Geometry: Proofs and Postulates

Definitions, Notes, & Examples

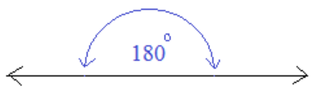
Statements	Reasons
1. $\overline{AD}$ and $\overline{BC}$ bisect each other	1. Given
2. $\overline{AM} \cong \overline{DM}$; $\overline{CM} \cong \overline{BM}$	2. Definition of bisector
3. $\angle AMC \cong \angle BMD$	3. Vertical angles are congruent
4. $\triangle AMC \cong \triangle DMB$	4. Side-Angle-Side (SAS) (2, 3, 2)
5. $\overline{AC} \cong \overline{BD}$	5. CPCTC (Corresponding Parts of Congruent Triangles are Congruent)

Topics include triangle characteristics, quadrilaterals, circles, midpoints, SAS, and more.

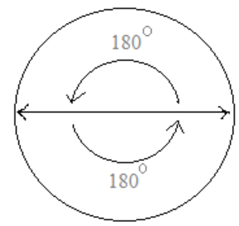
Proofs and Postulates: Triangles and Angles

Postulate: A statement accepted as true without proof.

I. A *Straight Angle* is 180°



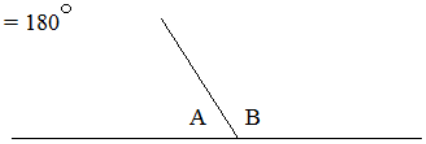
A circle has 360°



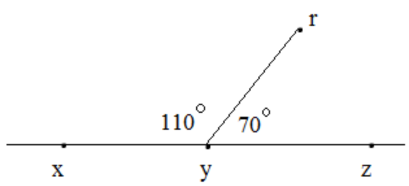
It follows that the semi-circle is 180 degrees.

II. *Supplementary Angles* add up to 180°

$m\angle A + m\angle B = 180^\circ$

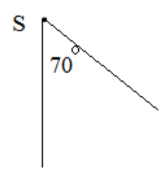


Example:



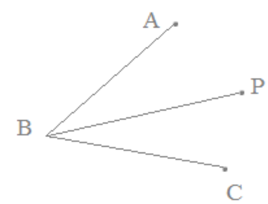
$\angle$xyr and $\angle$ryz are supplementary angles.

And, although they are not adjacent, $\angle$S and $\angle$xyr are supplementary as well.



Angle Addition Postulate: If point P lies in the interior of $\angle ABC$, then

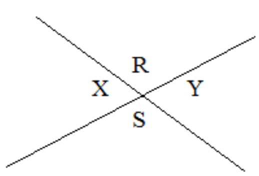
$m\angle ABP + m\angle CBP = m\angle ABC$



($\angle$ABP is *adjacent* to $\angle$CBP because they share a common vertex and side)

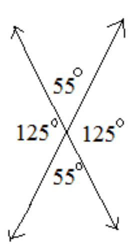
Theorem: A statement or assertion that can be proven using rules of logic.

III. *Vertical Angles* are congruent

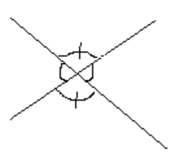


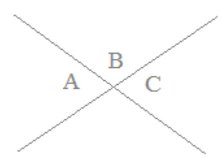
$\angle R \cong \angle S \qquad \angle X \cong \angle Y$

Examples:



(sample notation
for congruent angles)





Informal proof: $\angle A = \angle C$

A + B = 180 degrees (supplementary angles)

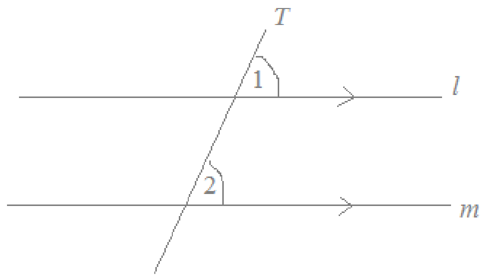
B + C = 180 degrees (supplementary angles)

A = C (substitution)

Using postulates and math properties, we construct a sequence of logical steps to prove a theorem.

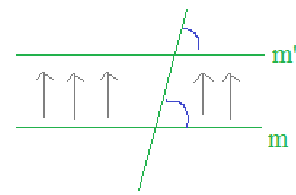
Proofs and Postulates: Triangles and Angles

Parallel Line Postulate: If 2 parallel lines are cut by a transversal, then their corresponding angles are congruent.



$\angle 1 \cong \angle 2$

A simple sketch can show the parallel line postulate.

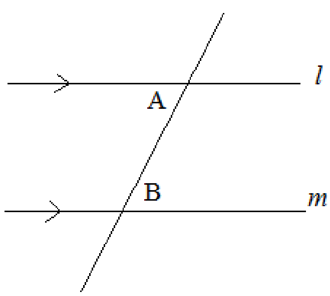


note: moving each point the same distance and direction will produce a parallel line (and a corresponding angle)

IV. If parallel lines are cut by a transversal, the *alternate interior* angles are congruent (Theorem)

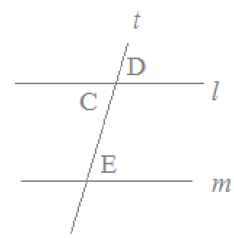
$l \parallel m$

$\angle A \cong \angle B$

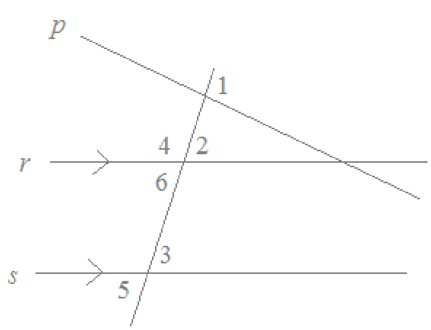


Proof of parallel lines/alt. interior angles:

Statement	Reason
1. $l \parallel m$	1. given
2. t is transversal	2. given (def. of transversal)
3. $\angle D \cong \angle E$	3. if parallel lines cut by transversal, then corresponding angles are congruent)
4. $\angle C \cong \angle D$	4. vertical angles congruent
5. $\angle C \cong \angle E$	5. substitution



Examples:



If $\angle 2 = 70^\circ$ and r is parallel to s,

$4 = 110^\circ$ (2 and 4 are supplementary)

$3 = 70^\circ$ (3 and 2 are corresponding)

$5 = 70^\circ$ (3 and 5 are vertical angles)

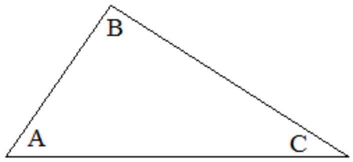
$6 = 70^\circ$ (3 and 6 are alt. interior angles)

$1 = ?$ (p is not parallel to r or s)

Proofs and Postulates: Triangles and Angles

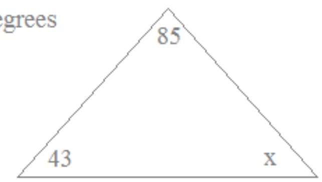
V. The sum of the *interior angles of a triangle* is 180° (Theorem)

$m\angle A + m\angle B + m\angle C = 180^\circ$



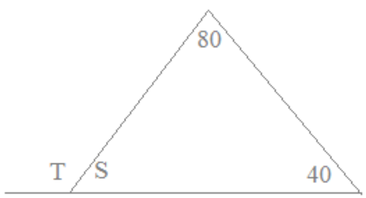
Examples:

x + 43 + 85 = 180 degrees
x = 52 degrees

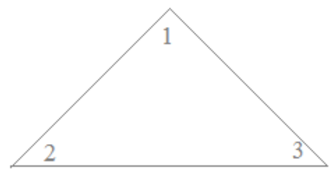


S + 40 + 80 = 180
S = 60 degrees

T + S = 180 degrees
T + 60 = 180
So, T = 120 degrees

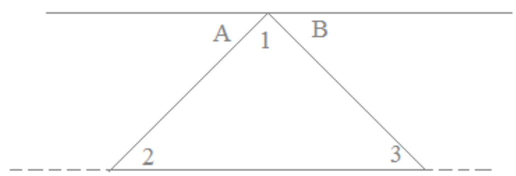


** Illustrates the *triangle (remote) exterior angle theorem*: the measure of an exterior angle equals the sum of the 2 non-adjacent interior angles.



Informal Proof: $1 + 2 + 3 = 180^\circ$

Add parallel line to one of the sides



A + 1 + B = 180 degrees (straight angle and addition postulate)

A = 2 and B = 3 (parallel lines cut by transversal, then alt. interior angles are congruent)

2 + 1 + 3 = 180 degrees (substitution)

Tools to consider in Geometry proofs:

1) Using CPCTC (Corresponding Parts of Congruent Triangles are Congruent) after showing triangles within the shapes are congruent.

Try

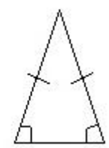
a) reflexive property
b) vertical angles are congruent
c) alternate interior angles (formed by parallel lines cut by a transversal) are congruent

Then, verify congruent triangles by SAS, SSS, ASA, AAS, HL

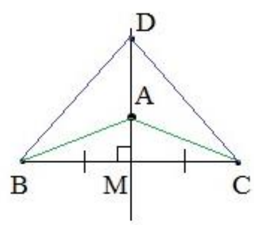
2) Common properties and theorems

a) Triangles are 180° ; Quadrilaterals are 360°

b) Opposite sides of congruent angles are congruent (isosceles triangle)



c) Perpendicular bisector Theorem
(All points on perpendicular bisector are equidistant to endpoints)

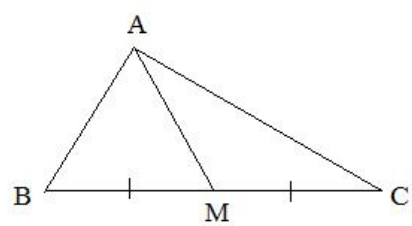


DM is perpendicular bisector of $\overline{BC}$ (B and C are the endpoints)

$\overline{BD} = \overline{CD}$
$\overline{BA} = \overline{CA}$

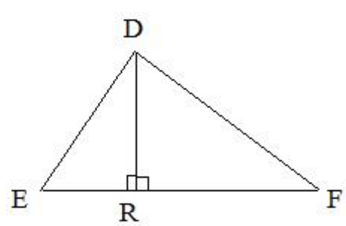
3) Other geometry basics:

a) All radii of a circle are congruent
b) Supplementary angles (180°); Complementary angles (90°)
c) Midpoints and medians divide segments into congruent parts



$\overline{AM}$ is median of $\triangle ABC$
M is midpoint of $\overline{BC}$

d) Altitudes form right angles

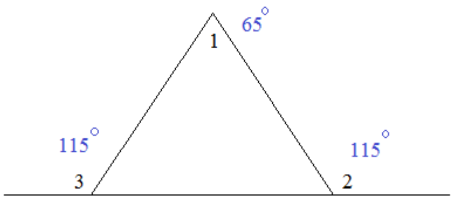


$\overline{DR}$ is altitude of $\triangle DEF$

$\angle DRE$ and $\angle DRF$ are right angles

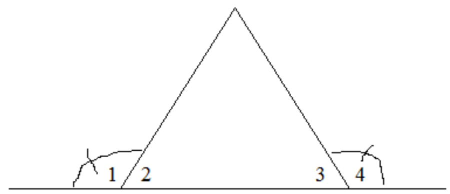
"Definition of Supplementary"

If 2 angles are supplementary to the same angle, then the angles are congruent.



Angle 3 is supplementary to angle 1...
Angle 2 is supplementary to angle 1...
Therefore, angles 3 and 2 are congruent!

If 2 angles are supplementary to congruent angles, then the angles are congruent.

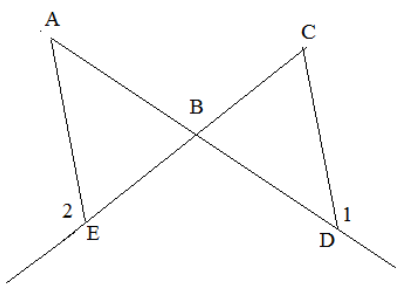


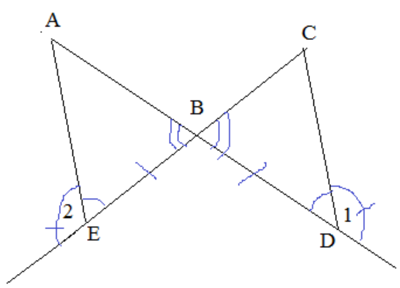
Angle 2 is supplementary to angle 1
Angle 3 is supplementary to angle 4
Since angles 1 and 4 are congruent, then therefore, angles 2 and 3 are congruent!

Given: $\angle 1 \cong \angle 2$

$\overline{BE} \cong \overline{BD}$

Prove: $\triangle ABE \cong \triangle CBD$





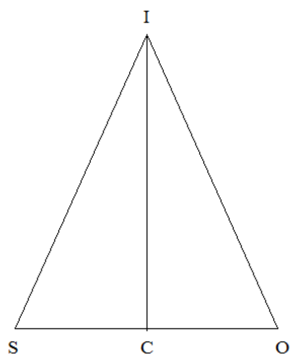
Statements	Reasons
1) $\overline{BE} \cong \overline{BD}$	1) Given
2) $\angle 1 \cong \angle 2$	2) Given
3) $\angle$ CDB is supp to 1	3) Definition of Supplementary
4) $\angle$ AEB is supp to 2	4) Def. of Supplementary (diagram)
5) $\angle CDB \cong \angle AEB$	5) If angles supp. to congruent angles, then angles congruent
6) $\angle CBD \cong \angle ABE$	6) Vertical angles congruent
7) $\angle ABE \cong \angle CBD$	7) Angle-Side-Angle (6, 1, 5)

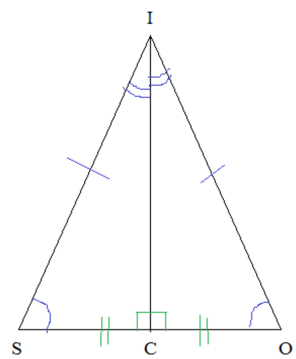
"Right Angle Theorem: If 2 angles are congruent and supplementary, then they are right angles."

Given: $\triangle$ISO is isosceles with base $\overline{SO}$

$\overline{IC}$ is an angle bisector

Prove: $\overline{IC}$ is an altitude



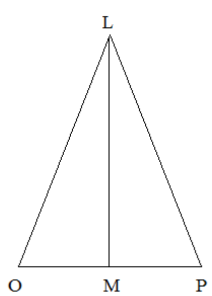


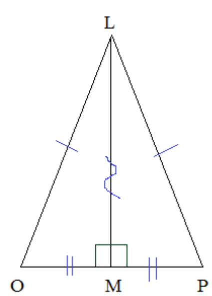
Statements	Reasons
1) ISO is an isosceles triangle	1) Given
2) $\overline{IS}$ = $\overline{IO}$	2) Definition of isosceles (2 or more sides congruent)
3) $\angle$ISC = $\angle$IOC	3) If congruent sides, then congruent angles (or, base angles of isosceles triangle congruent)
4) $\overline{IC}$ is an angle bisector	4) Given
5) $\angle$ SIC = $\angle$ OIC	5) Definition of bisector (segment divides angle into congruent angles)
6) $\triangle$ISC = $\triangle$ IOC	6) Angle-Side-Angle (5, 2, 3)
7) $\angle$ICS = $\angle$ICO	7) CPCTC (corresponding parts of congruent triangles are congruent)
8) $\angle$ICS and $\angle$ICO are supplementary	8) Definition of Supplementary angles (2 adjacent angles that for straight line/angle)
9) ICS and ICO are right angles	9) Right Angle Theorem (7, 8) (if angles are congruent AND supplementary than they are right angles...)
10) IC is an altitude	10) Definition of altitude (If segment for right angle with side of triangle, then it is an altitude)

Given: Isosceles triangle LOP

$\overline{OM}$ is a median drawn to base of triangle

Prove: $\overline{OP} \perp \overline{LM}$



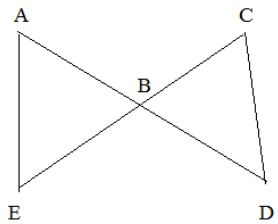


Statements	Reasons
1) LOP is isosceles $\triangle$	1) Given
2) $\overline{LO}$ = $\overline{LP}$	2) Definition of isosceles
3) $\overline{LM} = \overline{LM}$	3) Reflexive Property
4) $\overline{OM}$ is a median	4) Given
5) M is midpoint of $\overline{OP}$	5) Definition of median (segment drawn from vertex to midpoint of opposite side)
6) $\overline{OM} = \overline{PM}$	6) Definition of midpoint (a midpoint separates a segment into congruent halves)
7) $\triangle$LMO = $\triangle$LMP	7) SSS (Side-Side-Side) 3, 6, 2
8) $\angle$LMO = $\angle$LMP	8) CPCTC (corresponding parts of congruent triangles congruent)
9) Angles LMO and LMP are right angles	9) Right angle theorem (if angles are supplementary and congruent, then they are right angles)
10) $\overline{LM} \perp \overline{OP}$	10) definition of perpendicular (Right angle is formed by perpendicular segments)

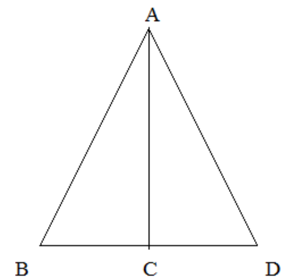
Explain the implication of each hypothesis... (Optional: label the diagrams, if possible.)

"If - Then Proof Parts"

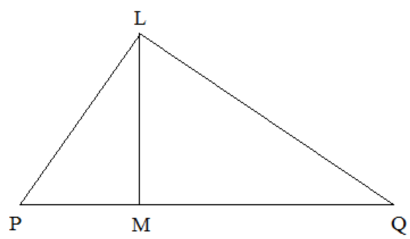
1) If $\overline{AD}$ bisects $\overline{EC}$, then ______________________



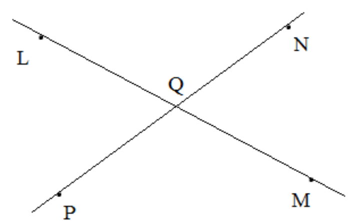
2) If $\overline{AC}$ bisects $\angle BAD$, then ______________________



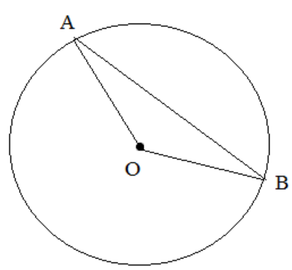
3) If $\overline{LM} \perp \overline{PQ}$, then ______________________



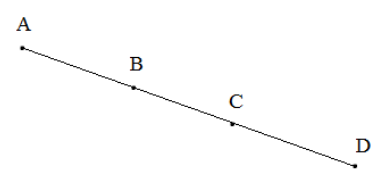
4) If lines LM and NP intersect at point Q, then ____________________



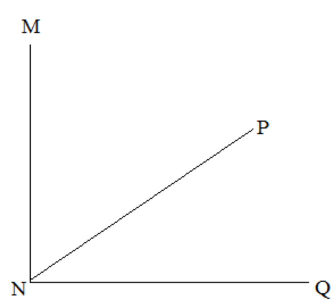
5) If O is the center of the circle, then ____________________



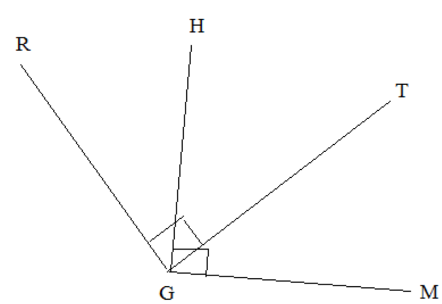
6) If $\overline{AC} \cong \overline{BD}$, then ______________________



7) $\angle MNP$ and $\angle MNQ$ are complementary angles, then ______________

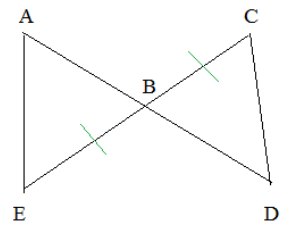


8) If $\angle RGT$ and $\angle HGM$ are right angles, then _______________

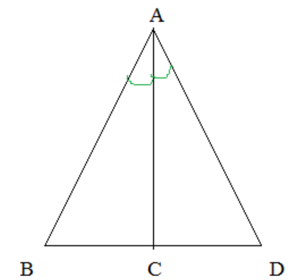


Explain the implication of each hypothesis... SOLUTIONS "If - Then Proof Parts"

1) If $\overline{AD}$ bisects $\overline{EC}$, then $\overline{BC} \cong \overline{BE}$

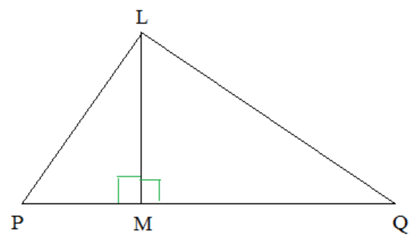


2) If $\overline{AC}$ bisects $\angle BAD$, then angles BAC and DAC are congruent

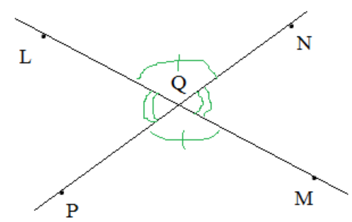


NOTE: It does not imply that $\overline{BC} = \overline{CD}$... It might be.. Or, they may not be..

3) If $\overline{LM} \perp \overline{PQ}$, then LMQ and LMP are right angles

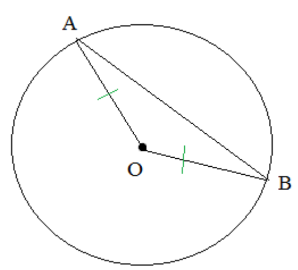


4) If lines LM and NP intersect at point Q, then angles NQM and LQP are congruent (by vertical angles)

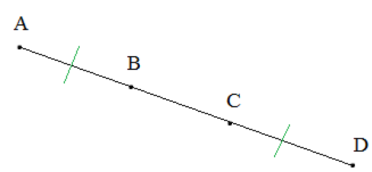


(also, $\angle LQN \cong \angle PQM$)

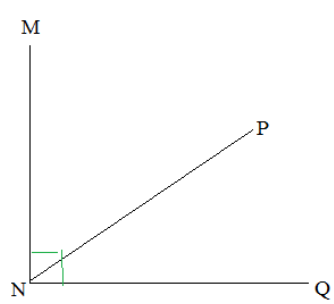
5) If O is the center of the circle, then $\overline{OA} \cong \overline{OB}$ (because all radii are congruent)



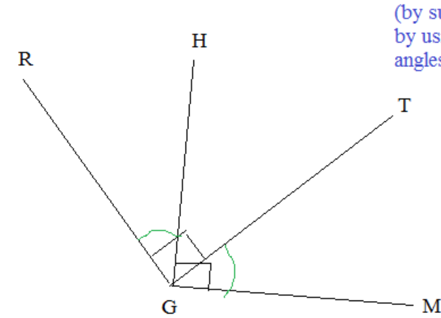
6) If $\overline{AC} \cong \overline{BD}$, then $\overline{AB} \cong \overline{CD}$ (by subtraction of $\overline{BC}$)



7) $\angle MNP$ and $\angle MNQ$ are complementary angles, then MNQ is a right angle (and $\overline{MN} \perp \overline{NQ}$)



8) If $\angle RGT$ and $\angle HGM$ are right angles, then $\angle RGH \cong \angle TGM$ (by subtraction OR by using complementary angles and transitive)

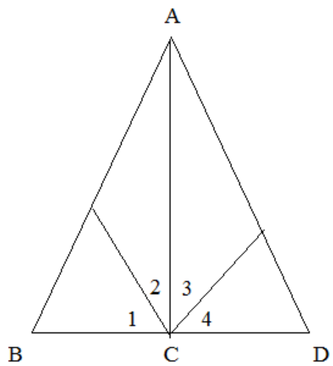


A) Given: $\angle 1$ is complementary to $\angle 2$

$\angle 3$ is complementary to $\angle 4$

$\overline{AC}$ is a median

Prove: $\triangle ACB \cong \triangle ACD$

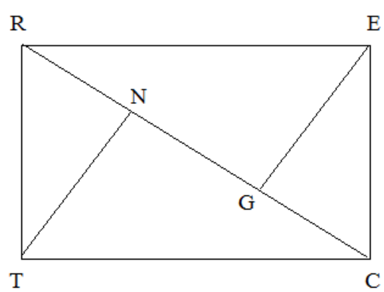


Statements	Reasons

B) Given: $\overline{RN} \cong \overline{GC}$ $\overline{NT} \cong \overline{GE}$

$\overline{EG} \perp \overline{RC}$ $\overline{NT} \perp \overline{RC}$

Prove: $\triangle EGR \cong \triangle TNC$



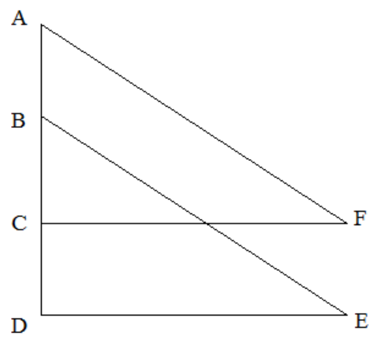
Statements	Reasons

C) Given: $\overline{AD} \perp \overline{DE}$ $\quad \overline{AC} \perp \overline{CF}$

$\angle E \cong \angle F$

$\overline{AB} \cong \overline{CD}$

Prove: $\triangle ACF \cong \triangle BDE$

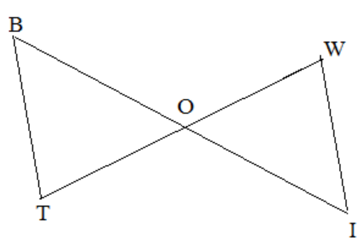


Statements	Reasons

D) Given: $\overline{BI}$ bisects $\overline{WT}$

O is the midpoint of $\overline{BI}$

Prove: $\triangle BOT \cong \triangle IOW$



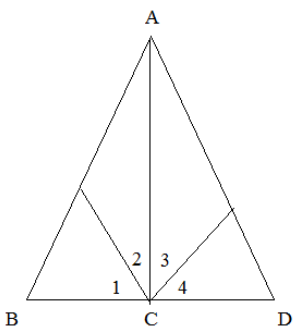
Statements	Reasons

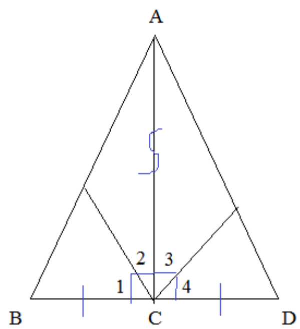
SOLUTIONS

A) Given: ∠1 is complementary to ∠2
∠3 is complementary to ∠4
$\overline{AC}$ is a median

Prove: △ ACB ≅ △ ACD

USES
-- Reflexive Property
-- Medians
-- Complementary Angles



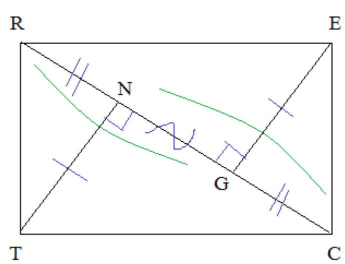


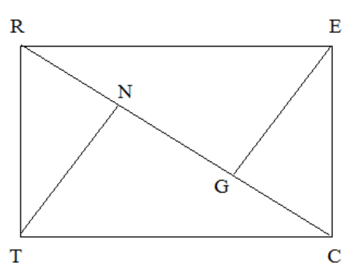
Statements	Reasons
1) ∠1 comp to ∠2 ∠3 comp to ∠4	1) Given
2) ∠ACB and ∠ACD are right angles	2) Complementary (adjacent) angles form right angles
3) ∠ACB = ∠ACD	3) All right angles are congruent
4) $\overline{AC}$ is a median	4) Given
5) $\overline{BC} \cong \overline{CD}$	5) Definition of Median (Median divides side into congruent segments)
6) $\overline{AC} \cong \overline{AC}$	6) Reflexive property
7) △ACB = △ACD	7) Side-Angle-Side (5, 3, 6)

B) Given: $\overline{RN} \cong \overline{GC}$ $\overline{NT} \cong \overline{GE}$
$\overline{EG} \perp \overline{RC}$ $\overline{NT} \perp \overline{RC}$

Prove: △EGR ≅ △TNC

USES
-- Addition Property
-- Perpendicular Segments and Right Angles





Statements	Reasons
1) $\overline{NT} \cong \overline{GE}$	1) Given
2) $\overline{RN} \cong \overline{GC}$	2) Given
3) $\overline{NG} \cong \overline{NG}$	3) Reflexive Property
4) $\overline{GR} \cong \overline{NC}$	4) Addition Property (If segment added to congruent segments, then sums are congruent)
5) EG ⊥ RC ; NT ⊥ RC	5) Given
6) ∠EGR and ∠TNC are right angles	6) Definition of Perpendicular (Perpendicular segments form right angles)
7) ∠EGR ≅ ∠TNC	7) All right angles are congruent
8) △EGR = △TNC	8) Side-Angle-Side (1, 7, 4)

SOLUTIONS

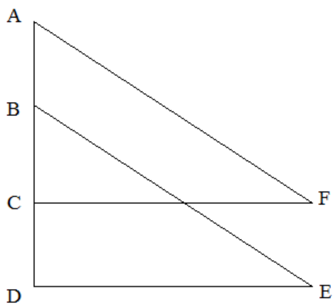
C) Given: $\overline{AD} \perp \overline{DE}$ $\quad \overline{AC} \perp \overline{CF}$

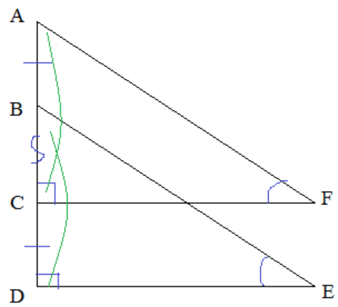
$\angle E \cong \angle F$

$\overline{AB} \cong \overline{CD}$

Prove: $\triangle ACF \cong \triangle BDE$

USES:
-- Right angles
-- Addition Property
-- Reflexive Property





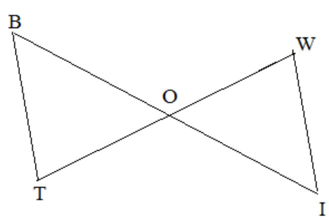
Statements	Reasons
1) $\overline{AD} \perp \overline{DE}$; $\overline{AC} \perp \overline{CF}$	1) Given
2) $\angle ADE$ & $\angle ACF$ are right angles	2) Definition of Perpendicular (perpendicular segments form right angles)
3) $\angle ADE \cong \angle ACF$	3) All right angles are congruent
4) $\angle E \cong \angle F$	4) Given
5) $\overline{AB} \cong \overline{CD}$	5) Given
6) $\overline{BC} \cong \overline{BC}$	6) Reflexive Property
7) $\overline{AC} \cong \overline{BD}$	7) Addition Property (If segment is added to congruent segments, then sums are congruent)
8) $\triangle ACF \cong \triangle BDE$	8) Angle-Angle-Side (4, 3, 7)

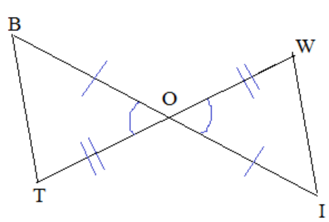
D) Given: $\overline{BI}$ bisects $\overline{WT}$

O is the midpoint of $\overline{BI}$

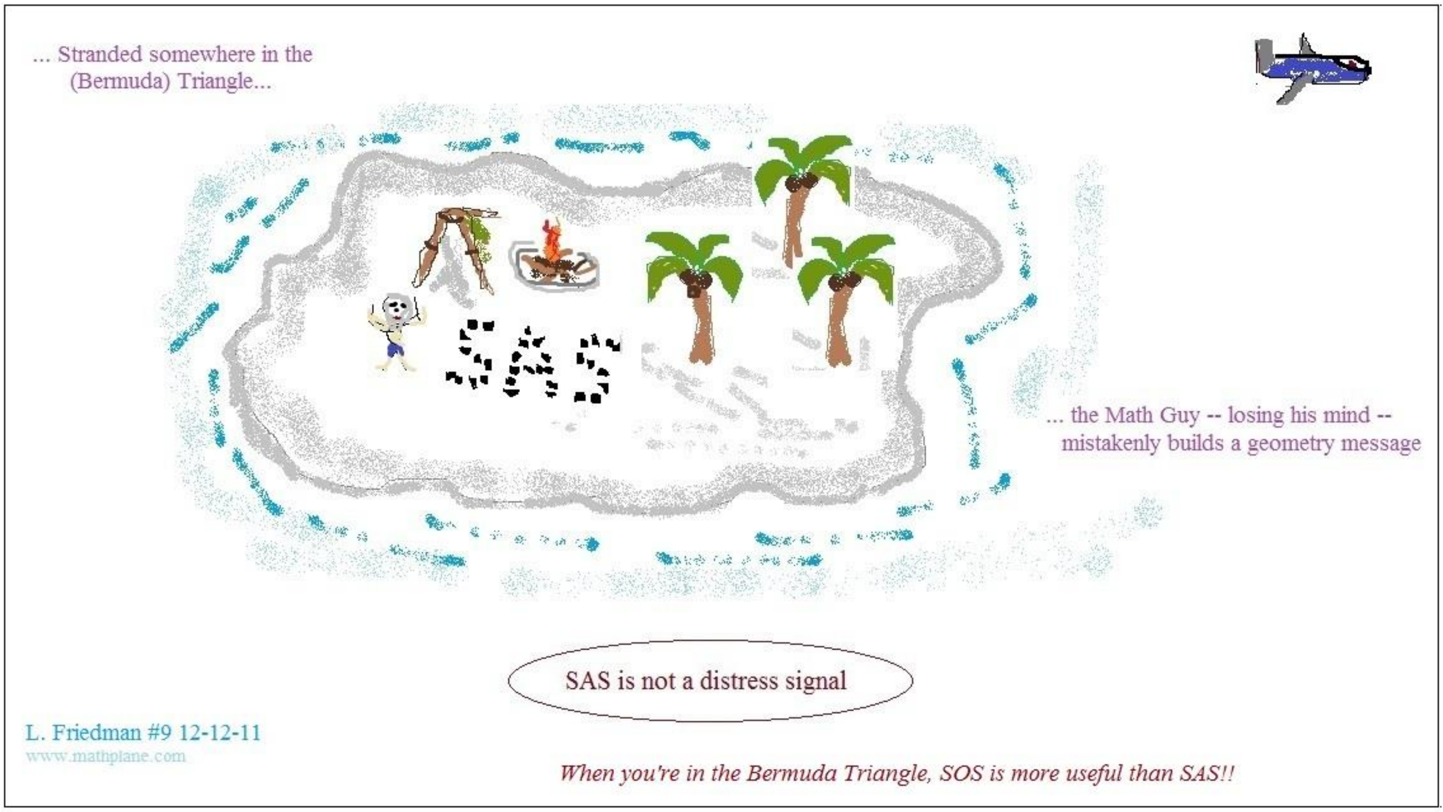
Prove: $\triangle BOT \cong \triangle IOW$

USES
-- Def. of Midpoint
-- Def. of Bisector
-- Vertical Angles





Statements	Reasons
1) $\overline{BI}$ bisects $\overline{WT}$	1) Given
2) $\overline{TO} \cong \overline{WO}$	2) Definition of bisector (a bisector divides a segment into congruent segments)
3) O is the midpoint of $\overline{BI}$	3) Given
4) $\overline{OB} \cong \overline{OI}$	4) Definition of Midpoint (midpoint divides a segment into congruent segments)
5) $\angle WOI \cong \angle TOB$	5) Vertical Angles are congruent
6) $\triangle BOT \cong \triangle IOW$	6) Side-Angle-Side (4, 5, 2)

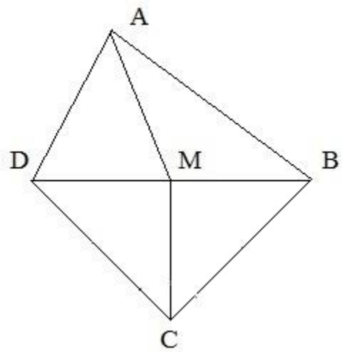

... Stranded somewhere in the (Bermuda) Triangle...
SAS
... the Math Guy -- losing his mind -- mistakenly builds a geometry message
SAS is not a distress signal
L. Friedman #9 12-12-11
www.mathplane.com
When you're in the Bermuda Triangle, SOS is more useful than SAS!!

Examples-→

Proving a Median of a Triangle: Example

Given: $\overline{CM}$ bisects $\angle BCD$
$\overline{DC} \cong \overline{BC}$

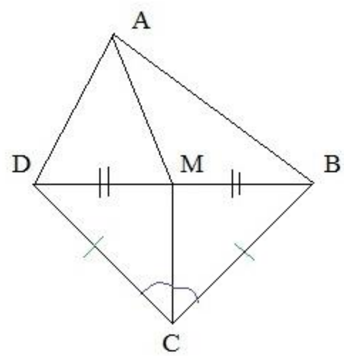
Prove: $\overline{AM}$ is a median of $\triangle BDA$



Step 1: "Label the picture."

$\overline{CM}$ bisects $\angle BCD$

$\overline{DC} \cong \overline{BC}$



What are we trying to prove? $\overline{DM} \cong \overline{MB}$
(definition of a median)

Step 2: Determine a Strategy

To prove a median, I need to show a segment bisects the opposite side. (def. of a median).

Notice triangles CMD & CMB. They include DM & MB. If I can show CMD = CMB, then I can use CPCTC to prove that DM = MB.

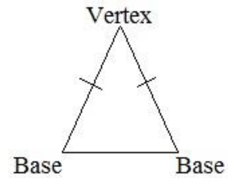
Step 3: Write the Proof (describing your approach and strategy)

Statements	Reasons
1) $\overline{DC} = \overline{BC}$ CM bisects $\angle BCD$	1) Given
2) $\angle DCM = \angle BCM$	2) Definition of Angle Bisector
3) $\overline{MC} = \overline{MC}$	3) Reflexive Property
4) $\triangle DCM = \triangle BCM$	4) Side-Angle-Side (SAS) postulate
5) $\overline{DM} = \overline{BM}$	5) Corresponding Parts of Congruent Triangles are Congruent (CPCTC)
6) AM is median of $\triangle ABD$	6) Definition of a Median (A line segment joining vertex of triangle to midpoint of opposite side)

Proof Example

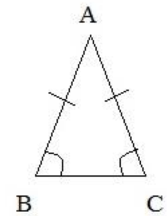
"Prove the base angles of an isosceles triangle are congruent."

Given: Isosceles Triangle
Prove: Base Angles are congruent



Step 1: Draw pictures and label..

What is given? A triangle with 2 sides of equal length.
(Definition of an Isosceles Triangle)



What are we trying to prove? $\angle B \cong \angle C$

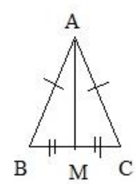
Step 2: Determine a strategy..

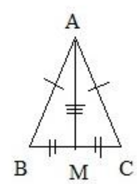
Try dividing into triangles.
Use properties of congruent triangles.
Then, CPCTC.

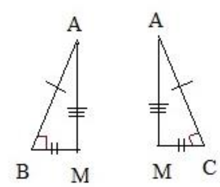
Step 3: Write the proof (describing your strategy!)

Statements	Reasons
1. $AB \cong AC$	1. Given/Definition of Isosceles Triangle
2. Draw a Median	2. An angle has one median
3. $BM \cong CM$	3. Definition of a median (A line segment joining a vertex and the midpoint of the opposite side)
4. $AM \cong AM$	4. Reflexive Axiom
5. $\triangle AMB = \triangle AMC$	5. SSS congruence postulate (If 3 sides of one triangle are congruent to corresponding sides of another triangle, then the triangles are congruent)
6. $\angle B \cong \angle C$	6. CPCTC (Corresponding Parts of Congruent Triangles are Congruent)

(Illustrations)

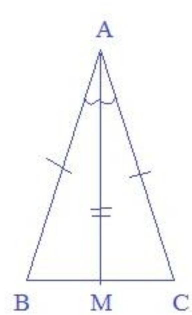






Alternate Proof: (Using angle bisector and Side-Angle-Side)

Statements	Reasons
1. $\triangle ABC$; $AB \cong AC$	1. Given; Def. of Isosceles triangle
2. $\overline{AM}$ is an angle bisector	2. An angle has one bisector
3. $\angle BAM \cong \angle CAM$	3. Def. of Angle Bisector
4. $AM \cong AM$	4. Reflexive Property
5. $\triangle BAM = \triangle CAM$	5. SAS postulate
6. $\angle B = \angle C$	6. CPCTC



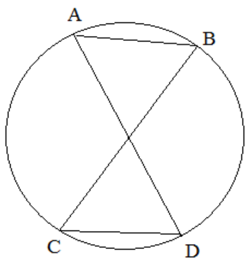
"Two Basic Circles Proofs"

NOTE: Both proofs use SAS (Side-Angle-Side)...

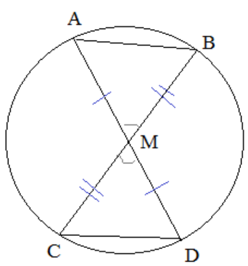
However, the first proof utilizes the midpoints to get congruent segments and, the second proof uses 'all radii congruent' to get congruent sides...

Given: M is the midpoint of $\overline{AB}$ and $\overline{CD}$

Prove: Triangles are congruent

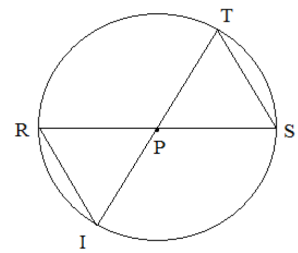


Statements	Reasons
1) M is midpoint of $\overline{BC}$ M is midpoint of $\overline{AD}$	1) Given
2) $\overline{BM} = \overline{CM}$ $\overline{AM} = \overline{DM}$	2) Definition of midpoint (midpoint divides into congruent segments)
3) $\angle AMB = \angle CMD$	3) Vertical angles congruent
4) $\triangle AMB = \triangle DMC$	4) side-angle-side

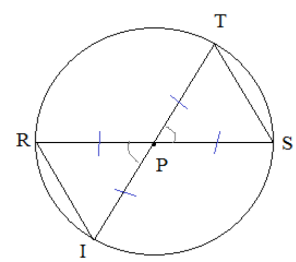


Given: Circle P

Prove: Triangles are congruent



Statements	Reasons
1) Circle P	1) Given
2) TP = IP = SP = RP	2) All radii are congruent
3) $\angle$ TPS = $\angle$ RPI	3) Vertical angles congruent
4) $\triangle$ TSP = $\triangle$ RIP	4) side-angle-side

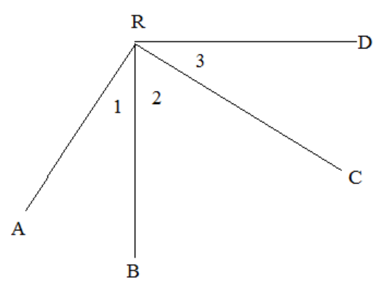


Applying "subtraction" or "complementary angles"

Given: $\overline{AR} \perp \overline{RC}$

$\overline{BR} \perp \overline{RD}$

Prove: $\angle 1 \cong \angle 3$



Statements	Reasons
1) $\overline{AR} \perp \overline{RC}$	1) Given
2) $\overline{BR} \perp \overline{RD}$	2) Given
3) $\angle$ARC and $\angle$BRD are right angles	3) Definition of Perpendicular (Perpendicular lines form right angles)
4) $\angle$1 and $\angle$2 are complementary angles	4) Complementary angles for right angles
5) $\angle$3 and $\angle$2 are complementary angles	5) Complementary angles for right angles
6) $\angle 1 \cong \angle 3$	6) If 2 angles are complementary to congruent angles, then the 2 angles are congruent (also, transitive property)

Using Complementary angles (and transitive property)

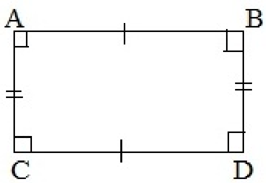
Statements	Reasons
1) $\overline{AR} \perp \overline{RC}$	1) Given
2) $\overline{BR} \perp \overline{RD}$	2) Given
3) $\angle$ARC and $\angle$BRD are right angles	3) Definition of Perpendicular (Perpendicular lines form right angles)
4) $\angle ARC \cong \angle BRD$	4) All right angles are congruent
5) $\angle 2 \cong \angle 2$	5) Reflexive property
6) $\angle 1 \cong \angle 3$	6) Subtraction Property (If congruent angles are subtracted from congruent angles, then the difference are congruent)

Using Subtraction Property...

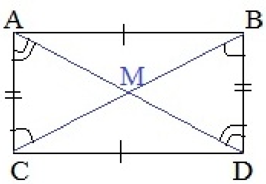
EXAMPLE: Prove diagonals of a rectangle are congruent and bisect each other.

Given: Rectangle ABDC
Prove: $\overline{BC} = \overline{AD}$ and $\overline{BC}, \overline{AD}$ bisect each other

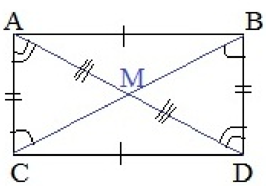
PROOF



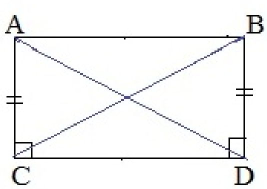
Statement	Reason	
1) AC = BD AB = CD AB || CD AC || BD	Definition of Rectangle	(opposite sides are congruent) (all angles are congruent; right angles)



Statement	Reason
2) $\angle CBD \cong \angle BCA$ $\angle CAD \cong \angle BDA$	Parallel lines cut by a transversal, then alternate interior angles are congruent
3) $\triangle ACM = \triangle DBM$	Congruent triangles Angle-Side-Angle



Statement	Reason	
4) $\overline{AM} = \overline{DM}$ $\overline{CM} = \overline{BM}$	CPCTC	
5) $\overline{BC}$ and $\overline{AD}$ bisect each other	Definition of Bisector	(A line, ray, or segment that cuts a segment into 2 congruent parts)



Statement	Reason
6) $\overline{CD} = \overline{CD}$	Reflexive Property
7) $\angle C = \angle D = 90^\circ$ $\overline{AC} = \overline{BD}$	Definition of Rectangle
8) $\triangle ACD = \triangle BDC$	Congruent triangles Side-Angle-Side
9) $\overline{BC} = \overline{AD}$	CPCTC

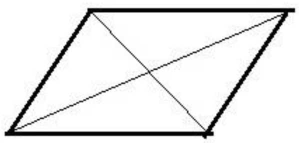
Note: Pythagorean theorem can show that diagonals are equal

Proof Example

Prove the "Parallelogram Diagonals Theorem"
(The Diagonals of a Parallelogram Bisect each other)

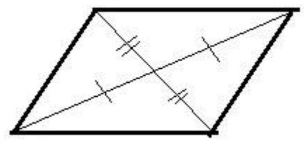
Given: Parallelogram
Prove: Diagonals Bisect Each Other

(Draw Picture)

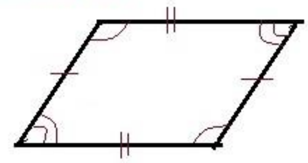


(Establish Strategy)

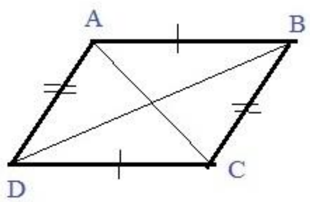
Diagonals bisecting each other implies that congruent line segments are inside the parallelogram.
(Also, notice that diagonals create triangles.)



(Label "things you know" about the Given (Parallelogram))



PROOF

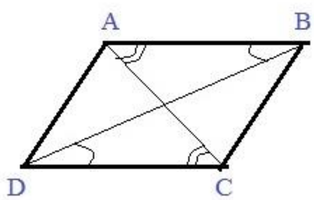


1) $AB \cong DC$
$AD \cong BC$
$AD \parallel BC$
$AB \parallel DC$

Definition of a Parallelogram

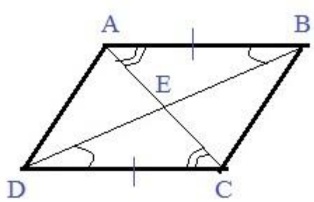
(opposite sides are congruent)

(opposite sides are parallel)



2) $\angle ACD \cong \angle CAB$
$\angle ABD \cong \angle CDB$

If two parallel lines are cut by a transversal, then alternate interior angles are congruent.



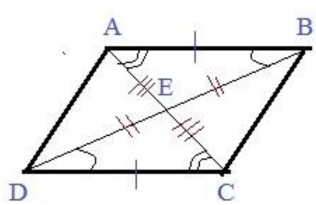
3) $\triangle ABE \cong \triangle CDE$

ASA (Angle-Side-Angle) Triangles are congruent

4) $\overline{BE} \cong \overline{DE}$

$\overline{AE} \cong \overline{CE}$

CPCTC
(Corresponding Parts of Congruent Triangles are Congruent)



5) AC bisects BD
and
BD bisects AC

Definition of a Bisector

(A line, ray, or segment that cuts a segment into 2 congruent parts)

("Diagonals of a Parallogram bisect each other)

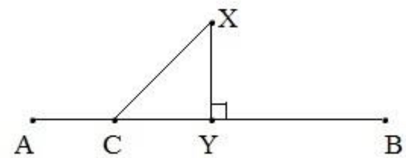
Prove that the shortest distance between a point and a line is a perpendicular line segment.

Step 1: Write out the Given and Prove statements

Given: Line AB with external point X
Line segment XY is perpendicular to AB
Segment XC is non-perpendicular to AB

Prove: Segment XY is shorter than segment XC

Step 2: Draw a diagram to clarify



Step 3: Determine a strategy

We need to prove that XY < XC.
We see that we're dealing with a right triangle
(angles X and C are both acute, and Y is 90 degrees)
And, we know that size of angles indicates size of opposite sides.

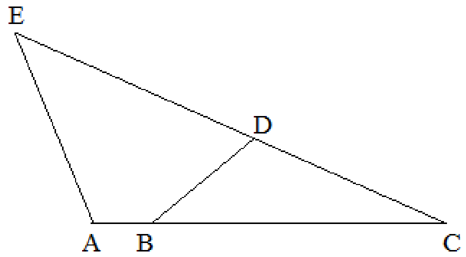
Step 4: Proof

Statements	Reasons
1. Line AB with external point X $XY \perp AB$	1. Given
2. $\angle XYC$ is a right angle	2. Perpendicular lines form right angle
3. $\angle XCY$ is an acute angle	3. (Interior angles of Triangle add up to 180°) Non-right angles of a right triangle (i.e. $\triangle XYC$) are always acute ($< 90^\circ$)
4. $\angle XCY < \angle XYC$	4. Definition of Acute Angle (Angles that are less than 90°)
5. XY is shorter than XC	5. In a triangle, the side opposite the smaller angle is shorter than a side opposite a larger angle $\angle XCY < \angle XYC$ so, $XY < XC$

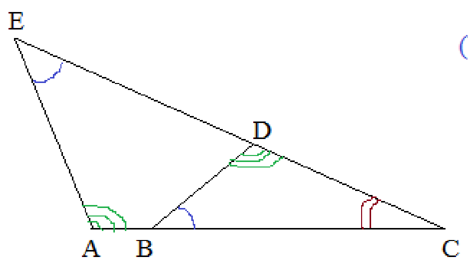
No-Choice Theorem Examples

Given: $\angle DBC \cong \angle E$

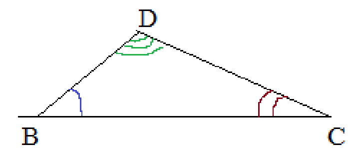
Prove: $\angle A \cong \angle BDC$

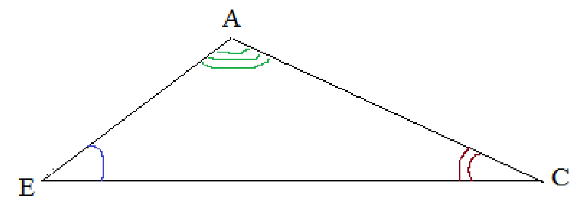


Statements	Reasons
1. $\angle DBC \cong \angle E$	1. Given
2. $\angle C \cong \angle C$	2. Reflexive Property
3. $\angle A \cong \angle BDC$	3. No-Choice Theorem
	(If 2 angles of one triangle are congruent to 2 angles of another triangle, then the 3rd angles of both triangles are congruent)



Note: The angles are congruent.
So, the triangles are *similar*.
(We need at least one pair of congruent sides for congruent triangles)

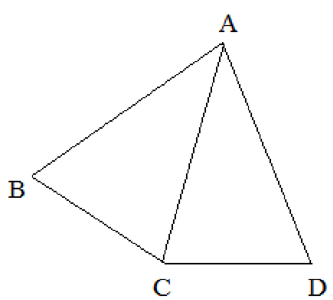


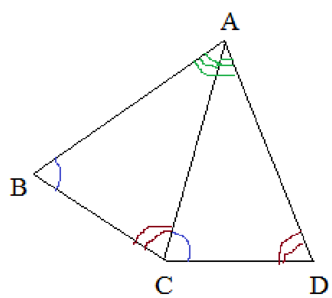


Given: $\angle ABC \cong \angle ACD$

$\angle ACB \cong \angle D$

Are the triangles congruent?



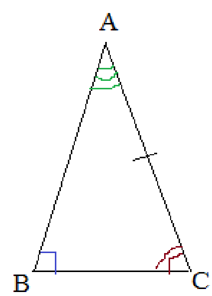


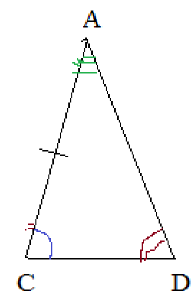
Since two angles are congruent, the 3rd angles must be congruent (no-choice theorem)

We have angle-angle-angle...

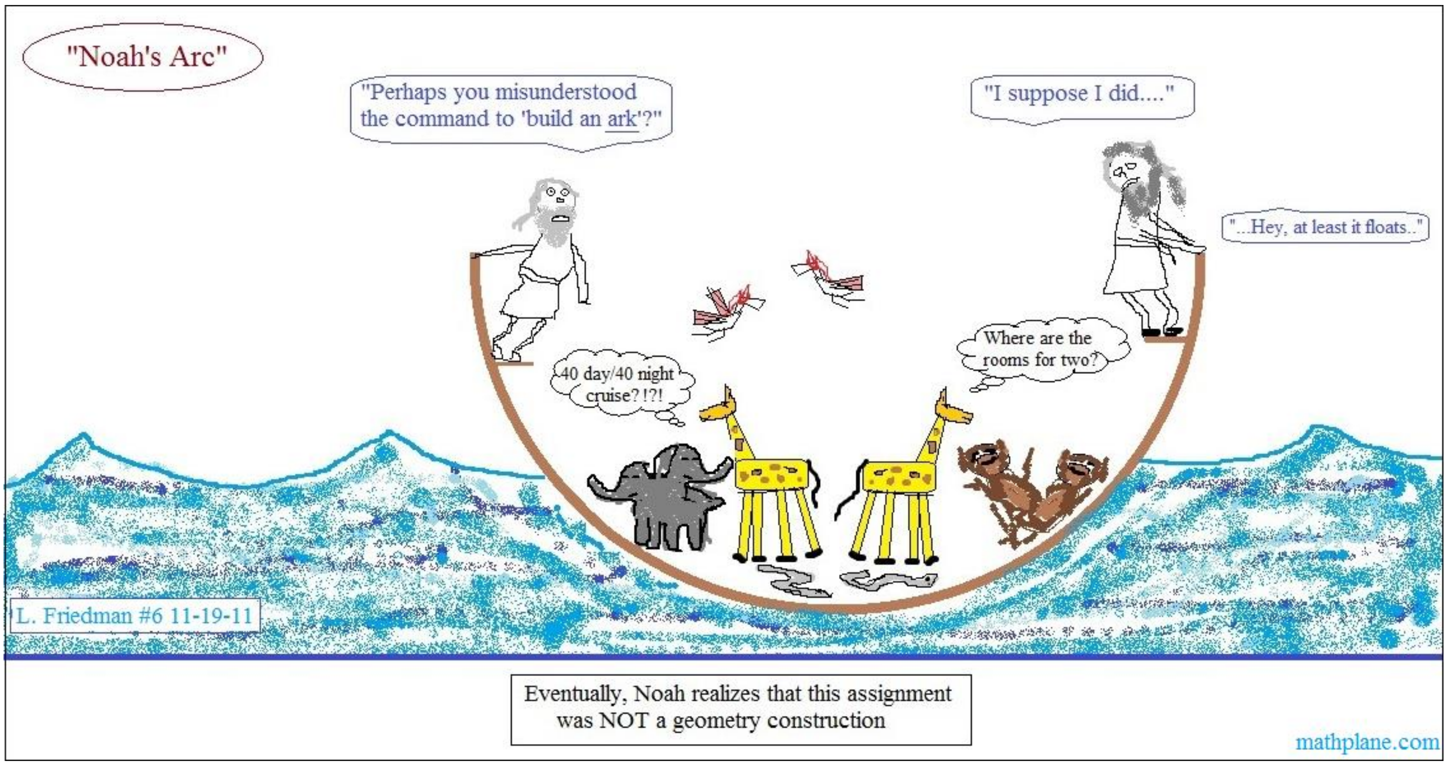
(Similar Triangles)

BUT, the triangles <u>may or may not</u> be congruent...





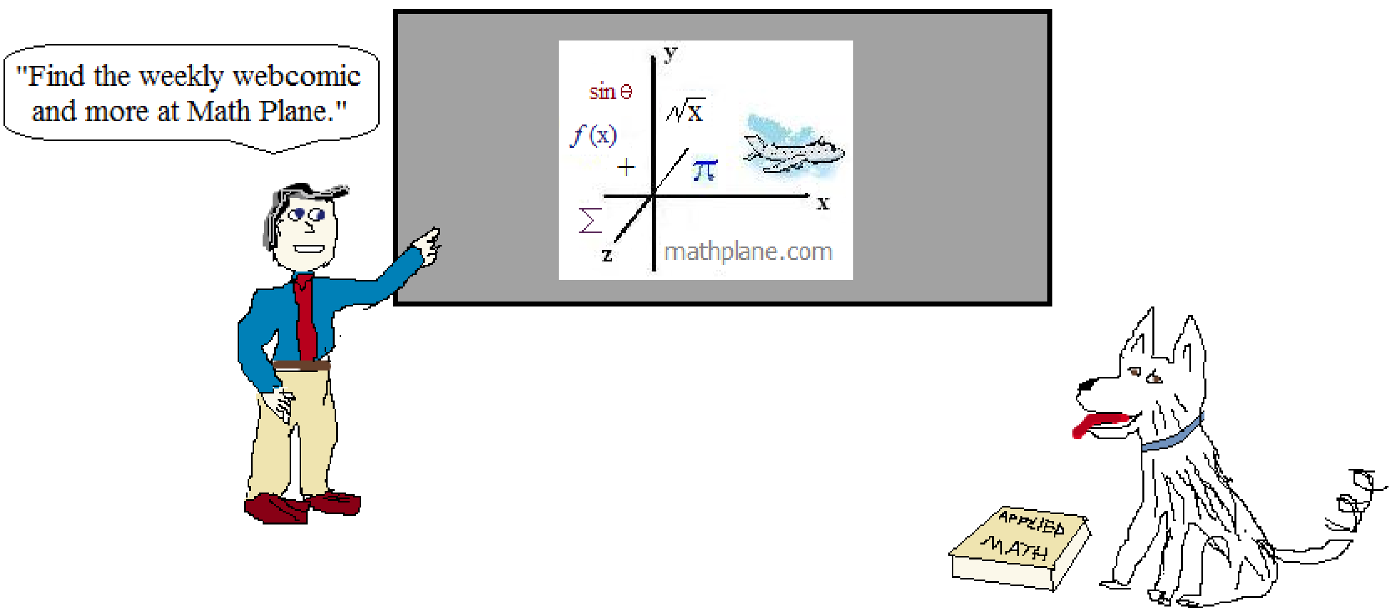
NOTE: $\overline{AC}$ in $\triangle ABC$ does not correspond to $\overline{AC}$ in $\triangle ACD$


"Noah's Arc"
"Perhaps you misunderstood the command to 'build an ark'?"
"I suppose I did...."
"...Hey, at least it floats.."
40 day/40 night cruise? !?!
Where are the rooms for two?
L. Friedman #6 11-19-11
Eventually, Noah realizes that this assignment was NOT a geometry construction
mathplane.com

Thanks for visiting. (Hope it helped!)

Find more proofs and geometry content at mathplane.com

If you have questions, suggestions, or requests, let us know.



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