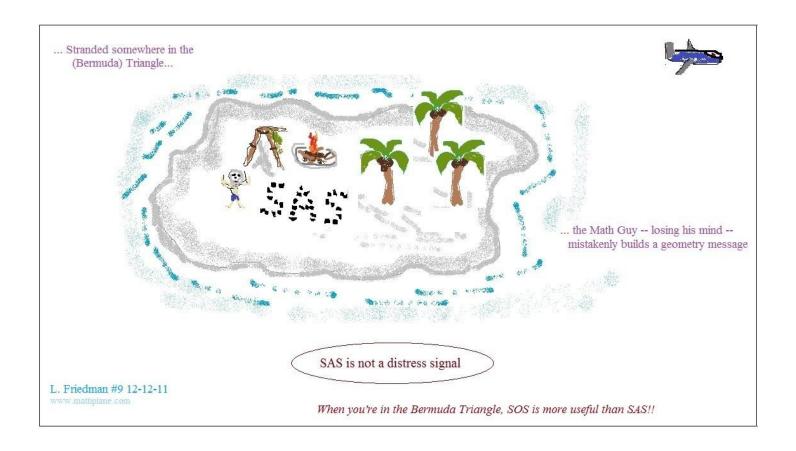
# Geometry: Proofs and Postulates Worksheet

Practice Exercises (w/ Solutions)

Statements	Reasons
1. AD and BC bisect each other	1. Given
2. $\overline{AM} \stackrel{\sim}{=} \overline{DM}$ ; $\overline{CM} \stackrel{\sim}{=} \overline{BM}$	2. Definition of bisector
3. ∠AMC≅ ∠BMD	3. Vertical angles are congruent
4. △AMC ≅ △ DMB	4. Side-Angle-Side (SAS) (2, 3, 2)
5. <del>AC</del> ≃ <del>BD</del>	5. CPCTC (Corresponding Parts of Congruent Triangles are Congruent)

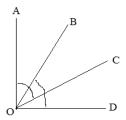
Topics include triangle characteristics, quadrilaterals, circles, midpoints, SAS, and more.



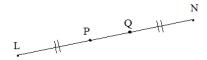
# PRACTICE EXERCISES -→

#### Introduction to proofs: Identifying theorems and postulates

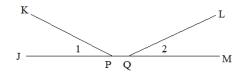
1) Why is  $\angle AOB \cong \angle COD$ ?



2) Why are  $\overline{LQ}$  and  $\overline{PN}$  congruent segments?

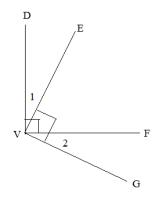


Angles 1 and 2 are congruent.
 Why are ∠ KPM and ∠LQJ congruent?

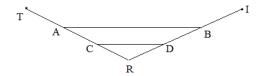


4) DV  $\perp$  VF and EV  $\perp$  VG

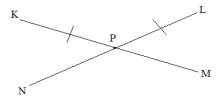
Why is angle 1 congruent to angle 2?



5) If TR = RI, and AB and CD are trisectors, why are CR and BD congruent segments?

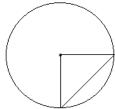


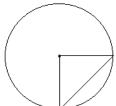
6) KM and LN bisect each other. Is KM congruent to NL?



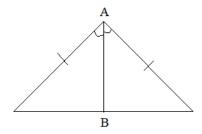
Explain using geometry concepts and theorems:

1) Why is the triangle isosceles?

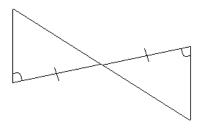




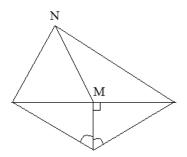
2) Why is  $\overline{AB}$  an altitude?



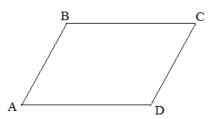
3) Why are the triangles congruent?



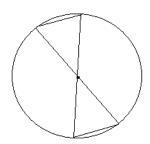
4) Why is  $\overline{NM}$  a median?



5) If ABCD is a parallelogram, why are ∠A and ∠C congruent?



6) Why are the triangles congruent?



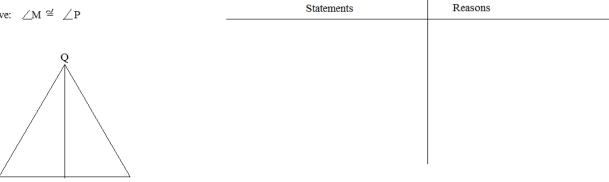
A) Given:  $\overline{AB} \stackrel{\text{def}}{=} \overline{EG}$   $\overline{CD} \stackrel{\text{def}}{=} \overline{EG}$ 

Prove:  $\overline{AC} \stackrel{\sim}{=} \overline{BD}$ 

Statements	Reasons

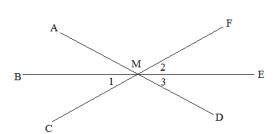
- B) Given:  $\angle M$  is the complement to  $\angle MQN$  $\angle P$  is the complement to  $\angle PQN$  $\overline{NQ}$  bisects  $\angle MQP$

Prove:  $\angle M \cong \angle P$ 



C) Given:  $\overline{BE}$  bisects  $\angle$  FMD

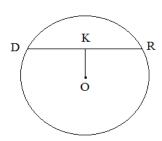
Prove:  $\angle 1 \cong \angle 3$ 



Statements	Reasons

1) Given:  $\odot$ O;  $\overline{DR} \perp \overline{OK}$ 

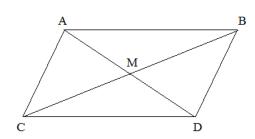
Prove:  $\overline{DK} \cong \overline{KR}$ 



Statements Reasons

2) Given:  $\overline{AD}$  and  $\overline{BC}$  bisect each other

Prove:  $\overline{AC} \cong \overline{BD}$ 

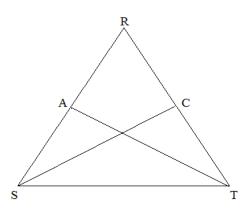


Statements Reasons

3) Given:  $\overline{RS} \cong \overline{RT}$ 

 $\overline{AT}$  and  $\overline{CS}$  are medians

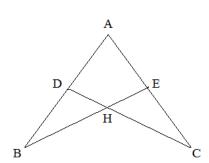
Prove:  $\overline{AT}$  and  $\overline{CS}$  are congruent



Statements Reasons

4) Given:  $\overline{AC} \cong \overline{AB}$ D and E are midpoints

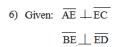
Prove:  $\angle B \stackrel{\sim}{=} \angle C$ 



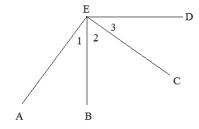
Statements	Reasons

5) Prove the diagonals of an isosceles trapezoid are congruent.

Statements	Reasons



Prove:  $\angle 1 = \angle 3$ 



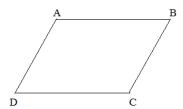
Statements	Reasons	

7) Given:  $\overline{AB} \parallel \overline{CD}$ 

 $\overline{AB} \cong \overline{CD}$ 

Prove:  $\overline{AD} || \overline{BC}$ 

(Hint: Use an auxilary line segment)



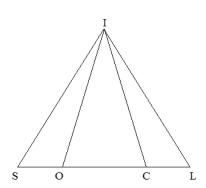
Statements	Reasons

8) Given:  $\triangle$  OIC is an isosceles triangle (with base OC)

Geometry Proofs

$$\overline{SO} \stackrel{\smile}{=} \overline{CL}$$

Prove:  $\triangle$  ISL is an isosceles triangle



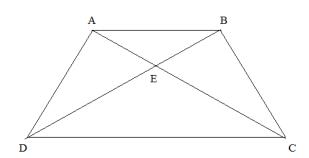
Statements	Reasons

9) Given:  $\overline{AD} = \overline{BC}$ 

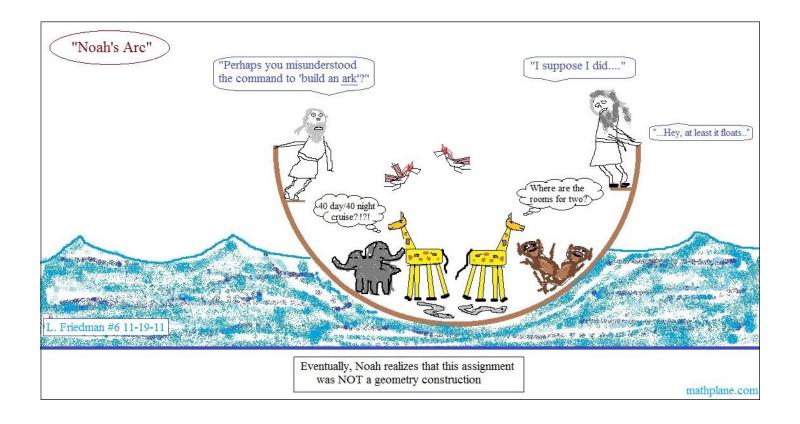
 $\triangle$  DEC is isosceles with base DC

 $\triangle$  ABE is isosceles with base AB

Prove:  $\angle ADC \stackrel{\sim}{=} \angle BCD$ 

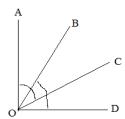


Statements	Reasons



# SOLUTIONS -→

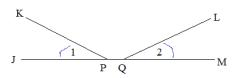
### 1) Why is $\angle AOB \cong \angle COD$ ?



Since AOC = BOD, and BOC = BOC (reflexive property), AOB = COD (subtraction property)

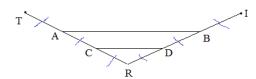
(If congruent angles are subtracted from congruent angles, then the differences are congruent)

# 3) Angles 1 and 2 are congruent. Why are ∠ KPM and ∠LQJ congruent?



If angles are supplementary to congruent angles, then they are congruent..

## 5) If TR = RI, and AB and CD are trisectors, why are CR and BD congruent segments?

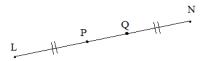


Division Property (If equal segments are divided by the same amount, the divided amounts are congruent)

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#### SOLUTIONS

#### 2) Why are $\overline{LQ}$ and $\overline{PN}$ congruent segments?



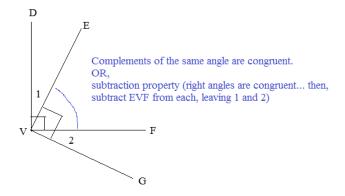
Since  $\overline{LP} = \overline{QN}$ , and  $\overline{PQ} = \overline{PQ}$  (reflexive property),

then  $\overline{LQ}$  and  $\overline{PN}$  are congruent (addition property)

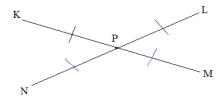
(If congruent segments are added to equal segments, then the sums are the same!)

#### 4) DV LVF and EV VG

Why is angle 1 congruent to angle 2?



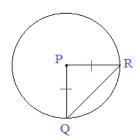
## 6) KM and LN bisect each other. Is KM congruent to NL?



Yes... Since segments bisect each other, KP = PM and NP = PL... Also, from diagram, KP = PL... So, using substitution and addition properties, KM = NL Or, use the multiplication property.. (Since KP = PL and both segments are doubled, the products are equal)

Explain using geometry concepts and theorems:

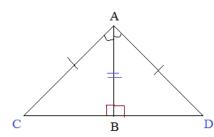
#### 1) Why is the triangle isosceles?



PR and PQ are radii of the circle. Therefore, they have the same length.

A triangle with 2 sides of the same length is isosceles.

#### 2) Why is AB an altitude?



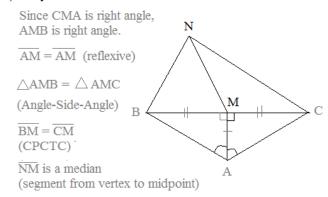
 $\overline{AB} = \overline{AB}$  (reflexive) therefore,  $\triangle CAB = \triangle DAB$ (side-angle-side)

If triangles are same, then ∠ABC = ∠ABD (CPCTC)

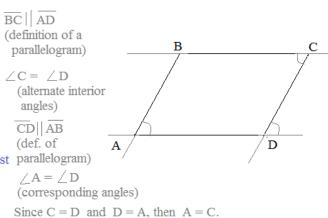
Since angles are same and must parallelogram) add up to 180, each is  $90^{\circ}$  / A = / D

Therefore, AB is altitude.

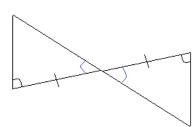
#### 4) Why is $\overline{NM}$ a median?



#### 5) If ABCD is a parallelogram, why are ∠A and ∠C congruent?



#### 3) Why are the triangles congruent? 6) Why are the triangles congruent?



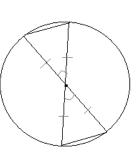
Vertical angles are congruent therefore, triangles are congruent (angle-side-angle)

the 4 marked sides are congruent.

Since they are radii of the circle,

Vertical angles

Triangles congruent by side-angle-side



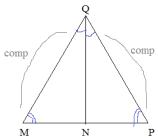
NOTE: CPCTC is "Corresponding Parts of Congruent Triangles are Congruent" A) Given:  $\overline{AB} \stackrel{\text{def}}{=} \overline{EG}$  $\overline{CD} \stackrel{\text{def}}{=} \overline{EG}$ 

Prove:  $\overline{AC} \stackrel{\checkmark}{=} \overline{BD}$ 

		E	G	
A	В		С	D

B) Given:  $\angle M$  is the complement to  $\angle MQN$  $\angle P$  is the complement to  $\angle PQN$  $\overline{NQ}$  bisects  $\angle MQP$ 

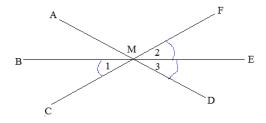
Prove:  $\angle M \cong \angle P$ 



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C) Given:  $\overline{BE}$  bisects  $\angle FMD$ 

Prove:  $\angle 1 \cong \angle 3$ 



Statements	Reasons
1) AB = EG	1) Given
2) CD = EG	2) Given
3) AB = CD	Transitive Property (Segments that are congruent to the same segment are congruent)
4) $BC = BC$	4) Reflexive Property
5) $AC = BD$	5) Addition Property (If a segment is added to congruent segments, the sums are congruent)

OR,

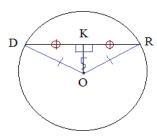
Statements	Reasons
1) AB = EG	1) Given
2) CD = EG	2) Given
3) AB = CD	Transitive Property (Segments that are congruent to the same segment are congruent)
4) AC = BD	4) Addition Property (If segment (BC) is added to congruent segments, then the sums are congruent)

Statements	Reasons
1) NQ bisects \( MQP \)	1) Given
2) ∠MQN ≃ ∠PQN	Definition of (Angle) Bisector     (If ray bisects an angle, the     angle halves are congruent)
3) Angles M and MQN are complementary	3) Given
4) Angles P and PQN are complementary	4) Given
5) ∠M = ∠P	Substitution (If angles are complementary to congruent angles, then they are congruent)     "Congruent Complements"

Statements	Reasons
1) BE bisects ∠FMD	1) Given
2) $\angle 2 = \angle 3$	Definition of Bisector     (If segment bisects an angle, the angle halves are congruent)
3) $\angle 1 = \angle 2$	3) Vertical angles are congruent
4) ∠1 = ∠3	Transitive Property (Angles congruent to the same angle are congruent)

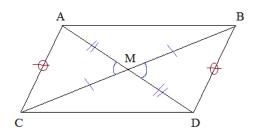
1) Given:  $\odot$ O;  $\overline{DR} \perp \overline{OK}$ 

Prove:  $\overline{DK} \cong \overline{KR}$ 



2) Given:  $\overline{AD}$  and  $\overline{BC}$  bisect each other

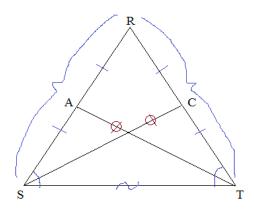
Prove:  $\overline{AC} \cong \overline{BD}$ 



3) Given:  $\overline{RS} \cong \overline{RT}$ 

 $\overline{AT}$  and  $\overline{CS}$  are medians

Prove:  $\overline{AT}$  and  $\overline{CS}$  are congruent



#### SOLUTIONS

Statements	Reasons
1. Circle O	1. Given
2. <del>DR</del> ⊥ <del>OK</del>	2. Given
3. ∠OKD and ∠OKR are right angles	3. Definition of perpendicular
4. ∠OKD ≅ ∠OKR	4. All right angles are congruent
5. Auxilary line segments $\overline{OR}$ and $\overline{OD}$	5. A segment joins 2 points
6. $\overline{OR} \stackrel{\sim}{=} \overline{OD}$	6. All radii are congruent
7. <del>OK</del> ≅ <del>OK</del>	7. Reflexive property
8. $\triangle$ DOK = $\triangle$ ROK	8. Hypotenuse Leg Theorem (HL)
9. $\overline{DK} \stackrel{\sim}{=} \overline{KR}$	9. CPCTC (4, 6, 7)

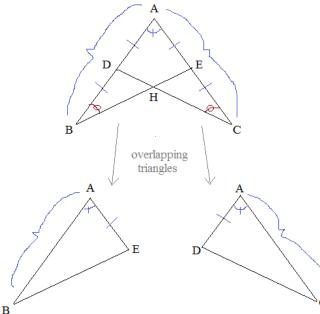
Statements	Reasons
1. AD and BC bisect each other	1. Given
2. $\overline{AM} \stackrel{\sim}{=} \overline{DM}$ ; $\overline{CM} \stackrel{\sim}{=} \overline{BM}$	2. Definition of bisector
3. ∠AMC≅ ∠BMD	3. Vertical angles are congruent
4. △AMC ≅ △ DMB	4. Side-Angle-Side (SAS) (2, 3, 2)
5. <del>AC</del> ≅ <del>BD</del>	5. CPCTC (Corresponding Parts of Congruent Triangles are Congruent)

Statements	Reasons
1. <del>RS</del> ≅ <del>RT</del>	1. Given
2. ∠RST ≅ ∠RTS	2. Sides-Angles Theorem (Isosceles Triangle)
3. $\overline{\text{AT}}$ and $\overline{\text{CS}}$ are medians	3. Given
4. A and C are midpoints	4. Definition of median
5. $\overline{RA} \stackrel{\checkmark}{=} \overline{SA}$ ; $\overline{RC} \stackrel{\checkmark}{=} \overline{TC}$	5. Definition of midpoint
6. <del>SA</del> ≝ <del>CT</del>	6. Division property (like division of congruent segments)
7. $\overline{ST} \cong \overline{ST}$	7. Reflexive property
8. $\triangle$ SAT $\stackrel{\hookrightarrow}{=}$ $\triangle$ TCS	8. Side-Angle-Side (SAS) (6, 2, 7)
9. $\overline{AT} \cong \overline{CS}$	9. CPCTC

4) Given:  $\overline{AC} \cong \overline{AB}$ 

D and E are midpoints

Prove:  $\angle B \stackrel{\sim}{=} \angle C$ 

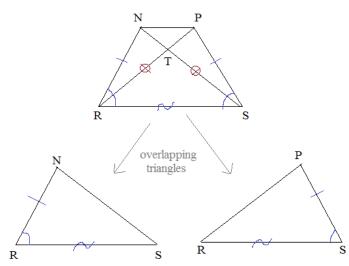


#### SOLUTIONS

Statements	Reasons
$1. \ \overline{AC} \cong \overline{AB}$	1. Given
2. $\overline{AE} \stackrel{\sim}{=} \overline{EC}$ (AE is 1/2 of AC)	2. Definition of midpoint
3. AD = DB (AD is 1/2 of AB)	3. Definition of midpoint
4. $\overline{AD} \cong \overline{AE}$	Division property     ("like division" of congruent segments)
5. <u>∠</u> A ≅ ∠ A	5. Reflexive property
6. △DAC ≅ △EAB	6. Side-Angle-Side (SAS) (4, 5, 1)
7. ∠B≅'∠C	7. CPCTC

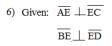
#### 5) Prove the diagonals of an isosceles trapezoid are congruent.

Definition of Isosceles Trapezoid: A trapezoid in which the base angles and  $\underline{\text{non-parallel}}$  sides are congruent

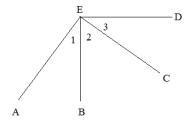


Statements	Reasons
RNPS is isosceles trapezoid with base RS	1. Given (diagram)
2. $\overline{NR} \cong \overline{PS}$	2. Definition of Isosceles Trapezoid
3. ∠NRS ≅ ∠PSR	3. Definition of Isosceles Trapezoid
4. $\overline{RS} \cong \overline{RS}$	4. Reflexive property
5. $\triangle NRS \cong \triangle PSR$	5. Side-Angle-Side (SAS) (2, 3, 4)
6. (diagonals) PR ≅ NS	6. CPCTC

#### SOLUTIONS



Prove:  $\angle 1 = \angle 3$ 



Proof 1: Using the Subtraction Property

Statements	Reasons
1) $\overline{AE} \perp \overline{EC}$	1) Given
2) $\overline{\text{BE}} \perp \overline{\text{ED}}$	2) Given
3) ∠AEC is right angle	Definition of Perpendicular     (Perpendicular lines form right angles)
4) ∠BED is right angle	4) Definition of Perpendicular
5) $\angle AEC \stackrel{\triangle'}{=} \angle BED$	5) All right angles are congruent
6) ∠2 ≅ ∠ 2	6) Reflexive property
7) ∠1 ≅ ∠3	7) Subtraction property (If equal angles are subtracted from congruent angles, then the differences are congruent)

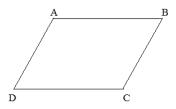
Proof 2: Using the Congruent Complements

Statements	Reasons
1) $\overline{AE} \perp \overline{EC}$	1) Given
2) $\overline{\text{BE}} \perp \overline{\text{ED}}$	2) Given
3) ∠AEC is right angle	Definition of Perpendicular     (Perpendicular lines form right angles)
4) ∠BED is right angle	4) Definition of Perpendicular
5) Angles 1 and 2 are complementary	5) Definition of complementary (Two angles that add up to 90 degrees i.e. form a right angle)
6) Angles 3 and 2 are complementary	6) Definition of Complementary
7) ∠1 ≅ ∠3	7) Congruent Complements (Complements of the same angle are congruent)

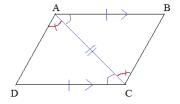
7) Given:  $\overline{AB} \parallel \overline{CD}$   $\overline{AB} \cong \overline{CD}$ 

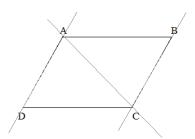
Prove:  $\overline{AD} \mid \mid \overline{BC}$ 

(Hint: Use an auxilary line segment)



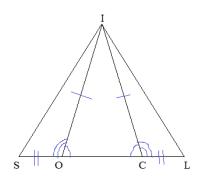
Statements	Reasons
1) $\overline{AB} = \overline{CD}$	1) Given
2) AC is a line segment	2) Auxilary line (2 points make a line)
3) AB    CD	3) Given
4) <u>/</u> BAC = <u>/</u> DCA	If    lines cut by transversal, then alternate interior angles congruent
5) $\overline{AC} \stackrel{\sim}{=} \overline{AC}$	5) Reflexive Property
6) $\triangle$ BAC $\stackrel{\sim}{=}$ $\triangle$ DCA	6) Side-Angle-Side (SAS) (1, 4, 5)
7) $\angle ACB = \angle CAD$	7) Corresponding Parts of Congruent Triangles are Congruent (CPCTC)
8) $\overline{AD} \parallel \overline{BC}$	If alternate interior angles are congruent, then lines are parallel





$$\overline{SO} \stackrel{\sim}{=} \overline{CL}$$

Prove: A ISL is an isosceles triangle



#### SOLUTIONS

Geometry Proofs

Statements	
1) OIC is an isosceles tria	ngle
with base OC	

Statements

- 2)  $\overline{IO} = \overline{IC}$
- 3)  $\overline{SO} = \overline{CL}$
- 4)  $\angle$  IOC =  $\angle$  ICO
- 5) ∠ IOS is supp. to ∠IOC ∠ICL is supp. to ∠ICO
- 6)  $\angle IOS = \angle ICL$
- 7)  $\triangle$  IOS =  $\triangle$  ICL
- 8)  $\angle S = \angle L$
- 9) △ ISL is isosceles

#### Reasons

- 1) Given
- 2) Definition of isosceles (2 congruent sides)
- 3) Given
- 4) If congruent sides (in triangle), then congruent angles..(or, base angles of isosceles triangle are congruent)
- 5) If adjacent angles form a straight angle, then angles are supplementary
- 6) If 2 angles are supplementary to congruent angles, then the 2 angles are congruent
- 7) Side-Angle-Side (2, 6, 3)
- 8) CPCTC (corresponding parts of congruent triangles are congruent)
- 9) If base angles of triangle are congruent, then triangle is isosceles

congruent angles, then the sums are

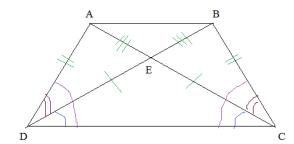
congruent)

9)	Given:	AD =	BC

△ DEC is isosceles with base DC

∧ ABE is isosceles with base AB

Prove:  $\angle ADC \stackrel{\sim}{=} \angle BCD$ 

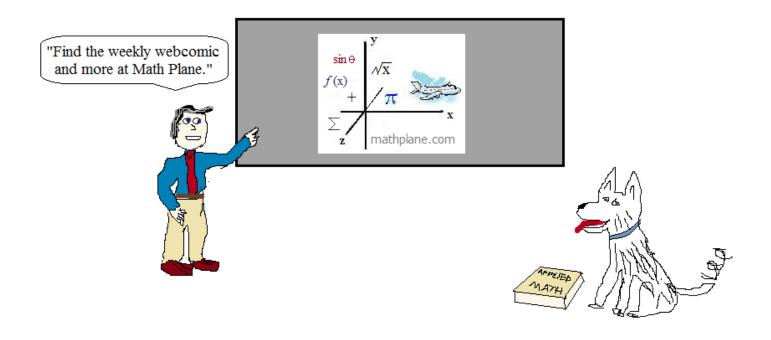


	1	
Statements	Reasons	
1) $\overline{AD} = \overline{BC}$	1) Given	
2) △DEC is isosceles	2) Given	
B) _EDC = _ECD	Base angles of isoseces triangle are congruent	
4) $\overline{DE} = \overline{EC}$	<ul> <li>4) If congruent angles (in triangle), then congruent sides</li> <li>(or, use def. of isoseles triangle 2 congruent sides)</li> </ul>	
5) ABE is isosceles	5) Given	
6) $\overline{AE} = \overline{BE}$	6) Definition of Isosceles (if isos., then at least 2 sides con	
7) $\triangle$ AED = $\triangle$ BEC	7) Side-Side-Side (6, 4, 1)	
8) \( \text{ADE} = \text{ BCE}	CPCTC (corresponding parts o congruent triangles are congruent triangles)	
9) $\angle$ ADC = $\angle$ BCD	Additional property     (if 2 congruent angles are added to	

Thanks for visiting. (Hope it helped!)

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