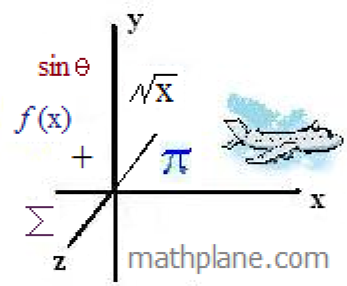
Factoring Quadratics

Introduction with notes, examples, and practice tests (with solutions)



Topics include linear binomials, greatest common factor (GCF), "when lead coefficient is > 1", quadratic formula and more.

Factoring Quadratics

Steps:
1) Find Greatest Common Factor
2) If Binomial, consider Difference of Squares
3) Search for 2 Linear Binomials
   EX: $(X + 3)(X - 5)$
4) Use Quadratic Formula
5) Check your Results!

Definitions:

Quadratic: A polynomial of degree 2

$$AX^2 + BX + C$$

Binomial: A polynomial with 2 terms (that are not 'like terms')

$X + 4$ $\quad$ $-4X + 3Y^3$ $\quad$ $\cancel{3X + (-5X)}$ $\quad$ $X^2 - 3X$

Linear Binomial: A binomial of degree 1

$(X - 6)$ $\quad$ $(-Y + 7)$ $\quad$ $\cancel{(X^2 + 2)}$

Examples:

Greatest Common Factor

$5X^2 + 20X \longrightarrow$ GCF is $5X \longrightarrow 5X(X + 4)$

Difference of Squares

$X^2 - 16$ $\quad$ X^2 and 16 are perfect squares $\quad$ $\sqrt{X^2} = X$ $\quad$ $\sqrt{16} = 4$

$(X + 4)(X - 4)$

(Square root of first term PLUS square root of second term) x (Square root of first term MINUS square root of second term)

2 Linear Binomials

$X^2 + 10X + 21$

$A = 1$
$B = 10$
$C = 21$

$3 \times 7 = 21$
$3 + 7 = 10$

$(X + 3)(X + 7)$

(If coefficient of first term is 1,) find 2 numbers whose product is the constant and whose sum is the coefficient of the middle term

Quadratic Formula

$X^2 + 6X - 10$

$A = 1$
$B = 6$
$C = -10$

$$\frac{-6 \pm \sqrt{36 - (-40)}}{2}$$

$-3 + \sqrt{19}$, $-3 - \sqrt{19}$

$(X + 3 - \sqrt{19})(X + 3 + \sqrt{19})$

Quadratic Formula

$$X = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

Methods of factoring quadratics: Examples

1)

$X^2 + 7X - 60$

$(X - 5)(X + 12)$

Find 2 numbers whose SUM is 7
and whose PRODUCT is -60

12 and -5

NOTE: when you're unable to figure out the factors, just use the quadratic formula!

2)

$6X^2 + 13X + 5$

$(? \ + 1)(? \ + 5)$

$(2X + 1)(3X + 5)$

Since the polynomial is + +, the terms will be + +
And, since the constant is 5, we hope the terms will be 1 and 5

Then, trial and error, we insert numbers whose product is 6...

3)

$3X^2 + 12X - 15$

$3(X^2 + 4X - 5)$

$3(X + 5)(X - 1)$

Greatest common factor is 3.. this will simplify the factoring process...

Now, find 2 numbers whose product is -5 and whose sum is 4

We get 5 and -1

4)

$X^2 + 7X = 0$

$X(X + 7) = 0$

$X = 0$
$X = -7$

Greatest Common Factor -- X

5)

$2X^2 + 13X + 15 = 0$

$(2X + \ ?)(X + \ ?) = 0$

$(2X + 3)(X + 5) = 0$

$2x + 3 = 0 \quad X = -3/2$
$X + 5 = 0 \quad X = -5$

Again, the polynomial is + +.
So, the terms should be + +

Since the coefficient of the first term is 2, we hope the factors will be 1X and 2X...

then, we try numbers whose product is 15...

Once we get the factors, we set each = to 0.. Solve...

Finally, check your answers...

$2(25) + 13(-5) + 15 = 0$ ✓

$2(9/4) + 13(-3/2) + 15 =$
$18/4 - 39/2 + 15 = 0$ ✓

Factoring Quadratics Practice Quiz 1

(w/ solutions)

Factoring Quadratics Quiz

Part I: Greatest Common Factor

Factor: 1) $X^2 + 3X$ 2) $12X^2 - 6XY$ 3) $14 - 7Z^3$

Solve: 4) $2Y^2 - 6Y = 0$ 5) $X^2 + 5X = 0$ 6) $4X^3 = 8X$

Part II: "Two Linear Binomials"

Factor: 1) $X^2 + 3X + 2$ 2) $X^2 - 7X + 6$ 3) $Y^2 + 5Y - 36$

Solve: 4) $X^2 + 11X - 26 = 0$ 5) $Y^2 - 5Y - 14 = 0$ 6) $Z^2 + 4Z = 5$

Factoring Quadratics Quiz (Continued)

Part III: Difference of Squares

Factor: 1) $X^2 - 36$ 2) $4Y^2 - 9Z^2$ 3) $X^2 + 4$

Solve: 4) $X^2 - 25 = 0$ 5) $4Z^2 - 25 = 0$ 6) $3X^2 + 2 = 11 - X^2$

Part IV: Quadratic Formula

Factor (using the quadratic formula)

1) $X^2 - 13X - 30$ 2) $4Y^2 + 17Y - 15$ 3) $3Z^2 - 13$

Solve (using the quadratic formula)

4) $X^2 + 7X - 60 = 0$ 5) $X^2 - 4X - 18 = 0$ 6) $5Z^2 + 6Z = 11$

Factoring Quadratics Quiz

SOLUTIONS

Part I: Greatest Common Factor

Factor:

1) $X^2 + 3X$

$X(X + 3)$

2) $12X^2 - 6XY$

$6X(2X - Y)$

3) $14 - 7Z^3$

$7(2 - Z^3)$

Solve:

4) $2Y^2 - 6Y = 0$

$2Y(Y - 3) = 0$

$Y = 0, 3$

Quick check: plug solutions into <u>original</u> equation!

$2(0)^2 - 6(0) = 0$ ✓ $2(3)^2 - 6(3) = 0$

$18 - 18 = 0$ ✓

5) $X^2 + 5X = 0$

$X(X + 5) = 0$

$X = 0, -5$

6) $4X^3 = 8X$

$4X^3 - 8X = 0$

$4X(X^2 - 2) = 0$

$4X = 0$

$X^2 - 2 = 0$

$\boxed{X = 0 \quad X = \sqrt{2} \quad -\sqrt{2}}$

Part II: "Two Linear Binomials"

Factor:

1) $X^2 + 3X + 2$

Note the signs: $+ \quad +$

$(X + 2)(X + 1)$

2) $X^2 \boxed{-} 7X \boxed{+} 6$

signs will be the same

must be $- \quad -$

$(X - 6)(X - 1)$

3) $Y^2 \boxed{+} 5Y \boxed{-} 36$

signs will be different

and, the larger number is +

$(Y + 9)(Y - 4)$

Solve:

4) $X^2 + 11X - 26 = 0$

$(X + 13)(X - 2) = 0$

$X + 13 = 0$

$X - 2 = 0 \qquad X = 2, -13$

to check: plug in solutions!

$(2)^2 + 11(2) - 26 = 4 + 22 - 26 = 0$ ✓

$(-13) + 11(-13) - 26 = 169 - 143 - 26$

$= 0$ ✓

5) $Y^2 \boxed{-} 5Y \boxed{-} 14 = 0$

must be $+ \quad -$

and, the larger number is $-$

$(Y - 7)(Y + 2) = 0$

$Y = 7, -2$

6) $Z^2 + 4Z = 5$

$Z^2 + 4Z - 5 = 0$

$(Z + 5)(Z - 1) = 0$

$Z = -5, 1$

Quick check: $(1)^2 + 4(1) = 5$

$1 + 4 = 5$ ✓

$(-5)^2 + 4(-5) = 5$

$25 - 20 = 5$ ✓

Factoring Quadratics Quiz (Continued)

SOLUTIONS

Quadratic Formula: If $ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Part III: Difference of Squares

Factor: 1) $X^2 - 36$

$(X + 6)(X - 6)$

2) $4Y^2 - 9Z^2$

$\sqrt{4Y^2} = 2Y$

$\sqrt{9Z^2} = 3Z$

$(2Y + 3Z)(2Y - 3Z)$

3) $X^2 + 4$

PRIME
Cannot be factored..
(Note: It is NOT a difference of squares)

Solve: 4) $X^2 - 25 = 0$

$(X + 5)(X - 5) = 0$

$\boxed{X = 5, -5}$

5) $4Z^2 - 25 = 0$

$(2Z + 5)(2Z - 5) = 0$

$2Z + 5 = 0 \quad \boxed{Z = -5/2}$

$2Z - 5 = 0 \quad \boxed{Z = 5/2}$

6) $3X^2 + 2 = 11 - X^2$

(move terms to left side) $4X^2 - 9 = 0$

(factor) $(2X + 3)(2X - 3) = 0$

(solve) $\boxed{X = -3/2 \quad X = 3/2}$

(check solutions) $3(-3/2)^2 + 2 = 11 - (-3/2)^2$

$3(9/4) + 2 = 11 - (9/4)$

$35/4 = 35/4$ ✓

$3(3/2)^2 + 2 = 11 - (3/2)^2$

$35/4 = 35/4$ ✓

Part IV: Quadratic Formula

Factor (using the quadratic formula)

1) $X^2 - 13X - 30$

a = 1
b = -13
c = -30

$$x = \frac{-(-13) \pm \sqrt{(-13)^2 - 4(1)(-30)}}{2(1)}$$

$$x = \frac{13 \pm \sqrt{169 + 120}}{2}$$

$\frac{13 + 17}{2} = 15 \qquad \frac{13 - 17}{2} = -2 \qquad \boxed{(X + 2)(X - 15)}$

2) $4Y^2 + 17Y - 15$

a = 4
b = 17
c = -15

$$y = \frac{-17 \pm \sqrt{(17)^2 - 4(4)(-15)}}{2(4)}$$

$$= \frac{-17 \pm \sqrt{289 + 240}}{8}$$

$= \frac{-17 + 23}{8} = 3/4$

and, $\frac{-17 - 23}{8} = -5$

$(Y + 5)(Y - 3/4)$

or

$\boxed{(Y + 5)(4Y - 3)}$

3) $3Z^2 - 13$

a = 3
b = 0
c = -13

$$\frac{0 \pm \sqrt{(0)^2 - 4(3)(-13)}}{2(3)}$$

$$\frac{\pm\sqrt{156}}{6} = \frac{\pm 2\sqrt{39}}{6}$$

$\boxed{(Z + \sqrt{39}/3)(Z - \sqrt{39}/3)}$

or

$(Z + \sqrt{13/3})(Z - \sqrt{13/3})$

Solve (using the quadratic formula)

4) $X^2 + 7X - 60 = 0$

$$X = \frac{-7 \pm \sqrt{(7)^2 - 4(1)(-60)}}{2(1)}$$

$= \frac{-7 + 17}{2} \qquad \frac{-7 - 17}{2}$

$\boxed{X = 5, -12}$

Quick check:

$(5)^2 + 7(5) - 60 =$
$25 + 35 - 60 = 0$ ✓

$(-12)^2 + 7(-12) - 60 =$
$144 - 84 - 60 = 0$ ✓

5) $X^2 - 4X - 18 = 0$

$$X = \frac{4 \pm \sqrt{(-4)^2 - 4(1)(-18)}}{2(1)}$$

$= \frac{4 \pm \sqrt{88}}{2} = \frac{4 \pm 2\sqrt{22}}{2}$

$= \boxed{2 + \sqrt{22},\ 2 - \sqrt{22}}$

6) $5Z^2 + 6Z = 11$

$5Z^2 + 6Z - 11 = 0$

a = 5
b = 6
c = -11

$$Z = \frac{-6 \pm \sqrt{(6)^2 - 4(5)(-11)}}{2(5)}$$

$= \frac{-6 + 16}{10} \qquad \frac{-6 - 16}{10}$

$\boxed{Z = 1, \frac{-11}{5}}$

Factoring Quadratic Trinomials when $A \neq 1$

General Form of Quadratic: $Ax^2 + Bx + C$

Distribute and grouping method

Steps:
1) Write Ax^2 in first binomial
   Write C in second binomial
2) List factor pairs of AC
3) Choose pair that adds up to B
4) Distribute the Bx term to the binomials
5) Factor and regroup

Logic method

Steps:
1) Consider the possible factors of A and C
2) Recognize the signs
3) Select values that add up to middle term

Example 1: $2x^2 + 11x + 5$

Method 1: $(2x^2 + \quad) + (\quad + 5)$

Factor pairs of AC $(2 \times 5 = 10)$

1	10	11
-1	-10	-11
2	5	7
-2	-5	-7

$(2x^2 + 1x) + (10x + 5)$

$x(2x + 1) + 5(2x + 1)$

$\boxed{(2x + 1)(x + 5)}$

Method 2: Since A is 2, its only factors are 1 and 2.

$(2x \quad\quad)(1x \quad\quad)$

The signs are + + $\quad 2x^2 \boxed{+} 11x \boxed{+} 5$

$(2x + \quad)(x + \quad)$

Since C is 5, its only factors are 1 and 5.

$\cancel{(2x + 5)(x + 1) = 2x^2 + 5x + 2x + 5}$

$\boxed{(2x + 1)(x + 5)} = 2x^2 + x + 10x + 5$

Example 2: $2x^2 - 11x + 12$

Method 1: Distribute and regroup

$(2x^2 \quad) + (\quad + 12)$

Factor pairs of AC (24)

1	24
-1	-24
2	12
-2	-12
3	8
-3	-8
4	6
-4	-6

multiplies to 24 and adds to -11

$(2x^2 + (-8)x) + ((-3x) + 12)$

$2x(x - 4) + (-3)(x - 4)$

$\boxed{(2x - 3)(x - 4)}$

FOIL to check:

First	Outer	Inner	Last
$2x^2$	-8x	-3x	12

$2x^2 - 11x + 12$ ✓

Method 2: Logic

Since A = 2 (a prime number), there are only 2 factors

$(2x \quad\quad)(x \quad\quad)$

The signs are − + $\quad 2x^2 \boxed{-} 11x \boxed{+} 12$

therefore, the factors will be negative!

$(2x - \quad)(x - \quad)$

Finally, we consider factors of 12 that'll fit..

$\cancel{(2x - 4)(x - 3) = 2x^2 - 3x - 4x + 12}$ close, but not correct...

$\boxed{(2x - 3)(x - 4)} = 2x^2 - 3x - 8x + 12$ ✓

Factoring Quadratic Trinomials when A $\neq$ 1

Solve the following:

$21x^2 - 20x - 9 = 0$ $\qquad$ $20x^2 - 19x + 3 = 0$ $\qquad$ $6x^2 + 11x = 10$

$21x^2 - 20x - 9 = 0$

Distribute and regroup: $\quad (21x^2 + \quad) + (\quad - 9) = 0$

Factor pairs of -189 (21 x -9)

1	-189
-1	189
3	-63
-3	63
$\boxed{7}$	$\boxed{-27}$
-7	27
9	-21
-9	21

multiply to -189
and add to
-20

$(21x^2 + 7x) + (-27x - 9) = 0$

$7x(3x + 1) + (-9)(3x + 1) = 0$

$(7x - 9)(3x + 1) = 0$

$\boxed{x = \frac{9}{7} \quad \text{or} \quad \frac{-1}{3}}$

Check: (plug into original equation)

$21\left(\frac{9}{7}\right)^2 - 20\left(\frac{9}{7}\right) - 9 = 0$

$\frac{21(81)}{49} - \frac{180}{7} - 9 = 0$

$\frac{243}{7} - \frac{180}{7} - \frac{63}{7} = 0$

$21\left(\frac{-1}{3}\right)^2 - 20\left(\frac{-1}{3}\right) - 9 = 0$

$\frac{21}{9} + \frac{60}{9} - \frac{81}{9} = 0$

$20x^2 - 19x + 3 = 0$

Logic Method: Since C = 3, a prime number, there are only 2 factors, 1 and 3..

$(\quad 1)(\quad 3)$

The signs are $-$ $+$ $\qquad 20x^2 \boxed{-} 19x \boxed{+} 3$

Therefore, the factor signs will be $-$ $-$

$(\quad -1)(\quad -3)$

Considering factors 1/20 2/10 4/5,
I'll try 4 and 5...

$(4x - 1)(5x - 3) = 20x^2 - \cancel{12x} + 3$

$\boxed{(5x - 1)(4x - 3) = 20x^2 - 19x + 3}$

Finally, solve:

$(5x - 1)(4x - 3) = 0$

5x - 1 = 0
4x - 3 = 0

$\boxed{\begin{matrix} x = 1/5 \\ x = 3/4 \end{matrix}}$

Check: (Plug into original equation)

$20(1/5)^2 - 19(1/5) + 3 =$

$\frac{4}{5} - \frac{19}{5} + \frac{15}{5} = 0$

$20(3/4)^2 - 19(3/4) + 3 =$

$\frac{45}{4} - \frac{57}{4} + \frac{12}{4} = 0$

$6x^2 + 11x = 10$

(Put equation into general form)

$6x^2 + 11x - 10 = 0$

$(6x^2 \quad) + (\quad - 10)$

Factors of -60

1	-60
-1	60
2	-30
-2	30
3	-20
-3	20
4	-15
$\boxed{-4}$	$\boxed{15}$

$(6x^2 + (-4x)) + ((15x) + 10) = 0$

$2x(3x - 2) + 5(3x + 2) = 0$

$(2x + 5)(3x - 2) = 0$

$\boxed{x = -\frac{5}{2}, \ \frac{2}{3}}$

Quick check:

$6(-5/2)^2 + 11(-5/2) = 10$

$6(25/4) - 55/2 = 10$

$20/2 = 10$

$6(2/3) + 11(2/3) = 10$

$6(4/9) + 22/3 = 10$

$24/9 + 66/9 = 10$

Another method of factoring: "Slide and Divide"

Example: Factor $10x^2 - 7x - 120$

"Slide"

$\boxed{10}x^2 - 7x - 120$ Remove the lead coefficient..

10 $x^2 - 7x - 120$ Factor the quadratic...

$(x - 15)(x + 8)$

"Divide" $(x - \frac{15}{10})(x + \frac{8}{10})$

$(x - \frac{3}{2})(x + \frac{4}{5}) \longrightarrow (2x - 3)(5x + 4)$

Example: $5x^2 + 37x - 72$ 5 x -72 = -360

What multiplies to -360 and adds to 37

$\boxed{5}x^2 + 37x - 72$ $+45$ and -8

slide $\longrightarrow$ $(x + 45)(x - 8)$

and

divide by 5

$(x + \frac{45}{5})(x - \frac{8}{5})$

$(x + 9)(x - \frac{8}{5}) \longrightarrow (x + 9)(5x - 8)$

Example: $7x^2 + 38x + 40$

Multiply the A and C values (i.e. the lead coefficient and the constant)

7 x 40 = 280

Find the factors (i.e. numbers that multiply to 280 and add to 38)

28 and 10

$(x + 10)(x + 28)$

Divide by the lead coefficient $\longrightarrow$ Simplify and rearrange...

$(x + \frac{10}{7})(x + \frac{28}{7})$

$(x + 10/7)(x + 4)$

$(7x + 10)(x + 4)$

Factoring quadratics: 4 methods

Example: $3x^2 - 14x - 5 = 0$

"Slide and Divide"

slide: $\boxed{3}x^2 - 14x - 5 = 0$

$x^2 - 14x - 15 = 0$

factor: $(x - 15)(x + 1) = 0$

divide: $(x - \frac{15}{3})(x + \frac{1}{3}) = 0$

simplify/ solve: $(x - 5)(3x + 1) = 0$

$x = 5, \frac{-1}{3}$

"Quadratic Formula"

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

a = 3
b = -14
c = -5

$$\frac{14 \pm \sqrt{196 - (-60)}}{6}$$

$$\frac{14 \pm 16}{6} = 5 \text{ or } -1/3$$

"Split and Regroup"

Split: What multipies to -15 and adds to -14?

-15 and 1

$3x^2 - 14x - 5 = 0$

$3x^2 - 15x + 1x - 5 = 0$

$3x(x - 5) + 1(x - 5) = 0$

Regroup: $(3x + 1)(x - 5) = 0$

$x = 5, \frac{-1}{3}$

"Logic Method"

$3x^2 - 14x - 5 = 0$

Since the lead coefficient is 3, the factors may be 1 and 3

$(x \quad)(3x \quad)$

Since the constant is negative 5, the sign will be different and the terms will be 1 and 5....
So, test 1, -5 -5, 1 -1, 5 and 5, -1

$(x - 1)(3x + 5)$ the middle term is 2x... try another.

$(x + 5)(3x - 1)$ the middle term is 14x... closer..

$(x - 5)(3x + 1)$ the middle term is -14x ✓

$(x - 5)(3x + 1) = 0$ $x = 5, \frac{-1}{3}$

Did you know?

If the discriminant is a perfect square, then you can find 2 linear binomials...

Example: $x^2 + 9x + 20$

$b^2 - 4ac$ $\quad (9)^2 - 4(1)(20)$

$81 - 80 = 1$

1 is a perfect square...

(both roots are rational) $\quad (x + 4)(x + 5)$

Example: $x^2 + 11x + 21$

$b^2 - 4ac$ $\quad (11)^2 - 4(1)(21)$

$121 - 84 = 37$

Since 37 is NOT a perfect square, you'll need to complete the square or use the quadratic formula to find factors/roots.

Example: $12x^2 - 7x - 10$

$b^2 - 4ac$ $\quad (-7)^2 - 4(12)(-10)$

$49 - (-480) = 529$

529 is a perfect square.

$\sqrt{529} = 23$ $\quad$ Therefore, the factors are linear binomials. $\quad (3x + 2)(4x - 5)$

Factoring Quadratics Quiz 2

(w/ solutions)

Factor/Solve each Quadratic Equation/Expression...

Factoring Quadratics Quiz

1) $x^2 + 9x + 8$

2) $x^2 - 7x + 12$

3) $x^2 + 7x - 18$

4) $x^2 + 13x + 30 = 0$

5) $2x^2 - 10x + 8 = 0$

6) $x^2 + 5x = 24$

7) $2x^2 + 15x + 7$

8) $3x^2 - 5x + 2$

9) $4x^2 + 21x + 5$

10) $6x^2 + 11x + 3 = 0$

11) $3x^2 + 13x - 10 = 0$

12) $8x^2 + 21x = 9$

Algebra Factoring Review

Factor the following polynomials:

1) $2x^2 + 11x + 5$

2) $x^2 - 25$

3) $x^2 + 13x - 48$

4) $2x^2 - 46x + 44$

5) $x^2 + 25$

6) $6x^2 + 23x + 20$

7) $x^2 - 22x - 75$

8) $3(x + 1)^2 + 5(x + 1) + 2$

9) $x^4 + 4x^2 - 5$

10) $49x^2 + 14x + 1$

11) $x^2 + 4xy + 3y^2$

12) $x^2 + 3x - 7$

Here are some questions to discuss....

Factoring Quadratics Questions

1) For what values of b is $x^2 + bx + 10$ factorable?

2) For what values of c is $x^2 - 5x + c$ factorable?

3) For these perfect square trinomials, what are the missing terms?

a) $x^2 +$ ______ $+ 64$

b) $x^2 - 12x +$ ______

c) $9x^2$ ______ $+ 16$

d) $4x^2$ ______ $- 9$

4) Find (at least 3) values of c, where $2x^2 - 5x + c$ factorable?

Factor/Solve each Quadratic Equation/Expression... SOLUTIONS

Factoring Quadratics Quiz

1) $x^2 + 9x + 8$

(x)(x) First terms must be x

(x +)(x +) signs are + +

$\boxed{(x+8)(x+1)}$ multiplies to 8 and adds to 9

2) $x^2 - 7x + 12$

(x)(x) First terms are x

(x −)(x −) signs are − −

$\boxed{(x-3)(x-4)}$ multiplies to 12 and adds to −7

3) $x^2 + 7x - 18$

(x)(x) First terms are x

(x +)(x −) signs are opposite

$\boxed{(x+9)(x-2)}$ multiplies to −18 and adds to 7
(notice: the B term is +, so the 9 is positive and 2 is negative)

4) $x^2 + 13x + 30 = 0$

(x)(x) First terms must be x

(x +)(x +) signs are + +

$(x+10)(x+3) = 0$ multiplies to 30 and adds to 13

$x + 10 = 0$ zero product property

$x + 3 = 0$ $\boxed{x = -3, -10}$

5) $2x^2 - 10x + 8 = 0$

$2(x^2 - 5x + 4) = 0$ Greatest Common Factor is 2...

2 (x)(x) Factor the trinomial

2 (x −)(x −) signs are + +

$2(x-1)(x-4) = 0$ multiplies to 4 and adds to 5

$x - 1 = 0$

$x - 4 = 0$ $\boxed{x = 1, 4}$ zero product property

6) $x^2 + 5x = 24$

$x + 5x - 24 = 0$ Set up the Quadratic

(x +)(x −) = 0 signs are opposite

What multiplies to -24 and adds to +5 ?

$(x+8)(x-3) = 0$

$\boxed{x = -8, 3}$

7) $2x^2 + 15x + 7$

(2x)(x) First terms are 2x and x

(2x +)(x +) signs are + +

the last terms are 7, 1 or 1, 7
(there are no other factor pairs of 7)

$\boxed{(2x+1)(x+7)}$

Check with FOIL

First	$2x^2$
Outer	14x
Inner	1x
Last	7

$2x^2 + 15x + 7$

8) $3x^2 - 5x + 2$

(3x)(x) First terms are 3x and x

(3x −)(x −) signs are − −

the last terms are 2, 1 or 1, 2
which order will get the desired result?

$\boxed{(3x-2)(x-1)}$

9) $4x^2 + 21x + 5$

(1)(5) Last terms must be 1 and 5

(+ 1)(+ 5) signs must be + +

the first terms are either 2, 2 or 4, 1 or 1, 4
which pair gets the desired result?

$\cancel{(2x+1)(2x+5)}$ ~~the middle B term is 12x~~

$\boxed{(4x+1)(x+5)}$ the middle B term is 21x

$\cancel{(x+1)(4x+5)}$ ~~the middle B term is 9x~~

10) $6x^2 + 11x + 3 = 0$

(3)(1) Last terms are 1, 3

(+ 3)(+ 1) signs are + +

the first terms are x, 6x or 6x, x or 2x, 3x or 3x, 2x
which order gets the desired result?

$(2x+3)(3x+1) = 0$

F	$6x^2$
O	2x
I	9x
L	3

$2x + 3 = 0$ $x = -3/2$

$3x + 1 = 0$ $x = -1/3$

11) $3x^2 + 13x - 10 = 0$

(3x)(x) First terms are 3x, x

signs are opposite; last terms can be

1, -10 -1, 10 -10, 1 10, -1

2, -5 -2, 5 -5, 2 5, -2

$(3x-2)(x+5) = 0$

$3x - 2 = 0$ $x = 2/3$

$x + 5$ $x = -5$

12) $8x^2 + 21x = 9$

$8x^2 + 21x - 9 = 0$

factors of A term are 1, 2, 4, 8

factors of C term are 1, 3, 9

what combination will get the desired result?

$(8x - 3)(x + 3) = 0$

First: 8x
Outer: 24x
Inner: -3x
Last: -9

$x = -3$

$x = 3/8$

Factor/Solve each Quadratic Equation/Expression...

SOLUTIONS USING "Split and Regroup"

Factoring Quadratics Quiz

1) $x^2 + 9x + 8$

A = 1
C = 8
B = 9

multiplies to 8 and adds to 9: 1 and 8

$x^2 + 8x + x + 8$

$x(x + 8) + 1(x + 8)$

$\boxed{(x + 1)(x + 8)}$

2) $x^2 - 7x + 12$

A = 1
C = 12
B = -7

multiplies to AC and adds to B ? -3 and -4

$x^2 \quad -3x - 4x \quad + 12$

factor by grouping:

$x(x - 3) + -4(x - 3)$

$\boxed{(x - 4)(x - 3)}$

3) $x^2 + 7x - 18$

What multiplies to (1)(-18) and adds to (7) ? -2 and 9

Split the 7x...

$x^2 + 9x - 2x - 18$

then, factor...

$x(x + 9) - 2(x + 9)$

$\boxed{(x - 2)(x + 9)}$

4) $x^2 + 13x + 30 = 0$

A = 1
C = 30
B = 13

multiplies to AC and adds to B?

3 and 10 -- mult. to 30 and add to 13

split the middle

$x^2 + 3x + 10x + 30 = 0$

factor by grouping

$x(x + 3) + 10(x + 3) = 0$

$(x + 10)(x + 3) = 0$

$\boxed{x = -3 \text{ or } -10}$

5) $2x^2 - 10x + 8 = 0$

GCF -- factor out 2

$2(x^2 - 5x + 4) = 0$

$2(x^2 - 1x - 4x + 4) = 0$

factor by grouping

$2(x(x - 1) - 4(x - 1)) = 0$

$2(x - 4)(x - 1) = 0$

$\boxed{x = 1 \text{ or } 4}$

6) $x^2 + 5x = 24$

$x^2 + 5x - 24 = 0$

What multiplies to -24 and adds to 5?

-3 and +8

$x^2 \quad -3x + 8x - 24 = 0$

$x(x - 3) + 8(x - 3) = 0$

$(x - 3)(x + 8) = 0$

$\boxed{x = -8 \text{ or } 3}$

7) $2x^2 + 15x + 7$

A = 2
C = 7

B = 15

AC = 14

What multiplies to 14 and adds to 15? 1 and 14

split the 15x:

$2x^2 + 1x + 14x + 7$

$x(2x + 1) + 7(2x + 1)$

$\boxed{(2x + 1)(x + 7)}$

8) $3x^2 - 5x + 2$

A = 3
C = 2

B = -5

AC = 6

What multiplies to 6 and adds to -5? -2 and -3

split the middle -5x:

$3x^2 -2x - 3x + 2$

$x(3x - 2) - 1(3x - 2)$

$\boxed{(x - 1)(3x - 2)}$

9) $4x^2 + 21x + 5$

AC = (4)(5) = 20
B = 21

Multiplies to 20 and adds to 21? 1 and 20

$4x^2 + 20x + 1x + 5$

$4x(x + 5) + 1(x + 5)$

$\boxed{(4x + 1)(x + 5)}$

10) $6x^2 + 11x + 3 = 0$

A = 6
B = 11
C = 3

Multiplies to 18 and adds to 11: 2 and 9

split the middle...

$6x^2 + 2x + 9x + 3 = 0$

factor by grouping...

$2x(3x + 1) + 3(3x + 1) = 0$

$(2x + 3)(3x + 1) = 0$

$\boxed{x = -3/2 \text{ or } -1/3}$

11) $3x^2 + 13x - 10 = 0$

Multiplies to -30 and adds to +13

+15 and -2

$3x^2 + 15x - 2x - 10 = 0$

$3x(x + 5) - 2(x + 5) = 0$

$(3x - 2)(x + 5) = 0$

$\boxed{x = 2/3 \text{ or } -5}$

12) $8x^2 + 21x = 9$

rewrite...

$8x^2 + 21x - 9 = 0$

multiplies to -72 and adds to 21 ---> 24 and -3

$8x^2 + 24x - 3x + 9 = 0$

$8x(x + 3) - 3(x + 3) = 0$

$(8x - 3)(x + 3) = 0$

$\boxed{x = 3/8 \text{ or } -3}$

Algebra Factoring Review

SOLUTIONS

Factor the following polynomials:

1) $2x^2 + 11x + 5$

"logic method"

first terms must be 2 and 1

$(2x \quad)(x \quad)$

signs must be + +

$(2x + \quad)(x + \quad)$

last terms must be 5 and 1

$\boxed{(2x + 1)(x + 5)}$

2) $x^2 - 25$

difference of squares

$\boxed{(x + 5)(x - 5)}$

3) $x^2 + 13x - 48$

find 2 linear binomials

$(x \quad)(x \quad)$

what multiplies to -48 and adds to 13?

-3 and 16

$\boxed{(x + 16)(x - 3)}$

4) $2x^2 - 46x + 44$

Greatest Common Factor

$2(x^2 - 23x + 22)$

Find 2 linear binomials

what multiplies to 22 and adds to -23?

-1 and -22

$\boxed{2(x - 1)(x - 22)}$

5) $x^2 + 25$

$\boxed{\text{PRIME}}$

6) $6x^2 + 23x + 20$

Use "split and regroup" or "divide and regroup"

$6x^2 \quad + 23x \quad + 20$

$6x^2 + 8x \quad + \quad 15x + 20$

$2x(3x + 4) \quad + \quad 5(3x + 4)$

$\boxed{(2x + 5)(3x + 4)}$

$AC = 6 \cdot 10 = 120$

what multiplies to 120 and adds to 23? 8 and 15

factor and regroup!!

7) $x^2 - 22x - 75$

find 2 linear binomials

what multiplies to -75 and adds to -22?

3 and -25

$\boxed{(x - 25)(x + 3)}$

8) $3(x + 1)^2 + 5(x + 1) + 2$

substitute and factor

let $a = (x + 1)$

$3a^2 + 5a + 2$

$(3a + 2)(a + 1)$

$(3(x + 1) + 2)((x + 1) + 1)$

$(3x + 3 + 2)(x + 2) = \boxed{(3x + 5)(x + 2)}$

9) $x^4 + 4x^2 - 5$

substitute and factor

let $b = x^2$

$b^2 + 4b - 5$

$(b + 5)(b - 1)$

$(x^2 + 5)(x^2 - 1)$

difference of squares

$\boxed{(x^2 + 5)(x + 1)(x - 1)}$

10) $49x^2 + 14x + 1$

use logic

last terms must be 1 and 1

$(\quad 1)(\quad 1)$

signs must be + +

$(\quad + 1)(\quad + 1)$

first terms must be 7x-7x or 49x-x

$\boxed{(7x + 1)(7x + 1) = (7x + 1)^2}$

perfect square polynomial

11) $x^2 + 4xy + 3y^2$

first term must be x and x

signs must be + +

$(x + \quad)(x + \quad)$

last term must have 1 and 3 as well as y and y...

$\boxed{(x + y)(x + 3y)}$

To check: FOIL the answer,

$x^2 + x(3y) + yx + 3y^2$

$x^2 + 4xy + 3y^2$ ✓

12) $x^2 + 3x - 7$

$\boxed{\text{PRIME}}$

Here are some questions to discuss.... ANSWERS

Factoring Quadratics Questions

1) For what values of b is $x^2 + bx + 10$ factorable?

To factor a quadratic trinomial, we seek numbers that multiply to 'c' and add to 'b'....
In this case, what 2 numbers multiply to 10 and add to b?

1 and 10
2 and 5

so, b could be 7 or 11...

2) For what values of c is $x^2 - 5x + c$ factorable?

Again, the coefficient of the first term ('a') is 1, so again, to factor,
we want numbers that multiply to 'c' and add up to 'b'..

In this case, what numbers multiply to c and add to -5?

1 and -6
3 and -8
0 and -5
14 and -19
etc...

the number pairs are x and -(x + 5)

so, c could be the product of any of those (and more!)...

-6
-24
0
-266

3) For these perfect square trinomials, what are the missing terms?

a) $x^2 +$ _____ $+ 64$ square root of 64 is 8... Must be 16x OR $-16x$

b) $x^2 - 12x +$ ______ B = -12 1/2 of (-12) squared is 36

c) $9x^2$ _______ $+ 16$ Two answers! $(3x + 4)(3x + 4)$ or $(3x - 4)(3x - 4)$

$+ 24x$ $-24x$

d) $4x^2$ _______ $- 9$ Zero.... Difference of two perfect squares, so there is no middle term..
Trick question! this is not a perfect square trinomial!

4) Find (at least 3) values of c, where $2x^2 - 5x + c$ factorable?

discriminant: $b^2 - 4ac$ must be perfect square...

$(-5)^2 - 4(2)(c) = 25 - 8c$

If $c = 2$, then the discriminant equals 9 ---> perfect square!

$2x^2 - 5x + 2 = (2x - 1)(x - 2)$

If $c = 3$, then the discriminant equals 1 ----> perfect square!

$2x^2 - 5x + 3 = (2x - 3)(x - 1)$

also, $c = 0, -3, -7..$

$2x^2 - 5x - 7 = (2x - 7)(x + 1)$

Thanks for visiting. (Hope it helped!)

If you have questions, suggestions, or requests, let us know.

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LanceAF #111 11/7/13
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Skip got a disappointing C+ on the math assignment...
(but, he did see 2 awesome reruns of Magnum P.I.)

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