14$

"split the absolute value"

$x + 2 = 14$ OR $x + 2 = -14$

$\boxed{x = 12}$ $\boxed{x = -16}$

b) $|2x + 6| = 4x$

"Split the absolute value"

$2x + 6 = 4x$ $2x + 6 = -4x$

$\boxed{x = 3}$ $x = -1$

"Check for extraneous solutions"

If $x = 3$:

$|2(3) + 6| = 4(3)$

$12 = 12$

If $x = -1$:

$|2(-1) + 6| = 4(-1)$

$4 \neq -4$

c) $|x + 5| = -x + 3$

$x + 5 = -x + 3$ OR $x + 5 = -(-x + 3)$

$2x = -2$ $x + 5 = x - 3$

$\boxed{x = -1}$ $5 \neq -3$

Check answer:

$|(-1) + 5| = -(-1) + 3$

$4 = 4$

d) $|x - 2| = |5 + x|$

$+\ +$ $x - 2 = 5 + x$

$-2 = 5$ NO solution

$+\ -$ $x - 2 = -(5 + x)$

$3 = -2x$ $x = -3/2$

$-\ +$ $-(x - 2) = 5 + x$

$-2x = 3$ $\boxed{x = -3/2}$

$-\ -$ $-(x - 2) = -(5 + x)$

$-x + 2 = -x - 5$ NO solution

e) $2|x + 1| = 5|x + 1| - 12$

"Collect 'like' terms"

$-3|x + 1| = -12$

$|x + 1| = 4$

"Split and solve"

$x + 1 = 4$ $x + 1 = -4$

$\boxed{x = 3 \text{ or } -5}$

f) $\dfrac{2|x + 7|}{5} = 12$

$2|x + 7| = 60$

$|x + 7| = 30$

$x + 7 = 30$ $x + 7 = -30$

$\boxed{x = 23}$ OR $\boxed{x = -37}$

(If you check both answers, you'll see that they both work!)

g) $|3x + 9| = x + 1$

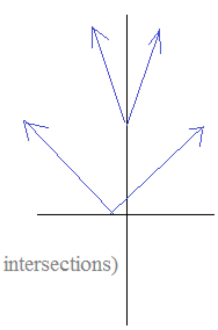
$3x + 9 = x + 1$ $3x + 9 = -(x + 1)$

$x = -4$ $3x + 9 = -x - 1$

$x = -\frac{5}{2}$

Both solutions are extraneous!

$\boxed{\text{NO SOLUTIONS}}$



(no intersections)

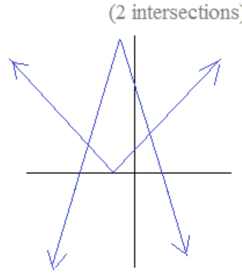
h) $3 - 10|x - 5| = -17$

$-10|x - 5| = -20$

$|x - 5| = 2$

$\boxed{x = 7 \text{ or } 3}$

(2 intersections)



i) $|x + 6| = -|2x + 10| + 20$

$+\ +$ $x + 6 = -(2x + 10) + 20$

$x + 6 = -2x + 10$

$\boxed{x = 4/3}$

$+\ -$ $x + 6 = -(-2x - 10) + 20$

$x + 6 = 2x + 30$ extraneous

$x = -24$

$-\ +$ $-(x + 6) = -2x - 10 + 20$

$-x - 6 = -2x + 10$ extraneous

$x = 16$

$-\ -$ $-(x + 6) = -(-2x - 10) + 20$

$-x - 6 = 2x + 30$

$\boxed{x = -12}$

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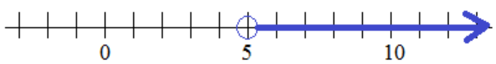
Quiz: Absolute Value, Inequalities, and the Number Line

SOLUTIONS

I. Graph the following on a number line

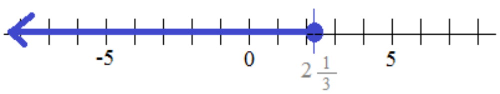
a) $x > 5$



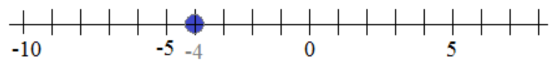
b) $2x \leq 4$ $x \leq 2$

c) $3x + 2 < 9$

subtract 2 from each side:
$3x < 7$
divide both sides by 3:
$x < 7/3$



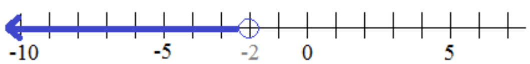
d) $x = -4$



e) $-3x > 6$

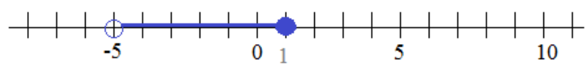
divide both sides by -3
(**flip the sign!)

$x < -2$



test points to check your answer!
try -5: $-3(-5) > 6$ $15 > 6$ yes
try 0: $-3(0) > 6$ $0 \not> 6$ no

f) $-5 < x \leq 1$

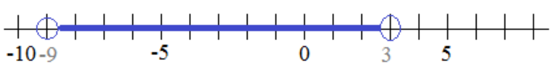


II. Absolute Values: Solve and graph the following

a) $|x + 3| < 6$

$x + 3 = 6$ $\boxed{x = 3}$
$x + 3 = -6$ $\boxed{x = -9}$

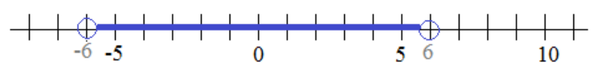
test pts: -10; $|-10 + 3| < 6$ no
0; $|0 + 3| < 6$ yes
5; $|5 + 3| < 6$ no



b) $|3x| < 18$

$3x = 18$ $x = 6$
$3x = -18$ $x = -6$

6 and -6 are critical points
since it is <, we use open circles.
(then, test a point in each region)

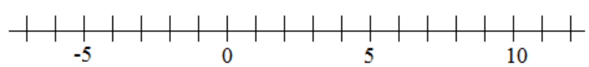


c) $2|x + 1| \geq 6$

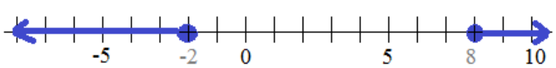
isolate the absolute value: (divide by 2)
$|x + 1| \geq 3$ then, find $x + 1 = 3$
$x + 1 = -3$
2 and -4

d) $|2x| + 8 < 4$

to isolate the absolute value, subtract 8
from both sides: $|2x| < -4$ $\boxed{\text{NO SOLUTION!!}}$
no absolue value is negative!



e) $|x - 3| + 4 \geq 9$



isolate the absolute value to solve:

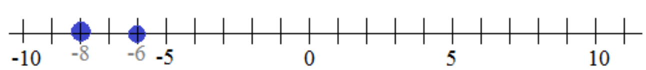
$|x - 3| \geq 5$

$x - 3 \geq 5$ $x \geq 8$
or
$x - 3 \leq -5$ $x \leq -2$

test pts to check answer:
-5: $|-5 - 3| + 4 \geq 9$ yes
0: $|0 - 3| + 4 \not\geq 9$ no
10: $|10 - 3| + 4 \geq 9$ yes

f) $|x + 7| = 1$

the points are
$x = -6$ or -8

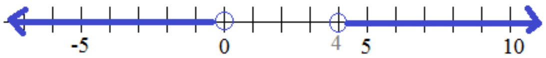


III. Solve and Graph

SOLUTIONS

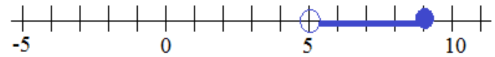
a) $x > 4$ or $x < 0$

note: all shaded values are EITHER greater than 4 OR less than 0



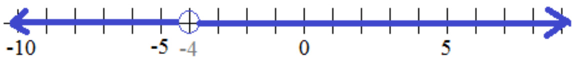
b) $x > 5$ and $x \le 9$

all shaded values are BOTH greater than 5 AND less than/equal to 9



c) $|x + 4| > 0$

at any value EXCEPT -4, the solution is greater than 0.. (at -4, it equals 0)

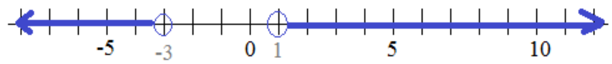


d) $2|x + 1| + 6 > 10$

isolate the absolute value: $2|x + 1| > 4$

$|x + 1| > 2$

then, solve...



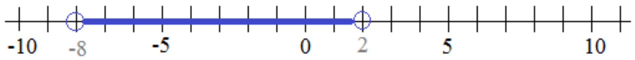
(test points in each region to check your answer)

e) $-|x + 3| > -5$

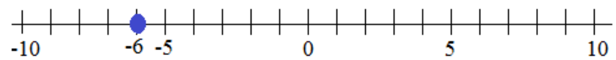
(divide by -1; flip sign)

$|x + 3| < 5$

$x < 2$
and
$x > -8$



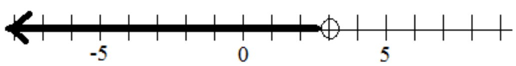
f) $|x + 6| \le 0$



At -6, the output is = 0;
at every other point, the absolute value is greater than 0..
therefore, there is only one solution..

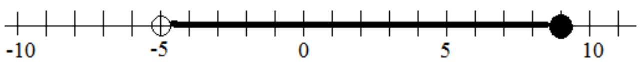
IV: Describe each graph with an equation

a)



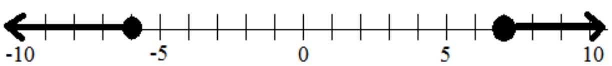
$x < 3$

b)



$-5 < x \le 9$

c)



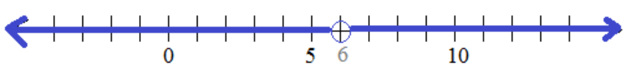
$x \ge 7$ or $x \le -6$

Also, an alternative answer (using absolute values): $|x - 1/2| \ge 6\ 1/2$

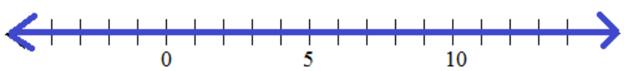
BONUS:

1) Graph the following on a number line:

a) $x \ne 6$



b) $x < 5$ or $x \ge -3$

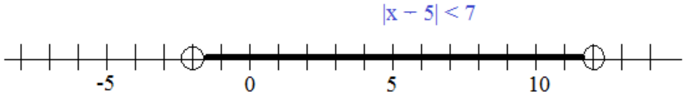


Any real number is either less than 5 or greater than -3!!

2) Use <u>Absolute Value Inequalities</u> to describe the following number line graphs:

a)

$|x - 5| < 7$



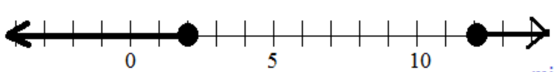
$-2 < x < 12$

midpoint of -2 and 12: 5
distance from 5 to either endpoint: 7 units

$|x - 5| < 7$

"the distance between x and 5 will be less than 7 units"

b)



$|x - 7| \ge 5$

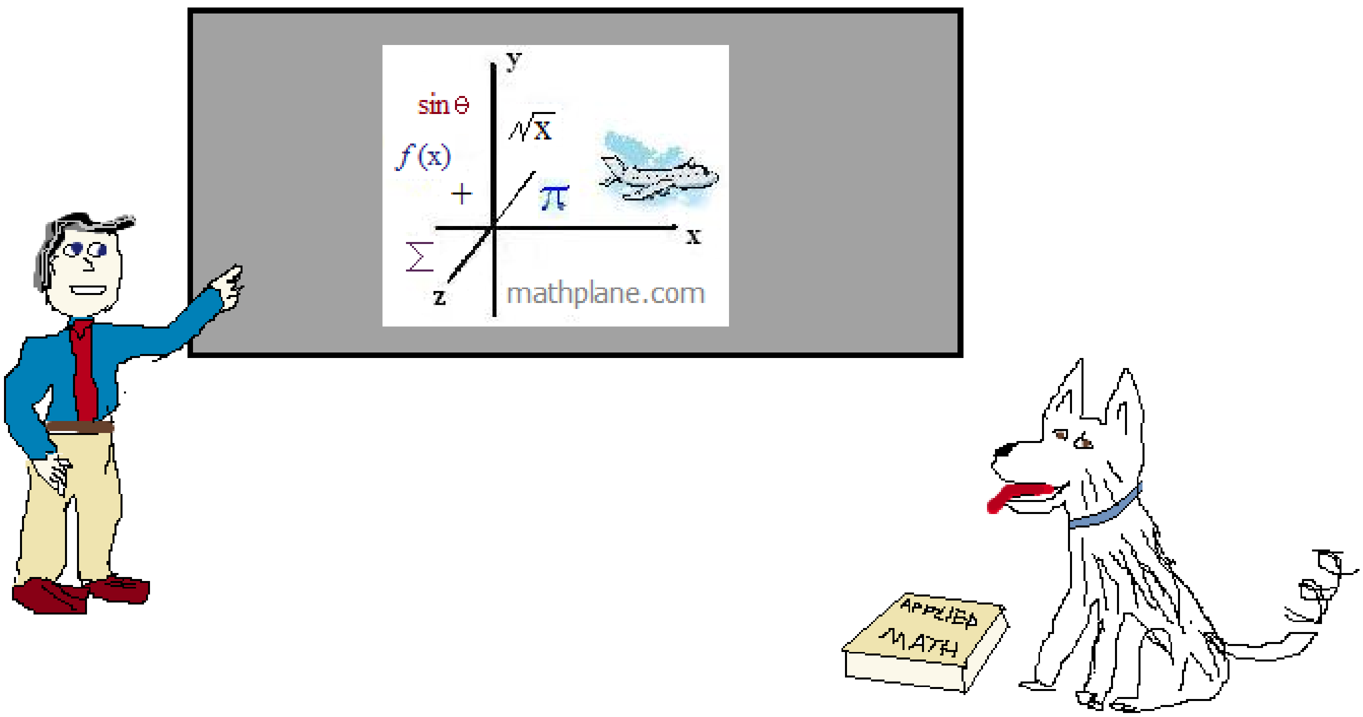
$x \le 2$ or $x \ge 12$

midpoint of 2 and 12: 7
distance from each endpoint: 5
"closed circles", arrows outward: $\ge$

Thanks for visiting. (Hope it helped!)

If you have questions, suggestions, or requests, let us know.

Cheers



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