

# Logic and Reasoning

## Notes and Exercises (with Solutions)

### What is the number?

If the number is divisible by 2, then it is between 50 and 59...

If the number is NOT divisible by 3, then it is between 60 and 69...

If the number is NOT divisible by 4, then it is between 70 and 79...

(ANSWER at the end!)

*Topics include truth tables, conditional statements, Venn and Euler Diagrams, and more.*



Valid or Invalid? Using truth statements and Euler diagrams.

*Example:* If you love baseball, then you watch baseball on TV.  
Dean watches baseball on TV.

Conclusion: Dean loves baseball.

"if you love baseball, then you watch TV"

$$B \Rightarrow TV$$

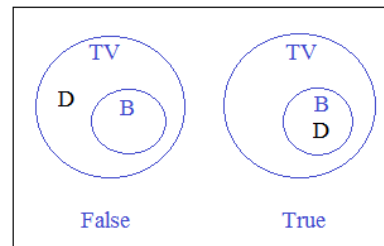
"Dean watches baseball on TV"  
(if you're Dean, then you watch baseball on TV)

$$D \Rightarrow TV$$

Conclusion: "Dean loves baseball."  
(If you're Dean, then you love baseball)

$$D \Rightarrow B$$

D: Dean  
B: Love baseball  
TV: Watch baseball



INVALID

There is no way to link the statements to get the conclusion!

*Example:* Students who turn in permission slips can go on the field trip.  
Jimmy turned in a permission slip.  
Jimmy is a student in the class.

Conclusion: Jimmy can go on the field trip.

"Students who turn in permission slips can go on field trip."  
(if you're a student AND you turn in slip, then you go on trip)

$$(S \wedge P) \Rightarrow FT$$

"Jimmy turned in permission slip."

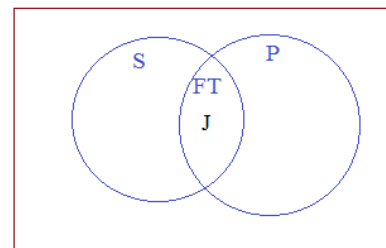
$$J \Rightarrow P$$

"Jimmy is a student."

$$J \Rightarrow S$$

Yes, Jimmy is a student and turned in a permission slip.. Therefore, he can go on the field trip.

S: Students  
FT: Field Trip  
J: Jimmy  
P: Permission Slip



VALID

$$J \Rightarrow (S \wedge P) \Rightarrow FT$$

*Example:*  $A \Rightarrow B$

$$\sim D \Rightarrow C$$

$$D \Rightarrow \sim B$$

$$C \Rightarrow E$$

Conclusion:  $A \Rightarrow E$

$B \Rightarrow \sim D$  (contrapositive has same truth value as conditional)

$$D \Rightarrow \sim B$$

$$A \Rightarrow B \Rightarrow \sim D \Rightarrow C \Rightarrow E$$

VALID

The Converse:

"If you like cool shoes,  
then get these."  
"If you get these,  
then you like cool shoes."

Mr. Taylor  
geometry

Room  
1212

This conditional statement was Chuck's favorite!

*(advertisement)*

*Make your statement with a Converse*

The unofficial shoes of  
mathletes and logicians.

"If good shoes, then these...  
If these, then good shoes..."

*Reorder right now!*



LanceAF #155 (9-10-14)  
mathplane.com

Exercises ->

Write the following, and determine the truth value.

1) If it is  $20^{\circ}\text{F}$ , then it is cold outside.

Converse:

Inverse:

Contrapositive:

2) You're in California when you visit Los Angeles.

Conditional Statement:

Converse:

Inverse:

Contrapositive:

3) In a coordinate plane, the slope of a horizontal line is always zero.

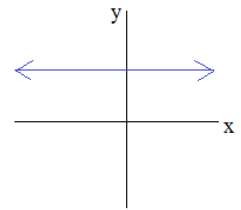
Conditional Statement:

Biconditional Statement:

Converse:

Inverse:

Contrapositive:



4) A quarter is a coin.

Conditional Statement:

Converse:

Inverse:

Contrapositive:

5) The girl is happy if she gets dessert.

Conditional Statement:

Converse:

Inverse:

Contrapositive:

6) The boy is happy if and only if he gets dessert.

Conditional Statement:

Converse:

Inverse:

Contrapositive:

A conditional statement is an if-then statement in which P is the hypothesis and Q is the conclusion.

$$P \Rightarrow Q$$

The conditional statement is defined as 'true', UNLESS a true hypothesis leads to a false conclusion.

Fill in the truth table:

*Conditional Statements*

P	Q	$P \Rightarrow Q$	$Q \Rightarrow P$
T	T		
T	F		
F	T		
F	F		

Conjunction: A compound statement formed by joining two statements with a connector ("AND") The symbol for 'and' is  $\wedge$   
 (similar to the 'intersection' or overlap of sets)

Disjunction: A compound statement formed by joining two statements with a connector ("OR") The symbol for 'or' is  $\vee$   
 (similar to the 'union' of sets)

Fill in the truth table:

*Conjunctions  
&  
Disjunctions*

P	Q	$P \vee Q$	$P \wedge Q$
T	T		
T	F		
F	T		
F	F		

In a 'conjunction', BOTH statements must be true for the compound to be true  
 (otherwise, the conjunction is false)

In a 'disjunction', BOTH statements must be false for the compound to be false  
 (otherwise, the disjunction is false)

The negation of a statement is the 'not'

$$\sim P \quad \neg P$$

Fill in the truth table:

*Negations  
&  
Compound Statements*

P	Q	$\neg P$	$\neg P \vee Q$	$\neg P \wedge Q$
T	T			
T	F			
F	T			
F	F			

Venn and Euler Diagrams

Match the diagram that best fits the statement:

1) If A, then B       $A \Rightarrow B$

2) If B, then A       $B \Rightarrow A$

3) A and B           $A \wedge B$

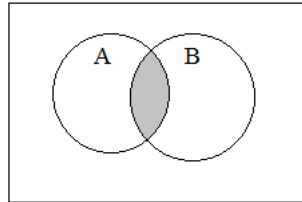
4) A or B           $A \vee B$

5) If  $\sim A$ , then  $\sim B$        $\sim A \Rightarrow \sim B$

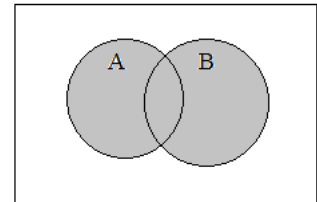
6) If A, then  $\sim B$        $A \Rightarrow \sim B$

7) B and  $\sim A$        $B \wedge \sim A$

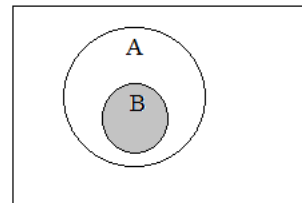
I



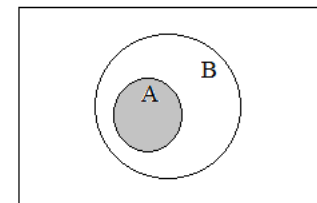
II



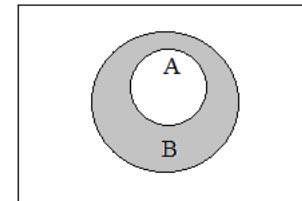
III



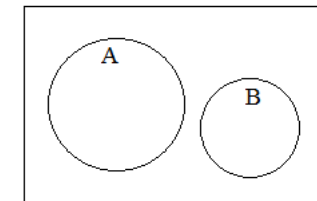
IV



V



VI



Conditional "if-then" statement: If a polygon is a square, then it is a \_\_\_\_\_.

Converse:

Inverse:

Contrapositive:

True or false?

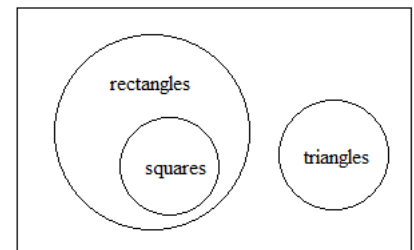
All squares are rectangles.

All rectangles are squares.

Triangles or Rectangles have 3 sides.

Triangles and Squares have 3 sides.

Polygons (sides and lengths)



Determine if the statement is 'valid' or 'invalid'. Then, sketch an Euler diagram to verify your answer.

- 1) All golfers have rain gear.  
JC has rain gear.  
 $\therefore$  JC is a golfer.

- 2) Some people love math.  
All people who love math love physics.  
 $\therefore$  Some people love physics.

- 3) No apples are citrus.  
All Fujis are apples.  
 $\therefore$  No fujis are citrus.

- 4) All rainy days are cloudy.  
Today is cloudy.  
 $\therefore$  Today is a rainy day.

- 5) Carl is 16 years old.  
Carl drives a car.

Conclusion: If you're 16 years old, then you drive a car.



Determine if the statement is 'valid' or 'invalid'. Then, sketch an Euler diagram to verify your answer.

- 6) Some plants are poisonous.  
Ivy is a plant.  
 $\therefore$  Ivy is poisonous.

- 7) All cordless phones have antennas.  
All things with antennas are insects.  
 $\therefore$  All cordless phones are insects.

- 8) All cordless phones have antennas.  
All insects have antennas.  
 $\therefore$  All cordless phones are insects.

- 9) If a lawyer passes the bar, then he/she can practice law.  
Cindy passed the bar.

Conclusion: Cindy can practice law.

"Mr. Kaffee, I'll answer the question...  
You want answers?? I'll give you answers!!!"

"If the code is red,  
then I ordered it!"

N. Jessup

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

p	$\neg p$
T	F
F	T

'Geez, the Colonel  
is intense...'

"I can't handle the truth tables!"

'I see why Santiago  
dropped this class.'

Logic

Geometry

A Few Good  
Math Students

SOLUTIONS-→

Write the following, and determine the truth value.

ANSWERS

Conditional Statements

1) If it is  $20^{\circ}\text{F}$ , then it is cold outside.

Converse: If it is cold outside, then it is  $20^{\circ}\text{F}$  (False. Counterexample: 23 degrees)

Inverse: If it is NOT  $20^{\circ}\text{F}$ , then it is NOT cold outside. (False)

Contrapositive: If it is not cold outside, then it is not  $20^{\circ}\text{F}$ . (True)

2) You're in California when you visit Los Angeles.

Conditional Statement: If you visit Los Angeles, then you're in California. (True)

Converse: If you're in California, then you're visiting Los Angeles. (False. Counterexample: San Francisco)

Inverse: If you're not visiting Los Angeles, then you're not in California. (False)

Contrapositive: If you're NOT in California, then you're NOT visiting Los Angeles. (True)

3) In a coordinate plane, the slope of a horizontal line is always zero.

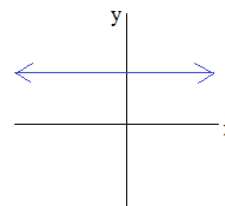
Conditional Statement: In a coordinate plane, if the line is horizontal, then its slope is zero. (True)

Biconditional Statement: In a coordinate plane, the slope of a line is zero IF AND ONLY IF it is horizontal. (True)

Converse: In a coordinate plane, if the slope is zero, then the line is horizontal. (True)

Inverse: (In a coordinate plane), if the line is NOT horizontal, then its slope is NOT zero. (True)

Contrapositive: (In a coordinate plane), if the slope is not zero, then the line is not horizontal. (True)



4) A quarter is a coin.

Conditional Statement: If it's a quarter, then it's a coin. (True)

Converse: If it's a coin, then it's a quarter. (False. Counterexamples: penny, dime, etc...)

Inverse: If it's not a quarter, then it's not a coin. (False)

Contrapositive: If it's not a coin, then it's not a quarter. (True)

5) The girl is happy if she gets dessert.

Conditional Statement: If the girl gets dessert, then she is happy. (true)

Converse: If the girl is happy, then she got/gets dessert. (false. she may or may not)

Inverse: If the girl doesn't get dessert, then she is not happy. (false)

Contrapositive: If she does not get dessert, then the girl is not happy. (true)

6) The boy is happy if and only if he gets dessert. **\*\*\*"if and only if" indicates that this is an 'equivalence statement'... all of the statements are true!**

Conditional Statement: If the boy gets dessert, then he is happy. (True)

Converse: If the boy is happy, he got dessert. (True)

Inverse: If the boy doesn't get dessert, then he is not happy. (True)

Contrapositive: If the boy is unhappy, then he did not get dessert. (True)

A conditional statement is an if-then statement in which P is the hypothesis and Q is the conclusion.

$$P \Rightarrow Q$$

The conditional statement is defined as 'true', UNLESS a true hypothesis leads to a false conclusion.

Fill in the truth table:

*Conditional Statements*

P	Q	$P \Rightarrow Q$	$Q \Rightarrow P$
T	T	T	T
T	F	F	F
F	T	T	T
F	F	T	T

Conjunction: A compound statement formed by joining two statements with a connector ("AND") The symbol for 'and' is  $\wedge$  (similar to the 'intersection' or overlap of sets)

Disjunction: A compound statement formed by joining two statements with a connector ("OR") The symbol for 'or' is  $\vee$  (similar to the 'union' of sets)

Fill in the truth table:

*Conjunctions  
&  
Disjunctions*

P	Q	$P \vee Q$	$P \wedge Q$
T	T	T	T
T	F	T	F
F	T	T	F
F	F	F	F

In a 'conjunction', BOTH statements must be true for the compound to be true (otherwise, the conjunction is false)

In a 'disjunction', BOTH statements must be false for the compound to be false (otherwise, the disjunction is true)

The negation of a statement is the 'not'

$$\sim P \quad \neg P$$

Fill in the truth table:

*Negations  
&  
Compound Statements*

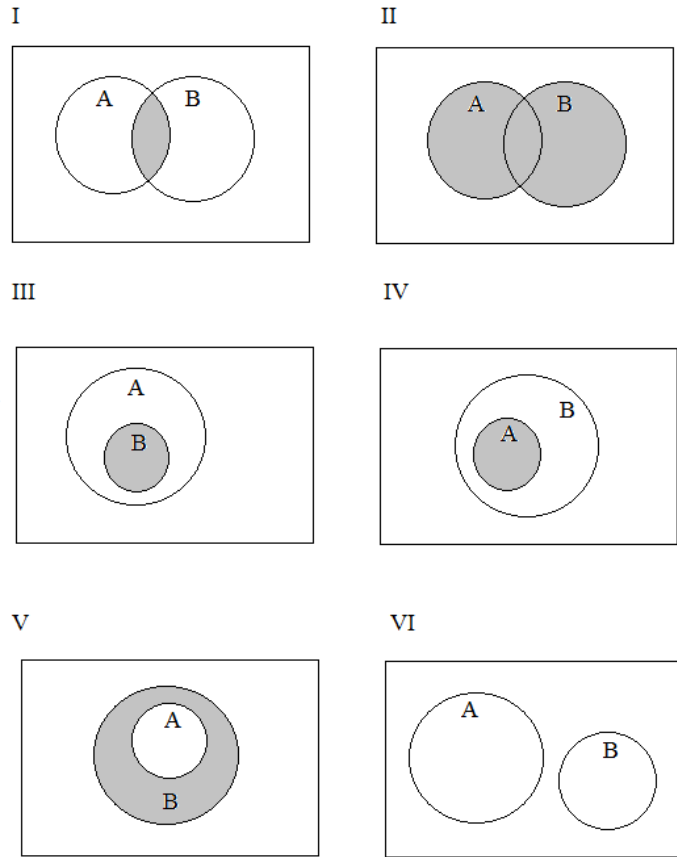
P	Q	$\neg P$	$\neg P \vee Q$	$\neg P \wedge Q$
T	T	F	T	F
T	F	F	F	F
F	T	T	T	T
F	F	T	T	F

Venn and Euler Diagrams

SOLUTIONS

Match the diagram that best fits the statement:

- 1) If A, then B       $A \Rightarrow B$       IV
- 2) If B, then A       $B \Rightarrow A$       III
- 3) A and B           $A \wedge B$           I
- 4) A or B             $A \vee B$           II
- 5) If  $\sim A$ , then  $\sim B$      $\sim A \Rightarrow \sim B$       III (contrapositive of 'if B, then A')
- 6) If A, then  $\sim B$        $A \Rightarrow \sim B$       VI
- 7) B and  $\sim A$          $B \wedge \sim A$         V



Conditional "if-then" statement: If a polygon is a square, then it is a rectangle.

Converse: If the polygon is a rectangle, then it is a square. (False... Only equilateral rectangles are squares)

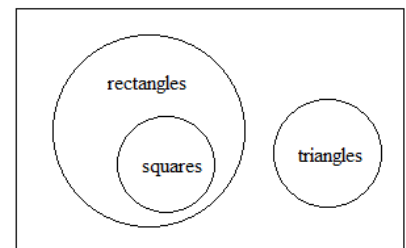
Inverse: If a polygon is not a square, then it is not a rectangle. (False)

Contrapositive: If a polygon is not a rectangle, then it is not a square. (True)

True or false?

- All squares are rectangles.      True
- All rectangles are squares.      False (some are.. some are not)
- Triangles or Rectangles have 3 sides.      True
- Triangles and Squares have 3 sides.      False

Polygons (sides and lengths)



Determine if the statement is 'valid' or 'invalid'. Then, sketch an Euler diagram to verify your answer.

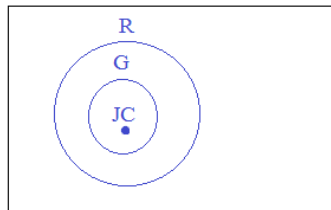
ANSWERS

Euler Diagrams and Truth Statements

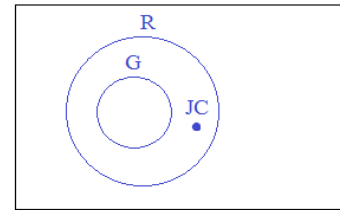
- 1) All golfers have rain gear.  
 JC has rain gear.  
 ∴ JC is a golfer.

It could be true..  
 But, it could be false..

Therefore, "invalid"



JC is a golfer

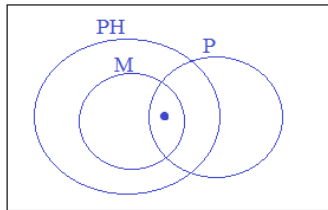


JC is not a golfer

- 2) Some people love math.  
 All people who love math love physics.  
 ∴ Some people love physics.

Anyone who loves math  
 will love physics...  
 Since some people love  
 math, there are some  
 people who love physics..

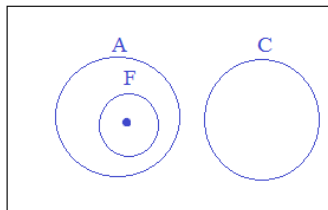
"Valid"



P: People  
 PH: Love Physics  
 M: Love Math

- 3) No apples are citrus.  
 All Fujis are apples.  
 ∴ No fujis are citrus.

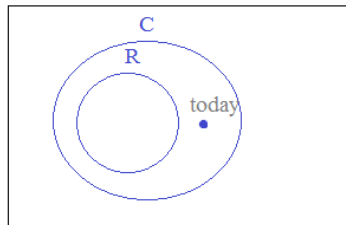
"Valid"



A: Apples  
 F: Fujis  
 C: Citrus

- 4) All rainy days are cloudy.  
 Today is cloudy.  
 ∴ Today is a rainy day.

"Invalid"



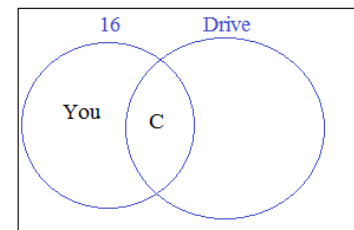
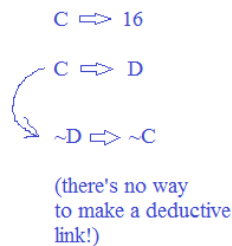
C: Cloudy days  
 R: Rainy days

Exception..

- 5) Carl is 16 years old.  
 Carl drives a car.

Conclusion: If you're 16 years old, then you drive a car.

"Invalid"



Counterexample

Determine if the statement is 'valid' or 'invalid'. Then, sketch an Euler diagram to verify your answer.

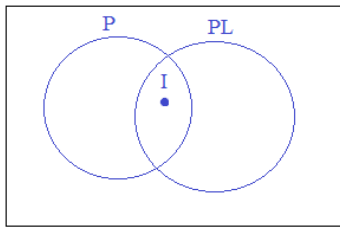
ANSWERS

Euler Diagrams and Truth Statements

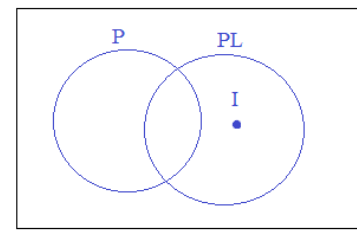
- 6) Some plants are poisonous.  
Ivy is a plant.  
∴ Ivy is poisonous.

Ambiguous, so  
"invalid"

P: Poisonous things  
PL: Plants  
I: Ivy



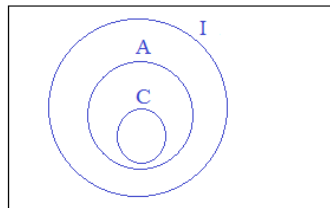
Yes



No

- 7) All cordless phones have antennas.  
All things with antennas are insects.  
∴ All cordless phones are insects.

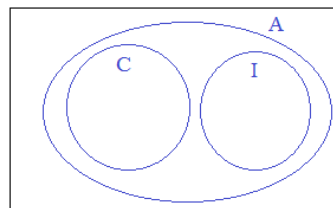
"Valid" (but, bizarre!)



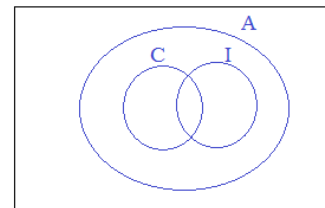
Maybe the cordless phones are shaped like insects? (cartoon characters)

- 8) All cordless phones have antennas.  
All insects have antennas.  
∴ All cordless phones are insects.

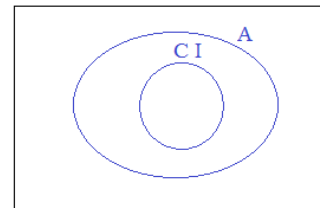
"Invalid"



False



False



true

- 9) If a lawyer passes the bar, then he/she can practice law.  
Cindy passed the bar.

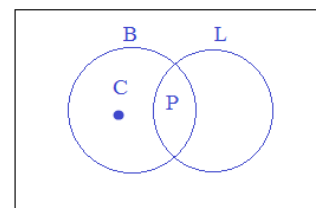
Conclusion: Cindy can practice law.

"Invalid"

Is Cindy a lawyer?  
Or, not?  
Maybe she was disbarred?

$$(L \wedge B) \Rightarrow PL$$

$$C \Leftrightarrow B$$

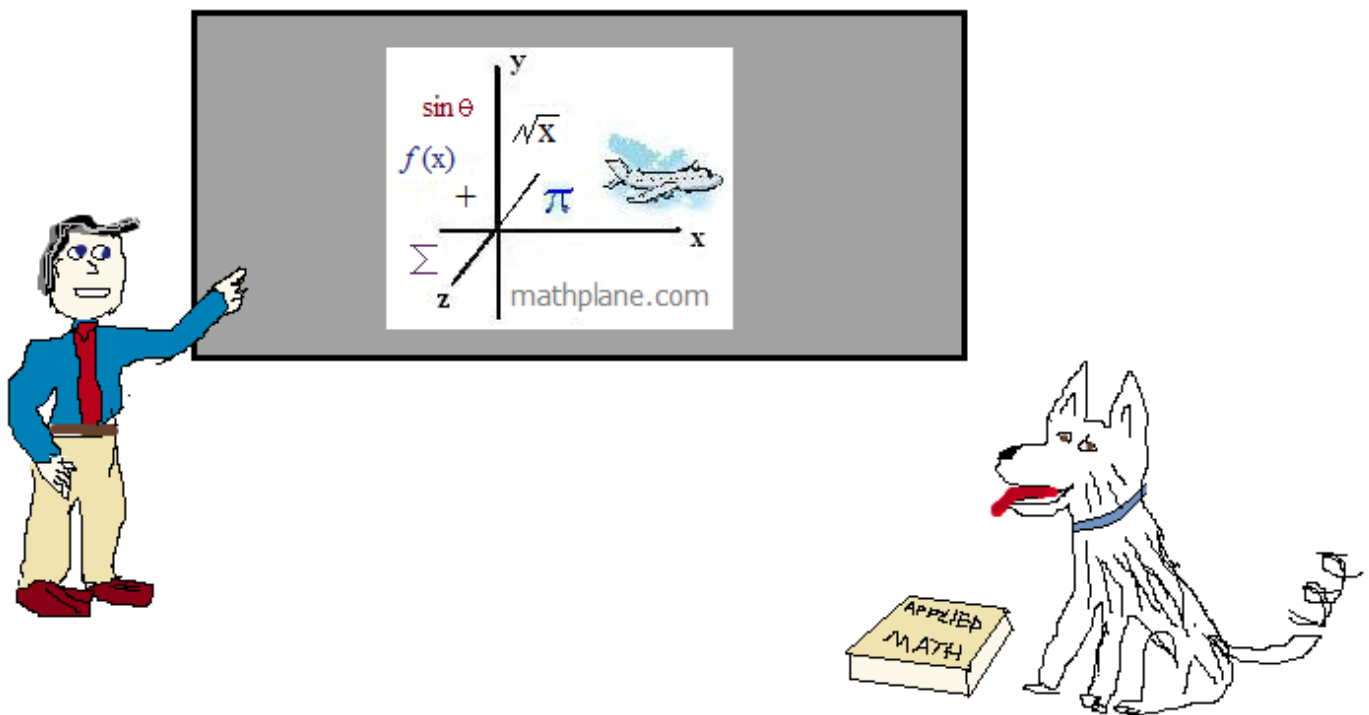


P: Practice Law  
B: Pass Bar  
C: Cindy  
L: Lawyers

Thanks for visiting. (Hope it helped!)

If you have questions, suggestions, or requests, then let us know.

Cheers



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Also, at Facebook, Google+, Pinterest, TES, and TeachersPayTeachers



## What is the number?

If the number is divisible by 2, then it is between 50 and 59...

If the number is NOT divisible by 3, then it is between 60 and 69...

If the number is NOT divisible by 4, then it is between 70 and 79...

SOLUTION

First, a quick look...

If the number is divisible by 2, then it is between 50 and 59...

Therefore, the number is ODD, unless it is between 50 and 59...

If the number is NOT divisible by 3, then it is between 60 and 69...

this eliminates 63, 66, 69...

If the number is NOT divisible by 4, then it is between 70 and 79...

this eliminates 72, 76...

Then, recognize that this question is implementing logic:

Recall: conditional statement, inverse, converse, and contrapositive!

\*\*\*Here's the key...

Contrapositive of 1st statement: "If NOT between 50 and 59, then NOT divisible by 2" ..

math property of factors: If NOT divisible by 2, then it can't be divisible by 4!!  
(any number divisible by 4 must be divisible by 2...)

3rd statement: If NOT divisible by 4, then it is between 70 and 79...

Conclusion: If NOT between 50 and 59, then it is between 70 and 79...

NOT 50-59  $\Rightarrow$  Not divisible by 2  $\Rightarrow$  Not divisible by 4  $\Rightarrow$  between 70-79

So, numbers can be 50 52 54 56 58 71 73 75 77 79

Contrapositive of 2nd statement: "If NOT between 60 and 69, then number IS divisible by 3..."

So, the numbers can be 54 or 75

But, 54 violates the 3rd statement!

So, the number must be 75....