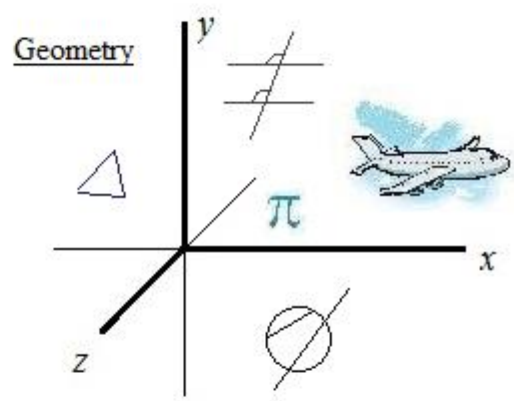
Geometry Proofs 2

Topics include congruent triangles, quadrilaterals, detours, CPCTC, diagram-less proofs, circles, equidistance theorem, and more.



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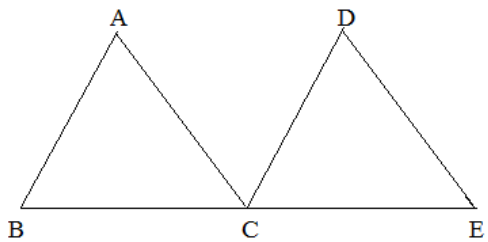
Geometry Proofs Review

1) Given: $\overline{AB} \parallel \overline{CD}$; $\overline{AB} \cong \overline{CD}$
C is the midpoint of $\overline{BE}$

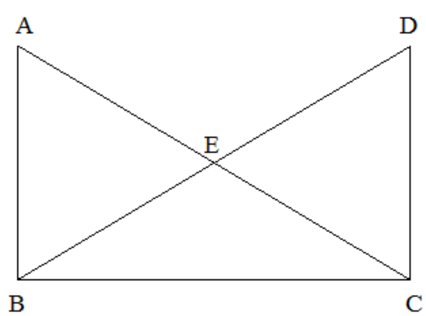
Prove: $\overline{AC} \parallel \overline{DE}$



Statements	Reasons

2) Given: $\overline{AB} \perp \overline{BC}$; $\overline{DC} \perp \overline{BC}$
$\overline{AC} \cong \overline{DB}$

Prove: $\overline{AE} \cong \overline{DE}$



Statements	Reasons

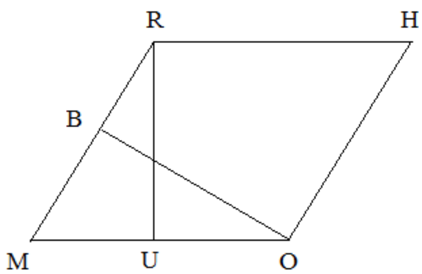
Geometry Proofs Review

3) Given: RHOM is a rhombus

$\overline{OB} \perp \overline{RM}$

$\overline{RU} \perp \overline{MO}$

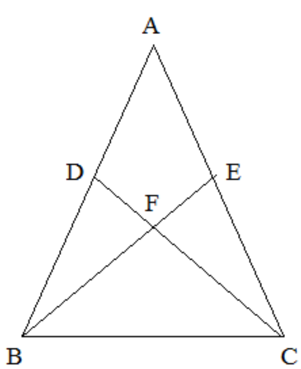
Prove: $\overline{OB} \cong \overline{RU}$



Statements	Reasons

4) Given: $\overline{AB} \cong \overline{AC}$; $\overline{AD} \cong \overline{AE}$

Prove: $\triangle$FBC is isosceles



Statements	Reasons

Geometry Proofs Review

5) Prove the diagonals of a rhombus are perpendicular.

Statements	Reasons

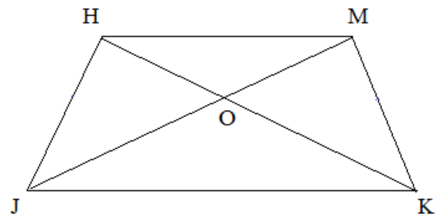
6) Prove the following: when the base of an isosceles triangle is extended in both directions, the exterior angles are congruent.

Statements	Reasons

7) Given: $\overline{HJ} \cong \overline{MK}$

$\angle HJK \cong \angle MKJ$

Prove: $\triangle$ JOK is isosceles

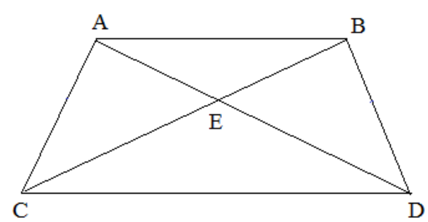


Statements	Reasons

8) Given: $\overline{AC} \cong \overline{BD}$

$\angle ACD \cong \angle BDC$

Prove: $\triangle$ AEB is isosceles



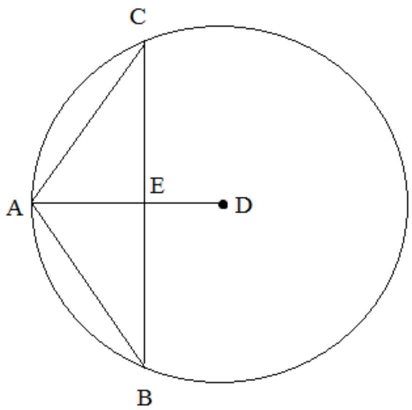
Statements	Reasons

Geometry Proofs Review

9) Given: Circle D

$\angle B \cong \angle C$

Prove: $\overline{AD} \perp \overline{BC}$



Statements	Reasons

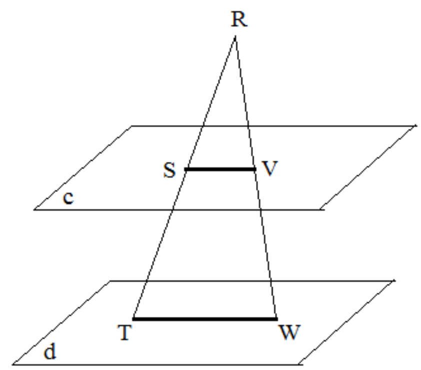
Geometry Proofs Review

10) Given: $c \parallel d$

$\triangle RTW$ is isosceles with base $\overline{TW}$

Prove: $\triangle RSV$ is isosceles

Statements	Reasons



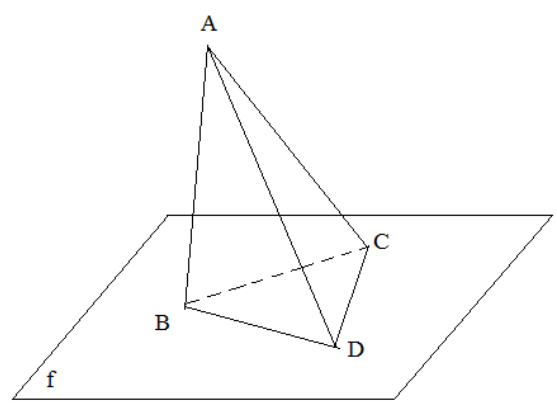
11) Given: $\overline{AB} \perp f$

$\triangle BCD$ is an equilateral triangle

B, C, D are coplanar

Prove: ACD is isosceles

Statements	Reasons



12) Given: Circle O

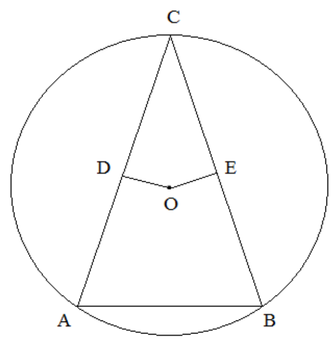
$\overline{AD} \cong \overline{BE}$

$\overline{AC} \perp \overline{OD}$

$\overline{CB} \perp \overline{OE}$

Prove: $\overline{AC} \cong \overline{BC}$

Statements	Reasons



13) Given: $\angle MPS \cong \angle MSP$

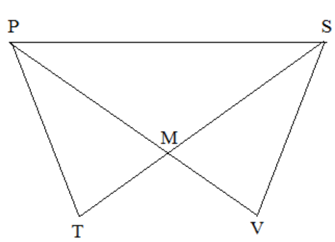
$\angle TPM \cong \angle VSM$

Prove: $\overline{PT} \cong \overline{SV}$

Statements	Reasons

Optional:
Can you prove using a different approach?!?!

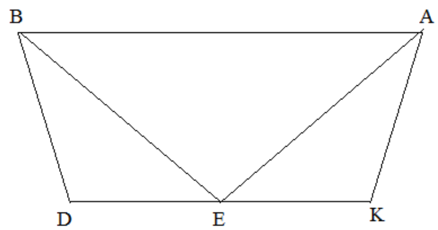
Statements	Reasons



14) Given: $\overline{AB} \parallel \overline{EC}$

$\angle AEK = \angle BED$

Prove: $\triangle ABE$ is isosceles

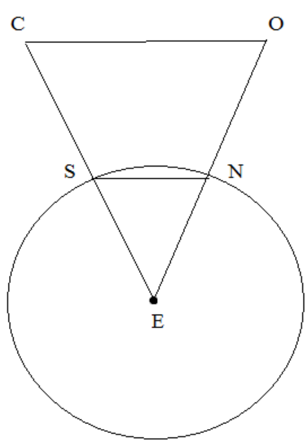


Statements	Reasons

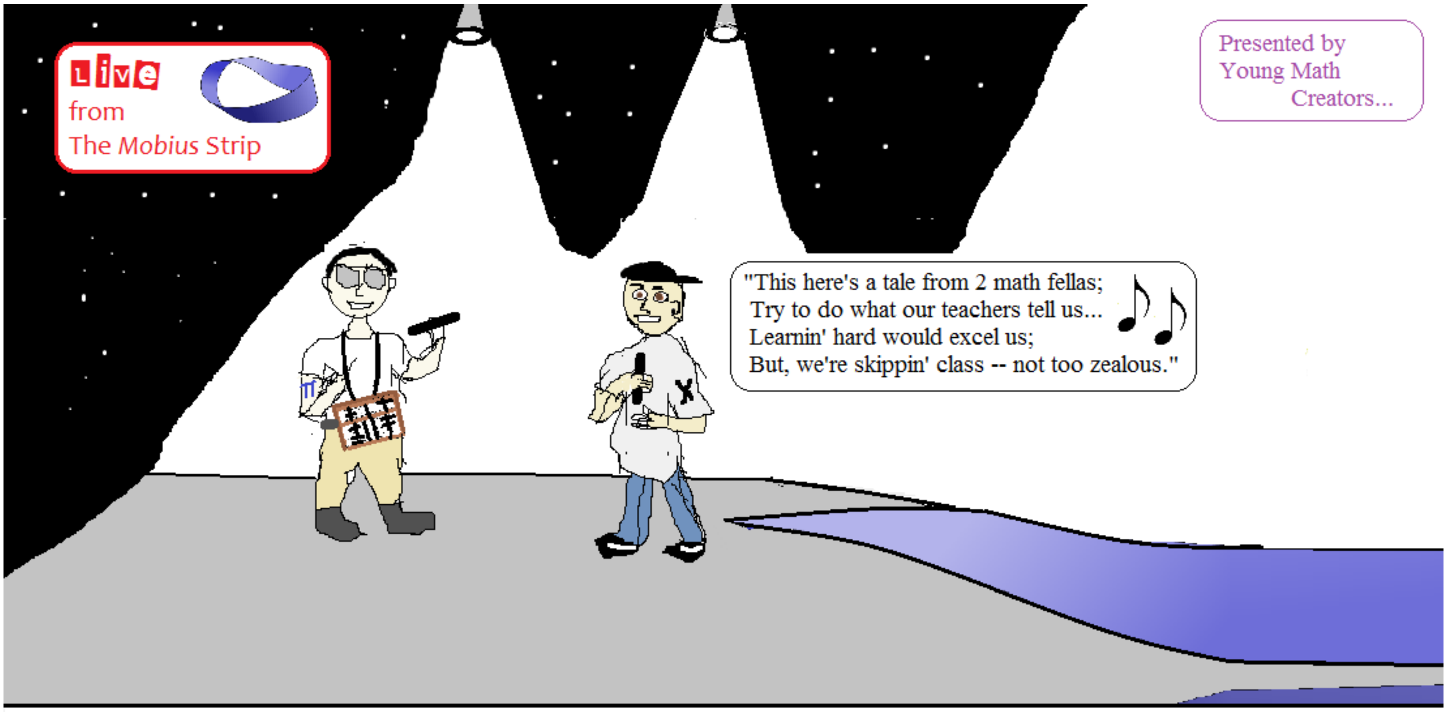
15) Given: Circle E

$\triangle$ COE is scalene

Prove: $\angle C \not\cong \angle ESN$

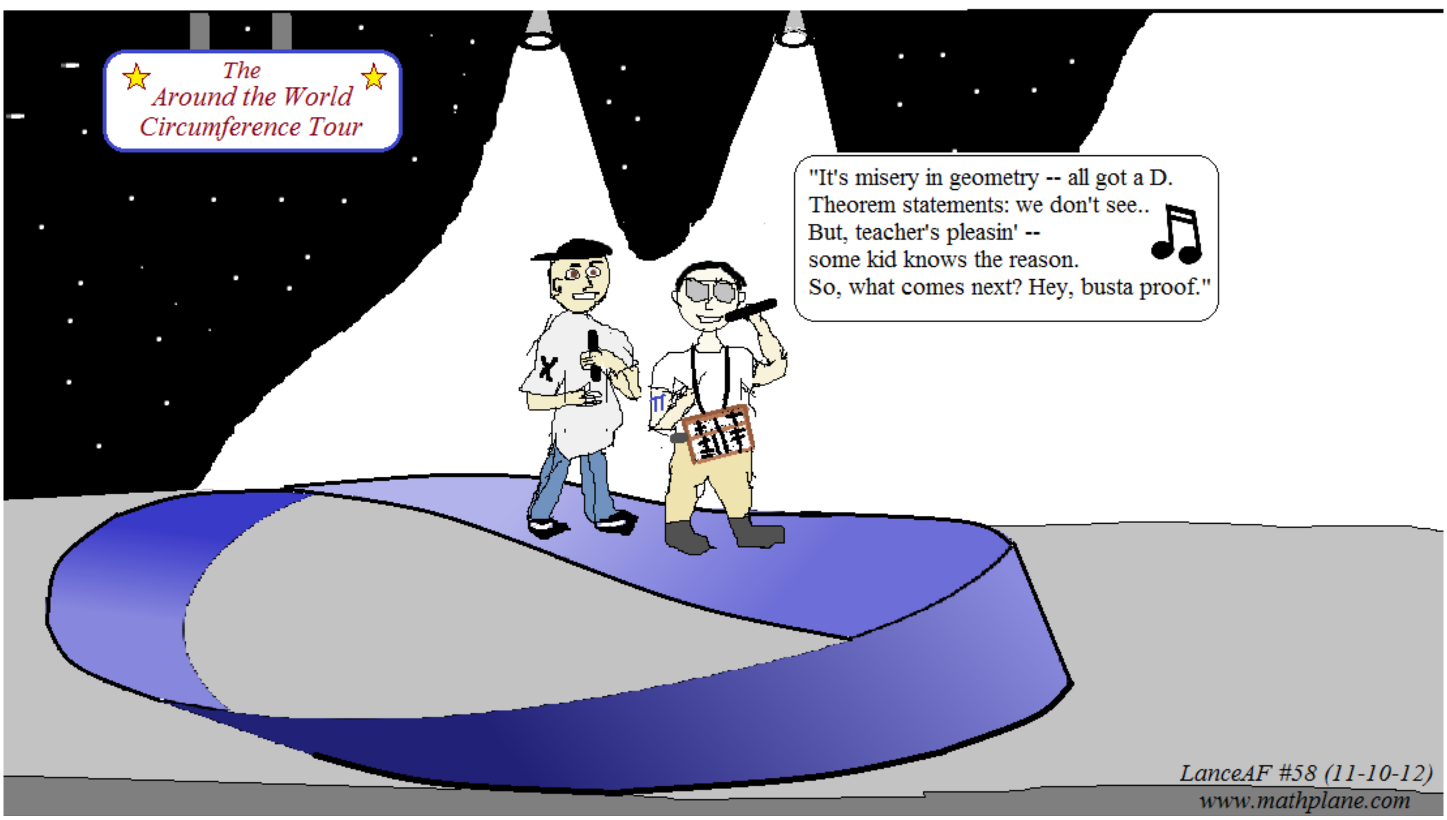


Statements	Reasons



2pi Crew and Xdx perform 'Busta Proof'
at the Mobius Strip

MATH
RAP

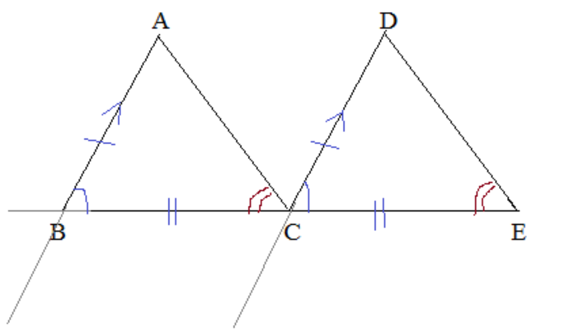


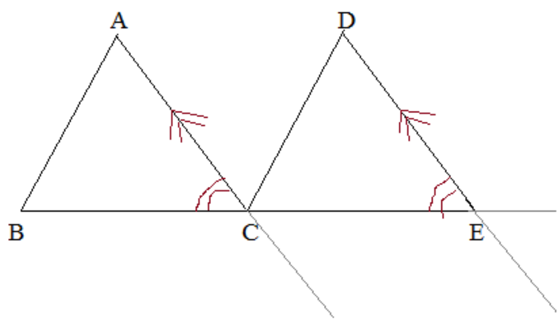
Geometry Proofs Review

SOLUTIONS

1) Given: $\overline{AB} \parallel \overline{CD}$; $\overline{AB} \cong \overline{CD}$
C is the midpoint of $\overline{BE}$

Prove: $\overline{AC} \parallel \overline{DE}$

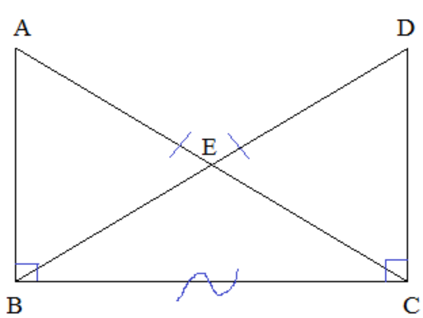


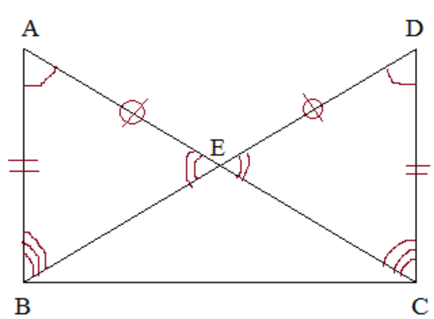


Statements	Reasons
1) $\overline{AB} \parallel \overline{CD}$	1) Given
2) $\angle B \cong \angle DCE$	2) If parallel lines cut by transversal then corresponding angles $\cong$
3) $\overline{AB} \cong \overline{CD}$	3) Given
4) C is midpoint of $\overline{BE}$	4) Given
5) $\overline{BC} \cong \overline{CE}$	5) Definition of midpoint
6) $\triangle ABC = \triangle DCE$	6) Side-Angle-Side (SAS) (3, 2, 5)
7) $\angle ACB = \angle E$	7) Corresponding Parts of Congruent Triangles are Congruent (CPCTC)
8) $AC \parallel DE$	8) If corresponding angles are $\cong$, then lines are parallel (converse of above theorem)

2) Given: $\overline{AB} \perp \overline{BC}$; $\overline{DC} \perp \overline{BC}$
$\overline{AC} \cong \overline{DB}$

Prove: $\overline{AE} \cong \overline{DE}$



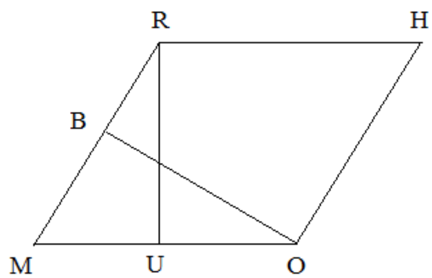


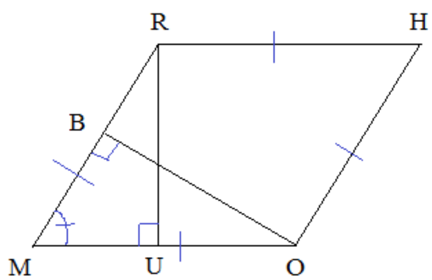
Statements	Reasons
1. $\overline{AB} \perp \overline{BC}$; $\overline{DC} \perp \overline{BC}$	1. Given
2. $\angle ABC$ and $\angle DCB$ are right angles	2. Definition of perpendicular
3. $\angle ABC \cong \angle DCB$	3. All right angles are congruent
4. $\overline{AC} \cong \overline{DB}$	4. Given
5. $\overline{BC} \cong \overline{BC}$	5. Reflexive property
6. $\triangle ABC = \triangle DCB$	6. HL (Hypotenuse-leg) (3, 4, 5)
7. $\overline{AB} \cong \overline{DC}$	7. CPCTC (Corresponding Parts of Congruent Triangles are Congruent)
8. $\angle A \cong \angle D$	8. CPCTC
9. $\angle AEB \cong \angle DEC$	9. Vertical Angles Congruent
10. $\angle ABE \cong \angle DCE$	10. No Choice Theorem (If 2 angle pairs congruent, then 3rd angle pair congruent)
11. $\triangle ABE = \triangle DCE$	11. AAS (Angle-Angle-Side) (8, 9, 7) or, ASA (Angle-Side-Angle) (8, 7, 10)
12. $\overline{AE} \cong \overline{DE}$	12. CPCTC

SOLUTIONS

Geometry Proofs Review

3) Given: RHOM is a rhombus
$\overline{OB} \perp \overline{RM}$
$\overline{RU} \perp \overline{MO}$
Prove: $\overline{OB} \cong \overline{RU}$



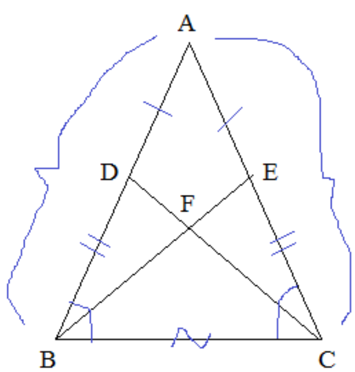


Statements	Reasons
1. RHOM is a rhombus	1. Given
2. $\overline{RM} \cong \overline{MO}$	2. Definition of Rhombus (all sides congruent)
3. $\overline{OB} \perp \overline{RM}$, $\overline{RU} \perp \overline{MO}$	3. Given
4. $\angle OBM$ and $\angle RUM$ are right angles	4. Definition of Perpendicular (perpendicular lines form right angles)
5. $\angle OBM \cong \angle RUM$	5. All right angles congruent
6. $\angle M = \angle M$	6. Reflexive property
7. $\triangle RUM = \triangle OBM$	7. AAS (Angle-Angle-Side) (5, 6, 2)
8. $\overline{OB} \cong \overline{RU}$	8. CPCTC (Corresponding Parts of Congruent Triangles are Congruent)

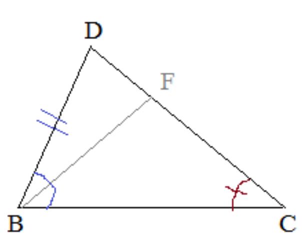
Strategy: label all the given parts
Rhombus: all sides congruent

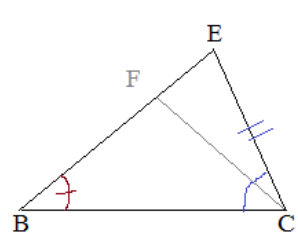
Look for congruent triangles (CPCTC)

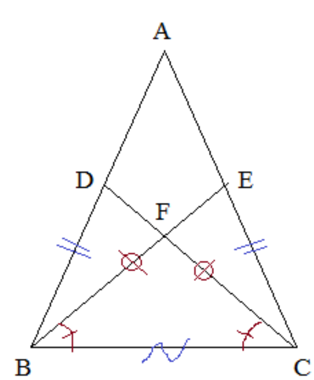
4) Given: $\overline{AB} \cong \overline{AC}$; $\overline{AD} \cong \overline{AE}$
Prove: $\triangle$FBC is isosceles



Statements	Reasons
1. $\overline{AB} \cong \overline{AC}$	1. Given
2. $\angle ABC \cong \angle ACB$	2. If congruent sides, then congruent angles
3. $\overline{AD} \cong \overline{AE}$	3. Given
4. $\overline{DB} \cong \overline{EC}$	4. Subtraction Property
5. $\overline{BC} = \overline{BC}$	5. Reflexive Property
6. $\triangle DBC \cong \triangle ECB$	6. Side-Angle-Side (SAS) (4, 2, 5)
7. $\angle DCB = \angle EBC$	7. CPCTC
8. $\overline{BF} \cong \overline{CF}$	8. If congruent angles, then congruent sides
9. $\triangle$ FBC is isosceles	9. Definition of Isosceles (2 or more congruent sides of triangle)







Geometry Proofs Review

SOLUTIONS

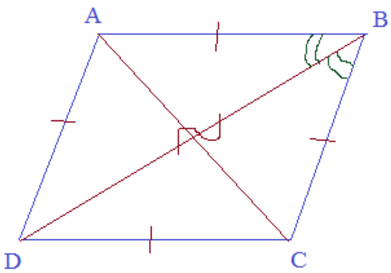
5) Prove the diagonals of a rhombus are perpendicular.

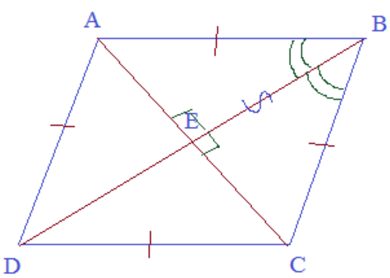
Given: Rhombus

(all sides of a quadrilateral are congruent)

Prove: Diagonals are perpendicular

(Finding right angles)





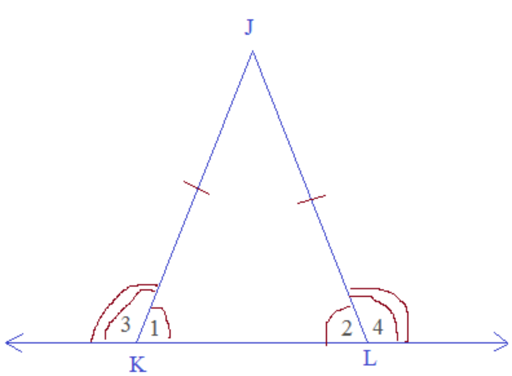
(detour)

Statements	Reasons
1. Rhombus ABCD	1. Given
2. $\overline{AB} \cong \overline{AD}$, $\overline{BC} \cong \overline{CD}$	2. Definition of a rhombus
3. $AC \cong AC$	3. Reflexive property
4. $\triangle ADB \cong \triangle CDB$	4. Side-Side-Side (2, 3, 2)
5. $\angle ABD \cong \angle CBD$	5. CPCTC (Corresponding Parts of Congruent Triangles are Congruent)
6. $\overline{BE} \cong \overline{BE}$	6. Reflexive property
7. $\triangle ABE \cong \triangle CBE$	7. Side-Angle-Side (2, 5, 6)
8. $\angle BEA \cong \angle BEC$	8. CPCTC
9. $\angle BEA$ & $\angle BEC$ are supplementary	9. def. of supplementary angles
10. $\angle BEA$ and $\angle BEC$ are right angles	10. If angles are supplementary and congruent, then they are right angles
11. $\overline{BD} \perp \overline{AC}$	11. Definition of Perpendicular (If lines form right angles, then they are perpendicular)

NOTE: Shortcut -- Utilizing the Equidistance Theorem!
Since $\overline{BA} = \overline{BC}$ and $\overline{DA} = DC$, $\overline{BD}$ is perpendicular bisector of $\overline{AC}$.. (if 2 points are each equidistant from the endpoints of a segment, then they determine the perpendicular bisector of the segment)

Note the detour! We used the large triangles (SSS) in order to get the congruent angles needed for the smaller triangles....

6) Prove the following: when the base of an isosceles triangle is extended in both directions, the exterior angles are congruent.



Statements	Reasons
1. Isoceles $\triangle JKL$	1. Given
2. $\overline{JK} = \overline{JL}$	2. Definition of Isosceles triangle
3. $\angle 1 \cong \angle 2$	3. If congruent sides, then the opposite angles are congruent
4. $\angle$'s 1 & 3 are supp. $\angle$'s 2 & 4 are supp.	4. Definition of supplementary
5. $\angle 3 \cong \angle 4$	5. Substitution - If angles are congruent, then their supplements are congruent

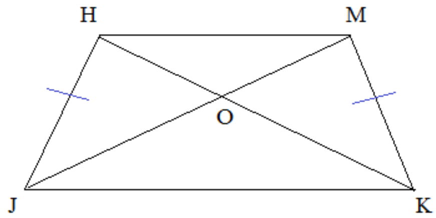
SOLUTIONS

Geometry Proofs Review

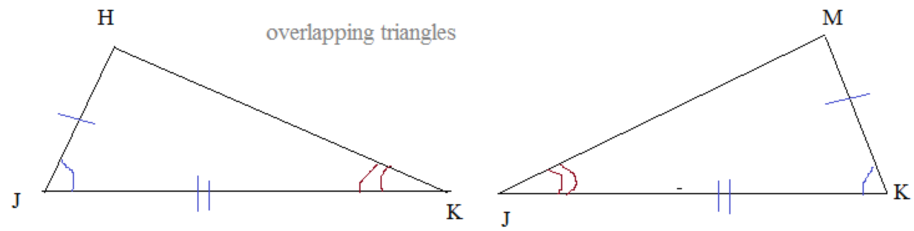
7) Given: $\overline{HJ} \cong \overline{MK}$

$\angle HJK \cong \angle MKJ$

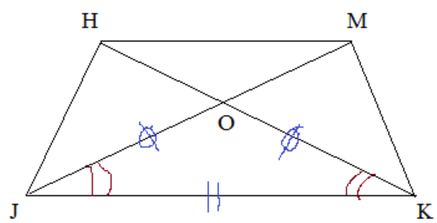
Prove: $\triangle$ JOK is isosceles



To prove JOK is isosceles, we want to prove that $\overline{JO} = \overline{KO}$



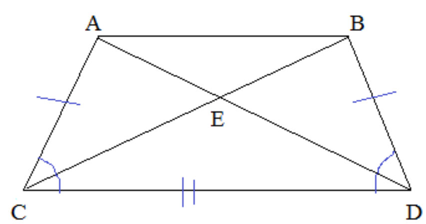
Statements	Reasons
1. $\overline{HJ} = \overline{MK}$	1. Given
2. $\angle HJK = \angle MKJ$	2. Given
3. JK = JK	3. Reflexive property
4. $\triangle HJK = \triangle MKJ$	4. SAS (1, 2, 3)
5. $\angle OKJ = \angle OJK$	5 CPCTC
6. JO = KO	6. If congruent angles, then congruent sides...
7. JOK is isosceles	7. Definition of isosceles



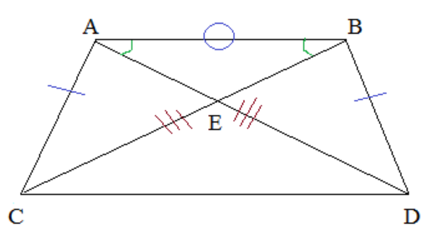
8) Given: $\overline{AC} \cong \overline{BD}$

$\angle ACD \cong \angle BDC$

Prove: $\triangle$ AEB is isosceles



Statements	Reasons
1. $\overline{AC} = \overline{BD}$	1. Given
2. $\angle ACD = \angle BDC$	2. Given
3. CD = CD	3. Reflexive property
4. $\triangle ACD = \triangle BDC$	4. SAS (1, 2, 3)
5. $\overline{AD} = \overline{BC}$	5 CPCTC
6. $\overline{AB} = \overline{AB}$	6. Reflexive property
7. $\triangle ABC = \triangle BAD$	7. SSS (1, 6, 5)
8. $\angle ABC = \angle BAD$	8. CPCTC
9. AE = BE	9 If congruent angles, then congruent sides
10. AEB is isosceles	10. Definition of isosceles



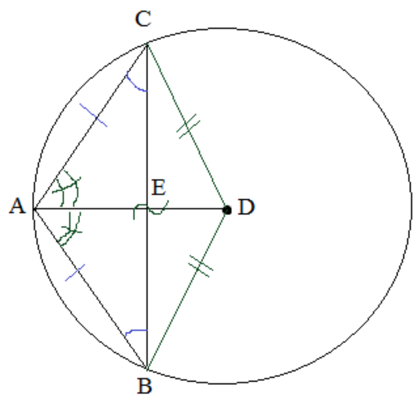
Geometry Proofs Review

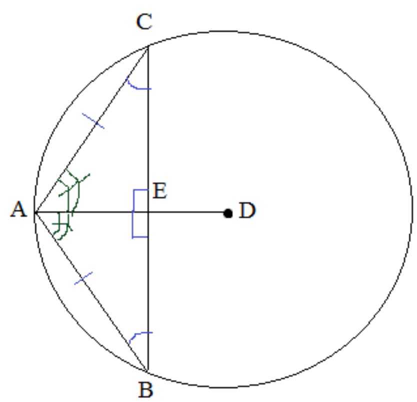
SOLUTIONS

9) Given: Circle D

$\angle B \cong \angle C$

Prove: $\overline{AD} \perp \overline{BC}$

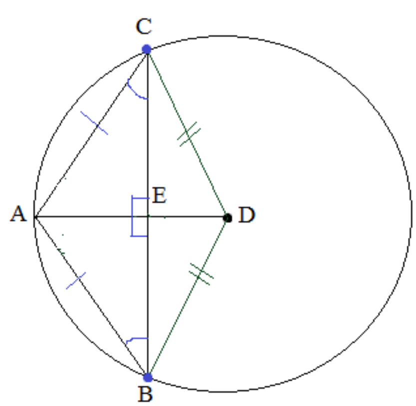




Statements	Reasons
1. $\angle B \cong \angle C$	1. Given
2. $\overline{AC} \cong \overline{AB}$	2. $\triangle$ ABC – If congruent angles, then opposite sides congruent
3. Circle D	3. Given
4. $\overline{CD} \cong \overline{BD}$	4. All radii are congruent
5. $\overline{AD} \cong \overline{AD}$	5. Reflexive property
6. $\triangle ACD \cong \triangle ABD$	6. Side-Side-Side (1, 4, 5)
7. $\angle CAD \cong \angle BAD$	7. CPCTC (corresponding parts of congruent triangles are congruent)
8. $\triangle CAE \cong \triangle BAE$	8. Angle-Side-Angle (7, 2, 1)
9. $\angle AEC \cong \angle AEB$	9. CPCTC
10. $\angle AEC$ & $\angle AEB$ are supplementary	10. Definition of supplementary (linear pair)
11. $\angle AEC$ & $\angle AEB$ are right angles	11. If angles are supplementary and congruent, then they are right angles
12. $\overline{AD} \perp \overline{BC}$	12. Definition of Perpendicular (if lines form right angles, then $\perp$)

Note the detour...
To prove lines are perpendicular, we want to show that CAE and BAE are congruent triangles...
But, to do that, we need to use triangles ACD and ABD to get one of the angles!

Utilizing the Equidistance Theorem



Statements	Reasons
1. $\angle B \cong \angle C$	1. Given
2. $\overline{AC} \cong \overline{AB}$	2. $\triangle$ ABC – If congruent angles, then opposite sides congruent
3. Circle D	3. Given
4. $\overline{CD} \cong \overline{BD}$	4. All radii are congruent
5. $\overline{AD}$ is perpendicular bisector of $\overline{CD}$	5. Equidistance Theorem (if 2 points are each equidistant from the endpoints of a segment, then they determine the perpendicular bisector of the segment)
6. $\overline{AD} \perp \overline{BC}$	6. Def. of perpendicular bisector

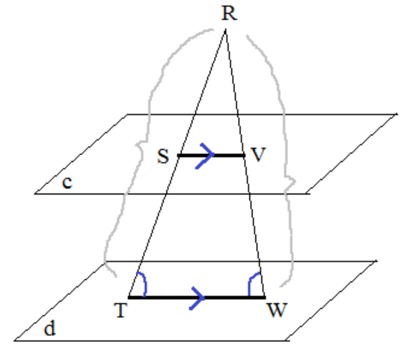
Geometry Proofs Review

SOLUTIONS

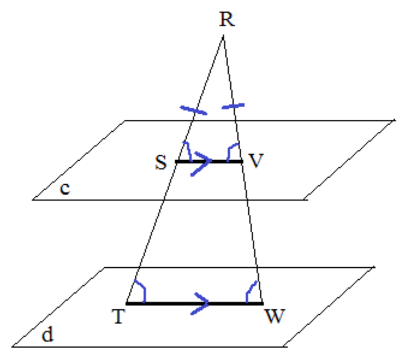
10) Given: c ∥ d

△RTW is isosceles with base $\overline{TW}$

Prove: △RSV is isosceles



Statements	Reasons
1) c ∥ d	1) Given
2) RTW is isosceles triangle	2) Given
3) $\overline{RT} \cong \overline{RW}$	3) Definition of isosceles
4) ∠T ≅ ∠W	4) If congruent sides, then congruent angles
5) RTW form a plane	5) 3 points form a plane
6) $\overline{SV} \parallel \overline{TW}$	6) If a plane intersects 2 parallel planes, then the intersected lines are parallel
7) ∠RSV ≅ ∠T ∠RVS ≅ ∠W	7) Corresponding angles
8) ∠RSV ≅ ∠RVS	8) Substitution
9) $\overline{RS} \cong \overline{RV}$	9) If congruent angles, then congruent sides
10) △ RSV is isosceles	10) Definition of Isosceles

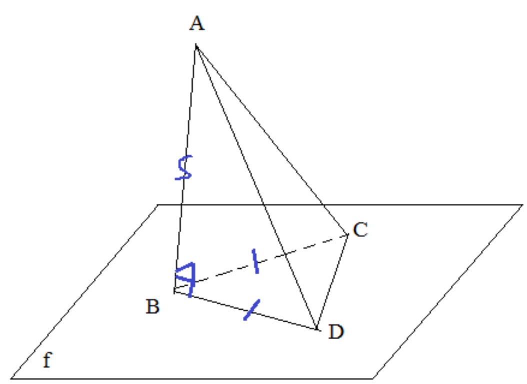


11) Given: $\overline{AB}$ ⊥ f

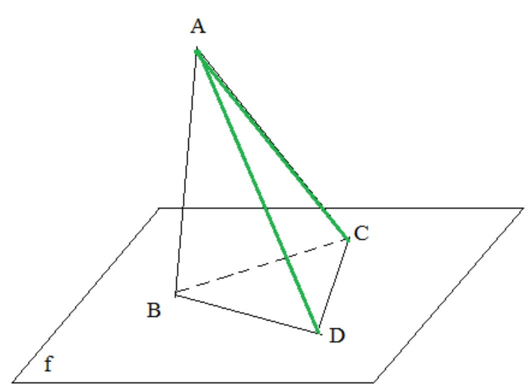
△BCD is an equilateral triangle

B, C, D are coplanar

Prove: ACD is isosceles



Statements	Reasons
1) AB ⊥ f	1) Given
2) ∠ABD and ∠ABC are right angles	2) If a line is perpendicular to a plane, it is perpendicular to any line passing through its foot; and, it forms right angles
3) ∠ABD ≅ ∠ABC	3) All right angles are congruent
4) △BCD is equilateral	4) Given
5) $\overline{BC} \cong \overline{BD}$	5) Definition of Equilateral
6) $\overline{AB} \cong \overline{AB}$	6) Reflexive property
7) △ABC ≅ △ABD	7) Side-Angle-Side (SAS) (6, 3, 5)
8) $\overline{AD} \cong \overline{AC}$	8) Corresponding parts of congruent triangles are congruent (CPCTC)
9) ACD is isosceles triangle	9) Definition of Isosceles



SOLUTIONS

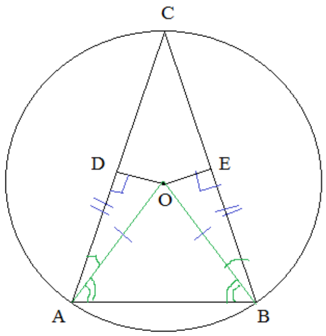
12) Given: Circle O

$\overline{AD} \cong \overline{BE}$

$\overline{AC} \perp \overline{OD}$

$\overline{CB} \perp \overline{OE}$

Prove: $\overline{AC} \cong \overline{BC}$

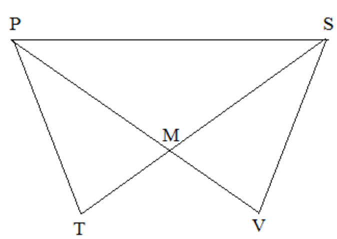


Statements	Reasons
1) Circle O	1) Given
2) $\overline{AC} \perp \overline{OD}$, $\overline{CB} \perp \overline{OE}$	2) Given
3) $\angle DOA$ and $\angle EOB$ are right angles	3) Definition of perpendicular (perpendicular segments form right angles)
4) $\angle DOA \cong \angle EOB$	4) All right angles are congruent
5) Auxilary line segments $\overline{OA}$ and $\overline{OB}$	5) A segment joins two points
6) $\overline{OA} \cong \overline{OB}$	6) All radii are congruent
7) $\overline{AD} \cong \overline{BE}$	7) Given
8) $\triangle DOA \cong \triangle EOB$	8) RHL (Right angle-Hypotenuse-Leg) 4, 6, 7
9) $\angle DAO \cong \angle EBO$	9) CPCTC (corresponding parts of congruent triangles are congruent)
10) $\angle OAB \cong \angle OBA$	10) In a triangle (AOB), if congruent sides, then opposite angles are congruent
11) $\angle DAB \cong \angle EBA$	11) Addition property (if congruent angles are added to congruent angles, then the sums are congruent)
12) $\overline{AC} \cong \overline{BC}$	12) In a triangle (ABC), if angles are congruent, then opposite sides are congruent

13) Given: $\angle MPS \cong \angle MSP$

$\angle TPM \cong \angle VSM$

Prove: $\overline{PT} \cong \overline{SV}$

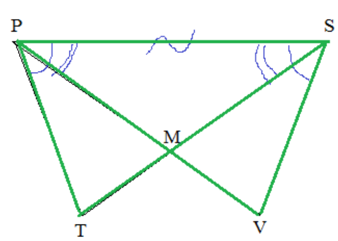


Approach 1:

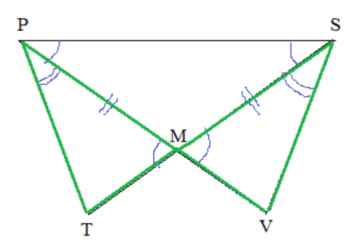
Statements	Reasons
1) $\angle MPS \cong \angle MSP$	1) Given
2) $\angle TPM \cong \angle VSM$	2) Given
3) $\angle TPS \cong \angle VSP$	3) Addition Postulate
4) $\overline{PS} \cong \overline{PS}$	4) Reflexive Property
5) $\triangle TPS \cong \triangle VSP$	5) ASA (Angle-Side-Angle) 3, 4, 1
6) $\overline{PT} \cong \overline{SV}$	6) CPCTC

Approach 2:

Statements	Reasons
1) $\angle MPS \cong \angle MSP$	1) Given
2) $\angle TPM \cong \angle VSM$	2) Given
3) $\overline{PM} \cong \overline{SM}$	3) If congruent angles, then congruent sides
4) $\angle PMT \cong \angle SMV$	4) vertical angles
5) $\triangle TPM = \triangle VSM$	5) ASA (Angle-Side-Angle) 2, 3, 4
6) $\overline{PT} \cong \overline{SV}$	6) CPCTC



(overlapping triangles)

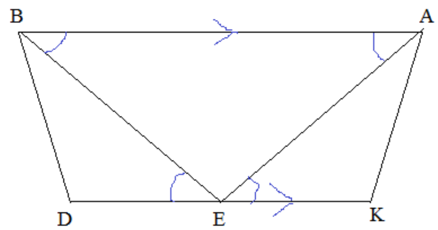


(adjacent triangles)

14) Given: $\overline{AB} \parallel \overline{EC}$

$\angle AEK = \angle BED$

Prove: $\triangle ABE$ is isosceles



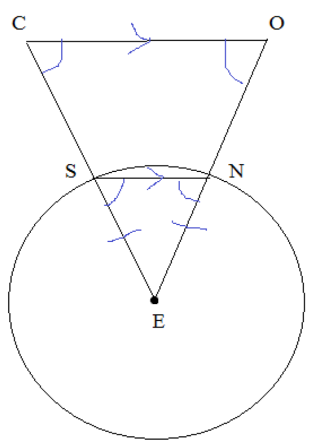
Statements	Reasons
1) $\overline{AB} \parallel \overline{EC}$	1) Given
2) $\angle AEK = \angle BED$	2) Given
3) $\angle BED = \angle EBA$ $\angle AEK = \angle BAE$	3) If parallel lines cut by transversal, then alternate angles are congruent
4) $\angle EBA = \angle BAE$	4) Transitive property
5) $\triangle ABE$ is isosceles	5) If base angles are congruent, then triangle is isosceles

C = ESN or C ≠ ESN

15) Given: Circle E

$\triangle COE$ is scalene

Prove: $\angle C \ncong \angle ESN$



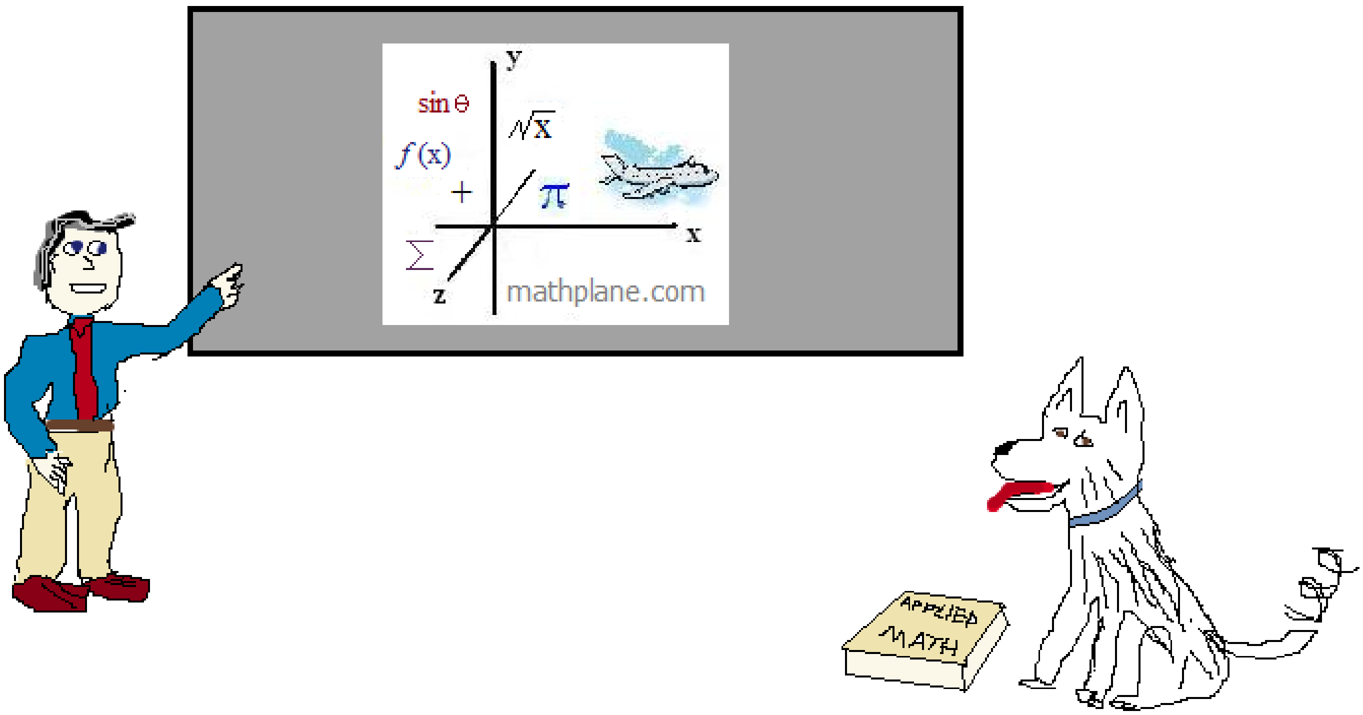
Statements	Reasons
1) Circle E	1) Given
2) $\triangle COE$ is scalene	2) Given
3) $\angle C = \angle ESN$	3) Assume for Contradiction
4) $\overline{ES} = \overline{EN}$	4) All radii are congruent
5) $\overline{CO} \parallel \overline{SN}$	5) If corresponding angles are congruent, then lines are parallel
6) $\angle O = \angle ENS$	6) If lines are parallel, then corresponding angles are congruent
7) $\angle ESN = \angle ENS$	7) If congruent sides, then congruent angles
8) $\angle O = \angle C$	8) Transitive property
9) $\triangle COE$ is isosceles	9) If base angles are congruent, then triangle is isosceles

However, 2) and 9) contradict each other

Thanks for visiting. (Hope it helps!)

If you have questions, suggestions, or requests, let us know.

Cheers.



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And, Mathplane *Express* for mobile at mathplane.ORG

ONE MORE FUN PROOF-→

What is wrong with this proof?!!?

Given: $a = b$

$a = b$	Given
$aa = ab$	Multiplication
$aa - bb = ab - bb$	Subtraction
$(a + b)(a - b) = b(a - b)$	Factoring
$a + b = b$	Division
$a + a = a$	Substitution
$2a = a$	Addition
$2 = 1$	Division

Therefore, $2 = 1$

ANSWER on the next page-→

What is wrong with this proof?!!?

Given: $a = b$

$a = b$	Given
$aa = ab$	Multiplication
$aa - bb = ab - bb$	Subtraction
$(a + b)(a - b) = b(a - b)$	Factoring
$a + b = b$	Division
$a + a = a$	Substitution
$2a = a$	Addition
$2 = 1$	Division

If $a = b$,
then $(a - b) = 0$...

And, you cannot
divide by zero!

Therefore, $2 = 1$