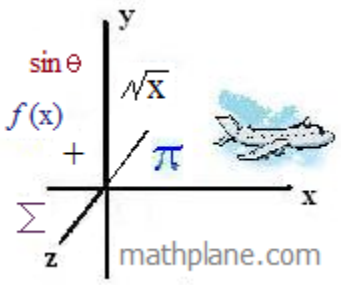
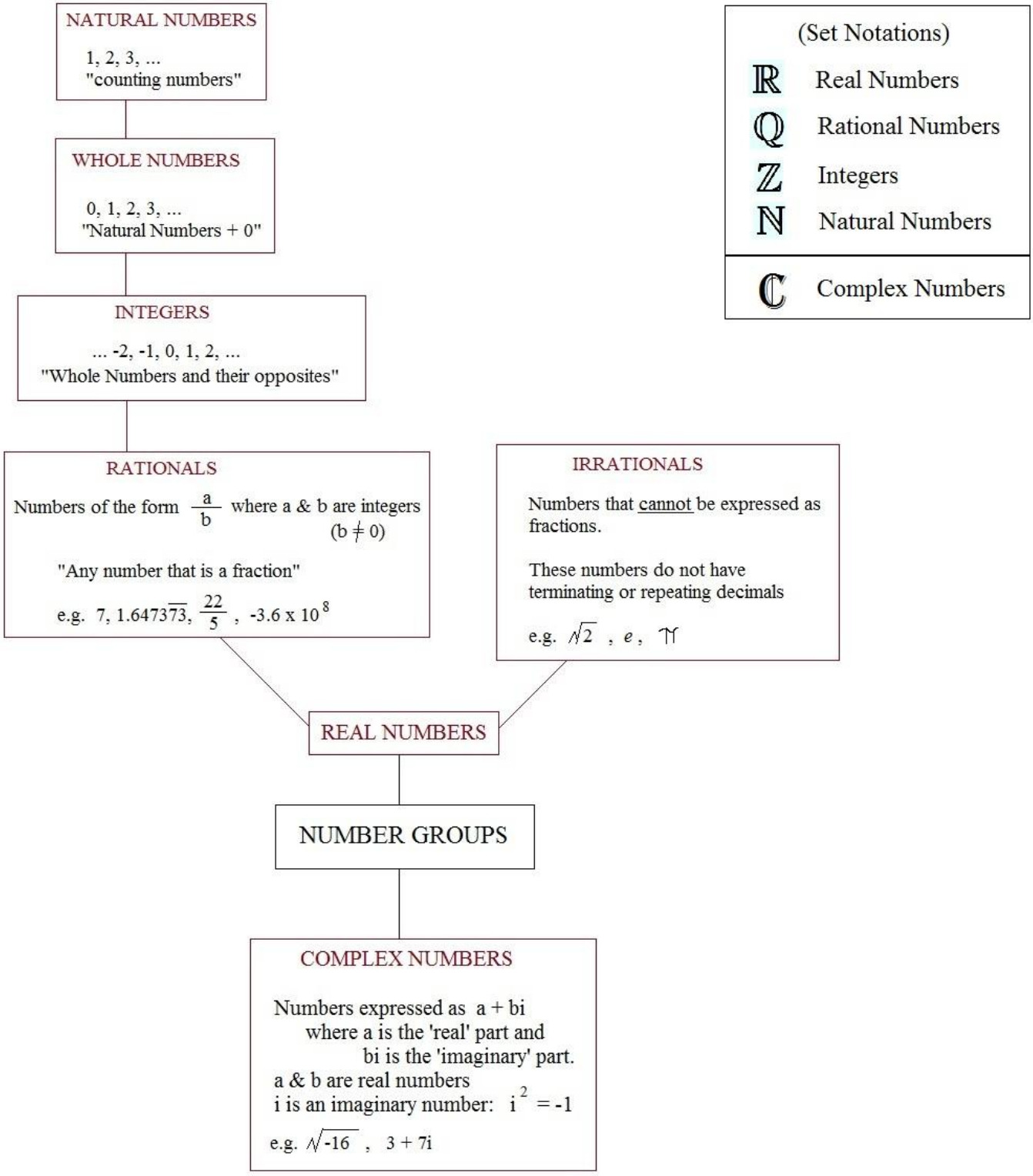
Number Classifications




NATURAL NUMBERS
1, 2, 3, ...
"counting numbers"
WHOLE NUMBERS
0, 1, 2, 3, ...
"Natural Numbers + 0"
INTEGERS
... -2, -1, 0, 1, 2, ...
"Whole Numbers and their opposites"
RATIONALS
Numbers of the form a/b where a & b are integers (b ≠ 0)
"Any number that is a fraction"
e.g. 7, 1.647373, 22/5, -3.6 x 10^8
IRRATIONALS
Numbers that cannot be expressed as fractions.
These numbers do not have terminating or repeating decimals
e.g. √2 , e , π
REAL NUMBERS
NUMBER GROUPS
COMPLEX NUMBERS
Numbers expressed as a + bi
where a is the 'real' part and
bi is the 'imaginary' part.
a & b are real numbers
i is an imaginary number: i^2 = -1
e.g. √-16 , 3 + 7i
(Set Notations)
ℝ Real Numbers
ℚ Rational Numbers
ℤ Integers
ℕ Natural Numbers
ℂ Complex Numbers

NUMBERS DEFINITIONS & EXAMPLES

PRIME NUMBER

A natural number ONLY divisible by itself or 1.
1 is not prime. 2 is the only even prime number.
e.g. 7, 23, 37

COMPOSITE NUMBER

A natural number that has a positive divisor other than 1 and itself.
e.g. 6, 9, 38

MIXED NUMBER

A mixed number is the sum of a fraction and a whole number.

e.g. $3\frac{4}{5}$ $2\frac{3}{7}$

RECIPROCAL

"A fraction with the numerator and denominator reversed"

e.g. 2/3 ---> 3/2
7 ---> 1/7

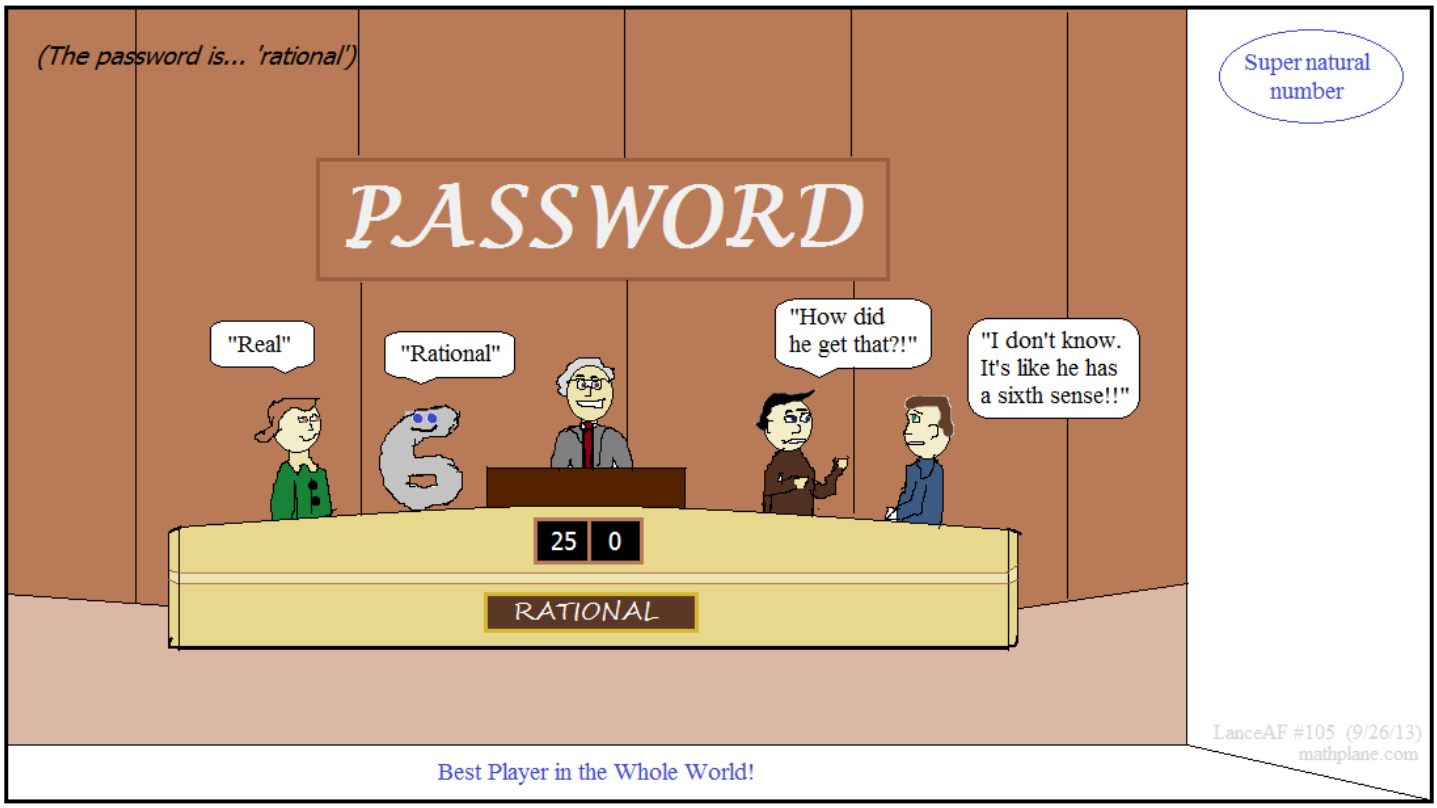
IDENTITY ELEMENTS

Additive Identity: 0 "Any number + 0 = the same number"
Multiplicative Identity: 1 "Any number x 1 = the same number"

PERFECT SQUARES

Any number that is the square of a rational number. (i.e. a "rational number times itself creates a perfect square")

e.g. 1, 4, 9, $\frac{9}{16}$, $\frac{100}{49}$



Practice Classifying Numbers…..

Place a check in any box that defines the number:

	natural	whole	integer	rational	irrational	real	imaginary
11							
$\sqrt{3}$							
-7							
$\frac{5}{2}$							
0							
$\sqrt{-9}$							
π							
$\frac{-8}{4}$							

What points (x, y) satisfy the equation $x + y = 10$, where x and y are *natural numbers*.

Answer always, sometimes, or never:

1) An integer is rational.

2) A whole number is natural.

3) A rational number is an integer.

4) An irrational number is a real number.

5) n and $-n$ are both natural numbers.

6) $\sqrt{x}$ is a real number.

7) a repeating decimal is a rational number.

Place a check in any box that defines the number:

Answers

	natural	whole	integer	rational	irrational	real	imaginary
11	X	X	X	X		X	
$\sqrt{3}$					X	X	
-7			X	X		X	
$\frac{5}{2}$				X		X	
0		X	X	X		X	
3i $\sqrt{-9}$							X
π					X	X	
-2 $\frac{-8}{4}$			X	X		X	

What points (x, y) satisfy the equation $x + y = 10$, where x and y are *natural numbers*.

natural numbers include: 1, 2, 3, 4, ...

(1, 9) (6, 4)
(2, 8) (7, 3)
(3, 7) (8, 2)
(4, 6) (9, 1)
(5, 5)

Answer always, sometimes, or never:

1) An integer is rational. Always.... (all integers are rational, because any integer can be expressed as n/1)

2) A whole number is natural. Sometimes.... 0 is whole, but NOT natural...

3) A rational number is an integer. Sometimes... 5 is rational integer... 1/2 is rational non-integer...

4) An irrational number is a real number. Always... (imaginary numbers are not rational nor irrational)

5) n and $-n$ are both natural numbers. Never... If n = 1, then -n = -1... natural numbers are positive..

6) $\sqrt{x}$ is a real number. Sometimes... If $x < 0$, then it is not...

7) a repeating decimal is a rational number. Always... Any repeating decimal can be expressed as a fraction -- ratio of 2 integers..

Vinculum (Bar) and "Repeating Decimals"

Vinculum: A horizontal bar drawn over multiple quantities to indicate they are grouped together.

Examples include: radicals $\sqrt{9X^3}$

line segments $\overline{AB}$ (joining points A & B)

repeating decimals $0.77\overline{676}$

Repeating Decimals: A decimal number that eventually becomes periodic (i.e. "the end repeats indefinitely")

Examples: $\frac{1}{3} = 0.333333\ldots = 0.\overline{3}$

$\frac{22}{7} = 3.142857142857\ldots = 3.\overline{142857}$

$12.0340353535\ldots = 12.034\overline{035}$

Converting Fractions to Decimals: Divide the numerator by the denominator

Examples: $\frac{42}{9}$

$$\begin{array}{r|l} & 4.6\overline{6} \\ 9 & 42 \\ & -36 \\ & \overline{60} \\ & -54 \\ & \overline{60} \\ & -54 \\ & \overline{} \\ & \text{etc....} \end{array}$$

$4.\overline{6}$

$\frac{3}{700}$

$$\begin{array}{r|l} & 0.00428571 \\ 700 & 3.0000000 \\ & -2.800 \\ & \overline{.2000} \\ & -.1400 \\ & \overline{6000} \\ & -5600 \\ & \overline{4000} \\ & -3500 \\ & \overline{5000} \\ & -4900 \\ & \overline{1000} \\ & -\ 700 \\ & \overline{3000}\ \text{etc...} \end{array}$$

$0.00\overline{428571}$

(repeats indefinitely)

Converting 'Repeating Decimals' to Fractions: Using algebra

Examples: $.\overline{7}$ let $n = .\overline{77}$ then, $10n = 7.\overline{77}$

$$\begin{array}{rr} 10n & 7.\overline{77} \\ -\ n & -.\overline{77} \\ \hline 9n & 7.0 \end{array}$$

substitution reveals that $9n = 7$

$n = \frac{7}{9}$

$11.\overline{18}$ let $m = .\overline{18}$ then, $100m = 18.\overline{18}$

$$\begin{array}{rr} 100m & 18.\overline{18} \\ -\ m & -\ .\overline{18} \\ \hline 99m & 18 \end{array}$$

$11.\overline{18} = 11 + m$

$m = \frac{18}{99}$

$11.\overline{18} = 11\frac{18}{99}$

$234.0017\overline{676}$

Separate the number into parts: $234 + .001 + .00076\overline{76}$

Convert the parts to fractions: $234 + .001 = \frac{1}{1000} + \frac{99}{99000}$

let $p = .0007676\overline{76}$

$1000p = .76\overline{76}$

$100000p = 76.\overline{76}$

$$\begin{array}{rr} 100000p & 76.\overline{76} \\ -\ 1000p & -\ .\overline{76} \\ \hline 99000p & 76 \end{array}$$

$p = \frac{76}{99000}$

Combine the Fractions: $234 + \frac{99}{99000} + \frac{76}{99000} = 234\frac{175}{99000}$

$.00076\overline{76} = \frac{76}{99000}$

Specifying number sets

Write the solution set:

1) $5x + 3 = 14$ $x \in$ {real numbers}

2) $3x + 7 = 20$ $x \in$ {integers}

3) $|3x + 4| > 10$ $x \in$ {natural numbers}

4) $|4x + 3| = 9$ $x \in$ {positive numbers}

5) $-5 < x + 2 \leq 7$ $x \in$ {whole numbers}

6) $x^2 = 64$ $x \in$ {real numbers}

SOLUTIONS

1) $5x + 3 = 14$ $x \in$ {real numbers}

$5x = 11$

$x = 11/5$ {11/5}

2) $3x + 7 = 20$ $x \in$ {integers}

$3x = 13$

$x = 13/3$ But, 13/3 is not an integer! $\emptyset$

3) $|3x + 4| > 10$ $x \in$ {natural numbers}

$3x + 4 > 10$ or $3x + 4 < -10$

$3x > 6$ $3x < -14$

$x > 2$ $x < -14/3$ {3, 4, 5, ...}

Since natural numbers are positive integers, the solution is 3, 4, 5, 6, ...

4) $|4x + 3| = 9$ $x \in$ {positive numbers}

$4x + 3 = 9$ or $4x + 3 = -9$

$4x = 6$ $4x = -12$ {3/2}

$x = 3/2$ ~~$x = -3$~~

5) $-5 < x + 2 \leq 7$ $x \in$ {whole numbers}

$-5 < x + 2$ and $x + 2 < 7$ {0, 1, 2, 3, 4, 5}

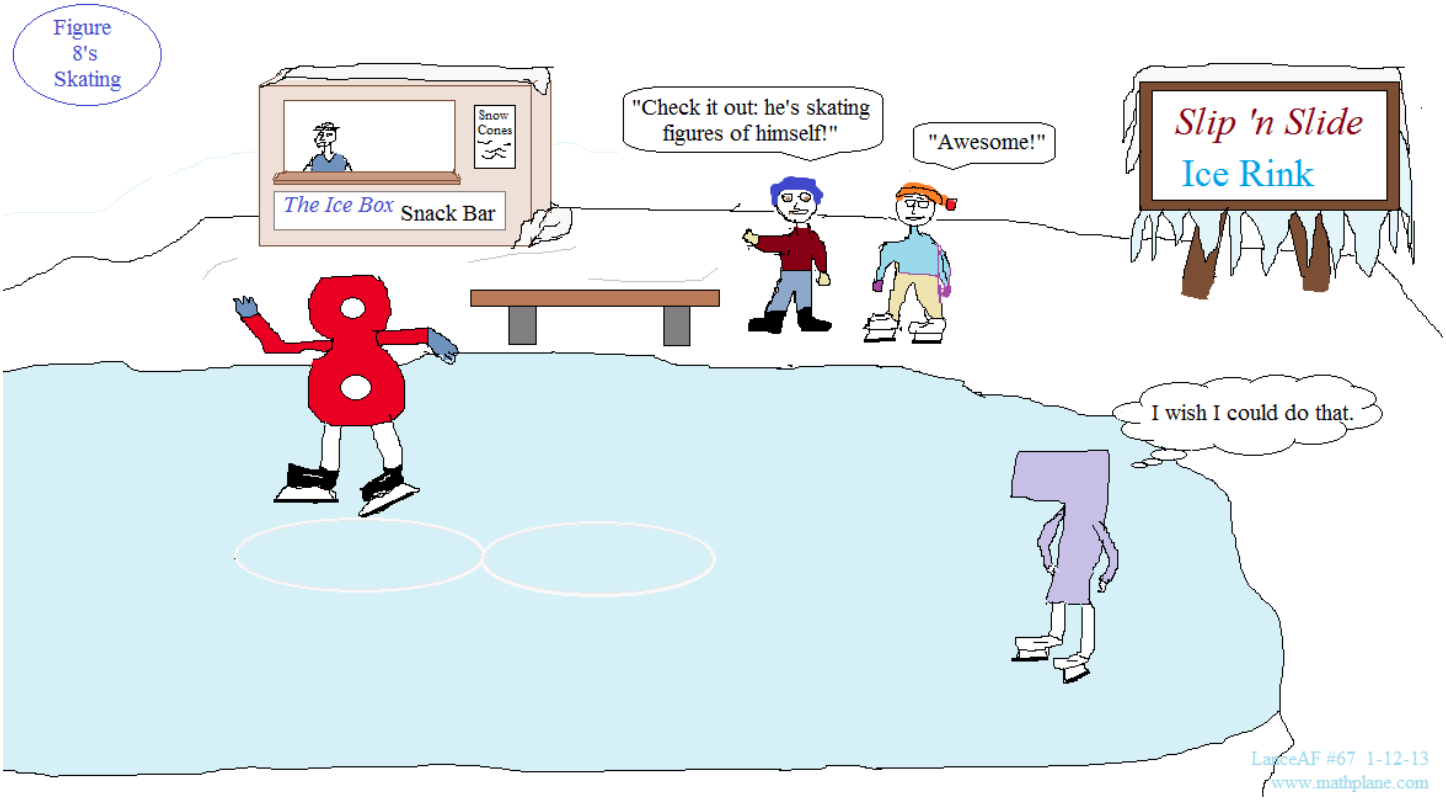
$-7 < x$ and $x \leq 5$

and, x must be a whole number!

6) $x^2 = 64$ $x \in$ {real numbers}

(square root both sides) {-8, 8}

$x = 8$ or -8



Quick Quiz (with solutions)

Nine Number Questions

1) List the 5 smallest prime numbers.

2) Is the product of two prime numbers ever prime?

3) {perfect squares < 100} $\cap$ {odd numbers}

4) How many rational numbers are between 2 and 9?

5) List the 4 integers closest to 1 on the number line.

6) Are natural numbers a subset of integers?

7) What is the product of a number and its reciprocal?

8) Give 3 examples of irrational numbers.

9) Express $.29\overline{29}$ as a fraction.

Nine Number Questions

ANSWERS

1) List the 5 smallest prime numbers.

2, 3, 5, 7, 11

2) Is the product of two prime numbers ever prime?

No.. The product of 2 primes will always produce a number which has a factor other than itself and one.

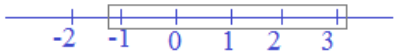
3) {perfect squares < 100} ∩ {odd numbers}

{1, 4, 9, 16, 25, 36, 49, 64, 81} ∩ {1, 3, 5, 7, 9, ... } = {1, 9, 25, 49, 81}

4) How many rational numbers are between 2 and 9?

There are an infinite number of rational numbers! notice: 2.001 2.00001 2.1001 are examples.. There are countless others..

5) List the 4 integers closest to 1 on the number line.



-1, 0, 2, and 3

6) Are natural numbers a subset of integers?

yes, every natural number is an integer..

7) What is the product of a number and its reciprocal?

one $a \cdot \frac{1}{a} = 1$

8) Give 3 examples of irrational numbers.

$\sqrt{2}$ e pi

9) Express $.2\overline{929}$ as a fraction.

Let $n = .2\overline{929}$

Then, let $100n = 29.2\overline{929}$

$100n - n = 99n$

Substitution: $29.2\overline{929} - .2\overline{929} = 29$

So, $99n = 29$

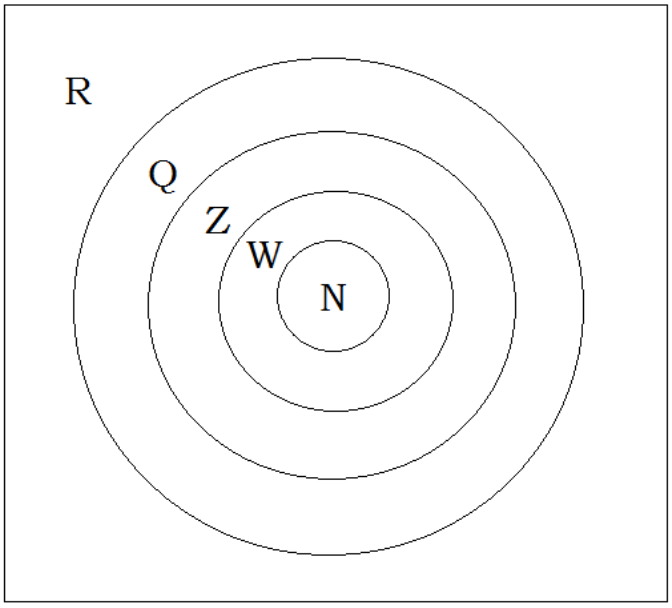
$n = \frac{29}{99}$

Quick review of Number Classes

Natural Numbers	"Count to 10 and Beyond"	1, 2, 3, 4, 5, ...	$\mathbb{N}$
Whole Numbers	"Add a zero to the Naturals"	0, 1, 2, 3, 4, 5, ...	$\mathbb{W}$
Integers	"Add the opposites to the Wholes"	.... -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, ...	$\mathbb{Z}$
Rational Numbers	"Use the Integers to make Fractions"	-.65,... -1/3, ..., 0, ... , 1/2, 3, 5.6, ...	$\mathbb{Q}$
Real Numbers	"Add the rest of the numbers on the number line"	$-\sqrt{3}$, ... -1, ... , 2, e, ... 8/3, ...	$\mathbb{R}$
Complex Numbers	"Add a perpendicular line to the number line"	$a + bi$ where a is any real number, and, b is any real number except 0 and, i is $\sqrt{-1}$	$\mathbb{C}$

$$\mathbb{R} - \mathbb{Q} = \mathbb{P}$$

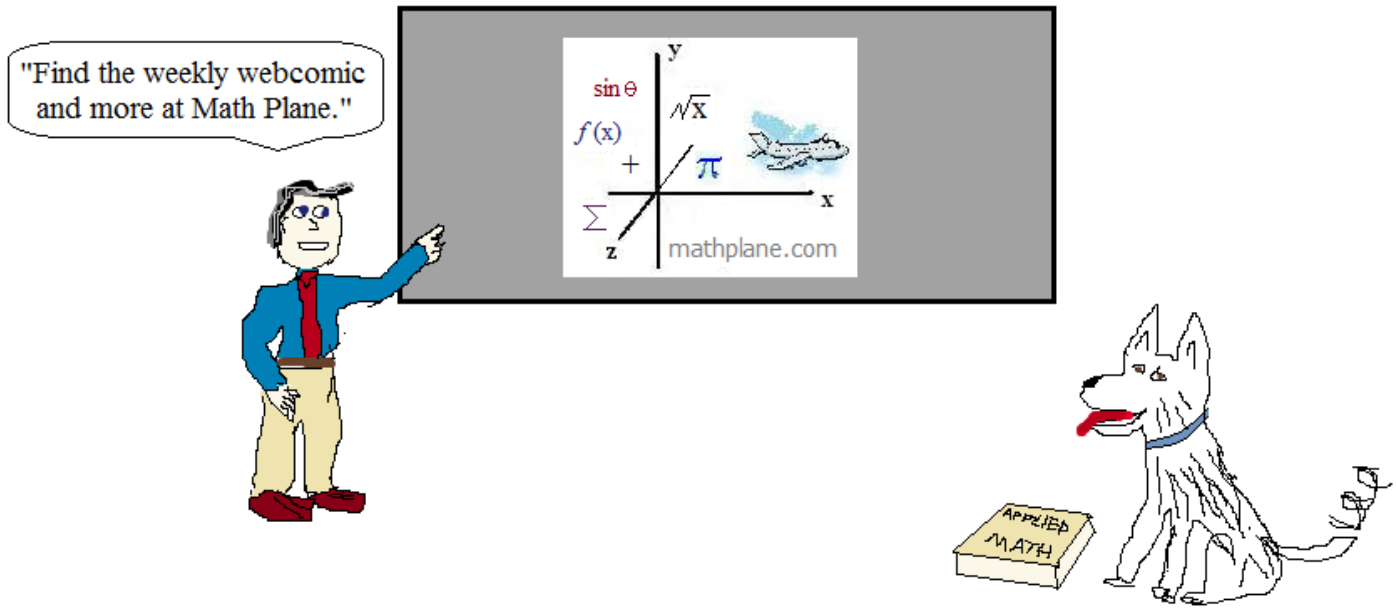
Reals + Rationals = Irrationals



Thanks for visiting. (Hope it helped!)

If you have questions, suggestions, or requests, let us know.

Cheers



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